The Black-Litterman Model for Structured Equity Portfolios

Using views based on factors to construct equity portfolios.

Robert Jones, Terence Lim, and Peter J. Zangari

The Black-Litterman model [1992] (B-L) is a process for developing better inputs for portfolio optimization. Prior to its development, investors were often frustrated by the seemingly unreasonable solutions produced by portfolio optimization techniques. This led many to either abandon the technology or forgo most of its benefits by applying so many constraints that the solution was largely predetermined.

Black's and Litterman's original insight was that these unreasonable solutions were not so much a problem with optimization per se, but rather the result of feeding inconsistent risk and return forecasts into an optimizer. Their solution was to combine an investor's private views with the market's implied views to produce blended views that were more consistent with risk forecasts. They realized that, to be effective in optimization, risk and return forecasts had to be consistent with one another.

The first step in the Black-Litterman process is to derive a set of equilibrium expected returns that would clear the market, assuming a given risk model. These equilibrium expected returns (or market-implied views) are the set of expected returns that would produce the market portfolio if fed into an optimizer with the specified risk model. In other words, the equilibrium alphas come from reverse-optimizing the market portfolio.

Next, the investor's private views are blended with the market's implied views using Bayesian mixed estimation techniques, where the relative weights on an investor's private views reflect the confidence the investor has in those views. If fed into an optimizer with the specified risk model, these blended views will produce balanced portfolios that are tilted toward the investor's private views, with the degree of tilt (for a given level of risk) depending on the investor's relative confidence in his or her views.

Robert Jones
is a managing director at Goldman Sachs Asset Management in New York City.
bob.jones@gs.com

Terence Lim
is a managing director at Goldman Sachs Asset Management in New York.
terence.lim@gs.com

Peter J. Zangari
is a managing director at Goldman Sachs Asset Management in New York.
peter.zangari@gs.com

The original Black-Litterman article uses the example of allocating assets among various global bond markets to illustrate how the model works. A follow-up article showed how to use the model for global asset allocation in various stock, bond, and currency markets. Both articles use the global capital asset pricing model (or global CAPM) as a basis for deriving the market's implied views.

We demonstrate here how structured equity portfolio managers who develop views based on factors (e.g., value or momentum) may use the Black-Litterman model to construct equity portfolios. Structured managers generally focus on returns relative to a benchmark.

Structured equity portfolio construction presents many theoretical and practical implementation issues that differ from those presented in the original Black-Litterman research. To name a few:

- How should we define equilibrium expectations in active risk-return space?
- How should we specify private views on literally thousands of securities?
- How should we specify and quantify our confidence in different views?

Nevertheless, as we show in this article, the basic Black-Litterman approach is robust to these issues and can easily be adapted to the problem at hand.

We first describe the basic B-L model. We also present a short survey of related research that suggests alternative ways to formulate views.

Then we focus on applying B-L to equity portfolios. We begin by deriving the optimal tilt portfolio (OTP), a portfolio that best expresses our investment insights. We next show how to derive stock-level Black-Litterman alphas by reverse-optimizing the OTP. We illustrate the ideas using a set of examples demonstrating how Black-Litterman alphas, when combined with the proper risk model, produce more attractive optimized portfolios. A separate technical appendix is available upon request.

Finally, we discuss the important role that the risk model plays in structured portfolio construction. We explain why it is important to have a risk model that includes both proprietary return factors and common risk factors, and why it is important to decompose risks using such a model in order to evaluate factor bets properly.

THE BLACK-LITTERMAN MODEL

Black and Litterman [1992] assume there are two distinct sources of information about future excess returns: investor views and market equilibrium (e.g., the CAPM). Both sources of information are assumed to be uncertain and are expressed as probability distributions. Market equilibrium serves as a reference point, or center of gravity, from which investors diverge, depending on their particular views. The expected excess returns that ultimately drive portfolio optimization combine both sources of information.

B-L use a Bayesian approach to combine an investor's subjective views (expected returns for one or more assets) with market equilibrium (the prior distribution of expected returns) to form a new, mixed estimate of expected returns. Specifically, the Black-Litterman formula for blended expected returns, α_{BL}, is:

$$\alpha_{BL} = \left[\left(\tau\Sigma\right)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[\left(\tau\Sigma\right)^{-1}\pi + P^T\Omega^{-1}q\right] \qquad (1)$$

where:

π is equilibrium expected returns;

Σ is an N by N covariance matrix of expected returns. This is what we refer to as the risk model;

τ is a scalar used to vary the level of confidence in the equilibrium expected returns;

P is a K by N matrix of view portfolio weights. Each row of P corresponds to an N-vector of portfolio weights for a particular view portfolio;

q is a K-vector of expected returns on view portfolios, also known simply as views; and

Ω is the covariance matrix of view portfolio returns (or confidence in views).

Equation (1) expresses what we refer to as Black-Litterman alphas. When used properly in optimization, they produce a portfolio with deviations from equilibrium (market) weights in the direction of an investor's views. The model balances the return expectations of the view (q) and market (π) portfolios against their respective contributions to overall portfolio risk.

He and Litterman [1999] show that the unconstrained optimal portfolio formed with B-L alphas and some risk model, Σ, can be written as:

$$w^* = w_{mkt} + P^T\lambda \qquad (2)$$

where w_{mkt} represents market portfolio weights, and λ is a K-vector of weights on different view portfolios. Since each column of P^T represents a particular view portfolio, we see that the unconstrained optimal portfolio is the market portfolio plus a weighted sum of the investor's views. If we re-express (2) as $w^* - w_{mkt} = P^T\lambda$, we see that the term $P^T\lambda$ represents our ideal active weights (relative to the market portfolio in this example).

The original Black-Litterman work inspired several subsequent articles. These seem to focus on three particular areas: 1) using alternative methods to rederive the Black-Litterman formula (De Santis [1999]); 2) extending the original model to include different specifications of priors and views; and 3) offering simple examples to help practitioners better understand how to apply B-L (Fabozzi et al [2006]).

Satchell and Scowcroft [2000] present a derivation of the Black-Litterman formula as well as two alternative formulations. The first takes into account prior beliefs about overall volatility. The second involves assigning prior probabilities to each view.

Idzorek [2002] offers a step-by-step guide to using B-L in practice. He presents a comprehensive example on how to use the Black-Litterman model to construct equity portfolios.

Herold [2003] provides a simple extension of B-L. He uses the Black-Litterman framework to construct portfolios with qualitative forecasts. His particular focus is on estimating alphas that incorporate directional views (bullish/bearish), with attached levels of conviction.

More recently, Bulsing, Scowcroft, and Sefton [2004] published a methodology for estimating alphas by combining portfolio forecasts. Their methodology is a straightforward extension of B-L that may prove useful for managers who seek to produce stock-level alpha forecasts that combine: 1) forecasts from a linear, cross-sectional factor model; 2) country or sector views; and 3) company-specific recommendations based on analyst research. The authors apply the Gauss-Markov theorem to derive their main results.

APPLYING BLACK-LITTERMAN IN STRUCTURED EQUITY PORTFOLIOS

Structured equity managers typically spend much of their research budget identifying and testing factors that can help forecast stock returns. Intuitively, they want to construct efficient portfolios that have positive exposures to their chosen factors. Unfortunately, they often get unbalanced or unintuitive portfolios when they combine their factor-based alpha forecasts with an inconsistent risk model in the optimization process. B-L provides a mechanism for structured portfolio managers to combine their factor views in a way that is more consistent with risk estimates.

We use numerical examples to illustrate this approach. We also describe how we use B-L to set factor weights and form individual stock alphas that are consistent with a prespecified risk model.

Views and View Portfolios

A view is simply a belief about the risk and return characteristics of a specific investment, called a *view portfolio*. Structured equity portfolio managers develop models based on views about broad *types* of stocks; they do not have views on *individual* stocks. That is, they might have a view that value stocks will outperform glamour stocks, not that a particular stock will outperform another stock (or the market).

B-L provides a consistent way to combine (weight) these views and incorporate them into the alpha-generating process. It does not provide guidance on how to develop views; it is up to the portfolio manager to derive relevant view portfolios and estimate their expected risks and returns. The views we use in our examples are motivated by the work of Fama and French [1992] and Carhart [1996]:

1. Small-cap stocks will outperform large-cap stocks.
2. Value (high B/P) stocks will outperform glamour (low B/P) stocks.
3. Stocks with strong prior-year price momentum will outperform stocks with weak momentum.

A common approach for quantifying the extent to which a particular stock is, say, a value stock or a glamour stock is to measure the number of cross-sectional standard deviations the stock's BE/ME (Book Value)/(Market Value) ratio is from the market average. This is called the stock's exposure to value. We can compute a stock's exposure to size and momentum using a similar methodology.

A view portfolio is simply a vector of stock weights that represent a particular view. There are various ways to build view portfolios. In the example below we use a linear cross-sectional factor model to demonstrate one way. The model posits a cross-sectional relationship between security returns and their exposures to factors:

$$R(t) = B(t-1)F(t) + e(t) \tag{3}$$

where:

$R(t)$ is an N-vector of stock returns;
$B(t-1)$ is an N by K matrix of stock exposures to factors;
$F(t)$ is a K-vector of returns to factors; and
$e(t)$ is an N-vector of idiosyncratic specific returns.

In Equation (3), each security's one-period return is expressed as a linear combination of factor exposures (B), times factor-based view portfolio returns (F), plus a stock-specific component (e). The estimated factor returns, F(t), have the nice feature that they they represent the return to a particular factor, holding all other factors constant. We will use this model to estimate view portfolio returns.

For example, the least-squares estimates of factor (view portfolio) returns are:

$$F(t) = PR(t) \tag{4}$$

where $P = [B(t-1)^T B(t-1)]^{-1} B^T(t-1)$

The K by N matrix P represents view portfolio weights, where each row is a separate view, and each column represents a stock's weight in the relevant view portfolio.

There are numerous ways to build factor-based view portfolios that maintain exposure to the relevant factor.[1] We can construct unit-exposure view portfolios that have minimum risk. Alternatively, view portfolios may have constant risk, or minimum position sizes (squared deviations), or simply be the stock exposure matrix: $P = B(t-1)$.

Optimal Tilt Portfolio

The first step in developing a single composite view is to combine our individual factor-based views (i.e., investment insights) into an optimal tilt portfolio (OTP), w_{OTP}. The OTP represents our ideal holdings in the absence of frictions.

Assume we have identified and measured stocks' exposures to a set of factors, $B = [B_1, B_2, \ldots, B_K]$, and we have determined some optimal factor weights, $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_K]$.[2] The matrix B includes exposures to both risk and return factors. Return factors drive alpha, while risk factors are used to control risk in the portfolio construction process.

Similar to view portfolios, there are various ways to define an optimal tilt portfolio. One definition is to let the OTP be an efficient zero-investment portfolio with the desired exposures to our factors.[3]

It is the solution to the problem:

$$\text{Min}_w \; w^T \Sigma w \text{ such that (i) } w^T B = \lambda \text{ and (ii) } w^T 1 = 0$$

The OTP is

$$w_{OTP} = P^T \lambda \tag{5}$$

where $P = (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1}$.

We see that the OTP is simply a weighted combination of view portfolios, P, and has the same form as (2), as discussed in He and Litterman [1999]. Our view portfolios are nothing but factor-mimicking portfolios, each with unit exposure to a particular factor and zero exposure to other factors.[4] Since each view portfolio is a zero-investment long-short portfolio, the OTP is as well. Thus, the OTP represents an ideal long-short portfolio that best captures our investment insights.

Deriving Black-Litterman Alphas: Reverse Optimization

In practice, however, we cannot invest directly in the OTP; unlike actual portfolios, it is completely free of constraints (including long-only), and is updated regularly without concern for transaction costs. (Remember: It represents the purest expression of our investment views at any one time.)

Nevertheless, we can use it to derive individual stock alphas that we can then use in actual portfolio optimizations that incorporate real-world constraints and costs. If we reverse-optimize the OTP, we get individual, stock-specific Black-Litterman alphas:[5]

$$\alpha_{BL} = \Sigma P^T \lambda \tag{6}$$

There are several important points worth noting about B-L alphas. First, by construction, they are entirely consistent with our risk model [Σ appears in Equation (6)]. This means the optimizer is unlikely to suggest extreme positions solely on the basis of inconsistent risk and return forecasts.

Second, B-L alphas are linear combinations of factor exposures. That is, we can express them as follows:[6]

$$\alpha_{BL} = B q_{BL} \tag{7}$$

where $q_{BL} = (P \Sigma P^T)\lambda$, and $P \Sigma P^T$ is the covariance matrix of the view portfolios.[7]

Note that q_{BL} is a K-vector of Black-Litterman alphas for our view portfolios. They are equal to the stock-level Black-Litterman alphas multiplied by the view portfolio weights matrix.[8]

Equation (7) provides some comforting intuition concerning the stock-level Black-Litterman alphas:

1. Higher exposure values (B) lead to higher stock alphas.

2. The weights on factor exposures are higher for higher-conviction (λ) views.
3. The risks of view portfolios ($P\Sigma P^T$) play an important role in determining the weights placed on factor exposures.[9]

Numerical Illustration

To illustrate the approach, we build factor exposures and a risk model for the U.S. stock market. At the end of each year from 1973 to 2002, we identify the 2,500 largest common stocks in the Center for Research in Security Prices database. Every month starting from June the next year, we construct size (ME), value (BE/ME), and momentum (prior one-year return skip one month) exposures for these stocks. These fundamental data are drawn from Compustat (and the ratios are constructed as described on the Kenneth French web site).

We estimate a factor risk model comprising 4 fundamental factors (the fourth factor is the stock's Scholes-Williams beta) and 12 industry factors, where the industry factors are dummy indicators based on the stock's membership in one of the 12 Fama-French industry groupings. The factor covariance matrix is calculated from the daily factor returns, estimated through daily cross-sectional regression of stock returns on these factors, using an exponential weighting scheme and accounting for cross-serial-correlation effects. For details of the factor covariance matrix estimation methodology, see Litterman [2003].

Assume for now that we want equal exposures to the size, value, and momentum factors. In the first example, we form naive alphas that are a simple equal-weighted average of exposures to these three factors: that is, where B is a matrix of stock exposures to size, value, and momentum, and λ is a three-element vector with constant weights equal to one-third.

We optimize to find a best long-short portfolio relative to cash with volatility of 4% (annual) and stock weight constraints of +/– 3%. Optimizations are conducted as of the end of the sample period.

Exhibit 1 shows that the optimized exposures are much greater for value than for the other factors. We have relatively little exposure to momentum. Because the alphas are not consistent with the risk model, the optimizer takes extreme positions, obtaining more exposure to factors it perceives to be less risky or uncorrelated with other factors.

Next, we form alphas following the Black-Litterman approach using Equation (6), setting each element of λ to be equal. Using Black-Litterman alphas, the same optimization yields the portfolio exposures presented in Exhibit 2. Now the optimal portfolio has balanced exposures, consistent with our desired (equal) weights on views.

EXHIBIT 1
Optimization with Naive Alphas

Exposures from Optimization with Naive Alphas	
Style	***Exposure***
Size	0.608
Value	1.681
Momentum	0.153

EXHIBIT 2
Optimization with Black-Litterman Alphas

Exposures from Optimization with Black-Litterman Alphas	
Style	***Exposure***
Size	0.619
Value	0.620
Momentum	0.621

Developing Views and Confidence Levels

In the first example, the desired weights on view portfolios were set to be equal; they did not depend on the relative strength of our views. In other words, the weights reflect only an uninformed prior view that they should be equal. Black-Litterman allows us to form alphas that blend more informed views (and confidence in those views) with a prior that the weights should be equal. Setting $w_{eq} = P\lambda_{eq}$, the Black-Litterman weights can be rewritten as:

$$\lambda = [\theta\Omega + (1-\theta)P\Sigma P']^{-1}[\delta\theta\Omega\lambda_{\text{equil}} + (1-\theta)q] \tag{8}$$

Note that:

- θ represents the weights placed on equilibrium (prior) views. In practice, we normally set $\theta = 0.5$ when we are uncertain as to how much weight to put on prior (equilibrium) versus proprietary (informed) views.
- λ_{equil} represents exposures to our factors in (uninformed) equilibrium. In practice, we may decide to

EXHIBIT 3
Standard Deviation and Mean Returns of View Portfolios

Portfolio	Standard Dev	Mean
Optimal Tilt	0.897	1.353
Size	4.201	0.241
Value	4.555	1.857
Momentum	8.740	4.909

give equal weights to our views in equilibrium. We would express this by setting λ_{equil} as a vector of equal weights.
- δ is a risk aversion parameter.

To see how a particular manager may develop proprietary views, note that the expected return on a view portfolio is simply:

$$q = PE(R) \tag{9}$$

where E(R) represents an N-vector of expected stock returns.

In practice, investors can estimate proprietary views as a time series average of realized returns on view portfolios. Suppose a hypothetical manager has three view portfolios representing value, size, and momentum. Starting with stock-level exposures for each of these factors, the manager can use Equation (2) to develop a time series of (multivariate) realized returns for each of these view portfolios.

From these time series, the manager can estimate the mean returns on the view portfolios, q, and their covariance matrix Ω. Thus, q is a three-element vector with mean returns for the value, size, and momentum view portfolios, and Ω is a 3 by 3 matrix that captures the variances and covariances of the returns on these view portfolios.

By applying estimates of q and Ω, and an equilibrium (prior) view of equal weights on the three view portfolios, we get the weights in Exhibit 4, expressed as a percentage of the sum of the weights. All else equal, the intuitive result is that B-L alphas put less weight on factors with lower expected returns (size) or higher risk (momentum).

EXHIBIT 4
Black-Litterman View Portfolio Weights (λ)

Views	Weight in view portfolios
Size	24.2%
Value	49.8
Momentum	26.0

Putting It All Together

Thus far, we have presented the key components of B-L as they apply to equity portfolio construction. We can summarize how we tie all this information together to build real-world portfolios:

- Decide on a factor exposure matrix, B, and estimate a risk model, Σ.
- Build view portfolios P.
- Use stock returns and view portfolios to generate a time series of view portfolio returns.
- Use these time series to produce mean estimates and confidence levels, q and Ω.
- Specify any prior information on factor weights, λ_{equil}.
- Estimate B-L factor weights, λ.
- Estimate stock-level B-L alphas, α_{BL}.

We then optimize using B-L alphas and the risk model, along with constraints and transaction costs, to produce an efficient and investable portfolio.

IMPORTANT ROLE OF THE RISK MODEL (Σ)

The risk model plays a key role in structured equity portfolio construction in general, and in the B-L approach in particular.[10] As structured equity managers typically cover thousands of stocks, they often use an equity factor model to measure and budget portfolio risk.[11]

As described here, the equity factor model is derived from (3), where there are N assets and K factors: $\Sigma = BSB^T + \Delta$, where B is an N by K matrix of stock exposures to factors, S is a K by K factor return covariance matrix, and Δ is an N by N specific variance matrix (variances along the diagonal, zeros elsewhere). A key assumption is that specific returns are uncorrelated.

For a structured manager, a good risk model will be customized and include both proprietary return factors and common risk factors. By proprietary, we mean that the definition of the factors used in the risk model will be the same as those used to estimate alphas. It is not possible to evaluate risks properly when there is a mismatch between the definition of factors in the risk model and the factors that drive alpha.

Return factors are the primary source of alpha. That is, our views on them drive our forecasts for stock returns. Risk factors, on the other hand, do not impact alphas—at least not directly (they do impact B-L alphas through $P\Sigma P^T$). Yet they are still important for investors who want to manage portfolio risk.

One can think of a risk factor as a mimicking portfolio that exhibits high return volatility, and accounts for substantial co-movement among stocks, but has an expected (mean) return close to zero. While risk and return factors are different, they both need to be included in a well-specified risk model.

Intuitive Risk Diagnostics

Ideally, structured portfolios should derive their entire alpha from return factors—that is, from view portfolios that have positive expected returns. Therefore, ex ante, the factor risk decomposition should show that most of the risk comes from these sources (as opposed to control factors or specific risk). Also, by analyzing diagnostics like contributions to risk, we can determine whether our risk budget is appropriate across factors.

Exhibits 5 and 6 show portfolio exposures and a risk decomposition for a portfolio constructed using Black-Litterman alphas. Here we allow views and confidence levels to modify equilibrium (equal) weights. As in the previous example, we optimize to find the highest-alpha long-short portfolio with volatility of 4% (annual) and stock position limits of +/– 3%.

The optimal portfolio in Exhibit 5 has the greatest exposure to value, with lower but still significant exposures to size and momentum. Exhibit 6 shows a risk decomposition for this portfolio. As expected, most of the portfolio's risk comes from return factors (87.4%); the primary sources of risk are size (18.9%), value (18.2%), and momentum (50.3%).

Momentum is the greatest contributor to risk, even though it has a much smaller exposure than value. This is because momentum is a much riskier view than value (more volatile *and* harder to hedge), as indicated in Exhibit 3. Thus, a lower exposure can still mean more risk. If, in fact, our equilibrium (prior) view is that the three factors should entail equal risk (instead of equal exposures), we could specify that in (8) when we develop B-L alphas.

EXHIBIT 5
Exposures upon Optimization with Black-Litterman Alphas

Style	Exposure
Size	0.523
Value	1.093
Momentum	0.566

EXHIBIT 6
Contribution to Risk (%)

Fundamental		87.4
Size	18.9	
Value	18.2	
Momentum	50.3	
Beta	–0.1	
Industry		–0.5
Specific		13.1

Choosing a Good Set of Risk Factors

Managers must ensure that their risk models are specified properly; they need to test and evaluate the model to make sure that it consists of a good set of factors. In a linear cross-sectional factor model, it should be noted that:

- Omitting a factor does not affect a stock's variance estimate. The omitted factor is absorbed into the error component of variance.

- Mistakenly omitting a factor will bias the covariance (and correlation) estimates. The impact of the omitted factor on the covariances depends on: 1) the exposures to the missing factor; 2) the volatility of the missing factor; and 3) the covariance between the missing factors and the remaining factors.
- Omitted factors can cause specific returns to be correlated. The covariance of the specific returns is a function of the variances and covariances of the omitted factors.

When we omit relevant factors, we observe the following:

- Biased estimates of portfolio volatility. This is because specific returns are no longer uncorrelated (although the formula for computing portfolio volatility assumes that they are).
- Biased estimates of contributions to risk by stock and by factor. The bias to the contribution to risk by stock is driven by the bias in portfolio volatility and the fact that we ignore the off-diagonal elements of the specific return covariance matrix (which are non-zero when we omit factors). The bias in the contribution to risk by factor is driven by the volatility bias and the missing factor return covariances (i.e., covariances that involve omitted factors are assumed to be zero).

How a Good Risk Model Can Enhance Portfolio Performance

Applying a good risk model can improve portfolio efficiency, i.e., a portfolio's alpha for a given level of risk. First and foremost, the risk model must do a good job of estimating risk. To this end, modelers should undertake extensive research to make sure the risk model produces timely and accurate risk estimates. The risk model can play an important role in estimating view portfolio returns.

Our view portfolios have been constructed using general least squares regression to have minimum risk (based on our simple risk model) while maintaining unit exposure to our factors. An alternative approach is to use (weighted) least squares, which minimizes squared deviations (position sizes) rather than risk, and therefore does not require a risk model.

Exhibit 7 reports the realized out-of-sample volatilities of portfolios formed using this approach and Exhibit 8 the results for view portfolios that minimize risk. We see that the minimum-risk portfolios indeed realize lower risk, while maintaining the same unit exposures to the size, value, and momentum factors.

Although both the approaches result in view portfolios with unit exposures to the underlying factors, the correlations between these view portfolios can be lower than one might expect (Exhibit 9). For example, the correlation between returns for the minimum-risk value portfolio and the least squares value portfolio is only slightly above 0.5. Thus, how we define the value view portfolio can have a significant impact on its performance, and ultimately on how much weight gets in the optimal tilt portfolio, and how much exposure it gets in actual portfolios.

Investors should define view portfolios that are relevant for their process. Since we use a risk model to

EXHIBIT 7
Weighted Least Squares View Portfolios (WLS)

	IR	Standard Dev	Mean
Size	0.06	4.02	0.24
Value	0.41	4.56	1.86
Momentum	0.56	8.74	4.91

EXHIBIT 8
Minimum-Risk View Portfolios (GLS)

	IR	Standard Dev	Mean
Size	0.27	3.66	1.00
Value	0.67	1.93	1.28
Momentum	0.57	6.08	3.49

EXHIBIT 9
Correlation of Monthly Returns with Minimum-Risk and Least Squares View

Size:	0.86
Value:	0.53
Momentum:	0.88

manage risk in actual portfolios, it makes sense to define view portfolios using this same risk model.

CONCLUSION

Black and Litterman [1992] pioneered a framework for expressing and combining investor views, and research since then has extended the work. We show how investors can use Black-Litterman in structured equity management. Specifically, we show that B-L provides a way for structured portfolio managers to: 1) define and combine different views (including equilibrium views); 2) improve factor weight estimation; and 3) construct alphas that are consistent with risk estimates. Providing informative diagnostics, the right risk model can also improve portfolio returns by helping investors allocate risk more efficiently across factors.

B-L offers a robust and theoretically rigorous method for incorporating investor views into portfolio optimization. As applied to structured portfolios, the model has some beneficial and intuitive properties.

An optimal tilt portfolio (OTP) captures our investment insights. This portfolio is the solution to an unconstrained optimization problem, where our view portfolios are the underlying assets. By construction, we can attribute all the OTP return to underlying factors (view portfolios). This is very useful because it provides a point of comparison for attribution on actual portfolios.

The OTP is a combination of view portfolios, with weights equal to λ, where λ is the result of a Bayesian forecasting approach that considers priors, views, and uncertainty of views. Thus, the OTP is the purest expression of our investment insights as it considers our priors, our views, and our conviction (confidence in views).

We reverse-optimize the OTP to generate B-L alphas using a factor risk model. Each B-L alpha is a weighted sum of exposures. This provides a very intuitive relationship between a stock's exposures and its B-L alpha.

Note that B-L alphas generate more balanced portfolios than other approaches without needing to apply unnecessary constraints. When we optimize with B-L alphas, our risk model, and no constraints, we produce back the optimal tilt portfolio, the ideal. In practice, we want the optimizer to consider trading costs and any required constraints (e.g., long-only). In such a constrained optimization, B-L alphas drive the solution toward the ideal portfolio, while balancing costs and constraints.

ENDNOTES

The approach the authors describe draws heavily on original insights from Robert Litterman and Steve Heston. The authors thank Andrew Alford, Kent Clark, and Giorgio De Santis for valuable contributions to this work. The opinions expressed here should not be taken to represent investment advice or the opinion of the writers' employer.

[1]Note that the Black-Litterman model says nothing about how we should define view portfolios. It is left completely up to the investor to determine views and view portfolios.

[2]For example, we choose view portfolio weights to maximize the net information ratio of the OTP.

[3]Note that since the OTP is a zero-investment portfolio, we ignore the scaling of λ since we would just lever the optimal tilt portfolio to match the desired scale.

[4]That is, PB = I.

[5]Reverse optimization yields: $\alpha \approx \Sigma w_{OTP}$.

[6]This follows from multiplying (6) by BP.

[7]Let R be an N-vector of stock returns with a covariance matrix, Σ. The returns on the K-vector of view portfolios can be written as: $\gamma = PR$. It follows that $\mathrm{cov}(\gamma) = P\Sigma P^T$.

[8]Actually, we derive q_{BL} independently, based on historical regressions, and derive B-L alphas through reverse optimization, as described in the text.

[9]Note that the weights we refer to here are what we multiply factor exposures by to get B-L alphas; they are conceptually different from the weights of view portfolios in the OTP.

[10]See Jorion [2003] for a detailed account of the role the risk model plays in determining portfolio weights.

[11]See Litterman [2003, chapter 20] for a complete exposition of equity risk factor models.

REFERENCES

Black, F., and R. Litterman. "Global Portfolio Optimization." *Financial Analysts Journal*, 48 (September/October 1992), pp. 28-43.

Bulsing, M., A. Scowcroft, and J. Sefton. "Understanding Forecasting: A Unified Framework for Combining Both Analyst and Strategy Forecasts." 2004.

Carhart, M. "On the Persistence in Mutual Fund Performance." *Journal of Finance*, 52 (1) (1997), pp. 57-82.

De Santis, G. "An Introduction to the Black-Litterman Asset Allocation Model." Goldman Sachs Asset Management. Mimeographed. 1999.

Fabozzi, F. J, Focardi, S. M., and Kolm, P. N. "Incorporating Trading Strategies in the Black-Litterman Framework." *Journal of Trading*, Spring, (2006), 28-36.

Fama, E. F., and K. R. French. "The Cross Section of Expected Stock Returns." *Journal of Finance*, 47 (2) (1992), pp. 427-465.

He, G., and R. Litterman. "The Intuition Behind the Black-Litterman Model Portfolios." Goldman Sachs Investment Management Series, 1999.

Herold, U. "Portfolio Construction with Qualitative Forecasts." *The Journal of Portfolio Management*, Fall 2003, pp. 61-72.

Idzorek, T. "A Step-by-Step Guide to the Black-Litterman Model." Working paper, 2002.

Jorion, P. "Portfolio Optimization with Tracking-Error constraints." *Financial Analysts Journal*, September/October 2003, Vol. 25, No. 5, pp. 70-82.

Litterman, R., ed. *Modern Investment Management: An Equilibrium Approach*. New York: Wiley, 2003.

Satchell, S., and A. Scowcroft. "A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Portfolio Construction." *Journal of Asset Management*, 1(2) (2000), pp. 138-150.

Scowcroft, A. "Using Bayesian Mixed Estimation Methods to Construct Enhanced Index Strategies." UBS Presentation. Oxford Centre for Computational Finance (OCCF), Third City Seminar Reuters'Thomas Moore Square offices: June (2003).