## Homework set #9

Based on lectures 15 – 17

1. Assume that $X_1, \ldots, X_n$ are i.i.d. Gamma$(r, 1/\lambda)$, where $r$ is known, i.e., $f(x|r,\lambda) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$. Find the posterior Bayes estimator of $\lambda$ for the $\Gamma(l, k)$ prior.

2. Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \sigma^2)$, where $\sigma^2$ is known and $\theta$ is unknown.

   (a) Find the posterior Bayes estimator for prior $\pi \sim N(a, b^2)$.

   (b) Find the posterior Bayes estimator for prior $\pi(\theta) = 1,\ \theta \in \mathbb{R}$. (This is called an "improper prior".)

3. Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli$(p)$. In what follows we will consider the loss function $L(p, a) = \frac{(p-a)^2}{p}$. (This is not the squared loss!)

   (a) Find the Bayes rule for the loss function $L(p, a)$ and Beta$(\alpha, \beta)$ prior. (Hint: Calculate the posterior risk $r = \int_0^1 L(p, a)\pi(p|\mathbf{x})dp$ and notice that this is a quadratic function in $a$. Then find $a$ that minimizes $r$.)

   (b) Find the MINIMAX estimator for this loss function.

4. Let $X_1, \ldots, X_n$ be i.i.d. Exp$(\lambda)$. (Use the parametrization where $EX = \lambda^{-1}$). Find the MINIMAX estimator for the loss $L(a, \lambda) = \lambda^2(a - \lambda^{-1})^2$.

5. (a) Prove or disprove: Any admissible estimator with a flat risk is MINIMAX.

   (b) Does a MINIMAX estimator have to be admissible?

6. From the book 7.23, 7.24, 7.26, (Hint: The calculations in this problem are complicated. Use of symbolic math package is recommended.), 7.65.