

Exam 2, Stat 435.1, Fall 2006

Name:

This exam consists of 9 questions. **To receive *any* credit you must show *all* work.**

1. **(5 points.)** The cumulative distribution function (c.d.f) of a random variable  $X$  is given as

$$F(x) = \begin{cases} 0 & \text{if } x < -2 \\ \frac{1}{8} & \text{if } -2 \leq x < 0 \\ \frac{1}{2} & \text{if } 0 \leq x < 3 \\ \frac{3}{4} & \text{if } 3 \leq x < 5 \\ 1 & \text{if } x \geq 5 \end{cases}$$

Find  $E(X^2)$ .

2. **(5 points.)** Suppose that  $X$  is a random variable with  $E(X) = 2$  and  $E(X(X + 2)) = 10$ . Find  $\text{Var}(5 - 3X)$ .

3. **(5 points.)** A particle is moving along the  $x$  axis. At each step it either moves one unit to the right or one unit to the left with probabilities  $1/4$  and  $3/4$  respectively. Suppose that the particle is at 0 initially. What is the probability that it will be at  $x = 2$  after 8 such steps?

**(Hint.** How many steps, in all, to the right the particle needs to make?)

4. **(5 points.)** Suppose that, cars pass a certain bridge at a Poisson rate of one every 12 minutes. What is the probability that 4 cars pass the bridge in the next 30 minutes?

5. The time it takes for a student to finish an exam (in hours) has a density function of the form:

$$f(x) = \begin{cases} 6(x-1)(2-x) & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) **(5 points.)** Find  $E(X)$ .

(b) **(5 points.)** Calculate the cumulative distribution function (c.d.f) of the time it takes for a randomly selected student to finish the exam.

6. **(5 points.)** Let  $X$  be a continuous random variable with density

$$f(x) = \begin{cases} cx^{11}e^{-3x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find  $c$ .

7. **(5 points.)** Let  $X \sim \text{Uniform}[0, 1]$ . Find the expected value of  $Y = \ln(X)$ . [Hint. Use integration by parts.]

8. **(5 points.)** Let  $T$  be a continuous random variable with density  $f_T(t) = \lambda e^{-\lambda t}$ ,  $t \geq 0$ . Let  $X$  be a discrete random variable defined as  $X = k$  if  $k - 1 \leq T < k$ ;  $k = 1, 2, \dots$ . Write the probability mass function of the discrete random variable  $X$ . What is the name of the distribution of  $X$ ? [Hint: Call  $1 - e^{-\lambda}$  as  $p$ ]

9. **(6 points.)** In each of the following say what is the name of the distribution of  $X$  and what are the parameters of the distribution.

(a) Buses arrive on a bus stop at a rate of 10 per hour.  $X$  represents the number of buses that will arrive in the next 10 minutes.

(b) Each bus that arrives on a bus stop is (independently of other buses) a bus that goes to downtown with probability .7. Let  $X$  be the number of buses I will have to see in order to get one that goes to downtown.

(c) As in (b) suppose that each bus that arrives on a bus stop is (independently of other buses) a bus that goes to downtown with probability .7. Let  $X$  represent the number of buses, out of the next 25 buses that arrive, that are going to downtown.

(d) Suppose in all 50 buses will stop at the station during a day. Of these 35 go to downtown. Let  $X$  represent the number of buses, (out of the next 25 buses that arrive), that are going to downtown.

(e) I will play a game in a casino until I lose four times. Suppose that my chance of winning on any game is .45.  $X$  represents the number of games I play before I leave.

(f) A dart is thrown randomly at a circular dart board of radius 10. Assuming that the dart will hit the board, let  $X$  denote the angle that the ray joining the location of the dart with the center of the board forms with the horizontal.