HOMEWORK SET #6

Based on lectures 9 - 10

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. with density $f(x) = \lambda e^{-\lambda(x-\beta)} I_{(\beta,\infty)}(x)$, where $\lambda > 0$, $\beta \in \mathbb{R}$ are unknown parameters.

   (a) Is this exponential family? Would the answer change if we assumed $\beta = 0$?

   (b) Find a minimal sufficient statistic.

   (c) Is $S(\mathbf{X}) = \frac{X_{(n)} - X_{(1)}}{X_{(n)} - \bar{X}_n}$ an ancillary statistics? Justify!
   (Hint: If $Y \sim \text{Exp}(1)$ and $X = \frac{Y}{\lambda} + \beta$, then the density of X is $f(x) = \lambda e^{-\lambda(x-\beta)} I_{(\beta,\infty)}(x)$.)

   (d) Assume $\beta = 0$. Calculate $E\left(\frac{X_{(1)}}{\bar{X}_n}\right)$. (Hint: Use Basu's theorem.)

   (e) Do not assume $\beta = 0$. Calculate $E\left(\frac{X_{(n)} - X_{(1)}}{\bar{X}_n - \beta}\right)$.

2. From the book: 6.9 (Hint: parts c and d use Fundamental Theorem of Algebra in the proof), 6.21, 6.27 (there is a typo $T = n/(\sum_{i=1}^{n}(1/X_i - 1/\bar{X}))$.