Homework set #10

1. Consider the following 150 sorted p-values:

   0.0003 0.0005 0.0009 0.0009 0.0012 0.0022 0.0025 0.0033 0.0035 0.0052
   0.0238 0.0263 0.0446 0.0470 0.0506 0.0564 0.0585 0.0660 0.0662 0.0685
   0.0805 0.0814 0.1084 0.1118 0.1217 0.1247 0.1288 0.1305 0.1447 0.1463
   0.1487 0.1541 0.1614 0.1896 0.1931 0.2181 0.2187 0.2218 0.2354 0.2389
   0.2485 0.2592 0.2976 0.3012 0.3050 0.3054 0.3122 0.3183 0.3202 0.3233
   0.3481 0.3491 0.3506 0.3543 0.3677 0.3738 0.3811 0.3872 0.3940 0.3992
   0.4033 0.4185 0.4240 0.4277 0.4361 0.4412 0.4436 0.4890 0.4894 0.4912
   0.4954 0.4972 0.5081 0.5193 0.5198 0.5199 0.5232 0.5254 0.5255 0.5290
   0.5292 0.5395 0.5397 0.5408 0.5444 0.5629 0.5638 0.5664 0.5767 0.5876
   0.5937 0.5960 0.6021 0.6203 0.6378 0.6396 0.6438 0.6513 0.6532 0.6671
   0.6857 0.6983 0.7085 0.7122 0.7302 0.7306 0.7426 0.7429 0.7454 0.7486
   0.7495 0.7534 0.7613 0.7633 0.7653 0.7681 0.7766 0.7806 0.7821 0.7828
   0.7866 0.7867 0.7870 0.7901 0.8039 0.8084 0.8116 0.8140 0.8159 0.8212
   0.8229 0.8304 0.8594 0.8698 0.8771 0.8874 0.8886 0.8973 0.9027 0.9043
   0.9066 0.9169 0.9208 0.9269 0.9330 0.9452 0.9454 0.9670 0.9781 0.9970

   (a) How many hypotheses would be rejected using without using any multiple test adjustment. How many would be rejected using Boinferroni adjustment?

   (b) How many hypotheses would be rejected using the step up and the step down method?

2. (a) Assume that $\mathcal{H}_0 = \mathcal{H} \neq \emptyset$ and $\text{FDR}(R) \leq \alpha$. What can you say about $P(\text{any hypothesis is rejected})$?

   (b) Assume that $\mathcal{H}_1 = \{h\}$, $\mathcal{H}_0 \neq \emptyset$ and $\text{FDR}(R) \leq \alpha$. What can you say about $P(\text{any hypothesis is rejected})$?

3. Let $X \sim \text{Binomial}(n, p)$.

   (a) If the prior $p \sim \text{Beta}(1/2, 1/2)$, find the Bayes factor for testing $\mathcal{H}_0 : p \in [0, 1/2]$ vs. $\mathcal{H}_1 : p \in (1/2, 1]$. Evaluate the Bayes factor and the p-value of the exact test for $n = 10$ and $x = 7$.

   (b) Modify the prior to test $\mathcal{H}_0 : p = 1/2$ vs. $\mathcal{H}_1 : p \neq 1/2$ and compute the Bayes factor? (Hint: Use $1/2$ for the point mass.) Evaluate the Bayes factor for $n = 10$ and $x = 7$.

(c) Propose a p-value for testing $\mathcal{H}_0 : p = 1/2$ vs. $\mathcal{H}_1 : p \neq 1/2$ and evaluate it for $n = 10$ and $x = 7$.

4. Let $X_1, \ldots X_n$ be iid Poisson(λ), and let λ have gamma(α, β) prior distribution. Consider a Bayesian test $\mathcal{H}_0 : \lambda \leq \lambda_0$ vs. $\mathcal{H}_1 : \lambda > \lambda_0$

   (a) Calculate the posterior probabilities for $\mathcal{H}_0, \mathcal{H}_1$.

   (b) Perform the Bayesian test using $\alpha = 2.5, \beta = 2$.