Homework 1

Calculate the following: (Show your work!)

1.
$$\sum_{i=1}^{\infty}\left(\sum_{j=i}^{\infty} p^{j+i}\right), \quad \text{where } |p|<1.$$

2.
$$\int_1^2 \frac{1}{x}dx.$$

3.
$$\int_{-1}^{1} \frac{1}{\sqrt{|x|}}dx.$$

4.
$$\int_0^2 xe^x dx.$$

5.
$$\int_0^2 xe^{x^2} dx.$$

6.
$$\iint\limits_{\substack{x+y<1,\\ x>0,\ y>0}} (x+y)^2 dxdy.$$

7. Prove or disprove the following:
$$1 \le \sum_{k=1}^{\infty} \frac{1}{k^2} \le 2.$$
(Hint: You do not have to evaluate the sum.)

8. Show that for any $n \ge 1$
$$\sum_{k=1}^{n} k^2 = \frac{n(1+n)(1+2n)}{6}.$$

9. Find all critical points of $f(x, y) = -x^2 - y^2 + 4y$.

10. Find the global maximum of $f(x, y) = -x^2 - y^2 + 4y$ over the region $D = \{(x, y) : x^2 + y^2 \leq 1\}$ and specify where it occurs.