Homework set #1

1. Prove or disprove: If X has a cdf F and $a \geq 0$ then $P(F(X) \leq a) \leq a$. Under what condition on F will you get $P(F(X) \leq a) = a$?

2. Consider a sequence of test statistics $\delta_a(X) \in \{0, 1\}$, $a \in [0, 1]$ such that the size $\sup_{\theta \in \Theta_0} E_\theta \delta_a(X) = a$; for $a_1 < a_2$ the tests $\delta_{a_1}(X) \leq \delta_{a_2}(X)$; and $\delta_0(X) = 0$, $\delta_1(X) = 1$. Set $p(X) = \inf\{a : \delta_a(X) = 1\}$. Prove or disprove $\sup_{\theta \in \Theta_0} P(p(X) \leq a) = a$.

3. Consider the following 150 sorted p-values:

0.0003	0.0005	0.0009	0.0009	0.0012	0.0022	0.0025	0.0033	0.0035	0.0052
0.0238	0.0263	0.0446	0.0470	0.0506	0.0564	0.0585	0.0660	0.0662	0.0685
0.0805	0.0814	0.1084	0.1118	0.1217	0.1247	0.1288	0.1305	0.1447	0.1463
0.1487	0.1541	0.1614	0.1896	0.1931	0.2181	0.2187	0.2218	0.2354	0.2389
0.2485	0.2592	0.2976	0.3012	0.3050	0.3054	0.3122	0.3183	0.3202	0.3233
0.3481	0.3491	0.3506	0.3543	0.3677	0.3738	0.3811	0.3872	0.3940	0.3992
0.4033	0.4185	0.4240	0.4277	0.4361	0.4412	0.4436	0.4890	0.4894	0.4912
0.4954	0.4972	0.5081	0.5193	0.5198	0.5199	0.5232	0.5254	0.5255	0.5290
0.5292	0.5395	0.5397	0.5408	0.5444	0.5629	0.5638	0.5664	0.5767	0.5876
0.5937	0.5960	0.6021	0.6203	0.6378	0.6396	0.6438	0.6513	0.6532	0.6671
0.6857	0.6983	0.7085	0.7122	0.7302	0.7306	0.7426	0.7429	0.7454	0.7486
0.7495	0.7534	0.7613	0.7633	0.7653	0.7681	0.7766	0.7806	0.7821	0.7828
0.7866	0.7867	0.7870	0.7901	0.8039	0.8084	0.8116	0.8140	0.8159	0.8212
0.8229	0.8304	0.8594	0.8698	0.8771	0.8874	0.8886	0.8973	0.9027	0.9043
0.9066	0.9169	0.9208	0.9269	0.9330	0.9452	0.9454	0.9670	0.9781	0.9970

   (a) How many hypotheses would be rejected using without using any multiple test adjustment. How many would be rejected using Bonferroni?

   (b) How many hypotheses would be rejected using the Holm's and Benjamini-Hochberg method?

4. Define $\tilde{r} = \max_r \{p_{(k)} \leq \alpha k/m$ for all $k \leq r\}$. Proof that the step down procedure, $R = \{p_{(1)}, \ldots, p_{(\tilde{r})}\}$ satisfies the condition $SC(\alpha, 1/m.r)$.

5. (a) Assume that $\mathcal{H}_0 = \mathcal{H} \neq \emptyset$ and $\mathrm{FDR}(R)) \leq \alpha$. What can you say about P(any correct hypothesis is rejected)?

(b) Assume that $\mathcal{H}_1 = \{h\}$, $\mathcal{H}_0 \neq \emptyset$ and $\mathrm{FDR}(R)) \leq \alpha$. What can you say about P(any correct hypothesis is rejected)?

6. Assume that $U \sim U(0,1)$, V is independent of U and $\beta(x) \leq x$. Prove or disprove: The dependency criterion $\mathrm{DC}(\beta)$ is satisfied for (U, V).