STOR 655 – April 30, 2012 Name:________________________

FINAL EXAM

All problem parts have equal weight. In budgeting your time expect that some problems will take longer than others.
Remember, answers without proper justification will not receive full credit!

1. Assume $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \dots \begin{pmatrix} X_n \\ Y_n \end{pmatrix}$ are i.i.d. bivariate normal$\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \rho & \rho \\ \rho & 1 \end{pmatrix}\right)$, $0 < \rho < 1$.

   (a) Consider $R_n = \frac{1}{n}\sum_{i=1}^n X_i Y_i$ and $V_n = \frac{1}{n}\sum_{i=1}^n X_i^2$. Are they consistent estimators of ρ?

   (b) Compute the asymptotic relative efficiency $ARE(R_n, V_n)$. Do you prefer one of the statistic over the other? Why?

   (c) Find the asymptotic distribution of $\frac{R_n^2}{V_n}$. Is it a consistent estimator of ρ?

   (d) Is any of the three estimators of ρ you computed above asymptotically efficient? If no, suggest an asymptotically efficient estimator; you do not have obtain a closed form expression. (Hint: $\frac{1}{\rho} + \frac{1}{1-\rho} = \frac{1}{\rho-\rho^2}$.)

More room for your solutions.

2. Let $X \sim \text{Binomial}(n, p)$ and consider the prior $p \sim \text{Uniform}(0, 1)$

   (a) Find the MLE and the Bayes estimator (using square loss) of p and evaluate them for $n = 50, x = 50$.

   (b) Find the 90% HPD credible interval for p if you observe $n = 50, x = 50$.

   (c) Find the Bayes factor for testing $\mathcal{H}_0 : p \in (0, 0.95]$ vs. $\mathcal{H}_1 : p \in (0.95, 1)$. Evaluate the Bayes factor for $n = 50, x = 50$.

   (d) Modify the prior to test $\mathcal{H}_0 : p = 0.95$ vs. $\mathcal{H}_1 : p \neq 0.95$ and find the Bayes factor. Evaluate the Bayes factor for $n = 50, x = 50$.

   (e) Find the likelihood ratio test for testing $\mathcal{H}_0 : p = 0.95$ vs. $\mathcal{H}_1 : p \neq 0.95$. Evaluate the value of the test statistic and find an approximate p-value for $n = 50, x = 50$.

More room for your solutions.

3. Let $X_1, X_2, \ldots$ be i.i.d. random variables so that $EX_i^6 < \infty$. Denote $\mu = EX_i$ and $\mu_j = E(X_i - \mu)^j$.

   (a) Find a U statistic for estimating μ_3.

   (b) Find the Hájek projection of your U-statistic.

   (c) Find the asymptotic distribution of your U statistic.
   (Hint: Use the the result of part b. If you did not find a solution to part b, you may use $n^{-1}\sum_{i=1}^{n}[(X_i - \mu)^3 - 3\mu_2(X_i - \mu)] - \mu_3$.)