

HOMEWORK SET #9
Based on lectures 15 – 18

1. Let X_1, \dots, X_n be an i.i.d. sample from $\text{Uniform}(\theta_1, \theta_2)$ distribution. ($n > 10$ and $\theta_1 < \theta_2$)
 - (a) Find the MM of (θ_1, θ_2) .
 - (b) Consider the estimators as two one-dimensional estimators. Are the estimators found in part a) unbiased?
 - (c) Find the MLE of (θ_1, θ_2) .
 - (d) Consider the estimators as two one-dimensional estimators. Are the estimators found in part c) unbiased? What are their MSE's?
2. Let X_1, \dots, X_n be an i.i.d. sample from $\text{Beta}(a, b)$ distribution.
 - (a) Find MM of (a, b) using the first and second raw moments.
 - (b) Find MLE of (a, b) . (Hint: If you are not able to get a formula. Explain what would you do.)
 - (c) You observe: 0.88, 0.66, 0.74, 0.65, 0.39, 0.87, 0.93, 0.11, 0.97, 0.99. Find the MLE and MM of (a, b) .
3. Assume that Y_1, \dots, Y_m are i.i.d. $\text{Uniform}(0, \theta_1]$ and Z_1, \dots, Z_n are i.i.d. $\text{Uniform}[\theta_1, \theta_2]$. (The Y 's and Z 's are mutually independent).
 - (a) Using \bar{Y} and \bar{Z} find MME of θ_1, θ_2 . Find also the MSE's of the estimators.
 - (b) Find MLE of θ_1, θ_2 . Find also the MSE's of the estimators.
 - (c) Does MLE seem to be better than MME in this case?
4. Let X_1, \dots, X_n be an i.i.d. sample from $N(0, \sigma^2)$. Find MLE of σ^2 .
5. Let X_1, \dots, X_n be an i.i.d. sample from Pareto distribution. (i.e. $f(x; \theta) = \frac{\theta}{(1+x)^{\theta+1}} I_{(0, \infty)}(x)$). Find MLE of θ .
6. Let X_1, \dots, X_n be an i.i.d. sample from a distribution with density. $f(x; a, b) = \frac{1}{b} e^{-\left(\frac{x-a}{b}\right)} I_{(a, \infty)}(x)$.
 - (a) Find MLE of (a, b) .

- (b) Find the conditional distribution of $X_1 - a, X_2 - X_1, \dots, X_n - X_1$ given $X_2 > X_1, \dots, X_n > X_1$.
- (c) Consider the MLEs as two one-dimensional estimators. Find their MSE. (Hint: Carefully consider what the result of part (b) mean for your calculations.)