

HOMEWORK SET #7

1. Let X_1, \dots, X_n be i.i.d. Bernoulli(p).
 - (a) Find the posterior Bayes estimator for the Beta(α, β) prior. (Hint: It is the posterior mean.)
 - (b) Find the asymptotic variance of the estimator you found. Is it asymptotically efficient?
2. Assume $\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \dots, \begin{pmatrix} X_n \\ Y_n \end{pmatrix}$ are i.i.d. bivariate normal $\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1, \rho \\ \rho, 1 \end{pmatrix} \right)$
 - (a) Prove that $T = (\sum_{i=1}^n X_i^2 + Y_i^2, \sum_{i=1}^n X_i Y_i)$ is a sufficient statistics.
 - (b) Consider $R_n(\mathbb{X}) = \frac{1}{n} \sum_{i=1}^n X_i Y_i$. Is it a consistent estimator of ρ .
 - (c) Find the asymptotic variance of $R_n(\mathbb{X})$
 - (d) Is it asymptotically efficient? If no could you suggest an asymptotically efficient estimator?
3. From the book 18.5, 18.7, 19.2, 19.3, 19.5.