

HOMEWORK SET #2
Based on lectures 1 – 2.

1. Let $X_1 \sim \Gamma(\alpha_1, 1)$ and $X_2 \sim \Gamma(\alpha_2, 1)$ be independent. Use the two-dimensional change of variables formula to show that $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$ are independent with $Y_1 \sim \Gamma(\alpha_1 + \alpha_2, 1)$ and $Y_2 \sim \beta(\alpha_1, \alpha_2)$.
2. (a) Using integration by parts, show that the gamma function $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$ satisfies the relation $\Gamma(t+1) = t\Gamma(t)$ for $t > 0$.
(b) Use the Hölder's inequality to show that $g(x) = \log \Gamma(x)$ is convex for $x \in (0, \infty)$.
3. Prove or disprove: If X has a cdf F and $a \geq 0$ then $P(F(X) \leq a) \leq a$. Under what condition on F will you get $P(F(X) \leq a) = a$?
4. Use Jensen's inequality to show that for $a, b > 0$ and $p \geq 1$,

$$(a + b)^p \leq 2^{p-1}[a^p + b^p].$$

Verify this inequality in case $p = 2$ by a direct calculation.

5. From the book 5.12.