Homework set #7

Based on lectures 11 – 12

1. (a) Let $X_1, \dots, X_n$ be i.i.d. $\mathrm{N}(\mu, \sigma^2)$. Find $c > 0$, possibly depending on n, that minimizes $MSE(cS_n^2)$.

   (b) What implications does this have for sample variance?

2. Let $X_1, \dots, X_n$ be i.i.d. Exponential(λ), i.e., $f(x|\lambda) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$. Find the UMVUE for estimating $e^{-10\lambda}$.

3. Let us assume that $X_1, X_2, \dots, X_{2n}$ be i.i.d. from some distribution with parameter θ and $S(X_1, \dots, X_{2n})$ is minimal sufficient complete statistics. Further assume that $T(X_1, \dots, X_n)$ is an unbiased estimator of $q(\theta)$. Is there a UMVUE of $q(\theta)^2$? Justify!

4. Let $X_1, \dots, X_n$ be an i.i.d. sample from geometric(p) distribution. (i.e. $f(x, p) = p(1-p)^{x-1} I_{\{1,2\dots\}}$, where $p \in (0, 1)$.)

   (a) Argue exponential family and deduce complete sufficient statistics.

   (b) Is there UMVUE of $\tau(p) = E_p X_1$? If so, find it.

   (c) Find the UMVUE of $1/p^2$.

   (d) Is there UMVUE of p? If so, find it.

5. Let $X_1, \dots, X_n$ be an i.i.d. sample from Pareto distribution. (i.e. $f(x; \theta) = \frac{\theta}{(1+x)^{\theta+1}} I_{(0,\infty)}(x)$).

   (a) Argue exponential family and find complete sufficient statistics.

   (b) Is there UMVUE of θ? If so, find it.

   (c) Is there UMVUE of $\tau(\theta) = 2^{-\theta}$? If so, find it.

6. From the book 7.50.