HOMEWORK SET #11

Based on lectures 20 – 21

1. Consider a sequence of test statistics $\delta_a(X) \in \{0,1\}$, $a \in [0,1]$ such that the size $\sup_{\theta\in\Theta_0} E_\theta \delta_a(X) = a$; for $a_1 < a_2$ the tests $\delta_{a_1}(X) \le \delta_{a_2}(X)$; and $\delta_0(X) = 0$, $\delta_1(X) = 1$. Set $p(X) = \inf\{a : \delta_a(X) = 1\}$. Prove or disprove $\sup_{\theta\in\Theta_0} P(p(X) \le a) = a$.

2. Let X be Geometric(p) and consider testing $\mathcal{H}_0 :\ p \ge p_0$ versus $\mathcal{H}_1 :$ $p < p_0$. Propose a p-value. Do you reject the null hypothesis for or $p_0 = 0.1$ and $x = 28$ at the $\alpha = 0.05$ level?

3. Consider the following 150 sorted p-values:
   0.0003 0.0005 0.0009 0.0009 0.0012 0.0022 0.0025 0.0033 0.0035 0.0052
   0.0238 0.0263 0.0446 0.0470 0.0506 0.0564 0.0585 0.0660 0.0662 0.0685
   0.0805 0.0814 0.1084 0.1118 0.1217 0.1247 0.1288 0.1305 0.1447 0.1463
   0.1487 0.1541 0.1614 0.1896 0.1931 0.2181 0.2187 0.2218 0.2354 0.2389
   0.2485 0.2592 0.2976 0.3012 0.3050 0.3054 0.3122 0.3183 0.3202 0.3233
   0.3481 0.3491 0.3506 0.3543 0.3677 0.3738 0.3811 0.3872 0.3940 0.3992
   0.4033 0.4185 0.4240 0.4277 0.4361 0.4412 0.4436 0.4890 0.4894 0.4912
   0.4954 0.4972 0.5081 0.5193 0.5198 0.5199 0.5232 0.5254 0.5255 0.5290
   0.5292 0.5395 0.5397 0.5408 0.5444 0.5629 0.5638 0.5664 0.5767 0.5876
   0.5937 0.5960 0.6021 0.6203 0.6378 0.6396 0.6438 0.6513 0.6532 0.6671
   0.6857 0.6983 0.7085 0.7122 0.7302 0.7306 0.7426 0.7429 0.7454 0.7486
   0.7495 0.7534 0.7613 0.7633 0.7653 0.7681 0.7766 0.7806 0.7821 0.7828
   0.7866 0.7867 0.7870 0.7901 0.8039 0.8084 0.8116 0.8140 0.8159 0.8212
   0.8229 0.8304 0.8594 0.8698 0.8771 0.8874 0.8886 0.8973 0.9027 0.9043
   0.9066 0.9169 0.9208 0.9269 0.9330 0.9452 0.9454 0.9670 0.9781 0.9970

   (a) How many hypotheses would be rejected using without using any multiple test adjustment. How many would be rejected using Boinferroni adjustment?

   (b) How many hypotheses would be rejected using the step up and the step down method?

4. Define $\tilde{r} = \max_r\{p_{(k)} \le \alpha k/m$ for all $k \le r\}$. Proof that the step down procedure, $R = \{p_{(1)}, \ldots, p_{(\tilde{r})}\}$ satisfies the condition $SC(\alpha, 1/m, r)$.

5. (a) Assume that $\mathcal{H}_0 = \mathcal{H} \ne \emptyset$ and $\mathrm{FDR}(R) \le \alpha$. What can you say about P(any hypothesis is rejected)?

(b) Assume that $\mathcal{H}_1 = \{h\}$, $\mathcal{H}_0 \neq \emptyset$ and $\mathrm{FDR}(R) \leq \alpha$. What can you say about P(any hypothesis is rejected)?

6. Assume that $U \sim U(0,1)$, V is independent of U and $\beta(x) \leq x$. Prove or disprove: The dependency criterion $\mathrm{DC}(\beta)$ is satisfied for (U, V).