HOMEWORK SET #5

Based on lectures 7 - 8

1. Let $X_1, \ldots, X_n$ be an i.i.d. sample from Uniform$(-\theta, \theta^2)$ distribution, $\theta > 0$. Find a one-dimensional sufficient statistics.

2. Let $X_1, \ldots, X_n$ be an i.i.d. sample from $\mathrm{N}(0, \sigma^2)$. Find a one-dimensional sufficient statistics.

3. Let $X_1, \ldots, X_n$ be an i.i.d. sample from Pareto distribution. (i.e. $f(x;\theta) = \frac{\theta}{(1+x)^{\theta+1}} I_{(0,\infty)}(x)$). Find a one-dimensional sufficient statistics.

4. Let $X_1, \ldots, X_n$ be an i.i.d. sample from a distribution with density. $f(x; a, b) = \frac{1}{b} e^{-\left(\frac{x-a}{b}\right)} I_{(a,\infty)}(x)$.

   (a) Show that $(X_{1:n}, \sum_{i=1}^n X_i)$ is sufficient.

   (b) Show that $(X_{1:n}, \sum_{i=2}^n (X_{i:n} - X_{1:n}))$ is sufficient.

5. Let $X_1, \ldots, X_n$ be an i.i.d. sample from exponential(β) distribution *censored* at $a > 0$. Find a sufficient statistics. (Hint: Use the “mixed” density $f(x; a, \beta) = \frac{1}{\beta} e^{-x/\beta} I_{(0,a)}(x) + e^{-a/\beta} I_{\{a\}}(x)$.)

6. From the book: 3.28bce, 3.29bce, 3.33, 6.16ab.