STOR 455
STATISTICAL METHODS I

Jan Hannig

Simultaneous Tests

- A company has developed four new additives that can be added into cement. An experiment has been carried out to determine if there are any differences between the average strength of cement blocks made with each additive.
- H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ H_A: there is a difference
- We are testing more than one difference at a time!
- H_0: $\theta_1=\mu_2-\mu_1=0$, $\theta_2=\mu_3-\mu_1=0$, $\theta_3=\mu_4-\mu_1=0$

Simultaneous Confidence Sets

- Similarly as for testing we can have one-at-a-time or simultaneous confidence intervals.
- One at a time CIs
  - $P(L_1 \le \theta_1 \le U_1)=1-\alpha, \ldots, P(L_p \le \theta_p \le U_p)=1-\alpha$
  - Each separately has confidence $1-\alpha$
- Simultaneous CIs
  - $P(L_1 \le \theta_1 \le U_1, \ldots, L_p \le \theta_p \le U_p)=1-\alpha$
  - They together have confidence $1-\alpha$

Bonferroni Adjustment

- Divide your α between the test or CIs.
- In our example we have 3 tests – use $\alpha/3$ as a significance level.

READ THE BOOK. (Conversation 1.6 in particular)

Example

- Certain small cars have a gas mileage forming a Gaussian population with unknown μ and σ. Obtain one-at-a time and simultaneous 95% Cis for μ and σ.
- Data

Car number	1	2	3	4	5	6	7	8	9	10
Gas mileage (mpg)	25.72	25.24	25.19	25.88	26.42	24.48	25.11	24.29	25.06	25.63

- One at a time

```
proc univariate data=TABLE161 alpha=.05 all; run;
Mean    [24.84470, 25.75930]
StD     [0.43971, 1.16705]
```

- Simultaneous

```
proc univariate data=TABLE161 alpha=.025 all; run;
Mean    [24.75921,25.84479]
StD     [0.41816, 1.28726]
```

Functions

- Let *D*, *R* be a sets of numbers. Function f(.) on *D* is a rule that assigns to each value in *D* another number in *R*.
- Examples:
  - $f(x)=3x+5$
  - $f(u)=\sin(u)$
  - $f(y)=2y^2-5y+10$
- We could use other letters $Y(.), \mu_Y(.), \ldots$

Variables

- In the equation $z=g(s)$
  - z is *dependent* variable
  - s is *independent* variable
- Functions can be defined piecewise
  - $v=r(u)$, where $r(u)=1$ if $u \geq 1$
    $r(u)=0$ if $u \leq 0$
    $r(u)=u$ if $0<u<1$
  - $z=\arcsin(x)$ for $-1<x<1$
    UNDEFINED for other values
- Graph of a function

Linear Function

- Functions can have more than one variable $f(x,y)=\sin(x)+y^2$
- Linear function
  - One variable $f(s)=a\ s+b$
    a,b called parameters, they represent fixed numbers.
  - Multiple variables $f(s_1,\ldots,s_p)=a_1s_1+\ldots+a_ps_p+b$
- Non-Linear function
  - Powers or more complicated functions of the variables
  - Functions could be linear in one variable and non-linear in another $\mu_Y(x_1,x_2)=10+4x_1-|x_2|$

Matrices and Vectors

- Matrix is a rectangular array of numbers

$$\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 & 3 \\ 4 & x & y \end{bmatrix}$$

$$\begin{bmatrix} 5 & x_1 & 4 \\ y_2 & log(x) & 6 \\ 5 & 9 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & \\ 7 & 8 & 9 \end{bmatrix}$$

Vector

- Vector is a column or row array of numbers
  - Row vector [12, 23, 453, -34.2]
  - Column vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$
- Notation:

  A, B,… stands for matrices

  a,b,… stand for vectors

Matrix Elements

- Consider matrix

$$A = \begin{bmatrix} 3 & 1 \\ 4 & -4 \\ 6 & 0 \end{bmatrix} \quad \text{or} \quad [a_{ij}] = \begin{bmatrix} 3 & 1 \\ 4 & -4 \\ 6 & 0 \end{bmatrix}$$

  – We have $a_{12}=1$ and $a_{31}=6$

- Similarly for vectors

$$a = [a_i] = \begin{bmatrix} 6 \\ 4 \\ -2 \\ 0 \end{bmatrix}$$

  – $a_3=-2$

- Size of a matrix $n \times m$
  (number of rows × number of columns)
- Column Vector is a $d \times 1$ matrix, row vector is a $1 \times d$ *matrix*

Matrix operations

- Two matrices are *equal* if they have the same size and same elements
- *Transposition* of a matrix ***A'*** – flip
- *Addition* and *subtraction* of matrices – must have the same size, add element by element
- Multiplication by a *scalar* (number) – multiply the each of the elements by a scalar.
- $(\boldsymbol{A}+\boldsymbol{B})'=\boldsymbol{A}'+\boldsymbol{B}'$, $(c\,\boldsymbol{C})'=c\,\boldsymbol{C}'$

Example

$$A=\begin{bmatrix}3 & -1\\ 0 & 4\\ 2 & 6\end{bmatrix}\quad B=\begin{bmatrix}4 & 0 & 2 & 3\\ 0 & 2 & 6 & 10\end{bmatrix}\quad C=\begin{bmatrix}5 & 9\\ -16 & 4\\ 0 & 6\end{bmatrix}\quad D=\begin{bmatrix}3 & 10 & 4\\ -1 & 0 & 6\end{bmatrix} \tag{1.8.7}$$

$$G=A+C=\begin{bmatrix}3 & -1\\ 0 & 4\\ 2 & 6\end{bmatrix}+\begin{bmatrix}5 & 9\\ -16 & 4\\ 0 & 6\end{bmatrix}=\begin{bmatrix}3+5 & -1+9\\ 0-16 & 4+4\\ 2+0 & 6+6\end{bmatrix}=\begin{bmatrix}8 & 8\\ -16 & 8\\ 2 & 12\end{bmatrix}$$

$$S=A-C=\begin{bmatrix}3 & -1\\ 0 & 4\\ 2 & 6\end{bmatrix}-\begin{bmatrix}5 & 9\\ -16 & 4\\ 0 & 6\end{bmatrix}=\begin{bmatrix}3-5 & -1-9\\ 0-(-16) & 4-4\\ 2-0 & 6-6\end{bmatrix}=\begin{bmatrix}-2 & -10\\ 16 & 0\\ 2 & 0\end{bmatrix}$$

Matrix Multiplications

- Complicated – see textbook
- Matrices have to fit $(n \times m).(m \times k)=(n \times k)$
- If $\boldsymbol{A}=[a_{\mathrm{ij}}]$, $\boldsymbol{B}=[b_{kl}]$, then
  $\boldsymbol{C}=\boldsymbol{A}.\boldsymbol{B}=[c_{pq}]$ where $c_{pq}=\Sigma_m a_{pm} b_{mq}$

$$\boldsymbol{P}=\boldsymbol{AB}=\begin{bmatrix} 3 & -1 \\ 0 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 0 & 2 & 3 \\ 0 & 2 & 6 & 10 \end{bmatrix} = \begin{bmatrix} 12 & -2 & 0 & -1 \\ 0 & 8 & 24 & 40 \\ 8 & 12 & 40 & 66 \end{bmatrix}$$

- The multiplication is associative but non commutative.
- Transpose: $(\boldsymbol{A}.\boldsymbol{B})'=\boldsymbol{B}'.\boldsymbol{A}'$

Special Matrices

- Square matrix
- Identity Matrix – ***I***
- Zero and One Matrix – ***0***, ***1***
- Diagonal Matrix
- Symmetric Matrix – ***A***'=***A***
  - See textbook for details

Inverse Matrix

- $\boldsymbol{A}^{-1}$ is a matrix such that $\boldsymbol{A}.\ \boldsymbol{A}^{-1}=\boldsymbol{A}^{-1}.\boldsymbol{A}=\boldsymbol{I}$

$$A = \begin{bmatrix} 7 & 2 \\ 10 & 3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 3 & -2 \\ -10 & 7 \end{bmatrix}$$

- Easy to find for 2×2

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a_{22}/d & -a_{12}/d \\ -a_{21}/d & a_{11}/d \end{bmatrix}$$

  – *Here* $d=\det(\boldsymbol{A})=a_{11}a_{22}-a_{12}a_{21}$

- Used for solving linear equation (see textbook)