STOR 655 – February 29, 2011 Name:________________________

Midterm

All problem parts have equal weight. In budgeting your time expect that some problems will take longer than others.
Remember, answers without proper justification will not receive full credit!

1. Consider the following 100 sorted p-values:

   0.0003 0.0005 0.0009 0.0012 0.0025 0.0035 0.0052 0.0068 0.0096 0.0097
   0.0114 0.0116 0.0260 0.0485 0.0805 0.1084 0.1118 0.1247 0.1288 0.1447
   0.1487 0.1541 0.1896 0.1931 0.2187 0.2218 0.2389 0.2592 0.2976 0.3012
   0.3050 0.3054 0.3183 0.3202 0.3481 0.3506 0.3543 0.3738 0.3811 0.3940
   0.3992 0.4185 0.4277 0.4361 0.4436 0.4890 0.4912 0.4954 0.5081 0.5198
   0.5199 0.5232 0.5254 0.5255 0.5292 0.5395 0.5408 0.5629 0.5638 0.5767
   0.5876 0.5960 0.6021 0.6378 0.6438 0.6513 0.6671 0.6857 0.7085 0.7122
   0.7306 0.7429 0.7454 0.7495 0.7534 0.7633 0.7653 0.7766 0.7821 0.7828
   0.7867 0.7870 0.8039 0.8084 0.8140 0.8212 0.8229 0.8304 0.8594 0.8698
   0.8874 0.8886 0.9027 0.9066 0.9169 0.9269 0.9330 0.9454 0.9670 0.9970

   How many hypotheses would be rejected at the 10% level using without using any multiple test adjustment. How many would be rejected using Bonferroni adjustment? How many hypotheses would be rejected at the 10% level using the step up and the step down method?

2. Let $X \sim \text{Poisson}(\lambda)$ and consider the prior $\lambda \sim \text{Exponential}(1)$

   (a) Find the 95% HPD credible interval for λ if you observe $x = 0$.

   (b) Find the Bayes factor for testing $\mathcal{H}_0 : \lambda \in (0, 3]$ vs. $\mathcal{H}_1 : \lambda \in (3, \infty)$. Evaluate the Bayes factor for $x = 0$.

   (c) Modify the prior to test $\mathcal{H}_0 : \lambda = 3$ vs. $\mathcal{H}_1 : \lambda \neq 3$ and find the Bayes factor. Evaluate the Bayes factor for $x = 0$.

3. Let $\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} \sim N\left(\begin{pmatrix} 2 \\ 4 \\ -6 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 16 \end{pmatrix}\right)$.

(a) Find the conditional distribution of $X_1 \mid X_2 + X_3 = 5$.

(b) Find the distribution of

$$V = X_1 + X_2/2 + X_3/3 + X_4/4.$$

(c) Find the distribution of

$$W = X_1^2 + X_2^2/4 + X_3^2/9 + X_4^2/16 - V^2/4.$$

(d) Are V and W independent?

4. (a) Let $X_n \xrightarrow{\mathcal{D}} X$ and $g(s)$ be a continuous function. Prove that $g(X_n) \xrightarrow{\mathcal{D}} g(X)$.

   (b) Let A be a symmetric matrix. Show that if $\mathbf{X} \sim N_p(\mu, \Sigma)$ and $\mathbf{Y} = \mathbf{X}^T A \mathbf{X}$ then $EY = \operatorname{tr}(A\Sigma) + \mu^T A \mu$.