

HOMEWORK SET #8

1. Consider the Kullback-Leibler divergence

$$KL(q, p) = -Q \log(p/q) = - \int \log(p(x)/q(x))q(x)dx.$$

- (a) Show that $KL(q, p) \geq 0$ with equality if and only if $p(x) = q(x)$ a.s.
- (b) Show that $KL(q, p) < \infty$ only if q is absolutely continuous with respect to p .
- (c) Is $KL(p, q) = KL(q, p)$?

2. Prove that for all $x > 0$

$$\left(\frac{1}{x} - \frac{1}{x^3}\right) \phi(x) < 1 - \Phi(x) < \frac{1}{x} \phi(x).$$

(Hint: Integrate the following inequalities: $(1 - 3y^{-4})e^{-y^2/2} < e^{-y^2/2} < (1 + y^{-2})e^{-y^2/2}$.)

3. From the book 20.2, 20.3.