

HOMEWORK 1

Calculate the following: (Show your work!)

1.

$$\sum_{i=1}^{\infty} \left(\sum_{j=i}^{\infty} p^{j+i} \right), \quad \text{where } |p| < 1.$$

2.

$$\int_1^2 \frac{1}{x} dx.$$

3.

$$\int_{-1}^1 \frac{1}{\sqrt{|x|}} dx.$$

4.

$$\int_0^2 x e^x dx.$$

5.

$$\int_0^2 x e^{x^2} dx.$$

6.

$$\iint_{\substack{x+y < 1, \\ x > 0, y > 0}} (x+y)^2 dx dy.$$

7. Prove or disprove the following:

$$1 \leq \sum_{k=1}^{\infty} \frac{1}{k^2} \leq 2.$$

(Hint: You do not have to evaluate the sum.)

8. Show that for any $n \geq 1$

$$\sum_{k=1}^n k^2 = \frac{n(1+n)(1+2n)}{6}.$$