HOMEWORK SET #8

Based on lectures 13 – 14

1. Let $X_1, \ldots, X_n$ be an i.i.d. sample from Uniform(θ_1, θ_2) distribution. $(n > 10$ and $\theta_1 < \theta_2)$
   (a) Find the MM of (θ_1, θ_2).
   (b) Consider the estimators as two one-dimensional estimators. Are the estimators found in part a) unbiased?
   (c) Find the MLE of (θ_1, θ_2).
   (d) Consider the estimators as two one-dimensional estimators. Are the estimators found in part c) unbiased? What are their MSE's?

2. Let $X_1, \ldots, X_n$ be an i.i.d. sample from Beta(a, b) distribution.
   (a) Find MM of (a, b) using the first and second raw moments.
   (b) Find MLE of (a, b). (Hint: If you are not able to get a formula. Explain what would you do.)
   (c) You observe: $0.88, 0.66, 0.74, 0.65, 0.39, 0.87, 0.93, 0.11, 0.97, 0.99$. Find the MLE and MM of (a, b).

3. Assume that $Y_1, \ldots, Y_m$ are i.i.d. Uniform$(0, \theta_1]$ and $Z_1, \ldots, Z_n$ are i.i.d. Uniform$[\theta_1, \theta_2]$. (The Y's and Z's are mutually independent).
   (a) Using $\bar{Y}$ and $\bar{Z}$ find MME of θ_1, θ_2. Find also the MSE's of the estimators.
   (b) Find MLE of θ_1, θ_2. Find also the MSE's of the estimators.
   (c) Does MLE seem to be better then MME in this case?

4. Let $X_1, \ldots, X_n$ be an i.i.d. sample from $N(0, \sigma^2)$. Find MLE of σ^2.

5. Let $X_1, \ldots, X_n$ be an i.i.d. sample from Pareto distribution. (i.e. $f(x; \theta) = \frac{\theta}{(1+x)^{\theta+1}} I_{(0,\infty)}(x)$). Find MLE of θ.

6. Let $X_1, \ldots, X_n$ be an i.i.d. sample from a distribution with density. $f(x; a, b) = \frac{1}{b} e^{-\left(\frac{x-a}{b}\right)} I_{(a,\infty)}(x)$.
   (a) Find MLE of (a, b).

(b) Find the conditional distribution of $X_1 - a, X_2 - X_1, \ldots, X_n - X_1$ given $X_2 > X_1, \cdots, X_n > X_1$.

(c) Consider the MLEs as two one-dimensional estimators. Find their MSE. (Hint: Carefully consider what the result of part (b) mean for your calculations.)