

## HOMEWORK SET #7

1. Consider the Kullback-Leibler divergence

$$KL(q, p) = -Q \log(p/q) = - \int \log(p(x)/q(x))q(x)dx.$$

- (a) Show that  $KL(q, p) \geq 0$  with equality if and only if  $p(x) = q(x)$  a.s.
- (b) Show that  $KL(q, p) < \infty$  only if  $q$  is absolutely continuous with respect to  $p$ .
- (c) Is  $KL(p, q) = KL(q, p)$ ?

2. Prove that for all  $x > 0$

$$\left(\frac{1}{x} - \frac{1}{x^3}\right)\phi(x) < 1 - \Phi(x) < \frac{1}{x}\phi(x).$$

(Hint: Integrate the following inequalities:  $(1 - 3y^{-4})e^{-y^2/2} < e^{-y^2/2} < (1 + y^{-2})e^{-y^2/2}$ .)

3. From the book 18.5, 18.7, 19.2, 19.3, 19.5.