# Homework set #9

1. Assume that $X_1, \dots, X_n$ are i.i.d. Uniform$(a_1, b_1)$, $Y_1, \dots, Y_m$ are i.i.d. Uniform$(a_2, b_2)$, and the $X$'s are independent of the $Y$'s. We will be testing $\mathcal{H}_0 : a_1 = a_2, b_1 = b_2$.

   (a) Find the likelihood ratio $\lambda(\mathbb{X})$.

   (b) Prove that under the null hypothesis the distribution of $\lambda(\mathbb{X})$ does not depend on the parameters.

   (c) Describe the likelihood ratio test of size $\alpha$.

2. Let $X_1, \dots, X_n$ be i.i.d. N$(\mu, 1)$. Consider a hypothesis $\mathcal{H} : 1 < \mu < 2$, and a test $\{$Reject $\mathcal{H}$ if either $\bar{X}_n < 1 - 2/\sqrt{n}$ or $\bar{X}_n > 2 + 2/\sqrt{n}\}$.

   (a) What is the function $\delta(\mathbb{X})$?

   (b) Find the power $\beta(\mu)$ and sketch it.

   (c) What is the size of this test?

3. Assume $X_1, \dots, X_{20}$ are i.i.d. Bernoulli$(p)$. We want to test $\mathcal{H}_0 : p = .8$ versus $\mathcal{H}_1 : p = .6$. Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size $\alpha = .1$.

4. Assume that $X_1, \dots, X_n$ are i.i.d. N$(\mu, \sigma^2)$ and consider testing the hypotheses $\mathcal{H}_0 : \mu = \mu_0, \sigma^2 = \sigma_0^2$ versus $\mathcal{H}_1 : \mu = \mu_1, \sigma^2 = \sigma_1^2$.

   (a) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size $\alpha$ if $\mu_0 < \mu_1$ and $\sigma_0^2 = \sigma_1^2$.

   (b) For the previous part, find the rejection region if $n = 25$, $\mu_0 = 0$, $\mu_1 = 10$, $\sigma_0^2 = \sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You will need to use normal tables.)

   (c) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size $\alpha$ if $\mu_0 = \mu_1$ and $\sigma_0^2 < \sigma_1^2$.

   (d) For the previous part, find the rejection region if $n = 25$, $\mu_0 = \mu_1 = 10$, $\sigma_0^2 = 9$, $\sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You will need to use chi-squared tables.)

   (e) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size $\alpha$ if $\mu_0 < \mu_1$ and $\sigma_0^2 < \sigma_1^2$.

(f) For the previous part, find the rejection region if $n = 25$, $\mu_0 = 0$, $\mu_1 = 10$, $\sigma_0^2 = 9$, $\sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You may need to use non-central chi-square tables.)

5. In what follows $\theta > 0$ is an unknown parameter.

   (a) Find the LRT for testing $\mathcal{H}_0 : a = 1$ versus $\mathcal{H}_1 : a \neq 1$ based on a sample $X_1, \ldots, X_n$ from $N(\theta, a\theta)$.

   (b) Find the LRT for testing $\mathcal{H}_0 : a = 1$ versus $\mathcal{H}_1 : a \neq 1$ based on a sample $X_1, \ldots, X_n$ from $N(\theta, a\theta^2)$.

6. Let $X_1, \ldots, X_n$ be iid Beta$(\mu, 1)$ and $Y_1, \ldots, Y_m$ be iid Beta$(\eta, 1)$; the $X$s are independent of the $Y$s.

   (a) Find an LRT for testing $\mathcal{H}_0 : \mu = \eta$ versus $\mathcal{H}_1 : \mu \neq \eta$.

   (b) Show that the test in part (a) can be based on the statistic

   $$T = \frac{\sum_{i=1}^n \log X_i}{\sum_{i=1}^n \log X_i + \sum_{j=1}^m \log Y_j}.$$

   (c) Find the distribution of $T$ under $\mathcal{H}_0$ and show how to find a test size $\alpha = 0.1$.

7. Consider a sequence of test statistics $\delta_a(X) \in \{0, 1\}$, $a \in [0, 1]$ such that the size $\sup_{\theta\in\Theta_0} E_\theta \delta_a(X) = a$; for $a_1 < a_2$ the tests $\delta_{a_1}(X) \leq \delta_{a_2}(X)$; and $\delta_0(X) = 0$, $\delta_1(X) = 1$. Set $p(X) = \inf\{a : \delta_a(X) = 1\}$. Prove or disprove $\sup_{\theta\in\Theta_0} P(p(X) \leq a) = a$.

8. Let $X$ be Geometric$(p)$ and consider testing $\mathcal{H}_0 : \ p \geq p_0$ versus $\mathcal{H}_1 : p < p_0$. Propose a p-value. Do you reject the null hypothesis for or $p_0 = 0.1$ and $x = 28$ at the $\alpha = 0.05$ level?