HOMEWORK SET #10

Based on lectures 17 – 19

1. Assume that $X_1, \ldots, X_n$ are i.i.d. Uniform(a_1, b_1), $Y_1, \ldots, Y_m$ are i.i.d. Uniform(a_2, b_2), and the X's are independent of the Y's. We will be testing $\mathcal{H}_0 : a_1 = a_2, b_1 = b_2$.

   (a) Find the likelihood ratio $\lambda(\mathbb{X})$.

   (b) Prove that under the null hypothesis the distribution of $\lambda(\mathbb{X})$ does not depend on the parameters.

   (c) Describe the likelihood ratio test of size α.

2. Let $X_1, \ldots, X_n$ be i.i.d. $\mathrm{N}(\mu, 1)$. Consider a hypothesis $\mathcal{H} :\ 1 < \mu < 2$, and a test {Reject $\mathcal{H}$ if either $\bar{X}_n < 1 - 2/\sqrt{n}$ or $\bar{X}_n > 2 + 2/\sqrt{n}$}.

   (a) What is the function $\delta(\mathbb{X})$?

   (b) Find the power $\beta(\mu)$ and sketch it.

   (c) What is the size of this test?

3. Assume $X_1, \ldots, X_{20}$ are i.i.d. Bernoulli(p). We want to test $\mathcal{H}_0 :\ p = .8$ versus $\mathcal{H}_1 :\ p = .6$. Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size $\alpha = .1$.

4. Assume that $X_1, \ldots, X_n$ are i.i.d. $\mathrm{N}(\mu, \sigma^2)$ and consider testing the hypotheses $\mathcal{H}_0 : \mu = \mu_0, \sigma^2 = \sigma_0^2$ versus $\mathcal{H}_1 : \mu = \mu_1, \sigma^2 = \sigma_1^2$.

   (a) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size α if $\mu_0 < \mu_1$ and $\sigma_0^2 = \sigma_1^2$.

   (b) For the previous part, find the rejection region if $n = 25$, $\mu_0 = 0$, $\mu_1 = 10$, $\sigma_0^2 = \sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You will need to use normal tables.)

   (c) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size α if $\mu_0 = \mu_1$ and $\sigma_0^2 < \sigma_1^2$.

   (d) For the previous part, find the rejection region if $n = 25$, $\mu_0 = \mu_1 = 10$, $\sigma_0^2 = 9$, $\sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You will need to use chi-squared tables.)

(e) Find the most powerful test of $\mathcal{H}_0$ vs. $\mathcal{H}_1$ of size α if $\mu_0 < \mu_1$ and $\sigma_0^2 < \sigma_1^2$.

(f) For the previous part, find the rejection region if $n = 25$, $\mu_0 = 0$, $\mu_1 = 10$, $\sigma_0^2 = 9$, $\sigma_1^2 = 25$, and $\alpha = .05$. (Hint: You may need to use non-central chi-square tables.)

5. From the book: 8.8, 8.17.