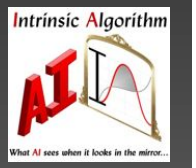
Better AI through Math

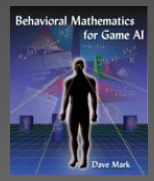
Dave Mark – Intrinsic Algorithm LLC

"Reducing the world to mathematical equations!"

Dave Mark



- President & Lead Designer of Intrinsic Algorithm LLC
  - Independent Game Studio
  - AI Consulting Company
- Author of *Behavioral Mathematics for Game AI*
- Contributed to:
  - *AI Game Programming Wisdom 4*
  - *Game Programming Gems 8*
- Co-advisor *GDC AI Summit* 2008, 2009, 2010
- Spoken at GDC, GDC Austin, GameX



"Reducing the world to mathematical equations!"

First, a word from Dr. Larry Fleinhardt

"Reducing the world to mathematical equations!"

First, a word from Dr. Larry Fleinhardt

"Let me guess, you tried to solve a problem involving human behavior and it blew up in your face.

Charles, you are a *mathematician*; you are always looking for the elegant solution. Human behavior is rarely, if ever, elegant.

Perhaps you need to make your equation, *less* elegant, more *complicated*, less *precise*, more *descriptive*. It's not going to be as pretty, but it might work a little better.

"Reducing the world to mathematical equations!"

Not to be pedantic…

- Many things you will see will seem obvious
  - Industry doesn't use them
  - Why not?
- Many things you will see will seem simple
  - Complicated isn't always the answer
  - Why does it need to be?
- Only the results matter.

What are we trying to do?

We aren't solving an equation.

We are trying to replicate behavior.

What makes a game a game?

"A game is a series of interesting choices."

- Sid Meier

Interesting Opponents Make for Interesting Choices

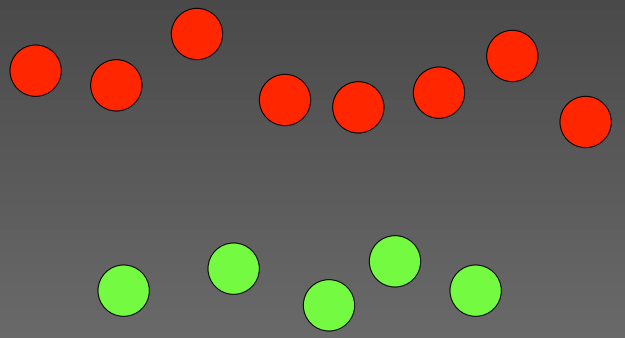
- Predictable vs. unpredictable
- Static vs. dynamic
- Shooting gallery vs. realistic

But how?

Know when to walk away…

- Design Decision:
  "Enemies don't always fight to the death"
- Enemies can sometimes retreat
  - Flat % chance
    - Is random… therefore looks random
    - Not realistic
  - Situational random
    - Based on circumstances
    - Circumstances are flexible and dynamic

Know when to walk away…



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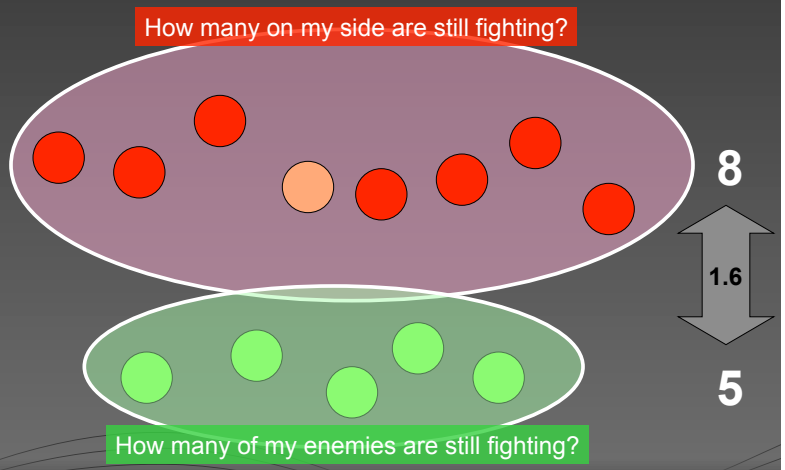
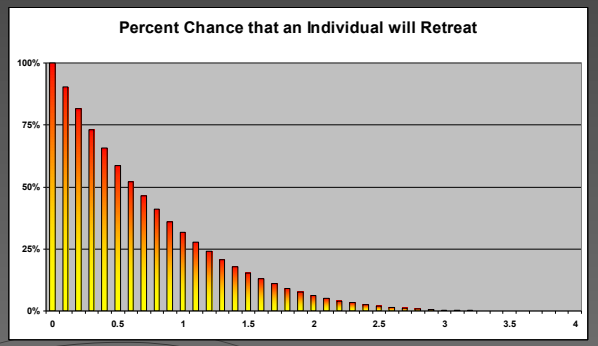


Know when to walk away…

$$\text{PercentChance} = (4 - \text{Ratio})^3 / (4^3)$$



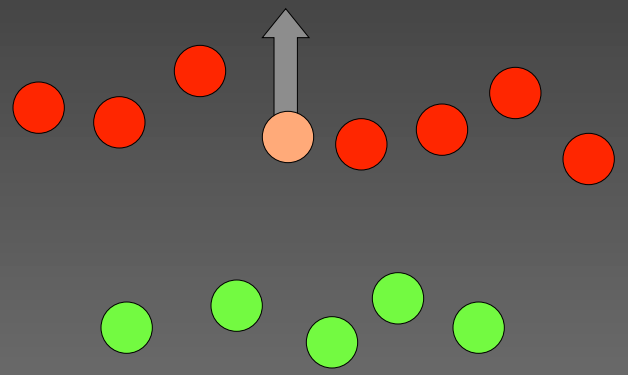
Know when to walk away…

$$PercentChance = (4 - Ratio)^4 / (4^4)$$

$$PercentChance = (4 - 1.6)^4 / (4^4)$$

$$PercentChance = 13\%$$

Know when to walk away…



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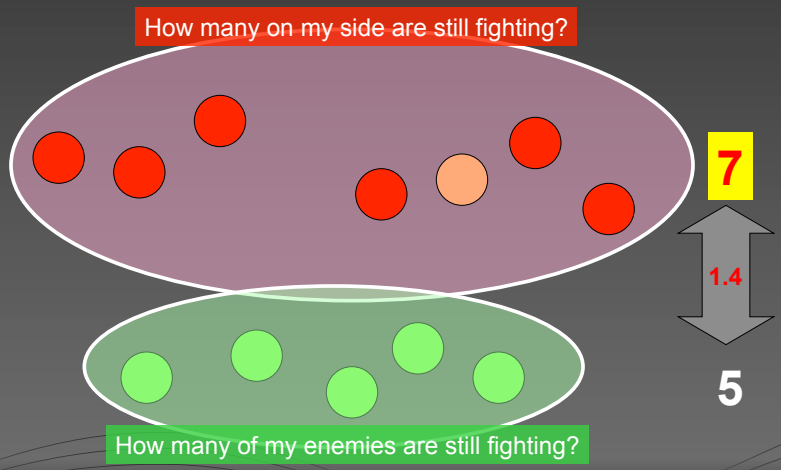


Know when to walk away…

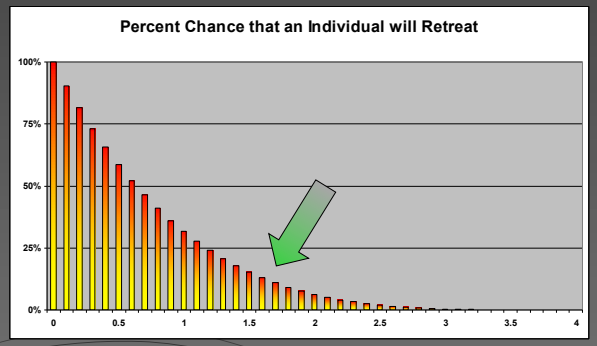
$$PercentChance = (4 - Ratio)^4 / (4^4)$$

$$PercentChance = (4 - 1.4)^4 / (4^4)$$

$$PercentChance = 18\%$$

Know when to walk away…

$$PercentChance = (4 \times Ratio)^4 / (4^4)$$



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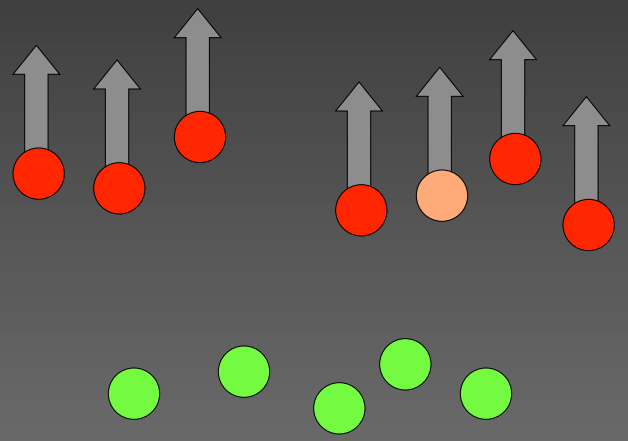

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Know when to walk away…

$$PercentChance = ((MaxRatio - Ratio)^k \times MaxPct) / (MaxRatio^k)$$

	MaxPct	k
In Field	1.00	4
Near Base	0.75	6
In Base	0.50	8

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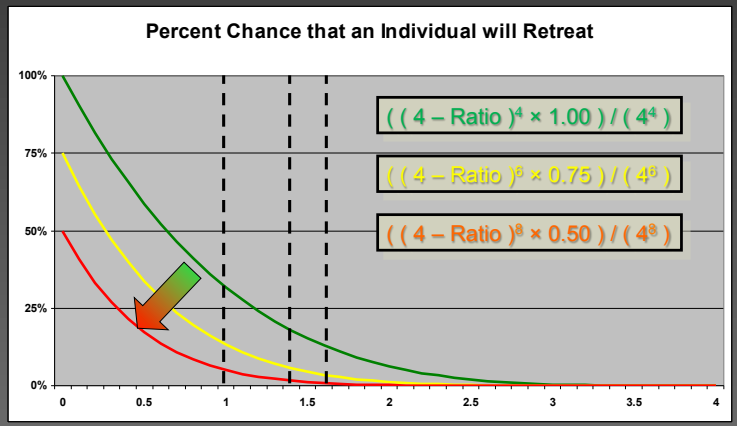
Know when to walk away…



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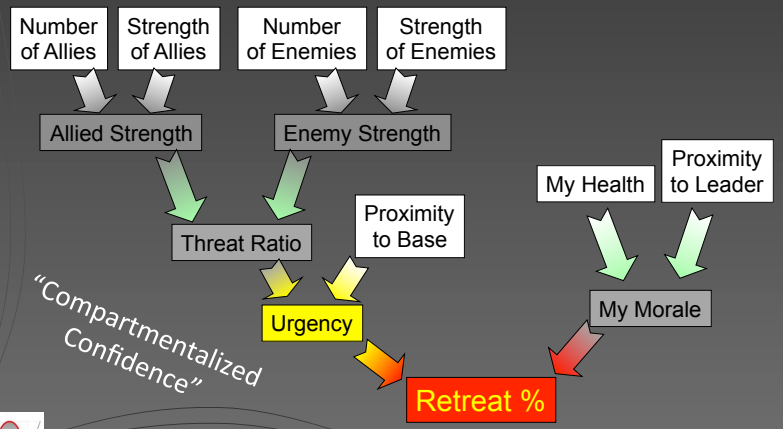
Know when to walk away…

- Factors to Consider
  - Number of allies
  - Number of enemies
  - Proximity to Base
  - Strength of allies
  - Strength of enemies
  - My own health
  - Proximity of my leader

Know when to walk away…



How does this look to the players?

- Enemies aren't completely fearless
- They are slightly unpredictable
- They are still reasonable
- Curiosity: "Where is he going?"
- "If we show force, they might break and run."
- "If we back them up, they are more aggressive."
- We have to react to *their* reactions.

Utility Theory

What is "Utility Theory"?

http://en.wikipedia.org/wiki/Utility

In economics, utility is a measure of the relative satisfaction from, or desirability of, consumption of various goods and services.

Given this measure, one may speak meaningfully of increasing or decreasing utility, and thereby explain economic behavior in terms of attempts to increase one's utility.

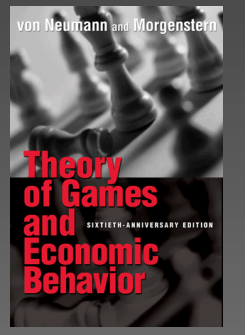
What is "Utility Theory"?

- How much is something worth to me?
- Not necessarily equal to "value"
  - E.g. $20 might mean more or less than $20
- Allows comparisons between concepts
- Allows decision analyses between competing interests
- "Maximization of expected utility"

What is "Utility Theory"?

- Related to…
  - Game theory
  - Decision theory
- Used by…
  - Economics
  - Business
  - Psychology
  - Biology

John von Neumann

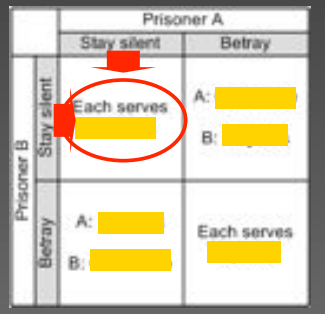


Value Allows Analysis

- Converting raw numbers to usable concepts
  - Distance
  - Ammo
  - Health
- Converting raw numbers to *useful* concepts
  - Distance → Threat
  - Ammo → Reload Necessity
  - Health → Heal Necessity

Value Allows Comparisons

- By assigning value to a selection, we can compare it to others
- Von Neumann and Morgenstern's game theory
- Without value, comparisons are difficult…
  or even impossible!



Marginal Utility

- Utility isn't always the same

Marginal Utility

- Decreas
  - Each
  - The ra
  - Utility

Marginal Utility

- Increasing Marginal Utility
  - Each additional unit is worth more than the one before
  - The rate of increase of the *total utility* increases
  - Utility of 20 Lego != 20 * Utility of 1 Lego

Converting Data to Concepts

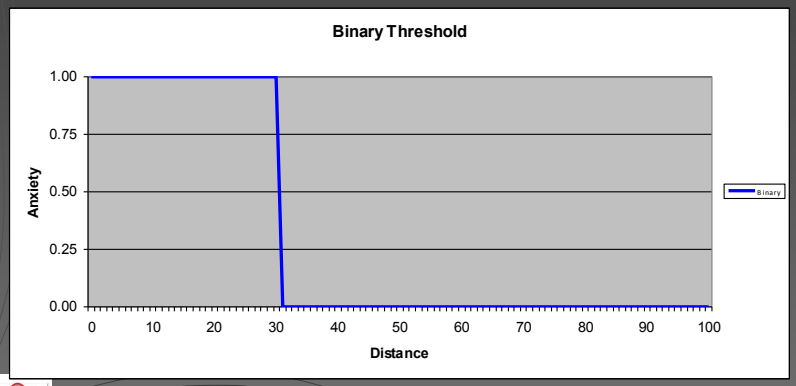
- What does the information say?
- Raw data doesn't mean much without context
- If data is ambiguous, we can't reason on it
- Various techniques to make sense of raw data
  - Conversion formulas
  - Response curves
  - Normalization (e.g. 0..1)

Processing One Piece of Info

As the distance changes,
how much anxiety do you have?

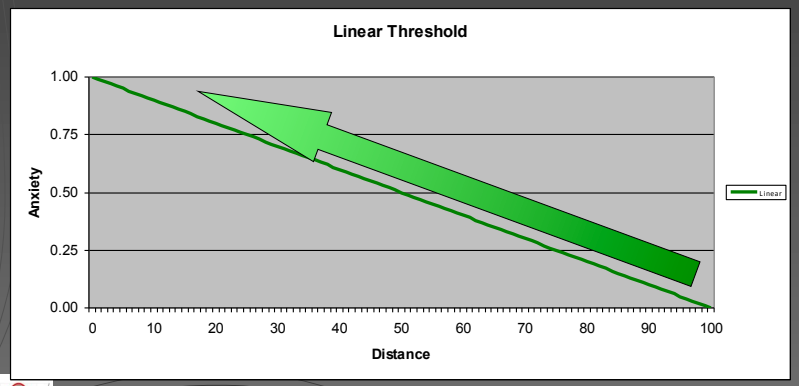
Simple Rule

If distance <= 30 then anxiety = 1



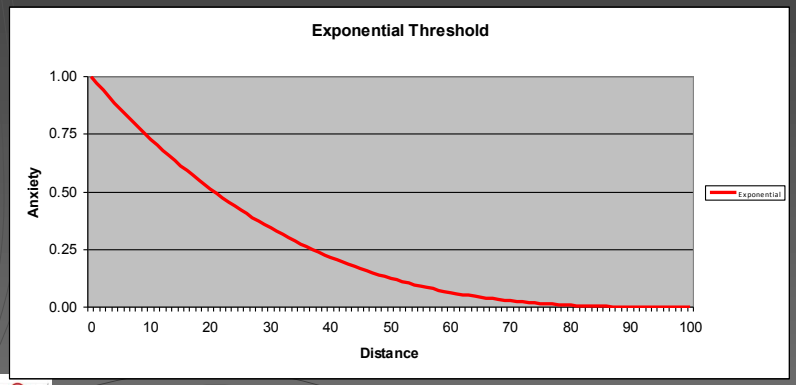
Linear Formula

Anxiety = (100 – distance) / 100



Exponential Formula

Anxiety = $(100 - \text{distance}^3) / (100^3)$



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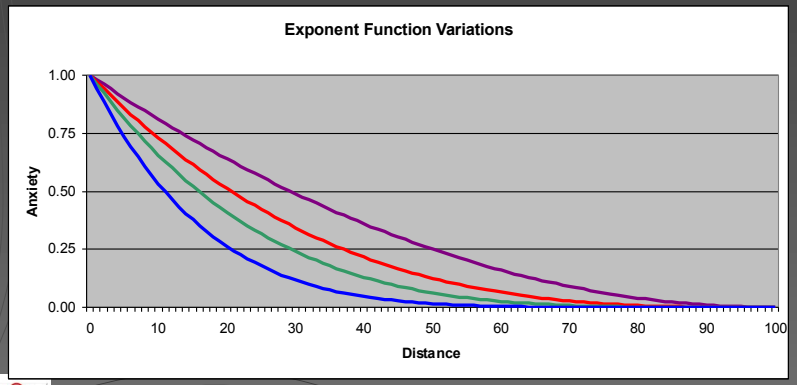
Changing Exponents

Anxiety = $(100 - \text{distance}^k) / (100^k)$

k = 2, 3, 4, 6



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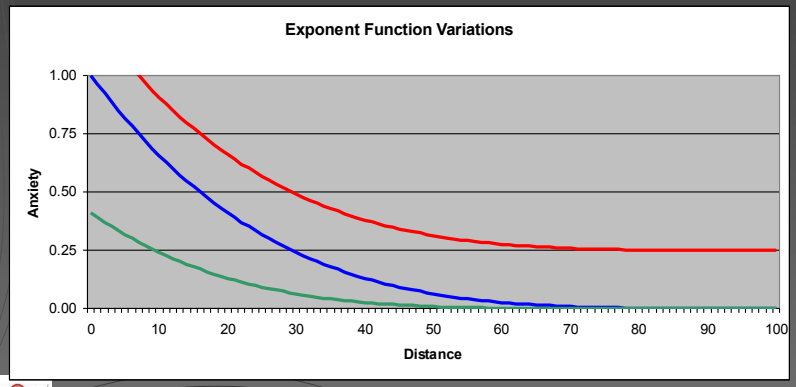
Shifting the Curve



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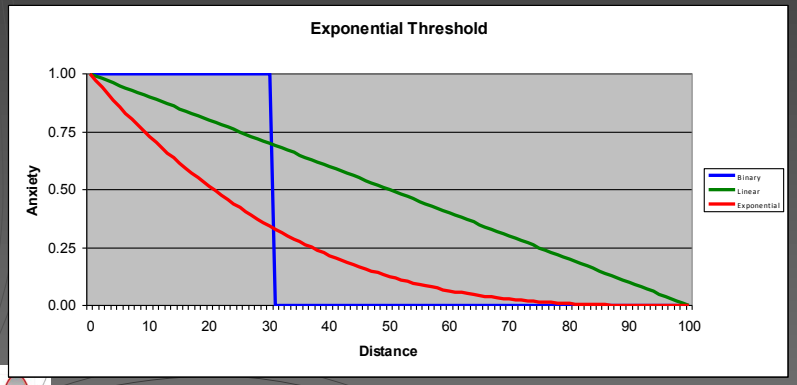
Threshold / Linear/ Exponential



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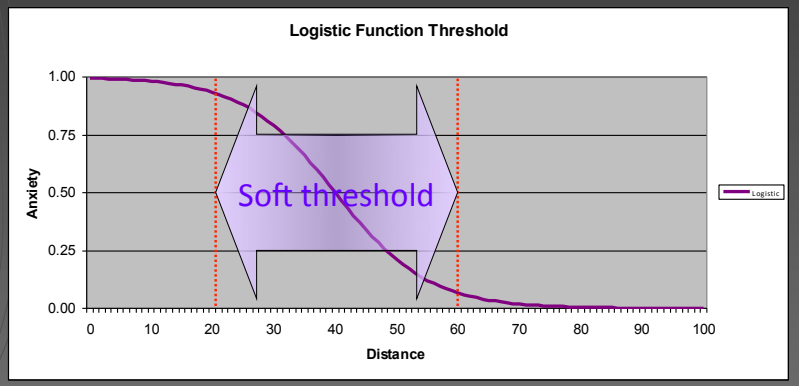
Logistic Function

(One of the sigmoid – or "s-shaped" – functions)

$$y = \frac{1}{1 + e^{-x}}$$

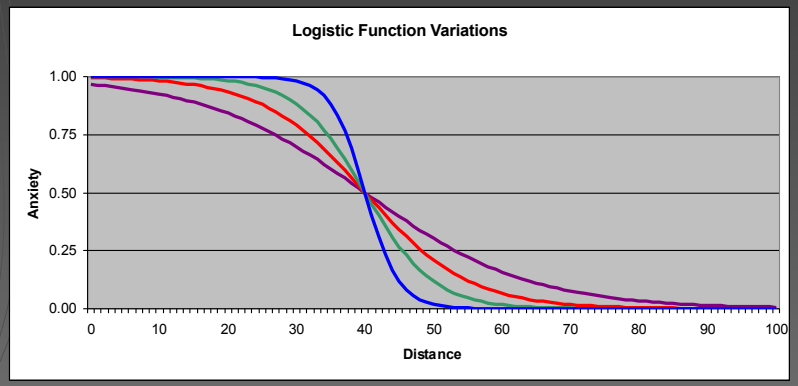
Logistic Function

Anxiety = $1/(1+(2.718 \times 0.45)^{distance+40})$



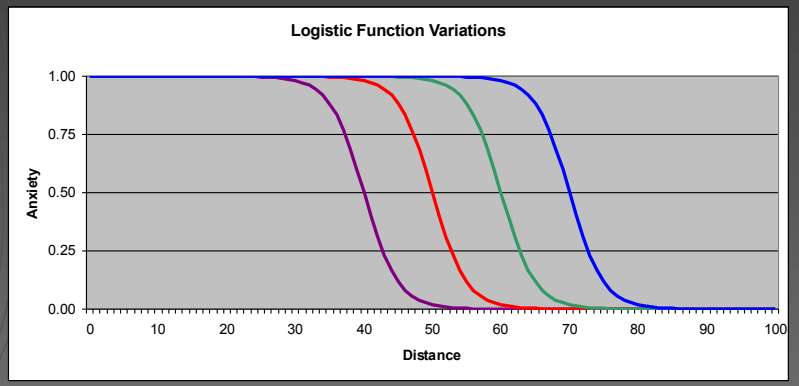
Variations on the Logistic Curve

Anxiety = $1/(1+(2.718 \times 0.45)^{distance+40})$

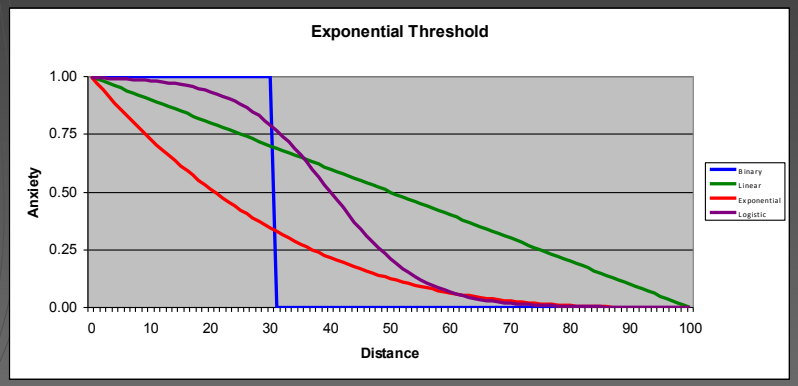


Shifting the Logistic Function

Anxiety = $1/(1+(2.718 \times 0.45)^{distance+40})$



Curve Comparison

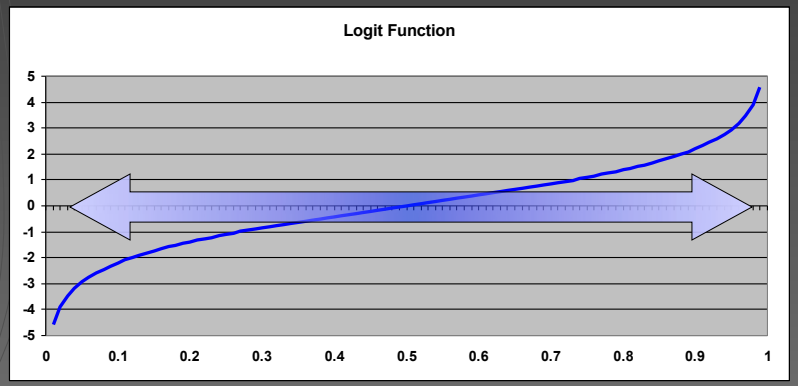


Logit Function

$$y = \log_e \frac{1}{1-x}$$

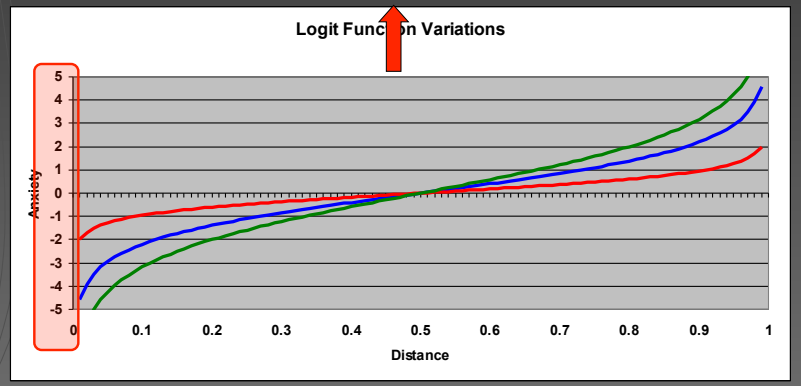
Logit Function

$y = \log_e(x/(1- x))$



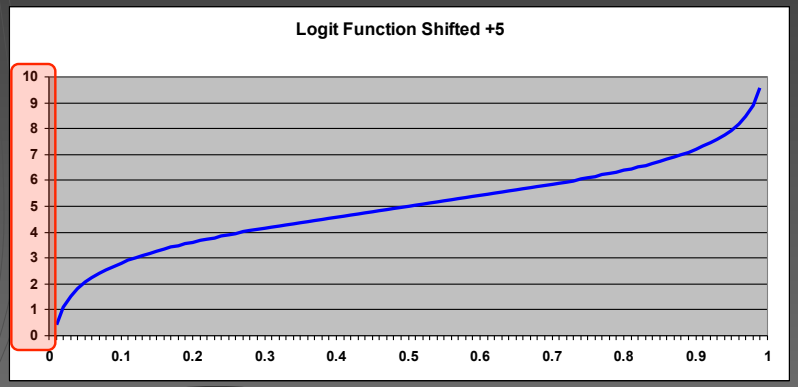
Logit Function

$y = \log_e(x/(1- x))$



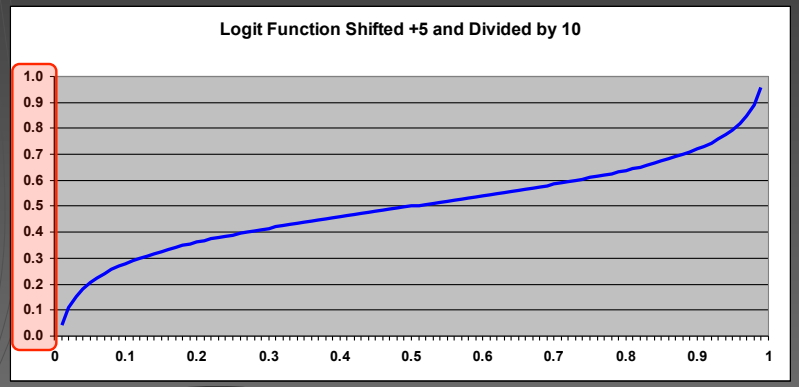
Logit Function

$y = \log_e(x/(1- x))+5$



Logit Function

$y = (\log_e(x/(1- x))+5)/10$

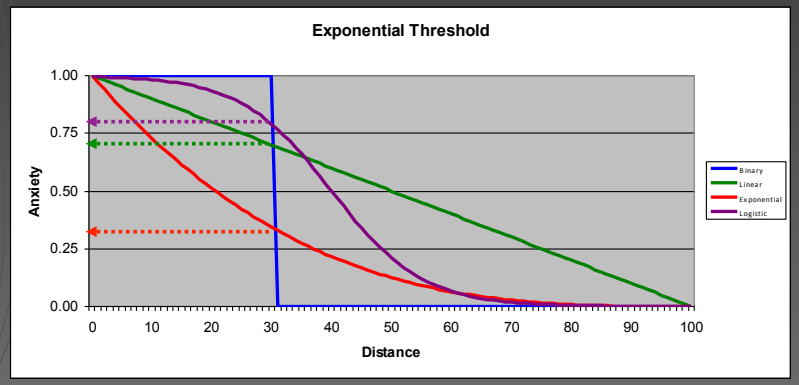


How Do We Model Our Information?

- Increasing or Decreasing?
- Rates of change
  - Steady or Variable?
  - Inflection Point?
- Amount of change
  - Constrained or Infinite?
  - Asymptotic?

But What Good Is It?

When Anxiety > *n* then…



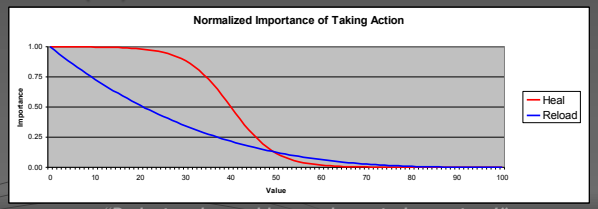
Comparing Apples and Ammo

- By using normalized utility values, we can define relationships and comparisons that otherwise would have been obscure
  - Risk vs. Reward (game theory)
  - Fear vs. Hate
  - Ammo vs. Health

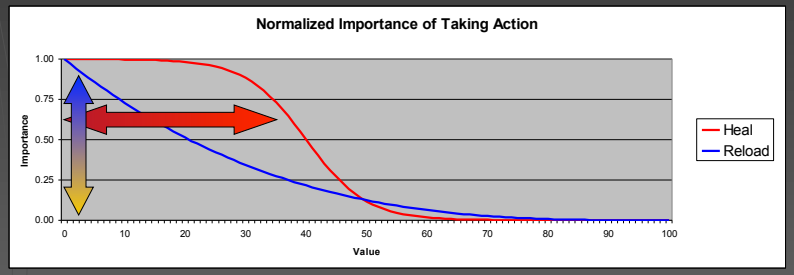
Comparing Apples and Ammo

- 100 Health (Max)
- 75 Health
- 50 Health
- 25 Health (??)
- 5 Health (!!!)

- 100 Ammo (Max)
- 75 Ammo
- 50 Ammo
- 25 Ammo
- 5 Ammo



Comparing Apples and Ammo



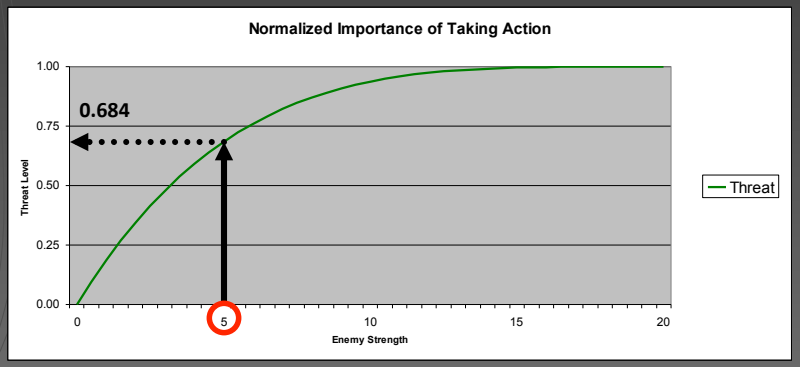
- As health decreases, urgency to heal increases
- Make sure we don't get too low on health!

- As ammo decreases, urgency to reload increases
- Urgency hits maximum when we are out of ammo

Comparing Apples and Ammo

- Collect current states of independent variables
- Normalize using response curves
- (Combine as necessary)
- Compare normalized values and select:
  - Highest scoring selection
  - Weighted random from all choices
  - Weighted random from top *n* choices

Comparing Apples and Ammo



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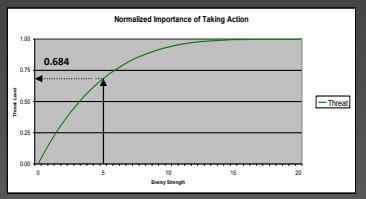
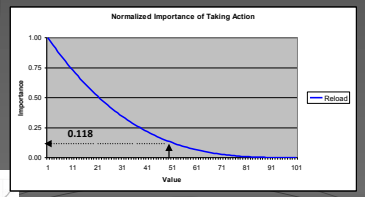
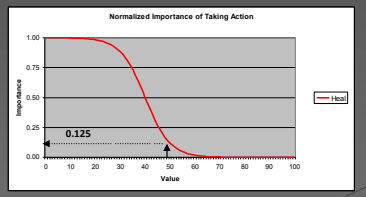
Comparing Apples and Ammo

	Enemies	Ammo	Health
Value	5	50	50
Utility	0.684	0.118	0.125







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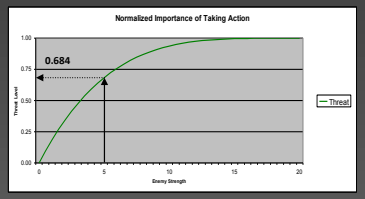
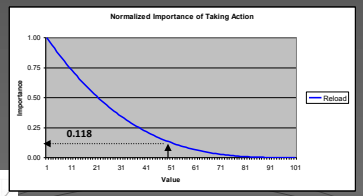
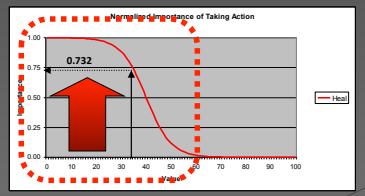
Comparing Apples and Ammo

	Enemies	Ammo	Health
Value	5	50	35
Utility	0.684	0.118	0.732







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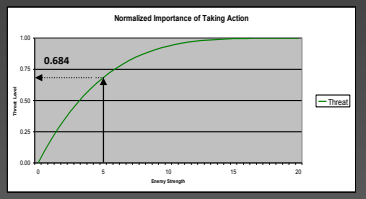
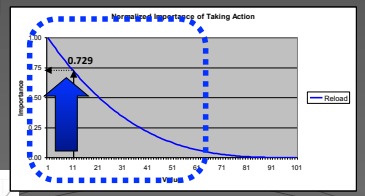
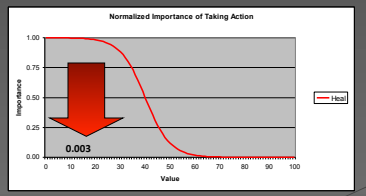
Comparing Apples and Ammo

	Enemies	Ammo	Health
Value	5	10	85
Utility	0.684	0.729	0.003







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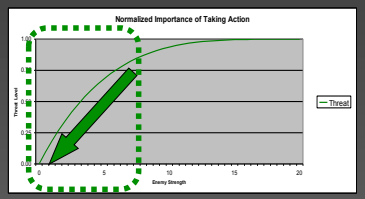
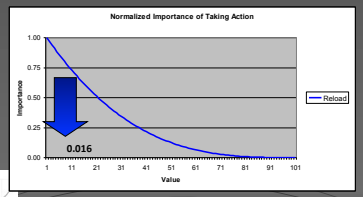
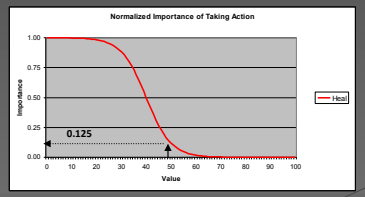
Comparing Apples and Ammo

	Enemies	Ammo	Health
Value	0	75	50
Utility	0.000	0.016	0.125







Comparing Apples and Ammo

- Don't simply process 1 potential action at a time
  - Should I attack?
  - Should I reload?
  - Should I heal?
  - Should I have a beer?
- Compare all potential actions to each other
  - Of all of the things I could do, which is the most important *at this moment?*

Beyond Apples and Ammo

- Utility measurements can model more than simply tangible data
- They can model abstract concepts:
  - Threat
  - Safety
  - Morale
  - Emotions

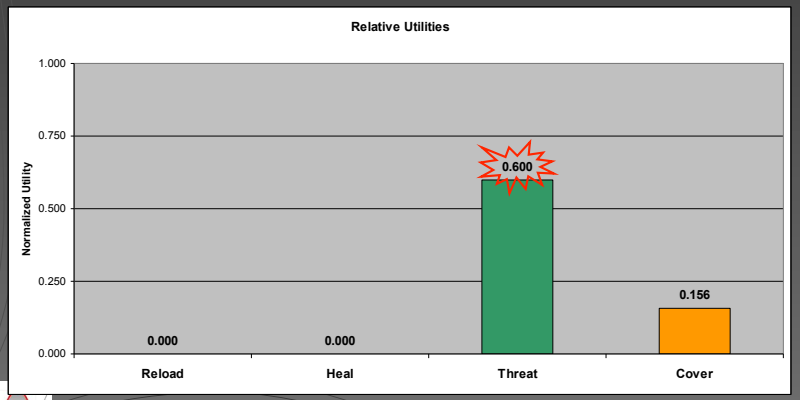
Stacking Apples and Ammo

- Individual utility value can be combined to form new conceptual utilities
- "Need to take cover"
  - Amount of fire being taken (Threat)?
  - Is it almost time to reload?
  - Is it almost time to heal?

Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)

Stacking Apples and Ammo

Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)



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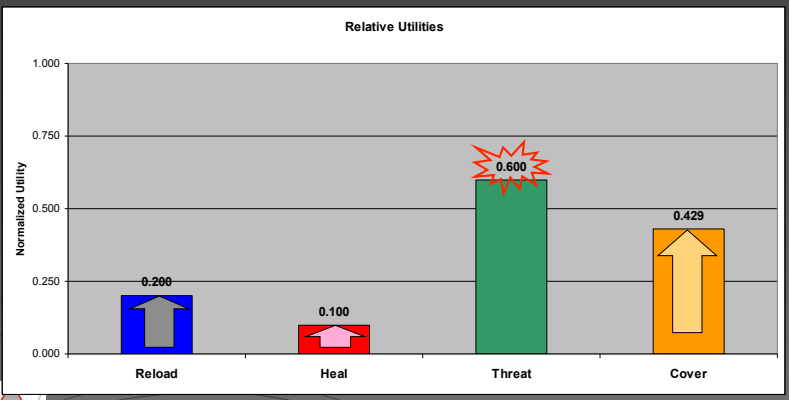

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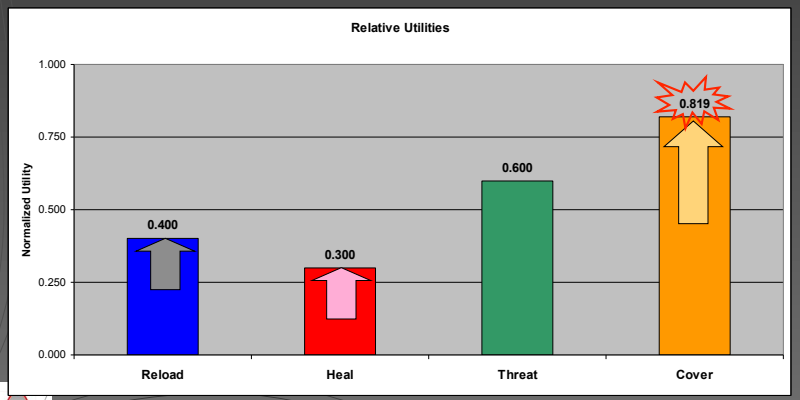

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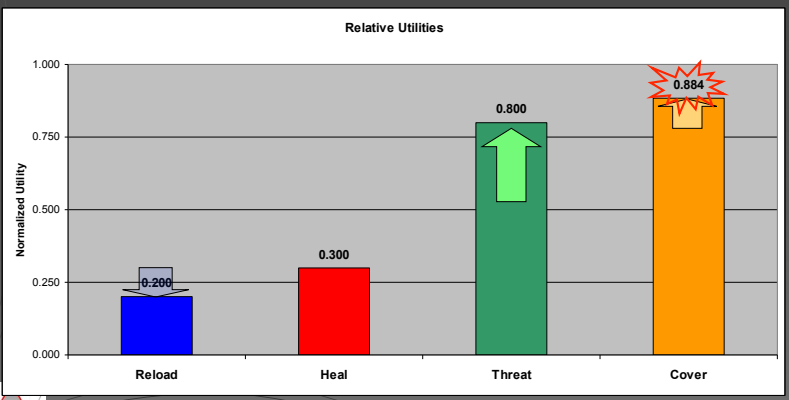
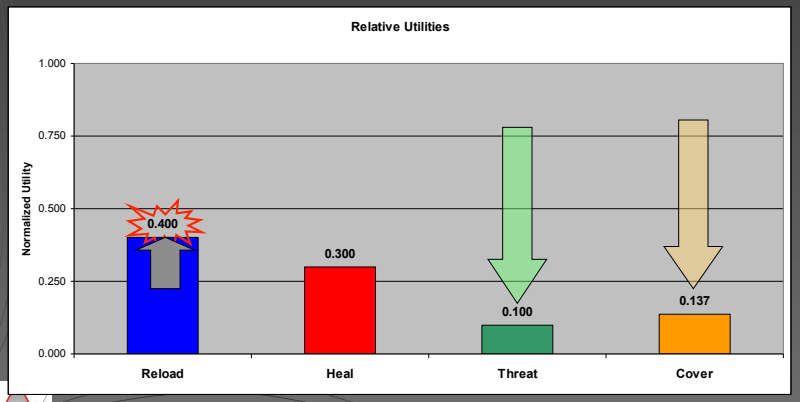


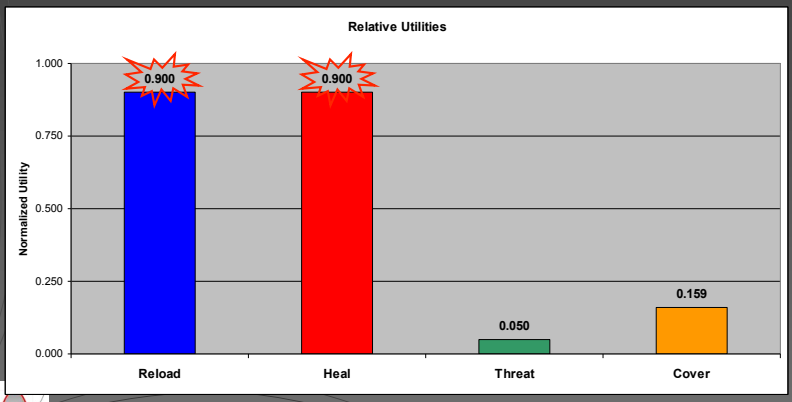
Stacking Apples and Ammo

Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)



Stacking Apples and Ammo

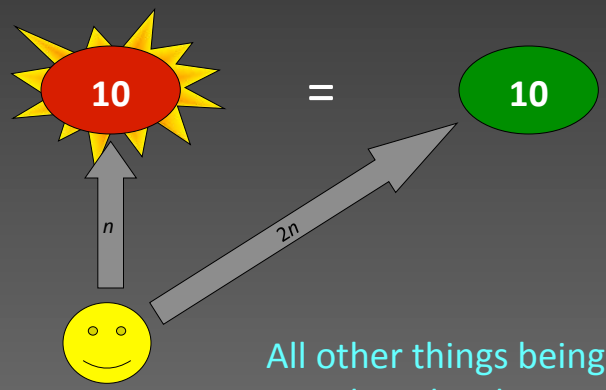
Cover = (0.2 + Reload + (Heal x 1.5)) x (Threat x 1.3)



Utility of Time

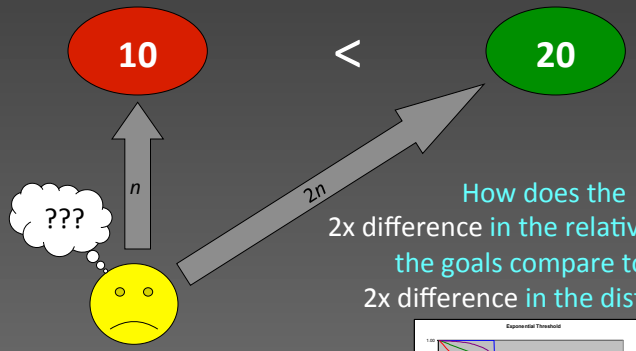
- Time can be converted into a utility value
  - Time to travel over distance
  - Time to complete something
- Utility of time can be used for comparisons
- Utility of time can modify other utilities

Utility of Time

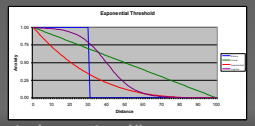


All other things being equal, select the closest goal

Utility of Time

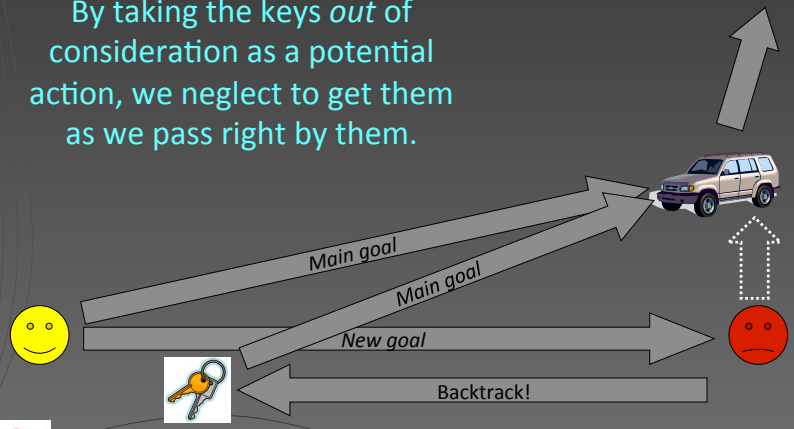


How does the
2x difference in the relative utility of the goals compare to the
2x difference in the distances?



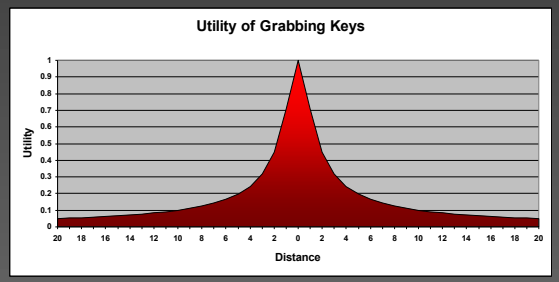
Utility of Time

By taking the keys *out* of consideration as a potential action, we neglect to get them as we pass right by them.



Utility of Time

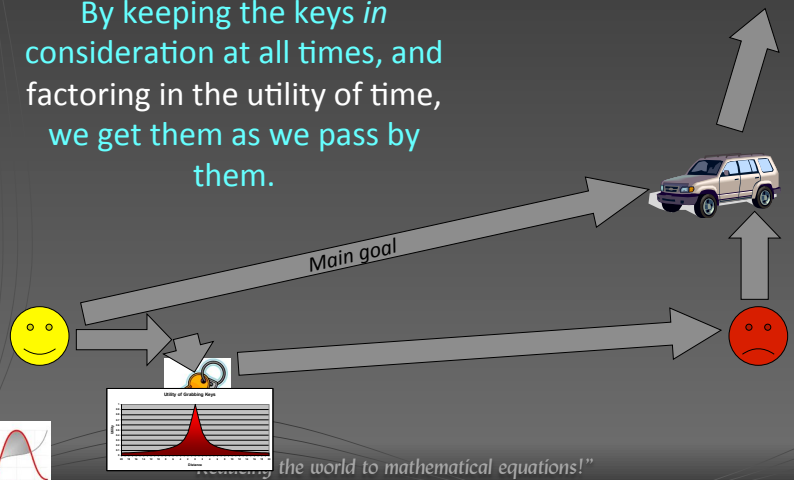
$$\frac{1}{Dist_{keys}}$$



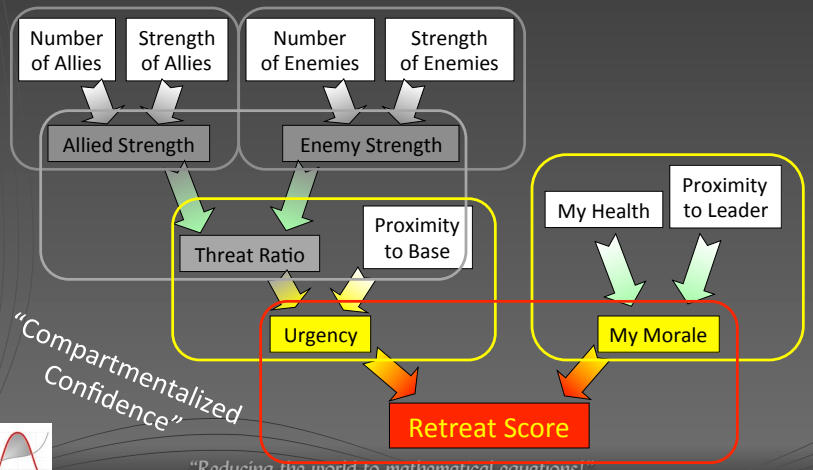
- Normalized distance utility as inverse of distance
- Use as coefficient to modify base utility of getting keys

Utility of Time

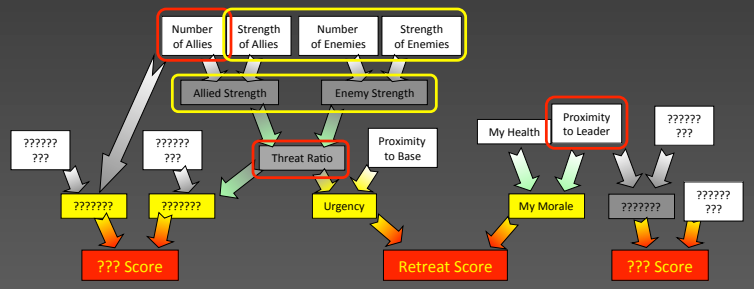
By keeping the keys *in* consideration at all times, and factoring in the utility of time, we get them as we pass by them.



Stacking It All Up



Spreading It All Out



Data processing != Decision processing

Managing Scalability

- Don't perform all calculations every frame
  - Every *n* frames
  - Use triggered updates
- Split data calculation off into separate processes
  - Used by multiple utility calculations for same agent
  - Used by decision calculations for multiple agents
  - Blackboard architecture to manage and store
- Lends itself well to multi-threading

Everything is Relative

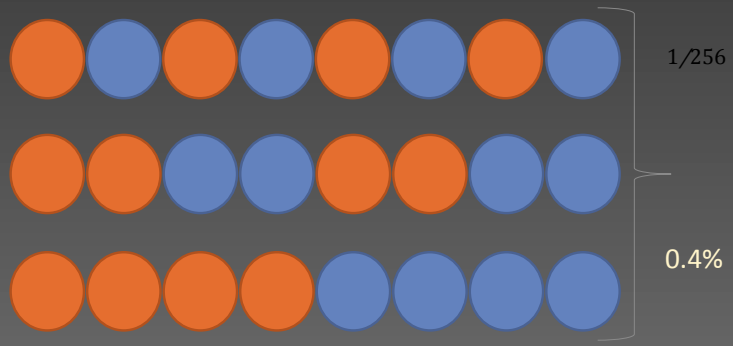
- Many AI decision processes (BTs, FSMs):
  - Examine one choice at a time and ask "should I do this one thing?"
  - Are certain parameters met to justify that choice?
  - If not, move on to the next one in a pre-specified order
- What happens if *no* options meet their criteria?
  - Fall back (idle) behavior may not be appropriate
  - Very susceptible to edge cases

Everything is Relative

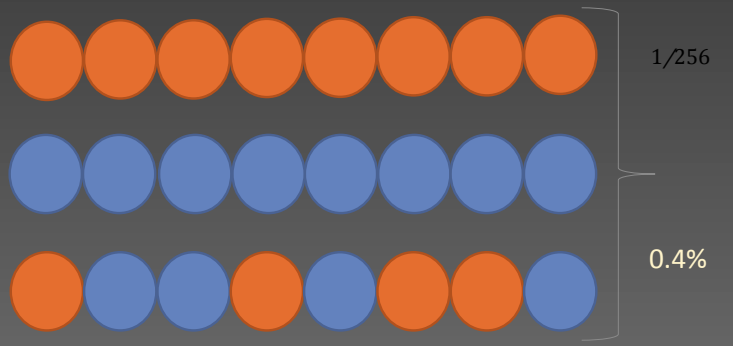
- Utility-based architectures:
  - Continuously analyze **all** options (rather than just one)
  - Rate **all** options based on their respective factors
  - Select the option that is **most appropriate** at the time
- Not based on arbitrary, independent thresholds
- Handles situational edge cases better
- Easier to manage potentially conflicting logic

Randomness

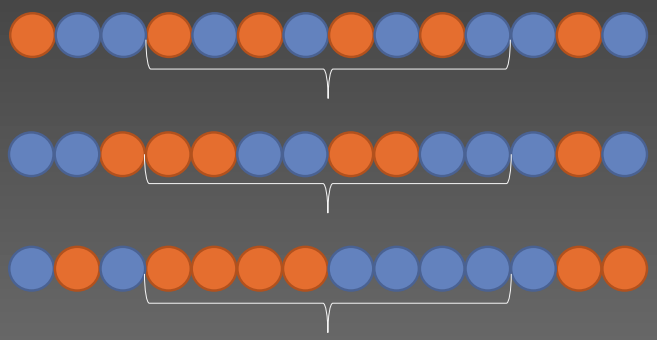
Eye of the Beholder



Eye of the Beholder



Eye of the Beholder

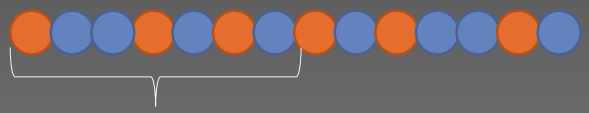


Eye of the Beholder

- People are conditioned to see patterns
  - Inflate their importance
  - Easier to recall
- Chunking
  - Packaging info into memorable chunks
- "The magical number 7, plus or minus 2"
  - George A. Miller, Princeton, 1956

Eye of the Beholder

- People can identify shorter patterns or groupings.
- Forget to consider the entire context



Opportunity != Outcome

- The long-term odds (or % chance) of an event happening can be a poor predictor of short term results.
- People *suck* at remembering that.
- People *suck* at calculating that.
- It can be used for good or eeevviiiiiiii!

Opportunity != Outcome

Sid Meier

"But I had *3*...
and he only had *1*!"

"But I had 1...
and he only had *3*!"

Opportunity != Outcome

7-game series (e.g. baseball, basketball)

Team A has a 55% chance of winning each game

Team B will win series 40% of the time

Team A has 66.6% chance of winning each game

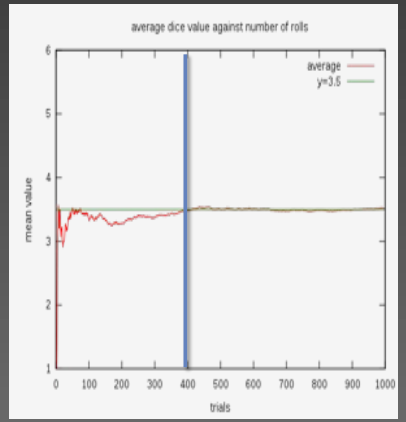
Team B will *still* win series 20% of the time

That's a lot of games…

- Statistically significant –
  i.e. weaker team wins < 5% of the time
  - Stronger team has 66.7% advantage
    - Best 12 out of 23
  - Stronger team has 55% advantage
    - Best of 135 out of 269!

Law of Large Numbers

As the number of trials increases, the outcome approaches the theoretically expected value.



When Is it "Large Enough"?

To achieve a 90% chance of getting within ± *n*%.

Statistical Significance	Minimum Trials
± 5%	370
± 2%	1000

(Think polls)

Law of… *small* numbers?

- Kahneman & Tversky
  - Judgment and decision-making
  - Behavioral economy
- Sarcastic reference to misconception that a small sample accurately reflects underlying probabilities.

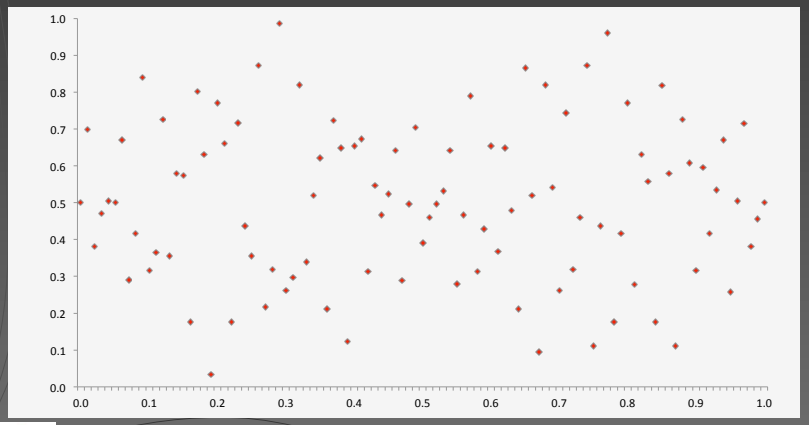
Gambler's Fallacy

- Idea that the odds of an independent event with fixed probability changing based on recent occurrences.
  - "Catching up."
  - "His luck will run out sooner or later."
  - "He's due!"

We all do it…

22	6	4
14	19	00
20	22	24
35	13	31
30	8	29
4	17	25
11	31	17
24	10	35

What do People Perceive?



Determinism that Looks Random

- 3 Sine waves
  - Different frequencies
  - Different magnitudes
- For each value of x
  - Sum y values
  - Normalize

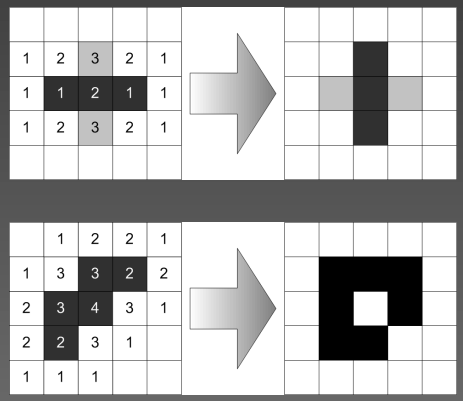
$$f(x) = \frac{\sum_{n=1}^{3} magnitude_n \times (\sin(x \times period_n \times \pi))}{\sum_{n=1}^{3} magnitude_n}$$

Period	50	133	271
Mag.	1.0	1.1	2.0

Complexity Obscures Patterns

- People can only keep track of a limited number of factors
- As factors increase, complexity increases
- People can no longer keep track of relationships
- Deterministic result looks "random"

Complexity Obscures Patterns



Complexity Obscures Patterns

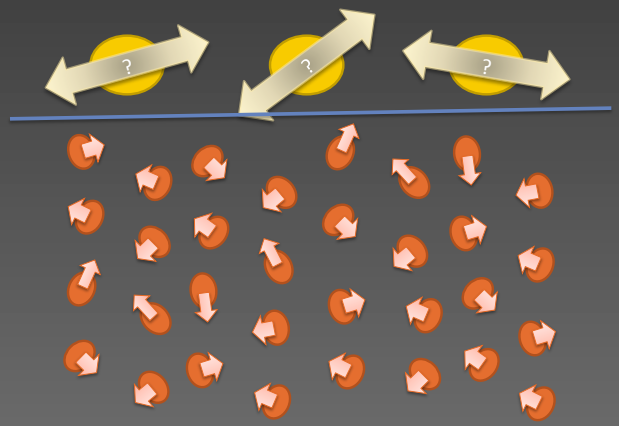
Hidden Factors Look Random

- People can only take into account what they *perceive*
- If input factors are hidden, people ascribe it to randomness

Brownian Motion

- Robert Brown – 1827
- Under a microscope – pollen grains moved in "jittery" manner
- Ascribed it to pollen being alive
- Heat-based molecular vibration bumping the grains

Brownian Motion



Brownian Motion

Further Reading

So what can we do with this?

- Random sequences that look rigged…
- Unseen deterministic factors that look random…
- Intentional randomness to fake complex determinism…

Two Ways to Use It

- Fuzzying Things Up
- Weighted Randoms

Fuzzying Things Up

Fuzzying Things Up

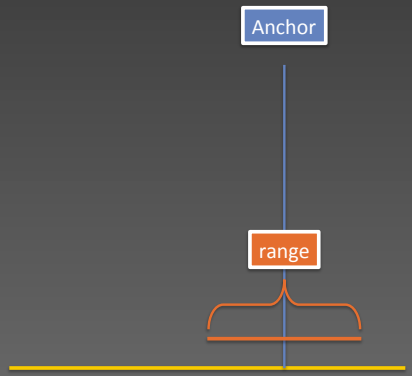
- Start with a defined anchor point
- Add "parametric noise" (i.e. ± *n*)

Fun with Distributions

- 1 die = flat ±
- 2 die = triangular distribution
- 3+ die = bell curve
  - More die → squished curve
- Fun tricks
  - Skewing dist. (e.g. 4 die, drop the lowest)
  - Combining different types of dice

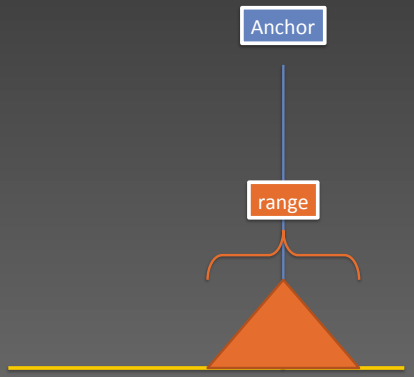
Single Die

- Adds variation
- Quick to process
  - Single random call
- Easy to visualize
- Easy to tweak

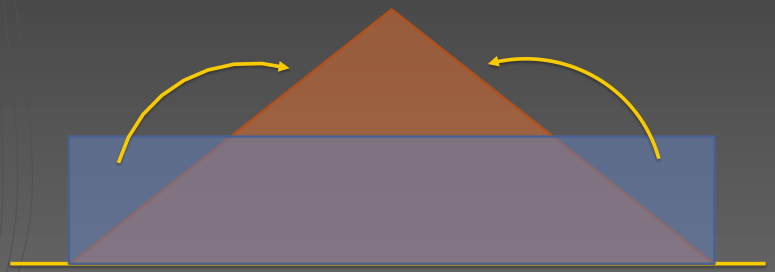


2 Dice

- Triangular distribution
- Mostly clustered towards center
- Fairly quick to process
  - Two random calls
- Easy to visualize
- Easy to tweak



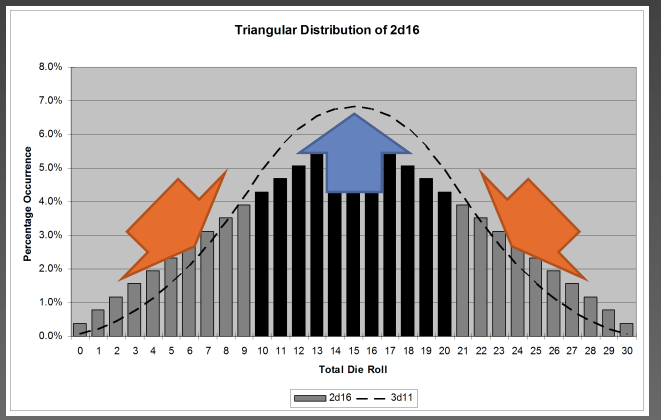
1 die vs. 2 dice



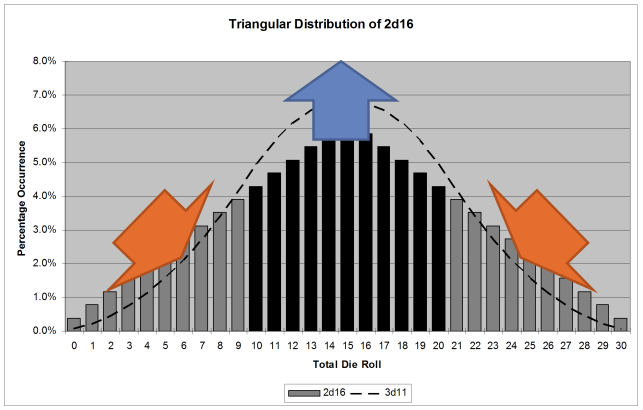
3+ Dice

- Normal distribution
  - Gaussian distribution
  - Bell curve
- Very clustered towards center
- Very expressive
- Slower to process
  - *n* random calls
- Harder to tweak

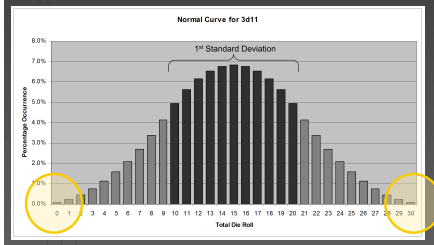
3+ Dice

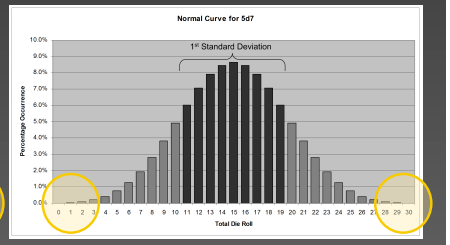


Increase the Number of Dice

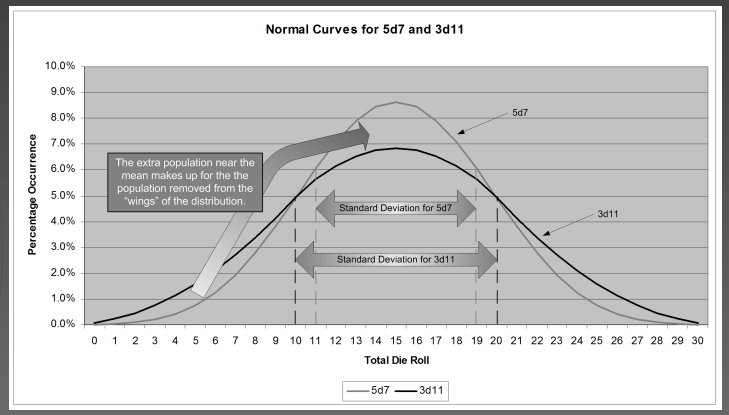


3 dice vs. 5 dice





3 dice vs. 5 dice



Where to Use Randomness

- Perception
- Decision
- Action

Where to Use Randomness

- Perception
- Decision
- Action

Randomizing Perception

- Range
  - I can see 30' ± 5' = [25-35']
- Accuracy
  - I have an 80% chance of seeing you ± 10%
- Frequency
  - Check senses every *n* ticks where *n* is 500-2000 ms

Where to Use Randomness

- Perception
- Decision
- Action

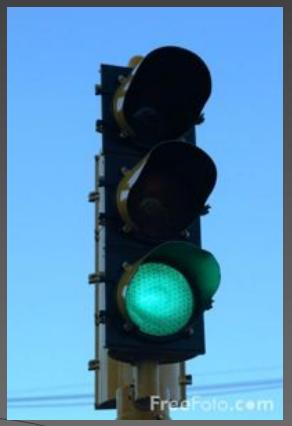
Where to Use Randomness

- Perception
- Decision
- Action

Randomizing Actions

- Delay before acting
- Speed of acting
- Acceleration into act

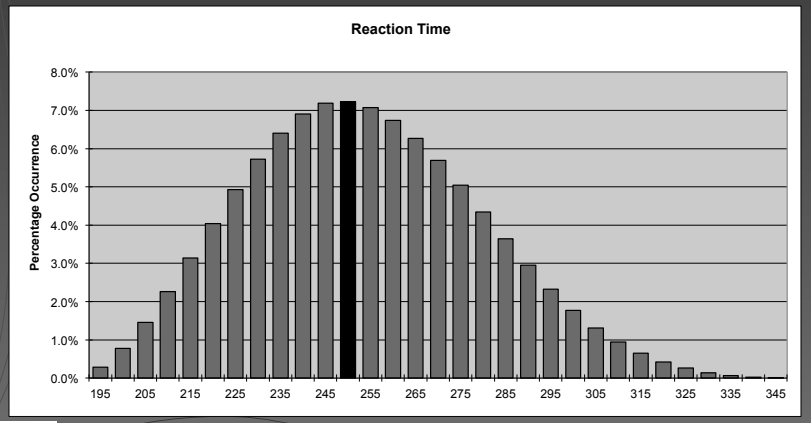
Clap when you see the light…



Human Response Times

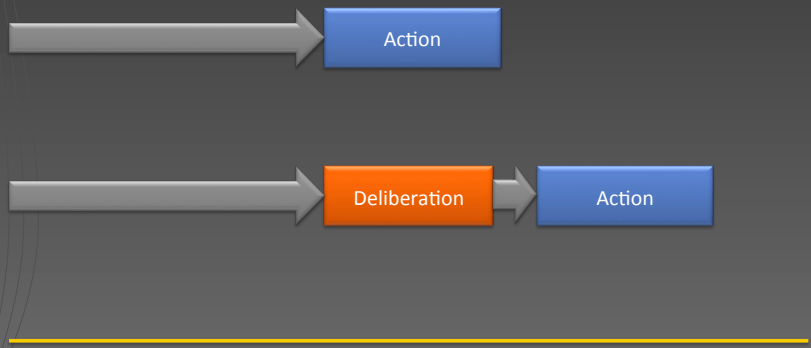
- Average is ≈250ms
- Not *everyone* clapped at 250ms
- Distributed over a range
- Everyone clapping at 250ms would be odd

Human Response Times



Deliberative Decisions Take Time

Deliberative Decisions Take Time



Action Parameters

- Reaction Time
  - Initial (startle)
  - With decision
- Movement
  - Max speed
  - Acceleration
  - Turn rate
- Combat
  - Acquisition time
  - Fire rate
  - Accuracy
  - Reload speed
- [Whatever]

What is different?

- Each person is different
  - People have different reflexes and reaction times
  - Why I can't play twitch games very well
- Each action is different
  - Every time you clap after the light, it will be different

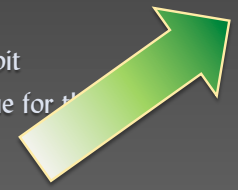
Instantiation vs. Action

Individual People

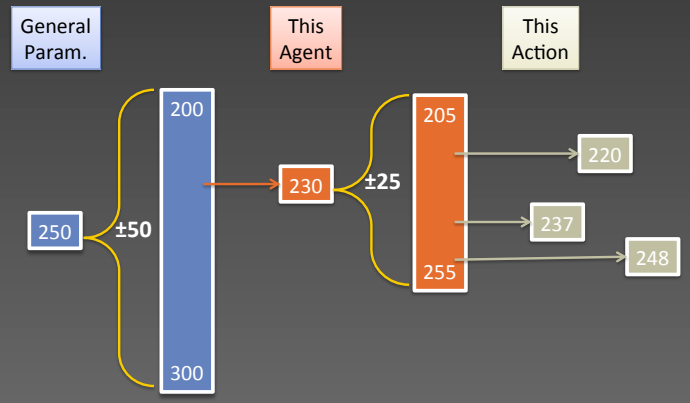
- Parameter's base value for all people
- Fuzzy it up a bit
- New base value for this person

Isolated Events

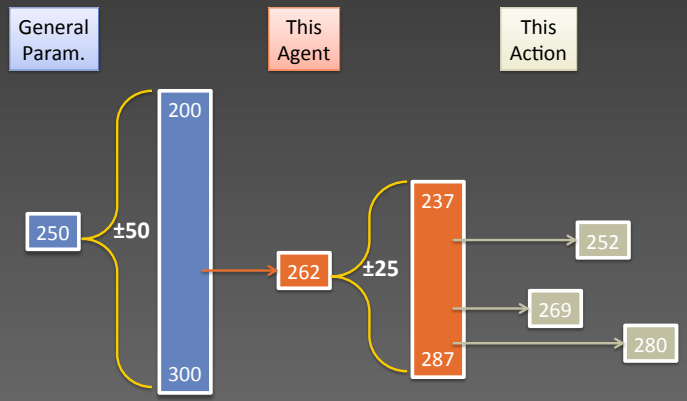
- Take base value for this person
- Fuzzy it up a bit
- New value for this event



Instantiation vs. Action



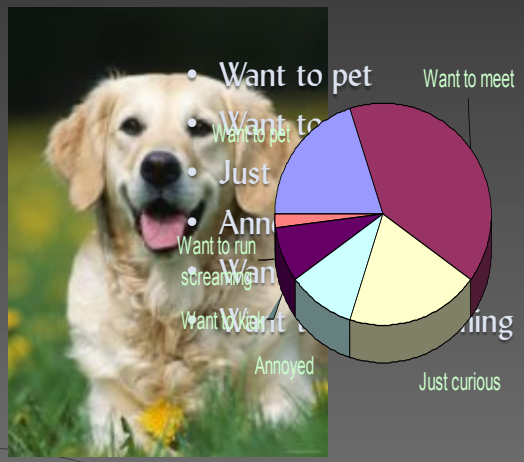
Instantiation vs. Action



Where to Use Randomness

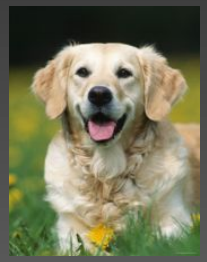
- Perception
- Decision
- Action

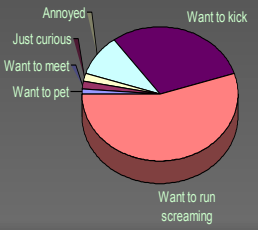
What is your reaction?



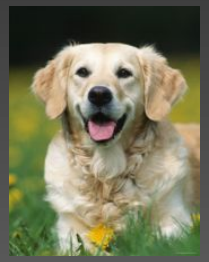
Now what is your reaction?

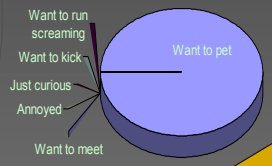
Varieties of Reactions

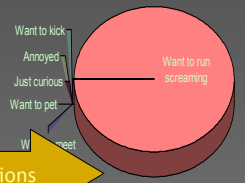




Same Model for All Agents

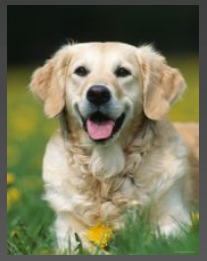


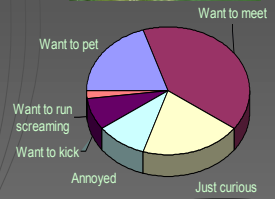




Extreme Reactions

Varieties of Reactions





- Differences exist
- Don't need to know why
- Simulate that differences do exist
  - Not completely random selection
  - Must be reasonable
  - Can be simulated with weighted randoms

Brownian Motion

Multiple Agents – Multiple Reactions

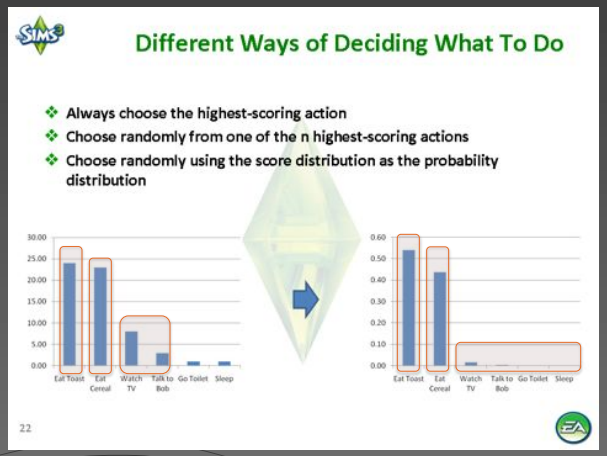
Straight Deterministic Method

- Same inputs
- Same decision algorithms
- Arrive at a decision
- Same results

Random Noise Method

- Same inputs
- Same decision algorithms
- Randomized selection
- Varied results

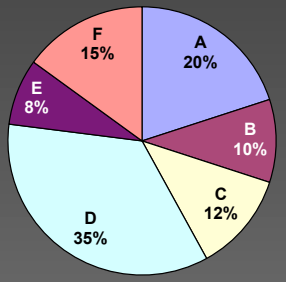
Weighted Randoms



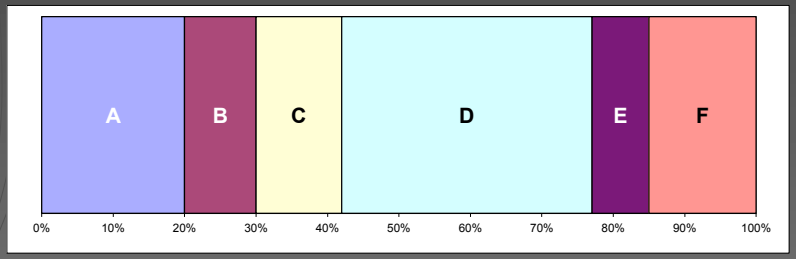
Weighted Randoms

- Score all the possible actions
  - Pre-set %
  - Contextual %
  - Dynamic %
- Calculate a weight
  - Normalized or not
- Line them up end-to-end
  - Covert weights into edges
  - Creates "buckets" sized based on weights
- Throw a random at it
  - Which bucket does it land in?

Throw a Dart at It



Let the bouncing ball choose!



The Payoff of Weighted Randoms

- People aren't all the same
- There are reasonable options
- Simulate how reasonable people may differ
- Utilize the scoring of actions that we already do
- Just do more than simply choose the "best"

We Aren't Solving an Equation

- Computers are good at finding "the best"
- People suck at it
- Pollen grains aren't alive
  - They shouldn't move
  - But they do (randomly?)
- People are alive
  - They should be different
  - List the likely decisions and pick (randomly)

Don't "solve" human behavior... Describe it!

Make Larry Proud!!

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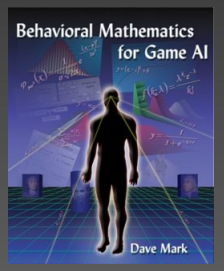

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