

History of MCMC – Gibbs sampler

Big picture: Session 3

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Idea of Gibbs sampling

- MCMC technique to estimate an unknown joint distribution by sampling one variable at a time from its conditional distribution where all other variables are fixed at their current value
- Useful when joint distribution is hard to derive or sample from directly
- Made many previously unsolvable problems solvable

Algorithm

- 1. Given unknown $f(\theta_1, \dots, \theta_p)$ but known $f(\theta_i \mid \theta_j)$
- 2. Start iteration with arbitrary values $\theta_1^0,\dots,\theta_p^0$
- 3. Sample θ_1^1 from $f(\theta_1 \mid \theta_2^0, \dots, \theta_p^0)$
- 4. Sample θ_2^1 from $f(\theta_2 \mid \theta_1^1, \theta_3^0, \dots, \theta_p^0)$
- 5. Continue until θ_p^1 sampled from $f(\theta_p \mid \theta_1^1, \dots, \theta_{p-1}^1)$
- 6. Start next iteration with values $\theta_1^1, \dots, \theta_p^1$
- 7. Repeat over n iterations
- 8. As $n \to \infty$, θ_1^n , ..., θ_p^n converges to $\sim f(\theta_1, \ldots, \theta_p)$



History

- 1902: Josiah Willard Gibbs' work in statistical physics
- 1984: Geman brothers formulated Gibbs sampler in their image restoration work with Gibbs random fields
- 1990: Gelfand and Smith demonstrated broad applicability of Gibbs sampler in (Bayesian) statistics
- 1991: BUGS (MRC Biostatistics Unit, Cambridge)
- 2007: JAGS (Martyn Plummer)



Thank you!



References

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