$k+\lambda$ is the combined precision, such that $\frac{1}{k+\lambda}$ is the combined variance.

The integral over μ of a normal PDF is 1, over whole range.
The integral removes μ-terms, leaving us with y, v, k and λ. The integral that was to solve is:

$$\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}(k+\lambda)\left(\mu-\frac{ky+\lambda v}{k+\lambda}\right)^2\right) d\mu$$ as we plug a back in

(a was found in eq. (2)).

This integral is the Gaussian kernel. The result is (where 2π is a normaliser): $\left(\frac{2\pi}{k+\lambda}\right)^{\frac{1}{2}}$. In addition to using 2π to normalise, we add the variance as well. In our case, the variance is $\frac{1}{\text{combined precision}} = \frac{1}{k+\lambda}$.

An increase in the combined precision causes a decrease in the variance, in turn tightening the spread of the distribution.

The normalisation factor, here $\left(\frac{2\pi}{k+\lambda}\right)^{-\frac{1}{2}}$ ensures that $f(y|H1)$ obeys the property of Gaussian PDFs that the integral over the range of all possible values sums up to 1.

Plugging this back in the normal distribution, we obtain:

$$f(y|H1) = \int_{-\infty}^{\infty} f(y,\mu) f(\mu)\, d\mu$$

$$f(y|H1) = \left(\frac{2\pi}{k+\lambda}\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\frac{k\lambda}{k+\lambda}(y-v)^2\right)$$