History of MCMC – Gibbs sampler

Big picture: Session 3

Group 2: Jan Hohenheim, Holly Vuarnoz, Sophie Haldemann, Andrea Staub, and Emanuel Mauch

Idea of Gibbs sampling

– MCMC technique to estimate an unknown joint
  distribution by sampling one variable at a time from its
  conditional distribution where all other variables are
  fixed at their current value

– Useful when joint distribution is hard to derive or sample
  from directly

– Made many previously unsolvable problems solvable

Algorithm

1. Given unknown $f(\theta_1, \ldots, \theta_p)$ but known $f(\theta_i \mid \theta_j)$

2. Start iteration with arbitrary values $\theta_1^0, \ldots, \theta_p^0$

3. Sample θ_1^1 from $f(\theta_1 \mid \theta_2^0, \ldots, \theta_p^0)$

4. Sample θ_2^1 from $f(\theta_2 \mid \theta_1^1, \theta_3^0, \ldots, \theta_p^0)$

5. Continue until θ_p^1 sampled from $f(\theta_p \mid \theta_1^1, \ldots, \theta_{p-1}^1)$

6. Start next iteration with values $\theta_1^1, \ldots, \theta_p^1$

7. Repeat over n iterations

8. As $n \to \infty, \theta_1^n, \ldots, \theta_p^n$ converges to $\sim f(\theta_1, \ldots, \theta_p)$

Animation of Gibbs Sampling

Feng, Chi (2016)

History

– 1902: Josiah Willard Gibbs' work in statistical physics

– 1984: Geman brothers formulated Gibbs sampler in their image restoration work with Gibbs random fields

– 1990: Gelfand and Smith demonstrated broad applicability of Gibbs sampler in (Bayesian) statistics

– 1991: BUGS (MRC Biostatistics Unit, Cambridge)

– 2007: JAGS (Martyn Plummer)

Thank you!

References

Casella, G. and George, E. I. (1992). Explaining the gibbs sampler. *The American Statistician*, 46(3):167–174. https://doi.org/10.2307/2685208.

Geman, S. and Geman, D. (1984). Stochastic relaxation, gibbs distributions, and the bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-6(6):721–741. https://doi.org/10.1109/tpami.1984.4767596.

Robert, C. P. and Casella, G. (2011). A short history of markov chain monte carlo: Subjective recollections from incomplete data. *Statistical Science*, 26(1):102–115. https://doi.org/10.1007/978-3-642-37887-4.