Neuromorphic engineering I

Photoreceptors; Photoreceptor Circuits

Created Oct 2020-Dec 2020 by Tobi Delbruck & Rui Graca

Group number: 4.5

Team members

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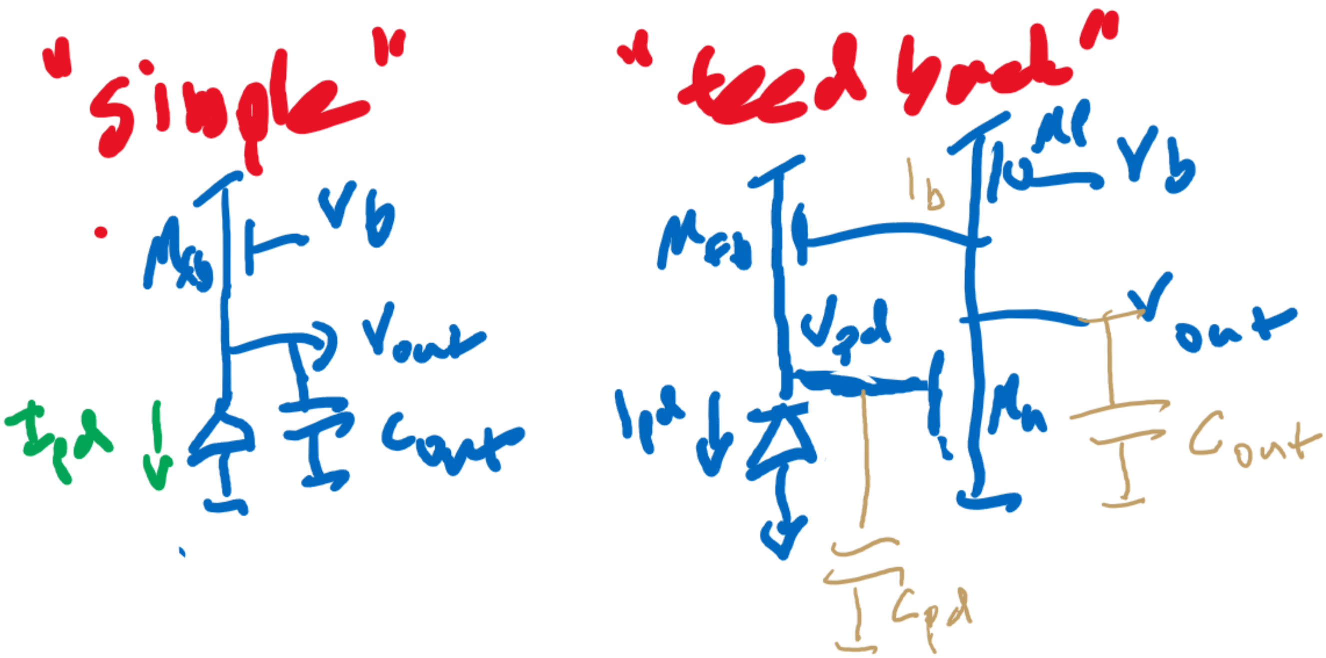
TA: ...

Objectives of this lab

You will compare the 2 circuits sketched below. The left one is the *source-follower* (**SF**) photoreceptor and the right one is the unity-gain active *transimpendance* feedback (**TI**) photoreceptor.



Exercise type and dates

COVID made it too difficult to set up a remote arrangement for testing the classchip photoreceptor circuits. Instead, we will do some circuit analysis and numerical evaluation to understand the concepts of feedback, loop gain, and transimpedence speedup.

The exercise spans 2 weeks with 2h per week. There will be two groups Thursday afternoon and Monday morning.

Excercise dates: Monday group: Nov 30, Dec 7, Thursday group: Dec 2, Dec 9, Monday group: Dec 6 and Dec 13.

Due date: Dec 20 2021

Running the notebook

You will run this exercise on your own computer using any available Jupyter server. If you have one already, you can use it. But you don't need to: https://www.dataschool.io/cloud-services-for-jupyter-notebook/ provides a list of free servers on the cloud that you can use after registration.

Requirements: libraries needed

python 3.7+

You might need to install libraries. You can install them from terminal into your python enviroment with

```
pip install jupyter matplotlib numpy  scipy engineering_notation
engineering_notation
```

Remember, when using any python, conda is your friend. Make a unique conda enviroment for each project to save yourself a lot of trouble with conflicting libraries. Here we will use only very standard libraries that are provided by all the Jupyter servers.

In [1]:

```
import matplotlib.pyplot as plt  # plotting
import numpy as np  # for math
from scipy.integrate import solve_ivp  #  - for timestepping ODEs
import os
from engineering_notation import (
    EngNumber as eng,
)  # useful library to e.g. format eng(1e-3) as 1m
from scipy.stats import linregress

### ------------------------------------------------------------------------------------------
```

In [2]:

```
import matplotlib as mpl

mpl.rcParams["figure.facecolor"] = "white"
mpl.rcParams["axes.facecolor"] = "white"
mpl.rcParams["savefig.facecolor"] = "white"
```

Define useful constants

In [3]:

```
I_0 = (
    1e-14  # FET off current - you measured it TODO check correct for classchip process
)
U_T = 25e-3  # you better know this
kappa = 0.8  # choose a reasonable value
vdd = 1.8  # power supply voltage
q = 1.6e-19  # charge of electron

V_e = 10  # Early voltage for the TI photoreceptor amplifier input FET that we will use la
```

Define useful functions

Let's define a function for subthreshold current that includes optional Early voltage for finite drain conductance:

In [4]:

```
def id_sub(V_g, V_s=0, V_d=1.8, U_T=U_T, I_0=I_0, kappa=kappa, V_e=V_e):
    """Computes the drain current from gate, source and drain voltage.
    At most one of V_g, V_s, V_d can be a vector in which case I_d is a vector

    :param V_g: gate voltage
```

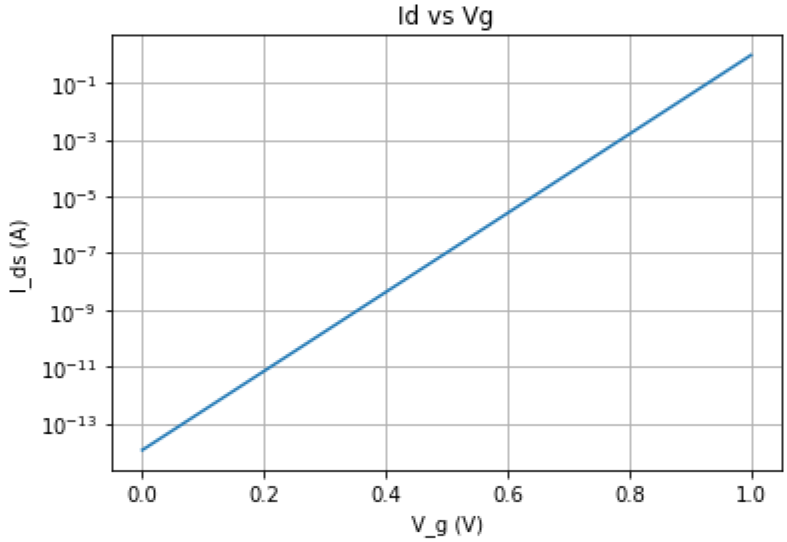
```
    :param V_s: source voltage, by default 0
    :param V_d: drain voltage, by default 1V
    :param: U_T: thermal voltage
    :param I_0: the off current
    :param V_e: the Early voltage; drain conductance is Idsat/V_e

    :returns: the drain current in amps
    """
    Vds = V_d - V_s
    Id_sat = I_0 * np.exp(((kappa * V_g) - V_s) / U_T)
    I_d = Id_sat * (1 - np.exp(-Vds / U_T))
    if V_e != np.infty:
        I_d = I_d * (1 + Vds / V_e)
    return I_d
```

Check that the subthreshold equation makes sense. Start by plotting the drain current versus gate voltage, and check that the slope is 1 e-fold per U_T/kappa.

In [5]:

```
import matplotlib.pyplot as plt  # plotting
import numpy as np  # for math

vg = np.linspace(0, 1, 100)
# drain current vs gate voltage (transconductance)
idvsvg = id_sub(vg)

plt.figure("idsat")
plt.semilogy(vg, idvsvg)
plt.xlabel("V_g (V)")
plt.ylabel("I_ds (A)")
plt.title("Id vs Vg")
plt.grid(True)

reg = linregress(np.log(idvsvg), vg)
efold_v_meas = reg[0]
efold_v_theory = U_T / kappa
print(
    f"Transconductance: Measured efold current gate voltage={eng(efold_v_meas)}V, predicte
)
if (
    np.abs((efold_v_meas - efold_v_theory) / (0.5 * (efold_v_meas + efold_v_theory)))
    > 0.01
):
    raise ValueError("Something wrong with subthreshold equations")
else:
    print("Transconductance OK")
```

```
Transconductance: Measured efold current gate voltage=31.25mV, predicted from U_T/kappa=3
1.25mV
Transconductance OK
```

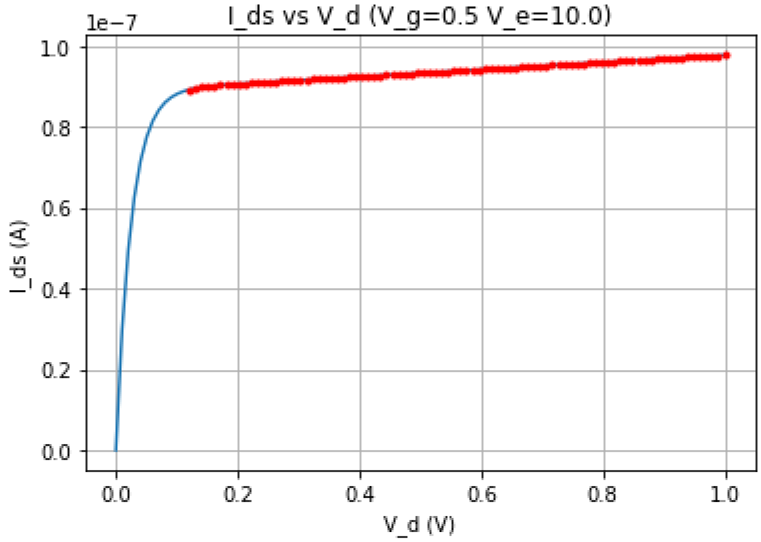


Now plot the drain current vs drain voltage and check that the actual drain conductance matches the expected value.

In [6]:

```
vd = np.linspace(0, 1, 100)
V_g = 0.5
idsat = I_0 * np.exp(kappa * V_g / U_T)
# drain current vs drain voltage (drain conductance)
idvsvd = id_sub(V_g=V_g, V_d=vd, V_e=V_e)

plt.figure("idsat2")
plt.plot(vd, idvsvd)
plt.xlabel("V_d (V)")
plt.ylabel("I_ds (A)")
plt.grid(True)
plt.title("I_ds vs V_d (V_g={:.1f} V_e={:.1f})".format(V_g, V_e))

r = [i for i in range(len(idvsvd)) if idvsvd[i] >= idsat]

plt.plot(vd[r], idvsvd[r], ".r")
reg = linregress(vd[r], idvsvd[r])
gout_meas = reg[0]
gout_pred = I_0 * np.exp(kappa * V_g / U_T) / V_e
print(
    f"Output conductance: Measured g_out={eng(gout_meas)}, predicted from g_out=Id_sat/Ve=
)
if np.abs((gout_meas - gout_pred) / (0.5 * (gout_meas + gout_pred))) > 0.05:
    raise ValueError("Something wrong with subthreshold equations")
else:
    print("Drain conductance OK within 5%")
```

```
Output conductance: Measured g_out=9.04n, predicted from g_out=Id_sat/Ve=8.89n
Drain conductance OK within 5%
```



It seems to make sense. Now we have an equation we can use in ODE equation for photoreceptors

Estimating actual photocurrent

Now we need to compute reasonable values for photocurrent and dark current. Let's take the interesting situation of operation in dark conditions at 1 lux scene illumination, which is about 10 times moonlight. The light falling onto the chip will be reduced by the optics according to the equation below.

We will also assume a photodiode area of 10um^2 which is a reasonably-large photodiode, and we will assume a not-so-great junction leakage "dark current" of 1nA/cm^2.

In [7]:

```
scene_flux_lux = 10  # 1 lux is about ten times moonlight
photodiode_area_um2 = 10  # photodiode area m^2
# optics reduces light intensity by square of aperture ratio
# we will assume a cheap f/3 lens with ratio focal length to aperture of 3
def optics_reduction(flux):
    f_number = 3
    return flux / (4 * f_number**2)


avg_reflectance = 0.18  # kodak's estimate of average scene reflectance
chip_flux_lux = optics_reduction(scene_flux_lux)
photons_per_um2_per_lux = 1e4  # you get about this many photons per lux falling on chip
photocurrent_e_per_sec = chip_flux_lux * (photons_per_um2_per_lux) * photodiode_area_um2
dark_current_amps_per_um2 = (1e-9 / 1e-4) * 1e-12  # junction leakage per m^2
dark_current_amps = photodiode_area_um2 * dark_current_amps_per_um2
dark_current_e_per_sec = dark_current_amps / q
photocurrent_amps = photocurrent_e_per_sec * q
photocurrent_total_amps = photocurrent_amps + dark_current_amps

print(
    f"scene illumination level {eng(scene_flux_lux)}lux\n"
    f"photodiode area: {photodiode_area_um2}um^2\n"
    f"DC photocurrent: {eng(photocurrent_total_amps)}A\n"
    f"dark current: {eng(dark_current_e_per_sec)}e/s or {eng(dark_current_amps)}A\n"
    f"I_0 off current: {eng(I_0/q)}e/s or {eng(I_0)}A\n"
)
```

```
scene illumination level 10lux
photodiode area: 10um^2
DC photocurrent: 4.54fA
dark current: 625e/s or 100aA
I_0 off current: 62.50ke/s or 10fA
```

Is the value smaller than the off-current? This is not surprising; under dark conditions, the photocurrent can be a small fraction of the FET off-current.

Making a photocurrent stimulus waveform

Now we will make a waveform input stimulus to drive our time-domain simulations of the photoreceptors. Let's define our input photocurrent waveform we will use it is a square wave with modulation of contrast signal_contrast and starts with bright DC level and then goes to dark DC level

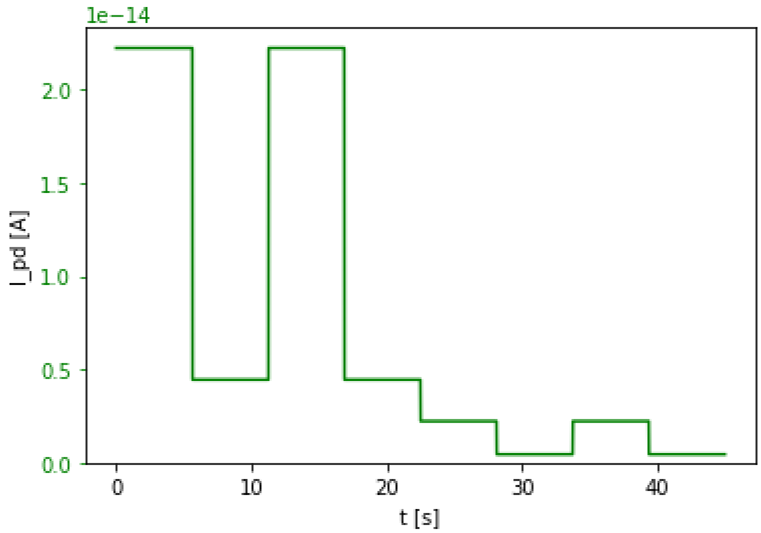
In [8]:

```
import numpy as np
from scipy.signal import square
import matplotlib.pyplot as plt  # plotting

dark = dark_current_amps  # dark current level
sigdc1 = photocurrent_amps  # DC photocurrent for bright half
sigdc2 = photocurrent_amps / 10  # and dark half
signal_contrast = 5  # contrast in each half, i.e. cont=10 means that the bright part wil
nper = 2  # how many periods to simulate for each half bright/dark
# to compute the period, let's make it so that half a period is 1 time constants for SF in
C_pd = 100e-15  # guesstimate about 100fF
tau_sf = C_pd * U_T / sigdc2
per = 2 * tau_sf  # period in seconds

print(
    f"source follower photodiode capacitance C_pd={eng(C_pd)}F and tau_sf={eng(tau_sf)}s\n
    f"Computed period: {eng(per)}s"
)
dt = per / 500  # timesteps per half period
time_basis = np.arange(
    0, 2 * nper * per, dt
)  # start,stop,step generate time basis that is nper long
npts = len(time_basis)
npts2 = int(npts / 2)
# generate square wave with period per using time basis t that has steps dt
# square(x) has period 2*pi, i.e. its output is 1 when input is 0-pi, then -1 for pi-2pi,
# thus if we want to have nper cycles in each half of our stimulus, we need to
# make its argument go to 2pi when the time goes to per
# Also, shift it up and divide by 2 to get 0-1 modulated square
sq = (square((2 * np.pi * time_basis) / (per)) + 1) / 2
# convolve with a short box filter to
# make the edges not perfectly square to simulate finite optical aperture
# sq=np.convolve(sq,np.ones(10)/10,mode='same') # causes some wierd transient, didn't debu

sig = np.zeros_like(sq)
sig[:npts2] = sigdc1 * (1 + (signal_contrast - 1) * sq[:npts2])
sig[npts2 + 1 :] = sigdc2 * (1 + (signal_contrast - 1) * sq[npts2 + 1 :])
sig[npts2] = sigdc2 * (1 + (signal_contrast - 1) * sq[npts2 + 1])
photocurrent_waveform = sig
# plt.plot(t,cur)

fig, ax1 = plt.subplots(sharex=True)
ax1.plot(
    time_basis,
    photocurrent_waveform,
    "g",
)
ax1.set_ylim([0, None])
ax1.set_yscale("linear")
ax1.set_xscale("linear")
ax1.tick_params(axis="y", colors="green")
ax1.set_xlabel("t [s]")
ax1.set_ylabel("I_pd [A]")
```

```
source follower photodiode capacitance C_pd=100fF and tau_sf=5.63s
Computed period: 11.25s
```

Out[8]:

```
Text(0, 0.5, 'I_pd [A]')
```



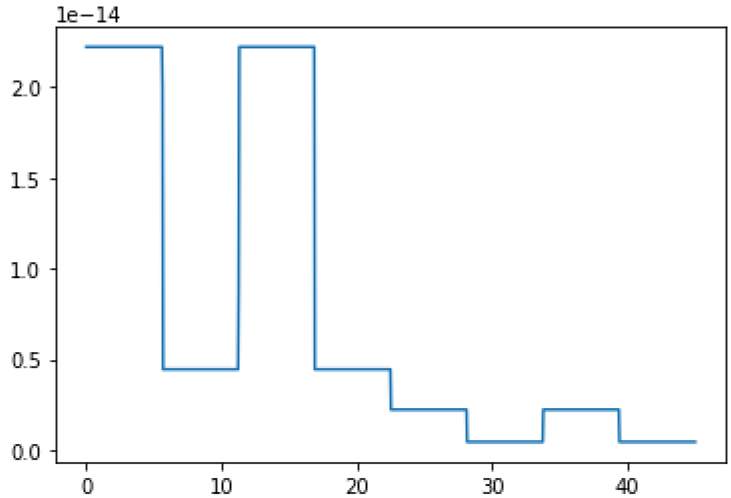
We need to make a function out of our nice I_pd vector so that we can get I_pd at any point in time

In [9]:

```
def find_nearest_idx(array, value):
    idx = np.searchsorted(array, value, side="right")
    return idx


def I_pd_function(time, time_basis, photocurrent_waveform):
    idx = find_nearest_idx(time_basis, time)
    idx -= 1  # go to next point just to left since search finds point just before time
    if idx < 0:
        return photocurrent_waveform[0]
    if idx >= (len(time_basis) - 1):
        return photocurrent_waveform[-1]
    t1 = time_basis[idx]
    i1 = photocurrent_waveform[idx]
    idx += 1
    t2 = time_basis[idx]
    i2 = photocurrent_waveform[idx]
    tfrac = (time - t1) / (t2 - t1) if t2 - t1 > 0 else 0.5
    i = (1 - tfrac) * i1 + tfrac * i2
    return i


I_pd = lambda t: I_pd_function(t, time_basis, photocurrent_waveform)
# test it
ttest = np.linspace(0, time_basis[-1], 1000)
itest = np.array([I_pd(t) for t in ttest])
plt.plot(ttest, itest)
```

Out[9]:

```
[<matplotlib.lines.Line2D at 0x12904c760>]
```



Exercise 1: Static vs. active unity-gain photoreceptors DC responses

First you will plot the theoretical DC responses of SF and TI to input photocurrent.

We will compute expressions for the DC response of the simple and unity gain feedback photoreceptor circuits.

We compute the SF output for you.

In [10]:
```
i_pd=np.logspace(-19,-6,100) # input photocurrent vector, log scale from well under I_0 ar


# equation for SF DC output, assuming gate voltage of 1.4V
v_g=1.4
v_sf= kappa*v_g-U_T*np.log((i_pd+dark_current_amps)/I_0)
```

(a) Computing the TI photoreceptor DC output

Let's define a function for the TI photoreceptor DC output. We can use that to plot it, and later on use it to define the initial condition for the TI photoreceptor voltage at the start of transient simulation.

Assume for the TI circuit that the gain of the feedback amplifier is infinite and that the amplifier is ideal, i.e. that the input FET never goes out of saturation.

You should fill in the expressions for the TI photodiode and output voltages:

In [11]:
```
def ti_dc(I_b, I_pd, I_0=I_0, V_e=V_e, U_T=U_T, kappa=kappa, I_dark=dark_current_amps):
    """Computes the theoretical DC operating point of TI photoreceptor given parameters
    :param I_b: bias current, should be scalar
    :param I_pd: photocurrent, can be vector
    :param V_e: amplfiier input FET Early voltage
    :param U_T: thermal voltage
    :param kappa: back gate coefficient of all FETs

    :returns: [V_pd, V_out] voltages in form suitable for solve_ivp initial condition
    """
    # TODO include effect of finite amplifier gain, not accounted for now
    # check that I_b is scalar
    if not np.isscalar(I_b):
        raise ValueError("I_b should be a scalar")
    # TODO compute the photodiode voltage.  It is determined by Ib, right?
    V_pd = np.log(I_b / I_0) * U_T / kappa
    # we need to handle that I_pd might be scalar or vector
    if not np.isscalar(I_pd):
```

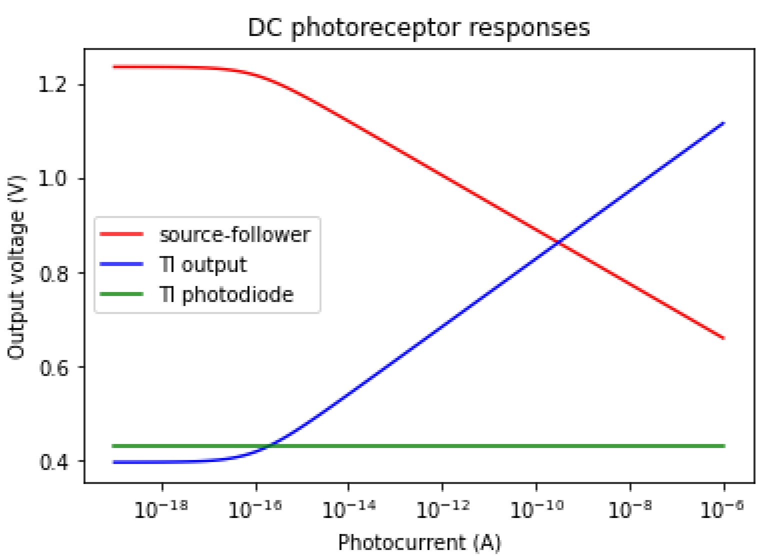
```
        V_pd = np.ones(len(I_pd)) * V_pd
    # TODO compute the TI output voltage expression
    V_out = (np.log((I_pd + I_dark) / I_0) * U_T + V_pd) / kappa
    return [V_pd, V_out]


# check DC output
I_b = 10e-9  # bias current for amplifier pullup Mp in TI photoreceptor
ip = I_pd(0)
vti0 = ti_dc(I_b, photocurrent_amps)
print(
    f"DC output of TI with bias current I_b={eng(I_b)} "
    f"and photocurrent I_p={eng(photocurrent_amps)} "
    f"are vpd={eng(vti0[0])} vout={eng(vti0[1])}"
)
```

```
DC output of TI with bias current I_b=10n and photocurrent I_p=4.44f are vpd=431.73m vout=
515.02m
```

(b) Plot the SF and TI DC output together on a log-linear plot of V_{out} versus I_{pd}, covering a range of photocurrents of I_{pd} from 0.01fA to 10nA. Assume I_0=1e-13A and that there is dark current of I_{dark}=0.1fA.

In [12]:
```
# compute the vector of TI outputs. The function returns a list of 2 vectors, [V_pd,V_out_
v_ti=ti_dc(I_b, i_pd)
plt.figure('DC responses')
plt.title('DC photoreceptor responses')
plt.semilogx(i_pd,v_sf,'r-', i_pd,v_ti[1],'b-', i_pd,v_ti[0],'g-')
plt.legend(['source-follower','TI output','TI photodiode'])
plt.xlabel('Photocurrent (A)')
plt.ylabel('Output voltage (V)')
```

Out[12]:
```
Text(0, 0.5, 'Output voltage (V)')
```



Preparation for large signal transient (time-domain) simulations

It seems to make sense. Now we have equations we can use in ODE equation for photoreceptors

Dynamical equations

Next we will write the dynamical equations for the source-follower and feedback photoreceptor using the id_sub equation for the currents. We need to write the right hand side equation for

```
dy / dt = f(t, y)
```

given initial condition

```
y(t0) = y0
```

The source follower only has one node so the output is a scalar derivative. The TI photoreceptor has 2 nodes (the photodiode and output), so the output is a vector of 2 deriatives w.r.t. time.

In [ ]:

```

```

Exercise 2: Large signal transient response of source follower and active photoreceptors

As an example, below we define the RHS for the SF. The time derivative of the output voltage is the current divided by the node capacitance:

In [13]:

```python
def sfdvdt(
    t, y, V_g=1.4, C_pd=100e-15
):  # fill in reasonable photodiode capacitance, e.g. 100fF
    vdot = (id_sub(V_g=V_g, V_s=y) - I_pd(t)) / C_pd
    return vdot
```

Exercise 2.1: define RHS of ODE for TI receptor

Now you should do the same thing, but for the vector of TI node voltages [vpd,vout] for the vpd input (photodiode) and vout output nodes:

In [14]:

```python
def tidvdt(t, y, Ib, I_pd, V_e=V_e, C_pd=100e-15, C_out=1e-15):
    """Compute time derivatives of TI photoreceptor node voltages

    :param t: the time in s
    :param y: the TI PD and output voltages vector [vpd,vout]
    :param V_e: the amplifier input n-fet Early voltage in V
    :param C_pd: the photodiode cap in Culombs
    :param C_out: the output capacitance

    :returns: the vector of photodiode/output voltage time derivatives
    """
    vpd = y[0]
    vout = y[1]
    # you fill in next parts from equations for TI photoreceptor.
    # TODO

    vpd_dot = (id_sub(V_g=vout, V_s=vpd, V_e=V_e) - I_pd(t) - dark_current_amps) / C_pd
    vout_dot = (Ib - id_sub(V_g=vpd, V_d=vout, V_e=V_e)) / C_out

    yout = [vpd_dot, vout_dot]
    return yout
```

Exercise 2.2: Timestepping transient simulation of photoreceptors

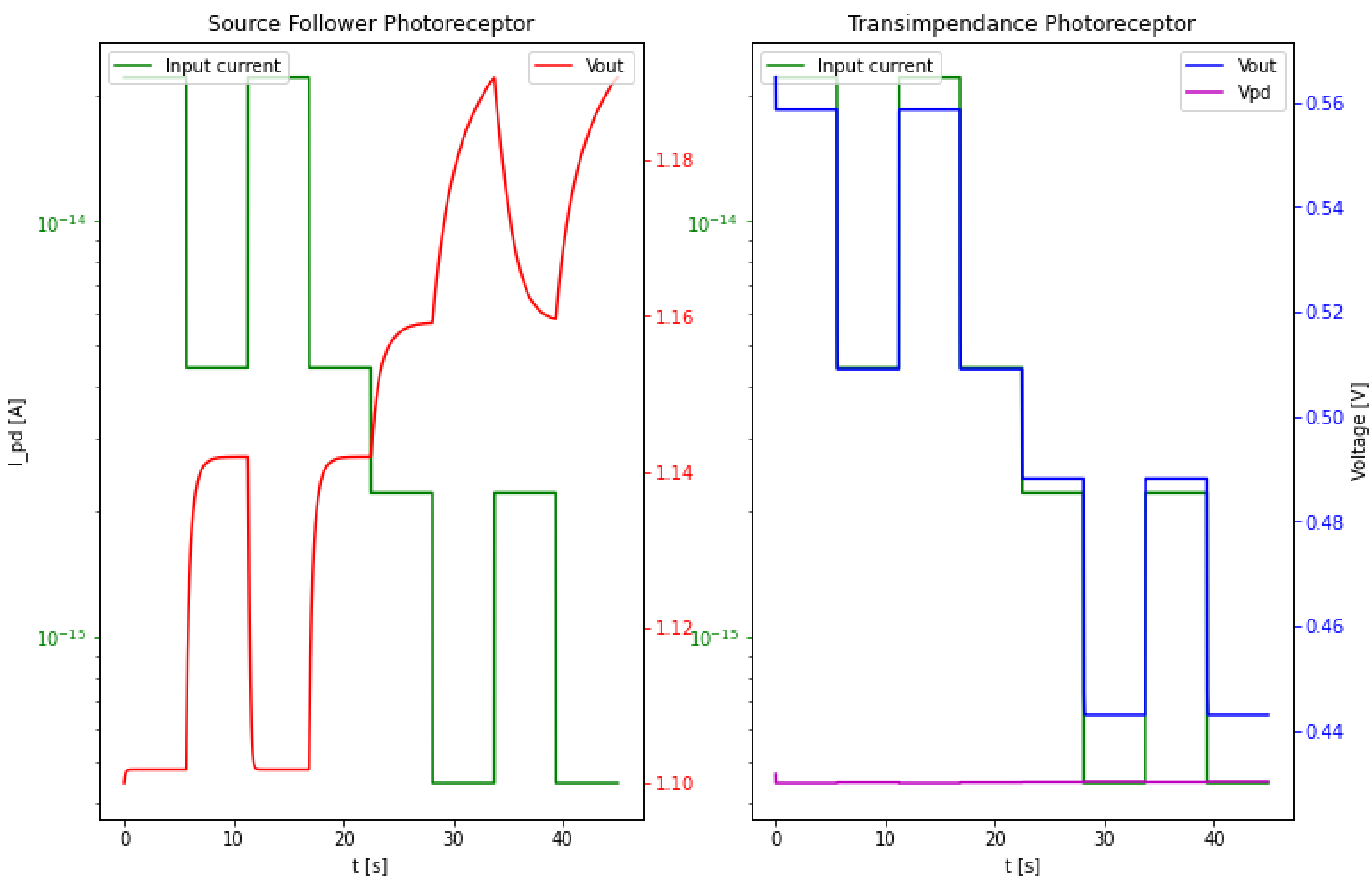
Below we have done it for the SF photoreceptor. You should add the TI photoreceptor to the simulation so you can compare them to each other. YOu may find issues with the simulator not responding to some of the edges in the photocurrent. If this is the case, you can try to decrease tolerance (rtol and atol), and also try different methods. Check solve_ivp() documentation for the different options.

In [126…

```python
import matplotlib.pyplot as plt  # plotting
import numpy as np  # for math
from scipy.integrate import solve_ivp  # for timestepping ODEs

V_sf0 = [
    1.1
]  # initial condition of v, just guess it to be approx Vg-a bit, e.g. 1.4-.3
sf_sol = solve_ivp(
    sfdvdt,
    (time_basis[0], time_basis[-1]),
    V_sf0,
    t_eval=time_basis,
    rtol=1e-10,
    atol=1e-20,
    method="LSODA",
    # rtol=1e-9,atol=1e-20, method='Radau',
    args=(1.4, C_pd),
)
# output is sol.t and sol.y
if sf_sol.message is not None:
    print(sf_sol.message)
v_sf = sf_sol.y[0]
t_sf = sf_sol.t


# TODO you can solve the TI by filling in below
# check DC output
ib = I_b
ip = I_pd(0)
V_ti0 = ti_dc(I_b, ip)
print(
    f"DC output of TI with I_b={eng(I_b)}A and I_pd={eng(ip)}A are vpd={eng(V_ti0[0])}V vo
)

C_out = 1e-15
ti_sol = solve_ivp(
    tidvdt,
    (time_basis[0], time_basis[-1]),
    V_ti0,
    t_eval=time_basis,
    rtol=1e-10,
    atol=1e-21,
    method="Radau",
    # rtol=1e-9, atol=1e-19, method='RK45',
    args=(I_b, I_pd, V_e, C_pd, C_out),
)

### output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti = ti_sol.y[1]
t_ti = ti_sol.t
v_pd = ti_sol.y[0]
t_pd = ti_sol.t

# use this plotting style to put several plots sharing same x-axis
# we will plot V_sf together with the input photocurrent
# using another axis since it is volts, not current, and linear not log
fig = plt.figure(figsize=(12, 8))
ax1 = plt.subplot(121)
ax2 = plt.subplot(122, sharex=ax1)

ax1.plot(time_basis, photocurrent_waveform, "g")
ax1.set_yscale("log")
ax1.set_xscale("linear")
ax1.tick_params(axis="y", colors="green")
```

```
ax1.set_xlabel("t [s]")
ax1.set_ylabel("I_pd [A]")
ax1.title.set_text("Source Follower Photoreceptor")
ax2.title.set_text("Transimpendance Photoreceptor")

ax3 = ax1.twinx()
ax3.plot(t_sf, v_sf, "r-")
ax3.tick_params(axis="y", colors="red")
ax1.legend(["Input current"], loc="upper left")
ax3.legend(["Vout"], loc="upper right")

ax2.set_yscale("log")
ax2.set_xlabel("t [s]")

ax2.tick_params(axis="y", colors="green")
ax2.set_xscale("linear")


ax2.plot(
    time_basis,
    photocurrent_waveform,
    "g",
)
ax2.legend(["Input current"], loc="upper left")

# TODO: Uncomment for TI photoreceptor
ax4 = ax2.twinx()
ax4.set_ylabel("Voltage [V]")
# ax4.set_ylim([0.4,0.8])
ax4.plot(t_ti, v_ti, "b-", t_pd, v_pd, "m-")
ax4.tick_params(axis="y", colors="blue")
ax4.legend(["Vout", "Vpd"], loc="upper right")
plt.show()
```

```
The solver successfully reached the end of the integration interval.
DC output of TI with I_b=10nA and I_pd=22.22fA are vpd=431.73mV vout=564.76mV
The solver successfully reached the end of the integration interval.
```



Note that in above transient solution, you might get a startup glitch particularly for the TI photoreceptor because your TI DC solution at the starting time is not quite correct. It means that the V_pd and V_out and not consistent, and so the circuit will go through a short period of adjustment to come to the steady-state level.

Exercise 3: Small signal modeling

You already did the most difficult part which is the large signal modeling. Now we will fix the operating point around some DC level and compute the small signal transfer functions. From these we can see the cutoff frequencies and stability.

Exercise 3.1: AC transfer functions (Bode plots) of static vs. active photorecptors

(a) Write the small-signal differential equations for the simple photoreceptor and the unity-gain feedback photoreceptor assuming photodiode capacitance C_{pd} and (for the feedback photoreceptor) output load capacitance C_{out}. For the feedback photoreceptor, you can assume that that amplifier has a output resistance g_{out}=I_b/V_e (recall that the DC voltage gain is A=-g_m/g_{out}).

> Simple photoreceptor:
>
> $$\begin{aligned} C_{out}\dot{v_{out}} &= g_m V_b - g_s V_{out} - i && (1)\\ &= \frac{\kappa I_{pd}}{U_T} V_b - \frac{I_{pd}}{U_T} V_{out} - i && (2)\\ &= \frac{I_{pd}}{U_T}(\kappa V_b - V_{out}) - i && (3)\\ &= -\frac{I_{pd}}{U_T} V_{out} - i && \kappa V_b \text{ is a constant DC value} \quad (4) \end{aligned}$$
>
> Unity-gain feedback photoreceptor:
>
> $$\begin{aligned} C_{pd}\dot{v_{pd}} &= g_{m_{M_{fb}}} v_{out} - g_{s_{M_{fb}}} v_{pd} - i && (5)\\ C_{out}\dot{v_{out}} &= -g_{m_{M_n}} v_{pd} - g_{out} v_{out} && (6) \end{aligned}$$

(b) From the differential equations, derive the transfer functions $H(s)$ for each circuit.

Your equations should end up with time constants τ_{in} (for the photodiode node) and τ_{out} for the feedback photoreceptor.

For this derivation, the input to the circuit is the small-signal photocurrent i_{pd} which is its deviation from the DC value I_{pd}. The output is the small signal output voltage v_{out}. But since the circuit is a log photoreceptor, a better way to express the transfer function is to write it as output voltage per log input current. Thus $H(s)$ will be the transimpedance 'gain' that transduces from i_{pd}/I_{pd} to v_{out}, i.e. the units of H(s) are volts/(fractional change in current).

> Simple photoreceptor:
>
> $$C_{out}\dot{v_{out}} = -\frac{I_{pd}}{U_T} V_{out} - i \quad (7)$$

$$\tau_{in} = \frac{C_{out}}{g_s} \tag{8}$$

$$g_s = \frac{I_{pd}}{U_T} \tag{9}$$

$$\tau_{in}\dot{v_{out}} = -V_{out} - \frac{i}{g_s} \tag{10}$$

$$\tau_{in} s v_{out} = -V_{out} - \frac{i}{g_s} \qquad \text{s plane} \tag{11}$$

$$(s\tau_{in} + 1)v_{out} = -\frac{i}{g_s} \tag{12}$$

$$= -U_T \frac{i}{I_{pd}} \tag{13}$$

$$H(s) = \frac{V_{out}}{\frac{i}{I_{pd}}} = -\frac{U_T}{s\tau + 1} \tag{14}$$

Unity-gain feedback photoreceptor:

$$\tau_{in} = \frac{C_{pd}}{g_{s_{M_{fb}}}} \tag{15}$$

$$\tau_{out} = \frac{C_{out}}{g_{out}} \tag{16}$$

$$A = \frac{g_{m_{M_n}}}{g_{out}} \tag{17}$$

$$\kappa = \frac{g_{m_{M_{fb}}}}{g_{s_{M_{fb}}}} \tag{18}$$

$$C_{pd}\dot{v_{pd}} = g_{m_{M_{fb}}} v_{out} - g_{s_{M_{fb}}} v_{pd} - i \tag{19}$$

$$\tau_{in}\dot{v_{pd}} = \kappa v_{out} - v_{pd} - \frac{i}{g_{s_{M_{fb}}}} \tag{20}$$

$$= \kappa v_{out} - v_{pd} - U_T \frac{i}{I_{pd}} \tag{21}$$

$$\tau_{in}\dot{v_{pd}} = \kappa v_{out} - v_{pd} - U_T \frac{i}{I_{pd}} \qquad \text{s plane} \tag{22}$$

$$(s\tau_{in} + 1)v_{pd} = \kappa v_{out} - U_T \frac{i}{I_{pd}} \qquad \text{Eq. 1} \tag{23}$$

$$C_{out}\dot{v_{out}} = -g_{m_{M_n}} v_{pd} - g_{out} v_{out} \tag{24}$$

$$\tau_{out}\dot{v_{out}} = -A v_{pd} - v_{out} \tag{25}$$

$$\tau_{out} s v_{out} = -A v_{pd} - v_{out} \qquad \text{s plane} \tag{26}$$

$$(s\tau_{out} + 1)v_{out} = -A v_{pd} \qquad \text{Eq. 2} \tag{27}$$

$$
\begin{aligned}
(\tau_{out}s+1)v_{out} &= -A\frac{\kappa v_{out} - U_T\frac{i}{I_{pd}}}{\tau_{in}s+1} && \text{Combine Eq. 1} \\
(\tau_{out}s+1)(\tau_{in}s+1)v_{out} &= -A\kappa v_{out} + AU_T\frac{i}{I_{pd}} \\
((\tau_{out}s+1)(\tau_{in}s+1)+A\kappa)v_{out} &= AU_T\frac{i}{I_{pd}} \\
H(s) = \frac{v_{out}}{\frac{i}{I_{pd}}} &= \frac{AU_T}{(\tau_{out}s+1)(\tau_{in}s+1)+A\kappa} \\
&= \frac{AU_T}{\tau_{in}\tau_{out}s^2+(\tau_{in}+\tau_{out})s+1+A\kappa}
\end{aligned}
$$

(c) The TI feedback should make the TI photoreceptor faster to respond to changes in photocurrent than the SF photoreceptor (and also noisier).

By setting τ_{out} to zero (taking the limit as τ_{out} goes to zero), compute the expected speedup from the feedback. I.e., what is the ratio of cutoff frequency of TI to SF circuit when I_b is really large? You will see if it true (at least in the model) in the Bode magnitude transfer function plots.

Simple photoreceptor:

$$
\begin{aligned}
H(s) &= -\frac{U_T}{\tau s+1} && (33) \\
\Rightarrow s_0 &= -\frac{1}{\tau} \qquad \text{pole} && (34) \\
-s_0 &= \frac{1}{\tau} && (35) \\
\omega_0 = |s_0| &= \frac{1}{\tau} && (36) \\
\Rightarrow f_{sf} &= \frac{1}{2\pi\tau} && (37)
\end{aligned}
$$

Unity-gain photoreceptor:

$$
\begin{aligned}
\omega &:= 2\pi f && (38) \\
\lim_{\tau_{out}\to 0} H(s) &= A\frac{U_T}{\tau_{in}s+1+A\kappa} && (39) \\
\Rightarrow s_0 &= \frac{-1-A\kappa}{\tau} \qquad \text{pole} && (40) \\
-s_0 &= \frac{1+A\kappa}{\tau} && (41) \\
\omega_c = |s_0| &= \frac{1+A\kappa}{\tau} && (42) \\
\Rightarrow f_{ti} &= \frac{1+A\kappa}{2\pi\tau} && (43)
\end{aligned}
$$

ratio of cutoff frequencies:

$$
\begin{aligned}
\frac{f_{ti}}{f_{sf}} &= \frac{\frac{1+A\kappa}{2\pi\tau}}{\frac{1}{2\pi\tau}} && (44) \\
&= 1+A\kappa && (45)
\end{aligned}
$$

(d) Plot the magnitude of the transfer functions versus frequency, assuming reasonable values for τ_{in}, τ_{out}, etc and an intermediate DC value of photocurrent, e.g. I_{pd}=1pA. You can assume that the bias current of the amplifier for the feedback photoreceptor is I_b=10nA and V_e=10V.

Remember that frequency in radians per second is w=$2\pi f$ where f is frequency in Hertz.

You can use numpy to compute the magnitude of the complex transfer function by using s=jw where j is $\sqrt{-1}$.

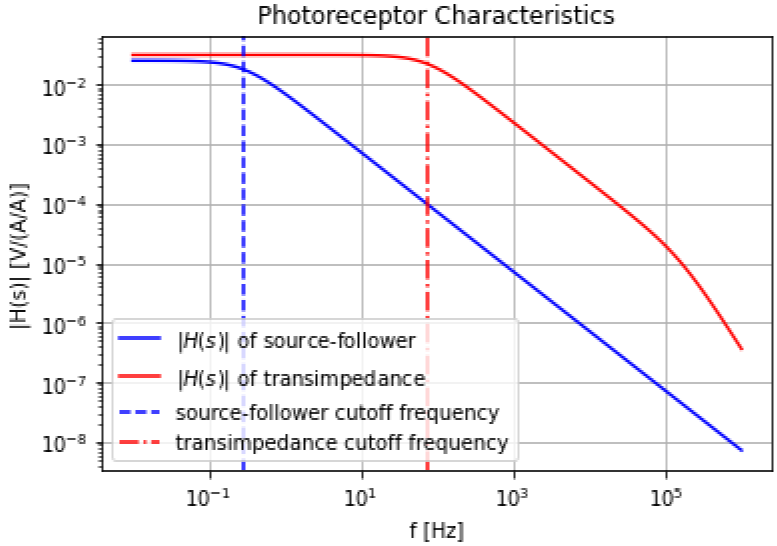
Assume the DC photocurrent is still photocurrent_amps from above.

In [16]:

```
import matplotlib.pyplot as plt  # plotting
import numpy as np  # for math

Ipd_dim = photocurrent_total_amps

C_out = 1e-15
freq = np.logspace(-2, 6, 100)  # plot from 1kHz to 1GHz
w = 2 * np.pi * freq
tau_sf = C_pd * U_T / Ipd_dim  # tau_sf = C_pd/gs_sf
H_sf = U_T / (np.sqrt(np.square(w * tau_sf) + 1))
sf_cutoff_hz = 1 / (2 * np.pi * tau_sf)

print(
    f"source follower photodiode capacitance C_pd={eng(C_pd)}F and tau_sf={eng(tau_sf)}s\n
    f"SF cutoff frequency: {eng(sf_cutoff_hz)}Hz"
)

# TODO TI photoreceptor
tau_in = C_pd * U_T / Ipd_dim
tau_out = C_out * V_e / I_b
A = kappa * V_e / U_T
H_ti = np.abs(
    (A * U_T)
    / (tau_in * tau_out * (1j * w) ** 2 + (tau_in + tau_out) * (1j * w) + 1 + A * kappa)
)
# Same as
# H_ti=(A*U_T)/(np.sqrt((tau_in + tau_out)**2*w**2 + (1 + A*kappa-tau_in*tau_out*w**2 )**2

ti_cutoff_hz = sf_cutoff_hz * (kappa * A + 1)

print(f"transimpedance TI cutoff frequency: {eng(ti_cutoff_hz)}Hz")


fig, ax1 = plt.subplots(sharex=True)
# ax1.plot(freq,H_sf,'b')
ax1.plot(freq, H_sf, "b", label="$|H(s)|$ of source-follower")
ax1.plot(freq, H_ti, "r", label="$|H(s)|$ of transimpedance")
ax1.axvline(sf_cutoff_hz, color='b', linestyle = '--', label="source-follower cutoff frequ
ax1.axvline(ti_cutoff_hz, color='r', linestyle = '-.', label="transimpedance cutoff freque
# ax1.set_ylim([0,None])
ax1.set_yscale("log")
ax1.set_xscale("log")
# ax1.tick_params(axis='y', colors='green')
ax1.set_xlabel("f [Hz]")
ax1.set_ylabel("|H(s)| [V/(A/A)]")
ax1.legend()
ax1.grid()
ax1.title.set_text("Photoreceptor Characteristics")
plt.show()
```

```
source follower photodiode capacitance C_pd=100fF and tau_sf=550.12ms
```

```
SF cutoff frequency: 289.31mHz
transimpedance TI cutoff frequency: 74.35Hz
```



(c) Comment on your results. Can you see the effect of feedback on the bandwidth in the TI circuit? Can you observe some ringing? I.e. is the circuit overdamped or underdamped at this I_b and I_{pd}?

> The feedback decreases τ, which increases the bandwidth since $f_c = \frac{1}{2\pi\tau}$ in a low pass filter. So the TI circuit can react faster to changes and follow along higher frequencies. From the bode plot we see that the circuit is overdamped, since there are no peaks in the frequency response.

Exercise 4: Root-locus plot of TI photoreceptor

As the photocurrent increases, it should be clear that the single pole of the SF photoreceptor moves farther away from the origin, as it speeds up.

Here you will compute the poles of the TI photoreceptor transfer function and then plot their locations on the complex plane as a function of the amplfier bias current I_b, given a fixed photocurrent I_{pd}.

The two poles of the quadratic demoninator $D(s)$ of $H(s)$ will either both be real or form a complex conjugate pair (since all the coefficients of the polynominal are real).

(a) First let's define a function to get the poles and the Q factor given the circuit parameters. Remember that a second order system (with no zeros) can be described by a transfer function of the type $H(s) = \frac{A}{\frac{1}{\omega_0^2}s^2 + \frac{1}{Q\omega_0}s + 1}$, where ω_0 (natural frequency) and Q (quality factor) are characteristics of the system.

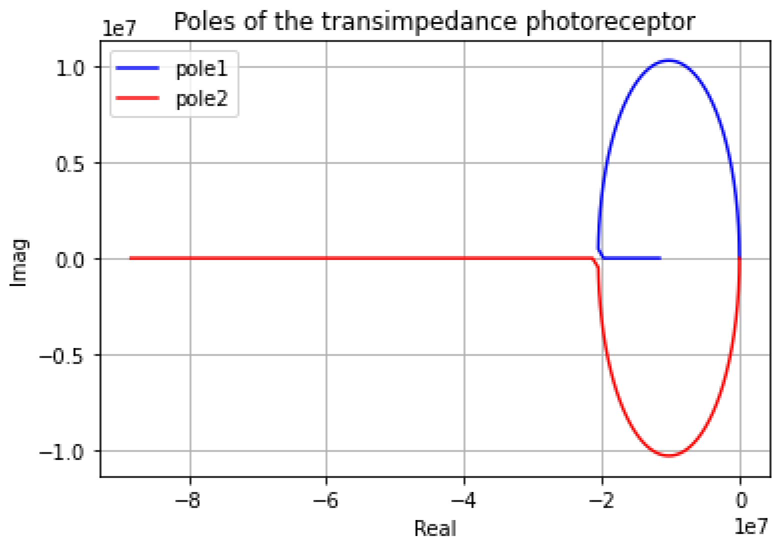
In [17]:
```python
def get_poles(Ipd,Ib,C_pd,C_out,V_e,U_T=U_T,kappa=kappa):
    # TODO:
    tau_in=C_pd * U_T / Ipd
    tau_out=C_out * V_e / Ib
    A = kappa * V_e / U_T
    canonical_tau = np.sqrt(tau_in * tau_out / (A * kappa))
    Q=np.sqrt(A*kappa*tau_in*tau_out)/(tau_in+tau_out)
    coef2=canonical_tau**2
    coef1=canonical_tau/Q
    coef0=1 + 1/(A*kappa)

    # apply quadratic formula to find roots of the transfer function denominator
    # 0j in sqrt forces result to be complex
```

```
    pole1=(-coef1+np.sqrt(0j+coef1*coef1-4*coef0*coef2))/(2*coef2)
    pole2=(-coef1-np.sqrt(0j+coef1*coef1-4*coef0*coef2))/(2*coef2)
    return [pole1,pole2,Q]
```

(b) Now let's plot the root locus. The resulting plot is a trajectory in the complex plane as a function of I_b

In [18]:

```
import matplotlib.pyplot as plt # plotting
import numpy as np # for math

Ib_sweep = np.logspace(-15,-6,10000) # define range of Ib, adjust it to show the loop
Ipd_bright = 100e-12 # Photocurrent of 100pA

pole1,pole2,Q=get_poles(Ipd=Ipd_bright,Ib=Ib_sweep,C_pd=C_pd,C_out=C_out,V_e=V_e)

# plot
fig,ax1=plt.subplots(sharex=True)
ax1.plot(np.real(pole1),np.imag(pole1),'b')
ax1.plot(np.real(pole2),np.imag(pole2),'r')
ax1.set_xlabel('Real')
ax1.set_ylabel('Imag')
ax1.legend(['pole1','pole2'])
ax1.title.set_text('Poles of the transimpedance photoreceptor')
ax1.grid()
plt.show()
```



As you should be able to see in the root locus plot, for some values of I_b both poles lie in the real axis, but by decreasing I_b, the poles become complex cojugates.

Let's now see the impact of pole location on the transient behavior of the circuit.

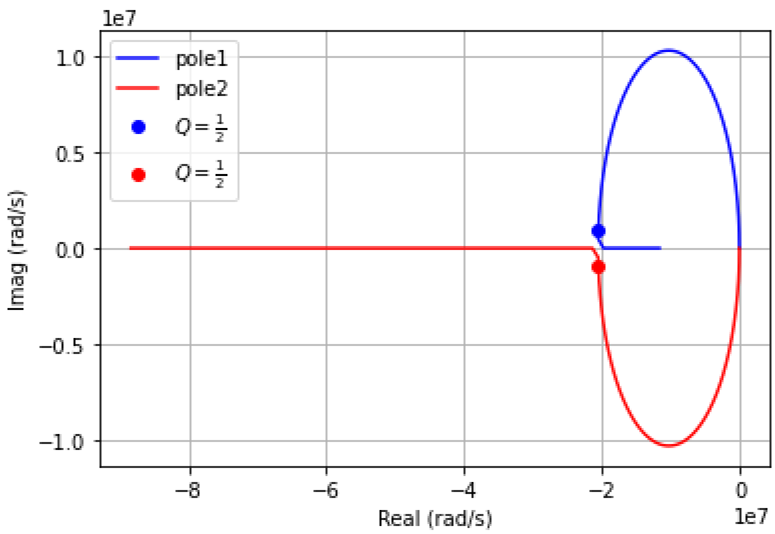
(c) First, for a particular photocurrent (which of course is not true in practice, it varies tremendously), find the minimum value of I_b that results in a critically damped circuit. In the exactly critically-damped condition, Q equals 0.5, and the transient response should show no ringing since it consists of 2 low pass filters in series with equal time constants. This bias current is also the minimum value of I_b that results in real valued poles at that photocurrent.

For the photocurrent, you can use the DC photocurrent under the illumination condition at the start of the waveform, which is called Ipd_bright.

In [19]:

```
# plot root locus again
fig, ax1 = plt.subplots(sharex=True)
ax1.plot(np.real(pole1), np.imag(pole1), "b")
ax1.plot(np.real(pole2), np.imag(pole2), "r")
```

```
ax1.set_xlabel("Real (rad/s)")
ax1.set_ylabel("Imag (rad/s)")
ax1.legend(["pole1", "pole2"])

# TODO fill expression Ib that results in a Q of 0.5
A = kappa * V_e / U_T
tau_in = C_pd * U_T / Ipd_bright
Ib_Qhalf = 4 * V_e * C_out * A * kappa / tau_in
pole1_Qhalf, pole2_Qhalf, Q_Qhalf = get_poles(
    Ipd=Ipd_bright, Ib=Ib_Qhalf, C_pd=C_pd, C_out=C_out, V_e=V_e
)
ax1.plot(np.real(pole1_Qhalf), np.imag(pole1_Qhalf), "bo")
ax1.plot(np.real(pole2_Qhalf), np.imag(pole2_Qhalf), "ro")
ax1.grid()
ax1.legend(["pole1", "pole2", "$Q=\\frac{1}{2}$", "$Q=\\frac{1}{2}$"])
plt.show()
print(f"Q: {Q_Qhalf}")
print(f"Ib_Qhalf: {Ib_Qhalf}A")
```




```
Q: 0.4995121951219512
Ib_Qhalf: 4.096e-07A
```

Note that you might have made some small approximations that result in the $Q = 1/2$ poles not quite coming together on the real axis.

I.e. in the transfer function $H(s)$, maybe you dropped a constant term?

Or maybe to find the $Q = 1/2$ condition, you simplified by assuming $\tau_2 << \tau_1$?

(d) Let's now look at the transient response of a TI photoreceptor operating under such conditions. First let's define a small signal transient input photocurrent

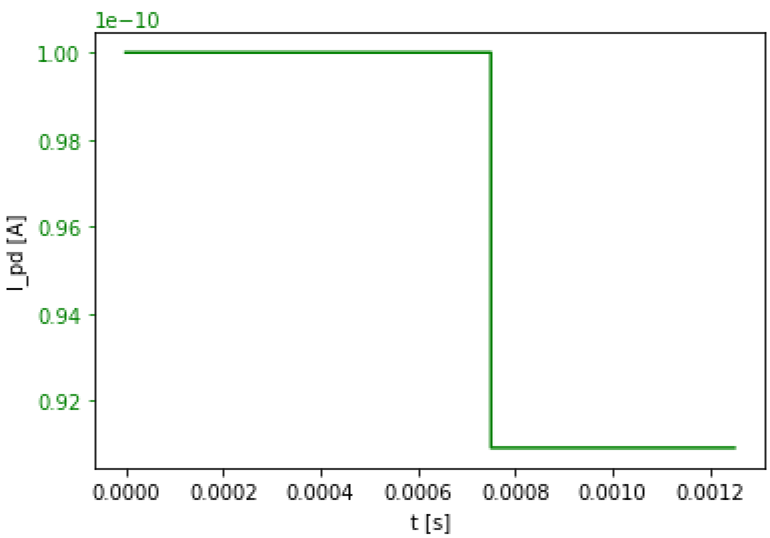
In [20]:

```
sigdc_ss=Ipd_bright
#sigdc=1e-15
signal_contrast_ss=1.1 # contrast in each half, i.e. cont=10 means that the bright part w
tau_ss=C_pd*U_T/sigdc_ss
t_warmup=tau_ss*30
t_total=t_warmup+tau_ss*20
Ipd_warmup=Ipd_bright
Ipd_final=Ipd_bright/signal_contrast_ss

def Ipd_step_func(t,I_t0=Ipd_warmup, I_t1=Ipd_final, t_warmup=t_warmup):
    if t<t_warmup:
        return I_t0
    else:
        return I_t1
```

```
dt_ss=tau_ss/1000
time_basis_ss=np.arange(0,t_total,dt_ss) # start,stop,step generate time basis that is npe
npts_ss=len(time_basis_ss)

# compute actual Ipd for timesteps
Ipd_ss=np.empty(npts_ss)
for t,i in zip(time_basis_ss,range(npts_ss)):
    Ipd_ss[i]=Ipd_step_func(t)
fig,ax1=plt.subplots(sharex=True)
ax1.plot(time_basis_ss,Ipd_ss,'g',)
#ax1.set_ylim([0,None])
ax1.set_yscale('linear')
ax1.set_xscale('linear')
ax1.tick_params(axis='y', colors='green')
ax1.set_xlabel('t [s]')
ax1.set_ylabel('I_pd [A]')
```

Out[20]:

```
Text(0, 0.5, 'I_pd [A]')
```



(e) Observe the transient response of the photoreceptor under such conditions. Does it behave as expected? Try also other values of Q, both above and below 0.5. Plot them in the root locus and observe the transient response. When is the system overdamped and when is it underdamped? Observe how the root locus trajectory changes with I_b.

In [21]:

```
# initial condition
V_ti0=ti_dc(Ib_Qhalf,Ipd_bright,I_0=I_0,V_e=V_e,U_T=U_T,kappa=kappa)
print(f'DC output of TI with Ib={eng(Ib_Qhalf)} and Ip={eng(Ipd_bright)} are vpd={eng(V_ti
ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='Radau',
                 args=(Ib_Qhalf,Ipd_step_func,V_e,C_pd,C_out))
# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qhalf=ti_sol.y[1]
t_ti_Qhalf=ti_sol.t
v_pd_Qhalf=ti_sol.y[0]
t_pd_Qhalf=ti_sol.t

ib_factor=2 # how much larger and smaller to try seeing how sensitive is the Q=1/2 condit

# now solve for a bit larger Ib
ib=Ib_Qhalf*ib_factor
V_ti0=ti_dc(ib,Ipd_bright,I_0=I_0,V_e=V_e,U_T=U_T,kappa=kappa)
print(f'DC output of TI with Ib={eng(ib)} and Ip={eng(Ipd_bright)} are vpd={eng(V_ti0[0])}
```

```
ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='Radau',
                 args=(ib,Ipd_step_func,V_e,C_pd,C_out))
# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qhalf1=ti_sol.y[1]
t_ti_Qhalf1=ti_sol.t
v_pd_Qhalf1=ti_sol.y[0]
t_pd_Qhalf1=ti_sol.t

# and solve for a bit smaller Ib
ib=Ib_Qhalf/ib_factor
V_ti0=ti_dc(ib,Ipd_bright,I_0=I_0,V_e=V_e,U_T=U_T,kappa=kappa)
print(f'DC output of TI with Ib={eng(ib)} and Ip={eng(Ipd_bright)} are vpd={eng(V_ti0[0])}
ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='Radau',
                 args=(ib,Ipd_step_func,V_e,C_pd,C_out))
# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qhalf2=ti_sol.y[1]
t_ti_Qhalf2=ti_sol.t
v_pd_Qhalf2=ti_sol.y[0]
t_pd_Qhalf2=ti_sol.t
```

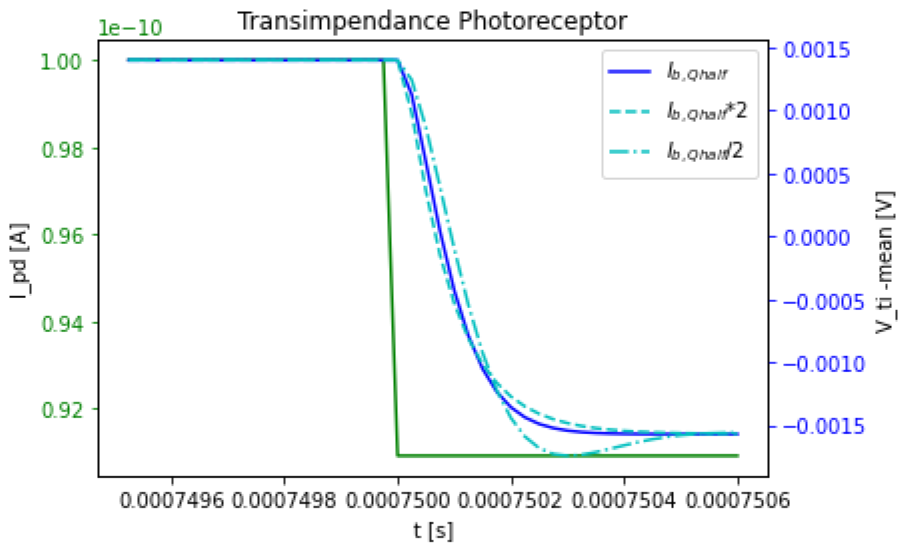
```
DC output of TI with Ib=409.60n and Ip=100p are vpd=547.75m vout=972.51m
The solver successfully reached the end of the integration interval.
DC output of TI with Ib=819.20n and Ip=100p are vpd=569.41m vout=999.59m
The solver successfully reached the end of the integration interval.
DC output of TI with Ib=204.80n and Ip=100p are vpd=526.09m vout=945.44m
The solver successfully reached the end of the integration interval.
```

In [153…

```
# use this plotting style to put several plots sharing same x-axis
# we will plot V_sf together with the input photocurrent
# using another axis since it is volts, not current, and linear not log

t_start = t_warmup-tau_ss/50
t_end = t_warmup+tau_ss/40
six=np.argmax(time_basis_ss>t_start)
eix=np.argmax(time_basis_ss>t_end)
r=range(six,eix) # range to plot, to eliminate startup transient

# because of transient, we need to limit the output voltage plotting range
# around the DC level before and after the step
fig=plt.figure(figsize=(12,8))
fig,ax1=plt.subplots(sharex=True)
# tlim=[t_start,t_end]
# lookup idx of time point just before step
# vlim=[v_ti_Qhalf2[len(time_basis_ss)//5],v_ti_Qhalf2[-1]*1.1]

ax1.plot(time_basis_ss[r],Ipd_ss[r],'g')
ax1.set_yscale('linear')
ax1.set_xscale('linear')
ax1.tick_params(axis='y', colors='green')
ax1.set_xlabel('t [s]')
ax1.set_ylabel('I_pd [A]')
# ax1.set_xlim(tlim)

ax2=ax1.twinx()
ax2.plot(t_ti_Qhalf[r],v_ti_Qhalf[r]-np.mean(v_ti_Qhalf[r]),'b-')
ax2.plot(t_ti_Qhalf1[r],v_ti_Qhalf1[r]-np.mean(v_ti_Qhalf1[r]),'c--')
ax2.plot(t_ti_Qhalf2[r],v_ti_Qhalf2[r]-np.mean(v_ti_Qhalf2[r]),'c-.')
# ax2.set_xlim(tlim)
ax2.set_ylabel('V_ti -mean [V]')
```

```
# ax2.set_ylim(vlim)
ax2.tick_params(axis='y', colors='blue')
ax2.legend(['$I_{b,Qhalf}$','$I_{b,Qhalf}$'+f'*{ib_factor}','$I_{b,Qhalf}$'+f'/{ib_factor}
ax2.title.set_text('Transimpendance Photoreceptor')
plt.show()
```

```
<Figure size 864x576 with 0 Axes>
```

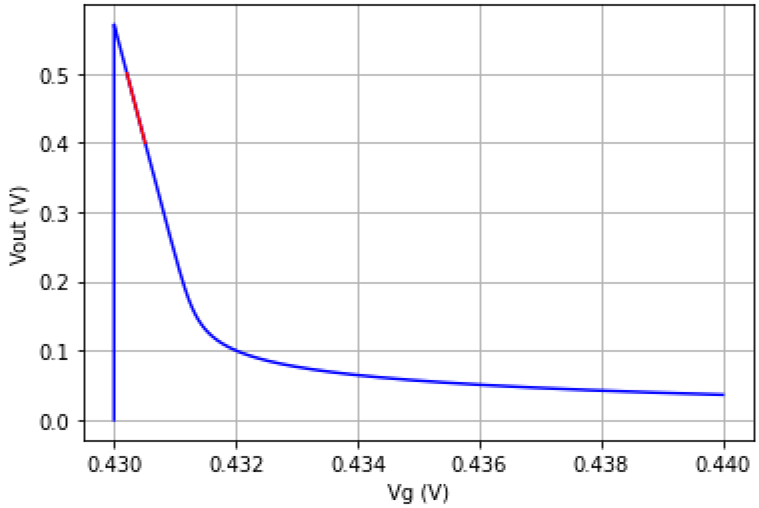


Let's simulate open loop amplifier with very slow ramp of input voltage to meausre the open loop voltage gain and compare it with theory

In [154…

```
tt=1
v0=.43
v1=.44
def ampdvdt(t,vd):
    vg=v0-(v0-v1)*t/tt
    id=id_sub(vg,0,vd,V_e=V_e,kappa=kappa)
    i=I_b-id
    vdot= i / 1e-15
    # print(f'vg={vg}V vd={vd}V id={id}A idiff={i}A')
    return vdot

t=np.linspace(0,tt,10000)
vd0=[0]
s=solve_ivp(ampdvdt,(t[0],t[-1]),
            vd0, t_eval=t, rtol=1e-9, atol=1e-12, method='Radau')
vd=s.y[0]
vg=v0-(v0-v1)*t/tt
plt.plot(vg,vd,'-b')
r=[i for i in range(len(vd)) if vd[i]> .4 and vd[i]<.5]
plt.xlabel('Vg (V)')
plt.ylabel('Vout (V)')
plt.plot(vg[r],vd[r],'r-')
plt.grid(True)
A_meas=linregress(vg[r],vd[r])[0]
A_pred=kappa*V_e/U_T
print(f'Amplifier gain: Measured {eng(A_meas)}, predicted from kappa*Ve/U_T={eng(A_pred)}'
```

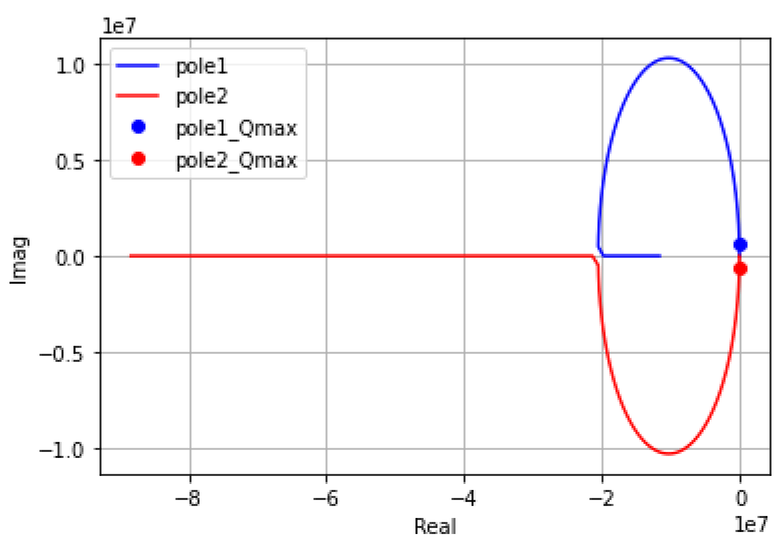
```
Amplifier gain: Measured -334.39, predicted from kappa*Ve/U_T=320
```



(f) If the poles are not purely real, the photoreceptor output will have a ringing behavior. The larger the Q, the more the system will ring. In the TI photoreceptor, Q is maximum when $\tau_{out} = \tau_{in}$. Find the value of I_b which results in maximum Q, and plot it on the root locus. Compare the value obtained for Q with the theoretical value.

In [26]:

```
# plot root locus
fig,ax1=plt.subplots(sharex=True)
ax1.plot(np.real(pole1),np.imag(pole1),'b')
ax1.plot(np.real(pole2),np.imag(pole2),'r')
ax1.set_xlabel('Real')
ax1.set_ylabel('Imag')

# TODO fill the expression with the value of Ib which maximizes Q
Ib_Qmax=Ipd_bright*V_e*C_out/(C_pd*U_T)
pole1_Qmax,pole2_Qmax,Q_Qmax=get_poles(Ipd=Ipd_bright,Ib=Ib_Qmax,C_pd=C_pd,C_out=C_out,V_e
ax1.plot(np.real(pole1_Qmax),np.imag(pole1_Qmax),'bo')
ax1.plot(np.real(pole2_Qmax),np.imag(pole2_Qmax),'ro')
ax1.legend(['pole1','pole2','pole1_Qmax','pole2_Qmax'])
ax1.grid()
plt.show()

print(f"Q: {Q_Qmax}")
print(f"Ib: {Ib_Qmax}")
```




```
Q: 8.0
Ib: 3.9999999999999996e-10
```

(g) Observe the transient response of the photoreceptor in such conditions. Does it behave as expected? Again, try other values of Q, both above and below the maximum. Plot them in the root locus and observe the transient response and how Q affects ringing.

In [23]:

```
# initial condition
V_ti0=ti_dc(Ib_Qmax,Ipd_bright,I_0=I_0,V_e=V_e,U_T=U_T,kappa=kappa)
ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='Radau',
                 args=(Ib_Qmax,Ipd_step_func,V_e,C_pd,C_out))
t_start = t_warmup-tau_ss/30
t_end = t_warmup+tau_ss*2
six=np.argmax(time_basis_ss>t_start)
eix=np.argmax(time_basis_ss>t_end)
r=range(six,eix) # range to plot, to eliminate startup transient

# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qmax=ti_sol.y[1][r]
t_ti_Qmax=ti_sol.t[r]
v_pd_Qmax=ti_sol.y[0][r]
t_pd_Qmax=ti_sol.t[r]

ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='LSODA',
                 args=(Ib_Qmax*2,Ipd_step_func,V_e,C_pd,C_out))
# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qmax1=ti_sol.y[1][r]
t_ti_Qmax1=ti_sol.t[r]
v_pd_Qmax1=ti_sol.y[0][r]
t_pd_Qmax1=ti_sol.t[r]

ti_sol=solve_ivp(tidvdt, (time_basis_ss[0],time_basis_ss[-1]),
                 V_ti0, t_eval=time_basis_ss, rtol=1e-9, atol=1e-19, method='LSODA',
                 args=(Ib_Qmax/2,Ipd_step_func,V_e,C_pd,C_out))
# output is sol.t and sol.y
if ti_sol.message is not None:
    print(ti_sol.message)
v_ti_Qmax2=ti_sol.y[1][r]
t_ti_Qmax2=ti_sol.t[r]
v_pd_Qmax2=ti_sol.y[0][r]
t_pd_Qmax2=ti_sol.t[r]
```

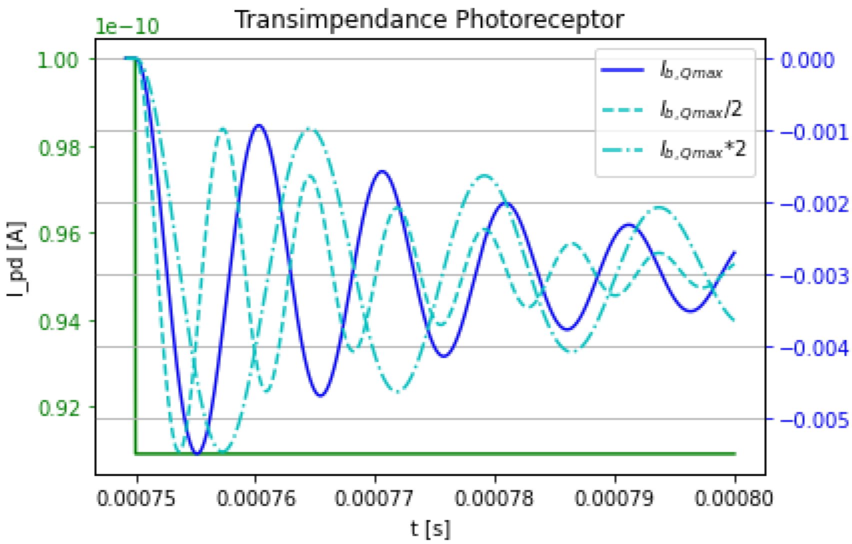
```
The solver successfully reached the end of the integration interval.
The solver successfully reached the end of the integration interval.
The solver successfully reached the end of the integration interval.
```

In [25]:

```
# use this plotting style to put several plots sharing same x-axis
# we will plot V_sf together with the input photocurrent
# using another axis since it is volts, not current, and linear not log
fig=plt.figure(figsize=(12,8))
fig,ax1=plt.subplots(sharex=True)


ax1.plot(time_basis_ss[r],Ipd_ss[r],'g')
ax1.set_yscale('linear')
ax1.set_xscale('linear')
ax1.tick_params(axis='y', colors='green')
ax1.set_xlabel('t [s]')
ax1.set_ylabel('I_pd [A]')

ax2=ax1.twinx()
#ax2.plot(t_ti_Qmax,v_ti_Qmax,'b-')
ax2.plot(t_ti_Qmax,v_ti_Qmax-v_ti_Qmax[0],'b-')
```

```
ax2.plot(t_ti_Qmax1,v_ti_Qmax1-v_ti_Qmax1[0],'c--')
ax2.plot(t_ti_Qmax2,v_ti_Qmax2-v_ti_Qmax2[0],'c-.')
#ax2.plot(t_pd,v_pd,'m-')
ax2.tick_params(axis='y', colors='blue')
ax2.legend(['$I_{b,Qmax}$','$I_{b,Qmax}$/2','$I_{b,Qmax}$*2'], loc='upper right')
ax2.title.set_text('Transimpendance Photoreceptor')
ax2.title.set_text('Transimpendance Photoreceptor')
ax2.grid()
plt.show()
```

```
<Figure size 864x576 with 0 Axes>
```



> By comparing the amplitudes at each peak, we see that the solid line has the highest ringing.