Calculus and Linear Algebra Workshop Notes and Problems - Basics of Derivatives and Differentiation

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1 Integration

Like the derivative, the definite integral is defined as a limit.

Definition 1. *Let f be a function defined on the interval $[a, b]$. We define the definite integral of f on $[a, b]$ as:*

$$\int_a^b f(x)dx = \lim_{\max(\Delta x_k)\to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

where $a = x_1 \leq x_2 \leq ... \leq x_n = b$ is a partition of the interval $[a, b]$, $\Delta_k = x_{k+1} - x_k$ for $k = 1, ..., n-1$ and $x_k \leq x_k^* \leq x_{k+1}$.

Each term in the sum above is the area of a rectangle, with height $f(x_k^*)$ and width $x_{k+1} - x_k$. In the limit, we make the width of the rectangles approach zero. The quantity we calculate is the area under the curve of the function $f(x)$ on the interval $[a, b]$.

1.1 Computing Definite Integrals