University of St.Gallen

EFMD EQUIS ACCREDITED

Methods: Statistics (4,120)

10. Two-Sample Procedure

Spring 2022

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Learning Objectives

After this lecture, you know how:

- Hypothesis tests are used to compare the means and proportions of **two dependent or independent samples.**
- Confidence intervals around the difference of two sample means or proportions are formed and interpreted.
- an *F-test* is applied and interpreted for the **comparison of two variances.**

Literature

Levine, D.M., K. A. Szabat, and D.F. Stephan. (2016). *Business Statistics: A First Course*, 7th ed. United States: Pearson, **Chapter 10**.*

Stinerock, R. (2018). *Statistics with R*. United Kingdom: Sage. **Chapter 11**.*

Shira, Joseph (2012). *Statistische Methoden der VWL und BWL*, 4th ed. Munich et al.: Pearson Studium, **Chapter 15**.

Weiers, R. M. (2011). *Introductory Business Statistics*,7th ed., Canada: Thomson South-Western, **Chapter 11**.

*Mandatory literature

I. Introduction Example

Cola Zero vs. Original:

In last week's blind test, students always had to taste *Cola Zero* first and were asked to try the *Original* afterwards. This procedure may have biased the results. Therefore, 40 new students are recruited and randomly devided into two groups. One group tastes and evaluates *Cola Zero*, while the other group tastes and rates the *Original* (on a 7-point scale from 1 = tastes bad to 7 = tastes great).

The test results show that the «*Cola Zero*-Sample» has a mean value of 5.2, while the «*Original*-Sample» has a mean value of 5.9. **Can you now conclude that *Cola Zero* does *not* taste as good as the original?**

I. Basic Idea

The introductory example aims to distinguish between **two unknown means** (up to now, we only tested statements about single parameters). In order to investigate such an assumption, appropriate samples must be taken from the population.

Samples are called **independent** if the selection of one sample does not affect the selection of the other sample. In contrast, one refers to **dependent samples** (or paired samples) if the selection process for one sample is related to the selection process for the other sample. For instance, samples are dependent if the observations of the first and second sample are taken from the same statistical unit (e.g., «before-after» comparisons).

Depending on the sample at hand (independent vs. dependent) one has to use **different test procedures** to examine differences between samples.

II. Comparing Means (Independent Samples)

There are three different methods for testing means of two **independent** samples:

- *z*-test
- *t*-test for equal variances
- *t*-test for unequal variances

The choice of the appropriate method depends on whether the population variances are **known/unknown** and whether they are **equal/different**. In addition, two-sample procedures are carried out in the same five steps as the hypothesis tests covered so far:

1. Hypotheses and significance level
2. Test statistic and test distribution
3. Critical value(s)
4. Calculation of the test statistic
5. Decision and interpretation

II. Comparing Means (Independent Samples)

Case 1: σ_1^2 and σ_2^2 known and equal!

A *z*-test is used if both independent samples originate from a **normally distributed** population. Further, variances are **known** and **variance homogeneity** is given.

Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma^2 \left(1/n_1 + 1/n_2\right)}}$$

with $\bar{x}_1, \bar{x}_2$ = means of sample 1 and 2

$(\mu_1 - \mu_2)_0$ = hypothesized difference between population means

σ^2 = population variance

n_1, n_2 = size of sample 1 and 2

II. Comparing Means (Independent Samples)

Case 2: σ_1^2 and σ_2^2 known and different!

A *z*-test is used when both independent samples originate from a **normally distributed** population but the variances are **known** and **different**.

Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$$

with $\bar{x}_1, \bar{x}_2$ = means of sample 1 and 2

$(\mu_1 - \mu_2)_0$ = hypothesized difference between the population means

σ_1^2, σ_2^2 = variances of population 1 and 2

n_1, n_2 = size of sample 1 and 2

II. Comparing Means (Independent Samples)

Case 3: Randomly distributed populations!

A *z*-test is used if both independent samples originate from an arbitrarily-distributed population and the samples are large enough ($n_1 \geq 30$ and $n_2 \geq 30$).

Test statistic:

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\hat{\sigma}_1^2 / n_1 + \hat{\sigma}_2^2 / n_2}}$$

$$\text{with } \hat{\sigma}_1 = \begin{cases} \sigma_1, & \text{if known } (\sigma_1 = \text{standard deviation of population 1}) \\ s_1, & \text{otherwise } (s_1 = \text{standard deviation of sample 1}) \end{cases}$$

$$\text{with } \hat{\sigma}_2 = \begin{cases} \sigma_2, & \text{if known } (\sigma_2 = \text{standard deviation of population 2}) \\ s_2, & \text{otherwise } (s_2 = \text{standard deviation of sample 2}) \end{cases}$$

II. Comparing Means (Independent Samples)

Case 4: σ_1^2 and σ_2^2 unknown and equal!

A *t*-test is used when both independent samples originate from a **normally distributed** population but the variances are **unknown.** However, it can be assumed that **variance homogeneity** prevails.

Test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_p^2 \left(1/n_1 + 1/n_2\right)}}$$

with $\bar{x}_1, \bar{x}_2$ = means of sample 1 and 2

$(\mu_1 - \mu_2)_0$ = hypothesized difference between population means

s_p^2 = estimation function for the population variance (see next slide)

n_1, n_2 = size of sample 1 and 2

II. Comparing Means (Independent Samples)

Estimate function for the population variance:

The estimation function for the population variance in the case of homogeneity of variances is called **«pooled variance»**. The *t*-test for equal variances is therefore also referred to as pooled-variances *t*-test.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

The above estimation function is determined by the corresponding variances and sizes of the two samples as well as the number of degrees of freedom:

$$df = n_1 + n_2 - 2$$

II. Comparing Means (Pooled-variances *t*-Test: Example)

Tutorial groups:

A lecturer randomly assigns 50 students to one of two equally-sized tutorial groups of two different tutors. After the exam, the lecturer computes the average number of points as well as the corresponding standard deviations:

Group 1: $\bar{x}_1 = 77.1$, $s_1 = 7.8$, $n_1 = 25$

Group 2: $\bar{x}_2 = 80.0$, $s_2 = 8.1$, $n_2 = 25$

Can a significant difference between the groups be detected under the assumption of a normally distributed population and variance homogeneity ($\alpha = 0.05$)?

1. **Hypotheses and significance levels:**

   H_0: $\mu_1 – \mu_2 = 0$; H_1: $\mu_1 - \mu_2 \neq 0$; $\alpha = 0.05$

II. Comparing Means (Pooled-variances *t-Test*: Example)

2. **Test statistic and test distribution:**

   σ^2 is unknown → *t*-distribution with 25 + 25 - 2 = 48 degrees of freedom

3. **Critical value(s):**

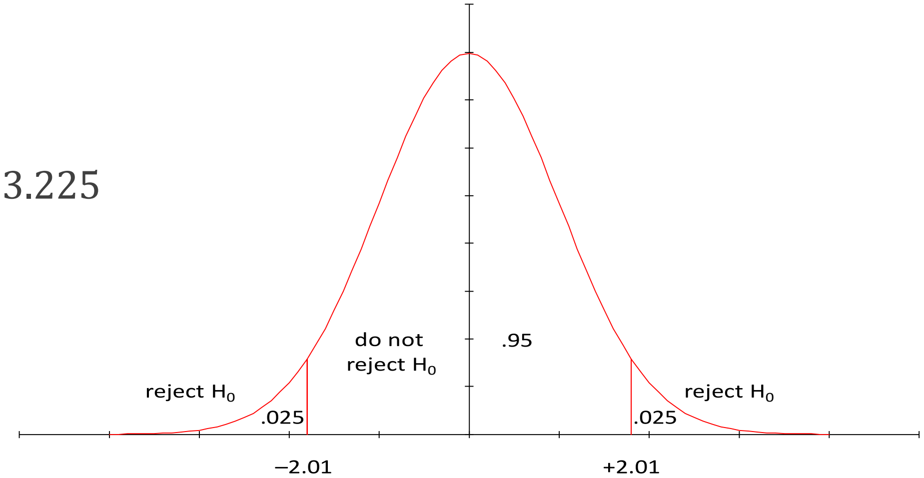
   Two-tailed hypothesis test with = 0.05 → $t_{\alpha/2}$ = ± 2.011

```
> qt(0.025,48)
[1] -2.010635
> qt(0.975,48)
[1] 2.010635
```

4. **Calculation of the test statistic:**

$$s_p^2 = \frac{24(7.8)^2 + 24(8.1)^2}{25 + 25 - 2} = \frac{1460.16 + 1564.64}{48} = 63.225$$

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{77.1 - 80}{\sqrt{63.225\left(\frac{1}{25} + \frac{1}{25}\right)}} = -1.289$$



3. **Decision and interpretation:**

   The test statistic *t* = -1,289 lies within $t_{\alpha/2}$ = ± 2,011. H_0 **cannot be rejected** at a significance level of 5%. No difference between the groups can be proven.

II. Comparing Means (Independent Samples)

Case 5: σ_1^2 and σ_2^2 unknown and different!

A *t*-test is used when both independent samples originate from a **normally distributed** population. Moreover, the variances are **unknown** and it can be assumed that **variance heterogeneity** prevails.

Test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{s_1^2 / n_1 + s_2^2 / n_2}}$$

with $\bar{x}_1, \bar{x}_2$ = means of sample 1 and 2

$(\mu_1 - \mu_2)_0$ = hypothesized difference between population means

s_1, s_2 = estimation of the standard deviation of sample 1 and 2

n_1, n_2 = size of sample 1 and 2

II. Comparing Means (Independent Samples)

Calculation of the degrees of freedom:

For the *t*-test for unequal variances the **degrees of freedom** of the *t*-distribution are calculated via the following formula:

$$df = \frac{\left(s_1^2 / n_1 + s_2^2 / n_2\right)^2}{\dfrac{(s_1^2 / n_1)^2}{n_1 - 1} + \dfrac{(s_2^2 / n_2)^2}{n_2 - 1}}$$

This formula is known as **«Welch correction»**, which is the reason why the *t*-test for unequal variances is also referred to as Welch's *t*-test. A non-integer value of *df* is rounded up or down.

II. Comparing Means (Welch correction: Example)

Sampling survey:

An analysis of two independent samples with normally-distributed populations and unequal variances ($\sigma_1^2 \neq \sigma_2^2$) resulted in the following values:

$$\bar{x}_1 = 120, \quad s_1 = 16, \quad n_1 = 25$$
$$\bar{x}_2 = 114, \quad s_2 = 12, \quad n_2 = 22$$

What is the appropriate number of degrees of freedom when applying the corresponding *t*-test?

$$df = \frac{\left[(s_1^2/n_1)+(s_2^2/n_2)\right]^2}{\dfrac{(s_1^2/n_1)^2}{n_1-1}+\dfrac{(s_2^2/n_2)^2}{n_2-1}} = \frac{\left[(16^2/25)+(12^2/22)\right]^2}{\dfrac{(16^2/25)^2}{24}+\dfrac{(12^2/22)^2}{21}} = 43.96$$

A *t-test* for unequal variances is used with **44 degrees of freedom**.

III. Comparing Means (Welch test: R-Example 1)

Open the file "L10-Example_1.R" in R-Studio and reproduce the R-Code.

```r
# example: rent comparison
#---------------------------------------
# the Portuguese resorts of Algarve and Cascais are very popular with tourists.
# so a travel agency would like to find out if the monthly rents for
# standard apartments in the locations differ significantly (alpha = 5%)

# import the CSV-file "Holidays".
holidays = read.csv("holidays.csv", header = TRUE)

# quick inspection of the new dataset:
head(holidays, 3)

# performing the Welch test under the assumption of unequal variances:
t.test(holidays$algarve, holidays$cascais, conf.level = 0.95)

# performing the Two Sample t-test of the assumption of equal variances:
t.test(holidays$algarve, holidays$cascais, conf.level = 0.95, var.equal = TRUE)

# how does assuming equal variances change the result and precision of the test?
```

III. Comparing Mean (Dependent Samples, Paired *t*-Test)

Two **dependent samples** are available for this test. **However,** the different **pairs of measured values are independent of each other** (e.g., pair 1 and pair 2 do not influence each other). The differences of the associated test values are normally distributed in the population.

Test statistic: $t = \dfrac{\bar{d}}{\dfrac{s_d}{\sqrt{n}}}$ $\quad t - distribution,\ df = n - 1$

with

d = difference between measurements of each individual unit: $x_1 - x_2$

$\bar{d}$ = average difference between measurements: $\dfrac{\sum d_i}{n}$

s_d = standard deviation of *d*: $\sqrt{\dfrac{\sum d_i^2 - n\bar{d}^2}{n-1}}$

n = sample size (number of pairs)

df = number of degrees of freedom: $n - 1$

III. Comparing Means (Paired *t*-Test: Example)

Keyboards:

A professor orders 12 ergonomic keyboards, which are given to a sample of doctoral students. If the new keyboards significantly improve productivity, the professor wants to replace all old keyboards.

For this reason, all 12 doctoral students have to type a standardized document - once with the old keyboard and once with the ergonomic keyboard. The number of words per minute is measured.

It turns out that the writing speed between the first and second measurement differs by -78.7 words per minute. Should the professor order the ergonomic keyboard for all doctoral students (α = 0.05)?

III. Comparing Means (Paired t-Test: Example)

Measurement overview:

Person	x_1, Words/Minute with Old Keyboard	x_2, Words/Minute with New Keyboard	Difference $d = (x_1 - x_2)$	d^2
1	25.5	43.6	−18.1	327.61
2	59.2	69.9	−10.7	114.49
3	38.4	39.8	−1.4	1.96
4	66.8	73.4	−6.6	43.56
5	44.9	50.2	−5.3	28.09
6	47.4	53.9	−6.5	42.25
7	41.6	40.3	1.3	1.69
8	48.9	58.0	−9.1	82.81
9	60.7	66.9	−6.2	38.44
10	41.0	66.5	−25.5	650.25
11	36.1	27.4	8.7	75.69
12	34.4	33.7	0.7	0.49
			−78.7 $= \sum d$	1407.33 $= \sum d^2$

1. **Hypotheses and significance levels:**

   H_0: $\mu_d \geq 0$; H_1: $\mu_d < 0$; $\alpha = 0.05$

III. Comparing Means (Paired *t-T*est: Example)

2. Test statistic and test distribution:

Variances are unknown → *t-distribution* with 12 - 1 = 11 degrees of freedom

3. Critical value(s):

One-tailed hypothesis test with α = 0.05 → t_α = - 1.796

```
> qt(0.05,11)
[1] -1.795885
```

4. Calculation of the test statistic:

$$\bar{d} = \frac{\sum d_i}{n} = \frac{-78.7}{12} = -6.558$$

$$s_d = \sqrt{\frac{\sum d_i^2 - n\bar{d}^2}{n-1}} = \sqrt{\frac{1407.33 - 12(-6.558)^2}{12-1}} = 9.001$$

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n}}} = \frac{-6.558}{\frac{9.001}{\sqrt{12}}} = -2.524$$

3. Decision and interpretation:

As the test statistic t = -2,524 is smaller than the critical value, the null hypothesis can be **rejected** at a significance level of 5%. An ergonomic keyboard should be ordered for all doctoral students.

IV. Confidence Interval for the Difference of Means

Instead of or in addition to the paired *t*-test, a confidence interval can be calculated for the difference between two means. This results from the following formula:

Confidence interval: $$\bar{d} \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

with d = difference between measurements of each individual unit: $x_1 - x_2$

$\bar{d}$ = average difference between measurements: $\frac{\sum d_i}{n}$

s_d = standard deviation of *d*: $\sqrt{\frac{\sum d_i^2 - n\bar{d}^2}{n-1}}$

n = sample size (number of pairs)

$t_{\alpha/2}$ = upper critical value of the *t*-distribution with *n - 1* degrees of freedom

V. Comparing Proportions of Independent Samples

The underlying binomial distribution can be approximated by the normal distribution if all of the following conditions are met:

$n_1 \cdot p_1 \geq 5$ | $n_2 \cdot p_2 \geq 5$

$n_1 \cdot (1 - p_1) \geq 5$ | $n_2 \cdot (1 - p_2) \geq 5$

Test statistic:

$$z = \frac{(p_1 - p_2)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad \overline{p} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

with p_1, p_2 = proportion of sample 1 and 2

n_1, n_2 = size of sample 1 and 2

$\overline{p}$ = pooled estimation of the population proportion

V. Comparing Proportions (R-Example 2)

Open the file "10-Example_2.R" in R-Studio and reproduce the R-Code.

```
# example: presidential election
#---------------------------------------
# in the 2012 US presidential election, 749 of 1337 women voted for Obama in a random sample.
# in a sample of 1214 men, 558 voted for Obama. Can we use these samples to determine
# a different voting behaviour of the sexes at a significance level of 5% ?

# defining the samples:
obama <- c(749, 558)
total <- c(1337, 1214)

# disable scientific notation
options(scipen = 999)

# testing the proportion differences
# H0: p1- p2 = 0
prop.test(obama, total, conf.level = 0.95, correct = FALSE)
```

VI. Confidence Interval for the Difference of Proportions

Instead of or in addition to an hypothesis test, a confidence interval can be calculated for the difference between two proportions. This interval results from the following formula:

Confidence interval: $$(p_1 - p_2) \pm z_{\alpha/2}\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

with $p_1,\ p_2$ = proportion of sample 1 and 2

$n_1,\ n_2$ = size of sample 1 and 2

$z_{\alpha/2}$ = upper critical value of the normal distribution

VII. Variance Test

In some situations, the question arises whether two independent samples have the same variances (H_0: $\sigma_1^2 = \sigma_2^2$). The prerequisites for this test are two independent samples from normally distributed populations with the variances σ_1^2 and σ_2^2.

Test statistic: $F = \frac{s_1^2}{s_2^2} \; oder \; \frac{s_2^2}{s_1^2}$, where the larger quotient is selected.

The critical values are determined by the **quantile $F(\alpha/2, \nu_1, \nu_2)$**:

- α = the specified significance level
- $\nu_1 = (n_1 - 1)$, the number of degrees of freedom in the counter
- $\nu_2 = (n_2 - 1)$, the number of degrees of freedom in the denominator

As the test statistic *F* follows an *F*-distribution, the variance test is called an ***F*-test**.

VII. Variance Test (Example)

(continued)

v_1 = *df*, numerator

v_2 = *df*, denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

VII. Variance Test (Example)

Tutorial groups:

For the previous example of two tutorial groups, it should be tested whether the assumption of variance homogeneity was justified ($\alpha = 0.05$).

1. **Hypotheses and significance levels:**

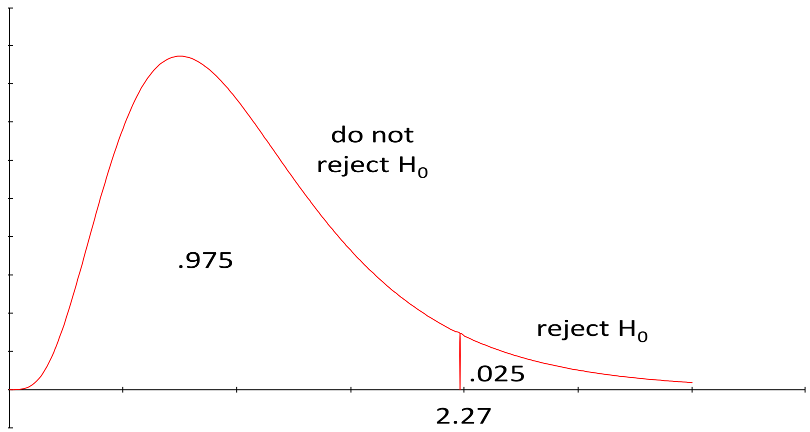
   H_0: $\sigma_1^2 = \sigma_2^2$; H_1: $\sigma_1^2 \neq \sigma_2^2$; $\alpha = 0.05$

2. **Test statistic and test distribution:**

   F-statistics with *df* = 24 in the numerator and *df* =24 in the denominator

3. **Critical value(s):**

   For this two-tailed test there is only one critical value, as the greater of the of both quotients is chosen.



= 0.05 → $F_{\alpha/2}$ = 2.27

```
> qf(0.975,24,24)
[1] 2.269277
```

VII. Variance Test (Example)

3. **Calculation of the test statistic:**

$$F = \frac{s_2^2}{s_1^2} = \frac{(8.1)^2}{(7.8)^2} = 1.0784$$

3. **Decision and interpretation:**

   As the test statistic $F = 1.08$ is smaller than the critical value $F_{\alpha/2} = 2.27$, the null hypothesis **cannot be rejected** at a significance level of 5%. Therefore, the assumption of equal variances in the previous example was justified.