Module 4 Assignment | Project: A Prescriptive Model for Strategic Decision-making, An Inventory Management Decision Model

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Introduction

Inventories are a significant investment for organizations, making proper management crucial. Excessive inventory indicates poor financial and operational control, while insufficient inventory can lead to business failure. Managers face two key inventory decisions: determining how much to order or produce in each interval and when to do so to minimize total inventory costs. These costs include holding costs, such as capital tied up, storage expenses, and potential losses, plus ordering costs, which involve expenses related to replenishing inventory. For this manufacturing company, inventory decisions for a key engine component are uncertain. The annual demand is 15,000 units with a constant cost of $80 per unit. Holding costs are estimated at 18% of the unit value, and each order costs $220. The company follows a reorder policy based on predetermined levels that "provide sufficient stock to meet demand until the supplier's order can be shipped and received; and then to order twice as many units." Effective inventory management is crucial to balance costs and ensure optimal stock levels for smooth operations. This analysis explores the tradeoff in inventory management, aiming to minimize the total cost by running a prescriptive model in determining the timing of orders and the order quantity. It considers various scenarios and examines the impact of different model parameters on cost-reduction strategies.

Analysis and Interpretation

Part I:

1. Define the data, uncontrollable inputs, model parameters, and the decision variables that influence the total inventory cost.
    - Model Parameters: Unit cost; Holding cost rate/ unit/ year; Ordering cost/ order.
    - Uncontrollable variable: Annual demand.
    - Decision (controllable) variable: *Order quantity*.

2. Develop mathematical functions:

   - Holding cost/ unit/ year = Holding cost rate/ unit/ year * Unit cost
   - Number of orders per year $= \frac{\text{Annual demand}}{Order\ quantity}$
   - Average inventory (re-order point) = $\frac{Order\ quantity}{2}$ (as per the company's policy)
   - Annual ordering cost = Number of orders per year * Ordering cost
   - Annual holding cost = Average inventory (re-order point) * Holding cost/ unit/ year

3. Implement the model:

   - Total inventory cost = Annual ordering cost + Annual holding cost

   $= \frac{\text{Annual demand}}{Order\ quantity}$ * Ordering cost + $\frac{Order\ quantity}{2}$ * Holding cost rate/ unit/ year * Unit cost

   - Model objective: minimize the total cost.

4. The one-way data table estimating the approximate order quantity that results in the smallest total cost is shown in Table 1 below.

Table 1: Estimation of the order quantity using data table

Order Quantity	**Total cost**
300	13,160
350	11,949
400	11,130
450	10,573
500	10,200
550	9,960
600	9,820
650	9,757
700	9,754
750	9,800
800	9,885
850	10,002
900	10,147
950	10,314
1000	10,500

5. We plot the Total Cost versus the Order Quantity as follows:

Figure 1: Total Cost and Order Quantity Plot

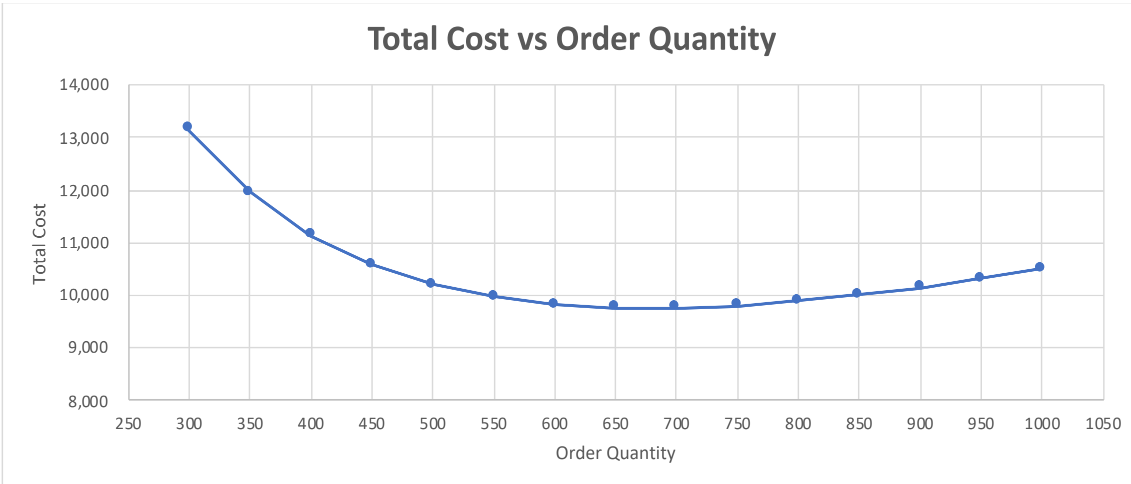


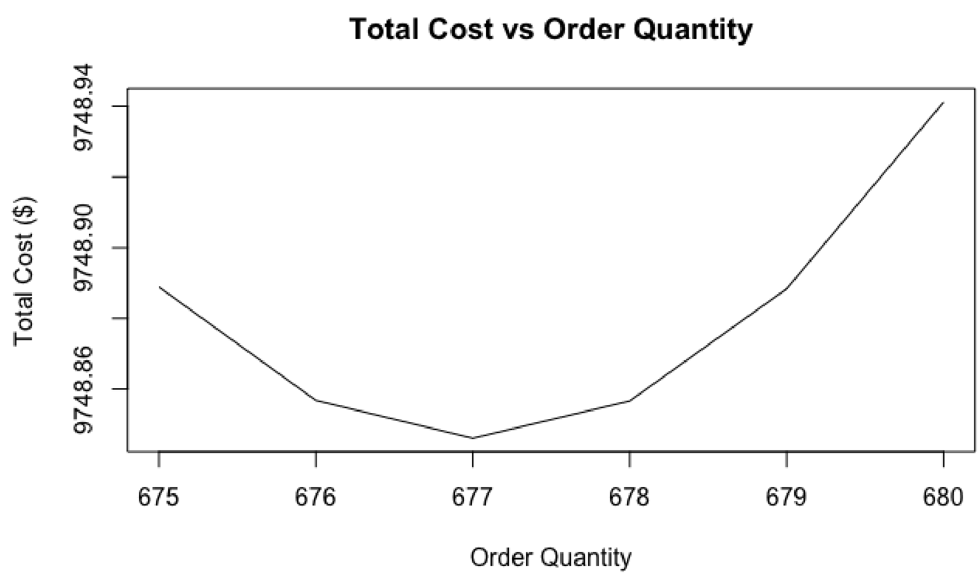
Figure 1 illustrates the nonlinear relationship between the variables of total cost and order quantity. It is observed that the optimal order quantity appears to fall within the range of 650 to 700, rather than precisely at 700 as indicated in Table 1. To further validate this result, we will continue utilizing the Excel Solver in the subsequent analysis.

6. The table below presents the results obtained from utilizing the Excel Solver to determine the precise order quantity that minimizes the total cost. Additionally, Figure 2 provides a zoom-in plot highlighting the optimal order quantity at that specific point.

Table 2: Finding exact order quantity using Excel Solver

Excel Solver	
Order quantity	677
Total cost	9,749

Figure 2: Zoom-in plot using R



7. In this part, we develop the sensitivity analysis of the total cost along with changes in the model parameters. Table 3 displays the results of a two-way what-if analysis, examining the impact of varying the inventory holding rate from 10% to 24% and the annual demand from $10,000 to $25,000. The analysis provides insights into how different combinations of these parameters affect the overall inventory management strategy, specifically the total cost.

Table 3: Two-way table of sensitivity analysis

Sensitivity Analysis									
		Holding cost rate							
	$ 9,749	10%	12%	14%	16%	18%	20%	22%	24%
Annual demand	10,000	$ 5,958	$ 6,499	$ 7,041	$ 7,582	$ 8,124	$ 8,666	$ 9,207	$ 9,749
	11,000	$ 6,283	$ 6,824	$ 7,366	$ 7,907	$ 8,449	$ 8,991	$ 9,532	$ 10,074
	12,000	$ 6,608	$ 7,149	$ 7,691	$ 8,232	$ 8,774	$ 9,316	$ 9,857	$ 10,399
	13,000	$ 6,933	$ 7,474	$ 8,016	$ 8,557	$ 9,099	$ 9,641	$ 10,182	$ 10,724
	14,000	$ 7,257	$ 7,799	$ 8,341	$ 8,882	$ 9,424	$ 9,965	$ 10,507	$ 11,049
	15,000	$ 7,582	$ 8,124	$ 8,666	$ 9,207	$ 9,749	$ 10,290	$ 10,832	$ 11,374
	16,000	$ 7,907	$ 8,449	$ 8,991	$ 9,532	$ 10,074	$ 10,615	$ 11,157	$ 11,699
	17,000	$ 8,232	$ 8,774	$ 9,316	$ 9,857	$ 10,399	$ 10,940	$ 11,482	$ 12,024
	18,000	$ 8,557	$ 9,099	$ 9,641	$ 10,182	$ 10,724	$ 11,265	$ 11,807	$ 12,349
	19,000	$ 8,882	$ 9,424	$ 9,965	$ 10,507	$ 11,049	$ 11,590	$ 12,132	$ 12,673
	20,000	$ 9,207	$ 9,749	$ 10,290	$ 10,832	$ 11,374	$ 11,915	$ 12,457	$ 12,998
	21,000	$ 9,532	$ 10,074	$ 10,615	$ 11,157	$ 11,699	$ 12,240	$ 12,782	$ 13,323
	22,000	$ 9,857	$ 10,399	$ 10,940	$ 11,482	$ 12,024	$ 12,565	$ 13,107	$ 13,648
	23,000	$ 10,182	$ 10,724	$ 11,265	$ 11,807	$ 12,349	$ 12,890	$ 13,432	$ 13,973
	24,000	$ 10,507	$ 11,049	$ 11,590	$ 12,132	$ 12,673	$ 13,215	$ 13,757	$ 14,298
	25,000	$ 10,832	$ 11,374	$ 11,915	$ 12,457	$ 12,998	$ 13,540	$ 14,082	$ 14,623

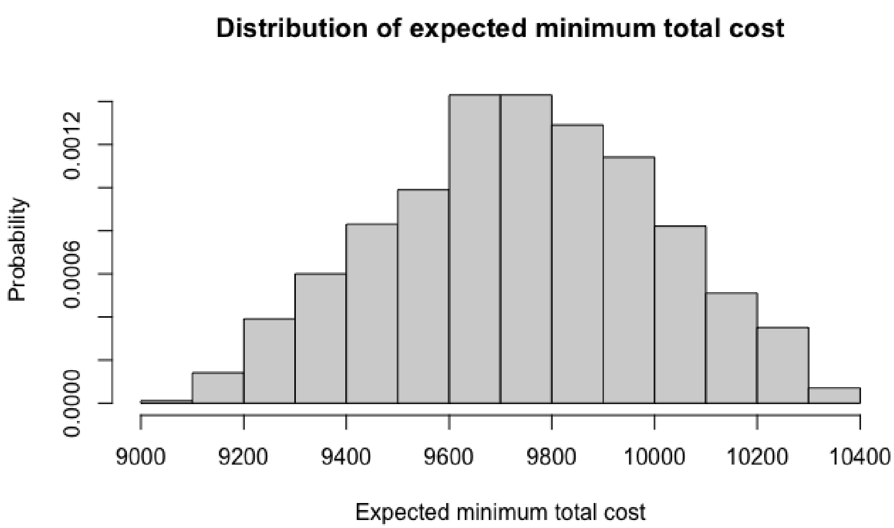
Part II:

For the annual demand, which follows a triangular probability distribution ranging from 13,000 to 17,000 units with a mode of 15,000 units, we will perform a Monte Carlo simulation comprising 1000 occurrences using [rtriangle] function and then calculate the minimum total cost for these occurrences by and adjusting the [optimize] function.

(i) The 95% confidence interval for the expected minimum total cost is: [$9726.369 - $9759.167]. The minimum total cost in Part I ($9749) also falls into this range.

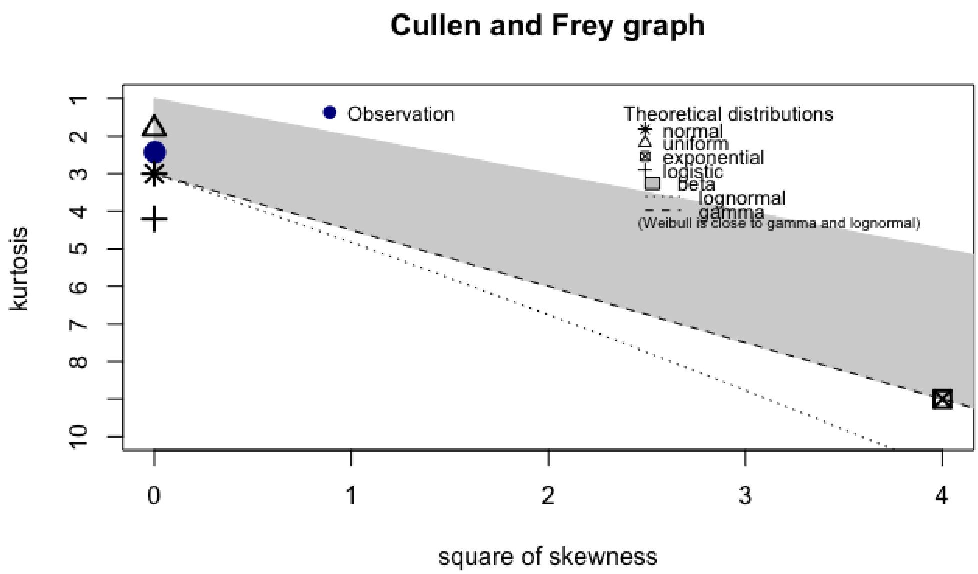
```
        One Sample t-test

data:  df$total_cost
t = 1165.9, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 9726.369 9759.167
sample estimates:
mean of x
 9742.768
```

Figure 3: Histogram of the expected minimum total cost



Besides, we use the function [descdist] to gain some ideas about possible candidate distributions:

Figure 4: Possible candidate distributions

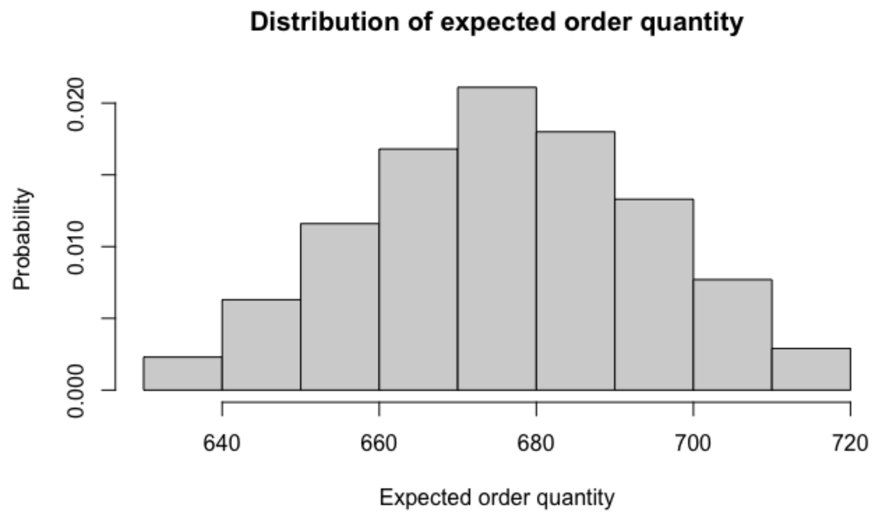


We wanted to test the normal frequency distribution, which appears to be the best fit for the total cost frequency distribution. After performing the chi-squared goodness of fit test, the p-value obtained is 0.03290265, less than the significant level of 0.05 (95% confidence level). So we reject the null hypothesis that the frequency distribution of the total cost is a normal frequency distribution. We do not have enough evidence to conclude it is a normal frequency distribution.

(ii) The 95% confidence interval for the expected order quantity is: [675.44 - 677.72]. The optimal order quantity in Part I (677) also falls into this range.

```
        One Sample t-test

data:  df$order_quantity
t = 1166.1, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 675.4405 677.7175
sample estimates:
mean of x
  676.579
```

Figure 5: Histogram of expected order quantity

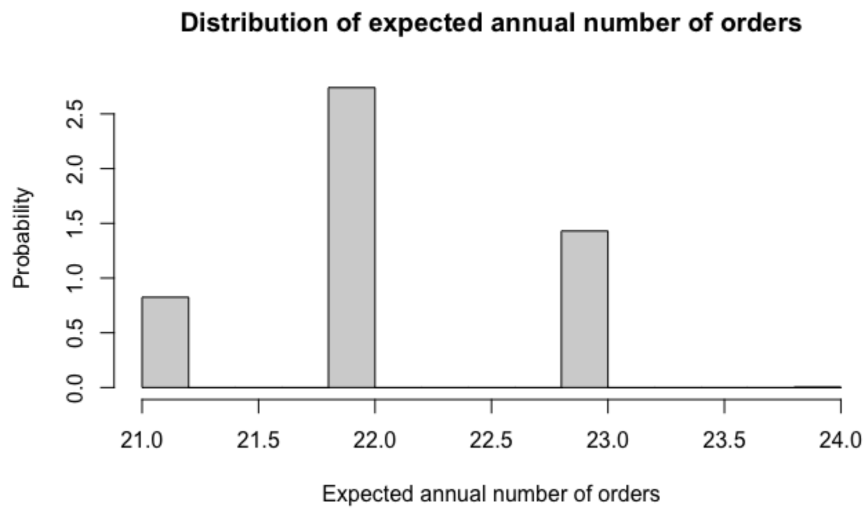


We wanted to test the Poisson frequency distribution since the order quantity is a discrete variable. After performing the chi-squared goodness of fit test, the p-value obtained is 0.02578235, less than the significant level of 0.05 (95% confidence level). So we reject the null hypothesis that the frequency distribution of the order quantity is a Poisson frequency distribution. We do not have enough evidence to conclude it is a Poisson frequency distribution.

(iii) The 95% confidence interval for the expected annual orders is: [22.08 - 22.16]. It doesn't make sense for the nature of this variable. We can presume it is similar to 22.

```
        One Sample t-test

data:  df$annual_orders
t = 1054.3, df = 999, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 22.08182 22.16418
sample estimates:
mean of x
   22.123
```

Figure 6: Histogram of expected annual orders



We wanted to test the Poisson frequency distribution since the annual orders is a discrete variable. After performing the chi-squared goodness of fit test, the p-value obtained is 0, less than the significant level of 0.05 (95% confidence level). So we reject the null hypothesis that the frequency distribution of the annual orders is a Poisson frequency distribution. We do not have enough evidence to conclude it is a Poisson frequency distribution.

Conclusion

In Part I, we analyzed the total inventory cost and its relationship with the order quantity variable. By formulating mathematical functions and implementing the model, we aimed to minimize the total cost. The analysis led to the identification of an optimal order quantity determined as 677 units with a total cost of $9749. Furthermore, we conducted a sensitivity analysis by exploring the impact of varying inventory holding rates and annual demand on the total cost. The two-way what-if analysis provided valuable insights into the interplay of these parameters and their influence on the overall inventory management strategy.

To determine the probability distributions that best fit the frequency distributions of the expected minimum total cost, expected order quantity, and expected annual orders in Part 2, we conducted chi-squared goodness of fit tests. The results indicated that their frequency distributions did not align with the assumed distributions (normal distribution and Poisson distribution, respectively). Further analysis and exploration may be required to identify the most suitable probability distributions for these variables.

References

1. Evans, J. R. (2013). Statistics, data analysis, and decision modeling. Pearson Education.

2. Eralda G. (January, 2021). Fitting Probability distribution in R. RPubs by Rstudio. https://rpubs.com/eraldagjika/715261

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