Module 6 Assignment | Project: Two Optimization Problems

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Introduction

In this analysis, we have employed non-linear programming methods to enhance the resolution of complex problems that go beyond the scope of linear programming. The objective of this report is to address two optimization problems: a transshipment problem and a risk-minimizing problem. Both of the problems have practical applications in the business and investment sectors, providing valuable insights into the utilization of non-linear models.

The first problem pertains to the Rockhill Shipping & Transport Company project, where the company offers shipping services for waste transfer to disposal sites. Our goal is to minimize the overall shipping cost by optimizing the shipping route and the quantity of waste picked up and transferred between the client's plants and waste sites. We explore two different strategic plans, with one plan involving the implementation of intermediate shipping points.

The second optimization problem focuses on identifying the best portfolio allocation structure, allowing investors to achieve a certain expected return while minimizing the associated risk. Furthermore, we aim to uncover the relationship between the minimized risk and the expected portfolio return in the later part of this analysis.

Analysis and Interpretation

Part I: Rockhill Shipping & Transport Company

Plan A: Ship directly from the plants to the waste sites.

Three constraints must be considered in this plan. Firstly, the total number of barrels from each plant must be equal to the total waste generated each week. Secondly, the total barrels shipped to each waste disposal site cannot exceed its maximum capacity. Lastly, all optimal values of decision variables need to be integers. By employing the [Solver] tool in Excel and minimizing the total shipping cost, we have determined the optimal number of barrels to be shipped from six plants to three waste sites, as shown in the table below. In this plan, the total cost amounts to $2988.

Figure 1: Plan A's optimal result

Number of barrels	Waste Proposal Site				
Plant:	*Orangeburg*	*Florence*	*Macon*	*Total barrels of each plant*	*Total cost*
Denver	40	0	5	45	$ 2,988
Morganton	0	0	26	26	
Morrisville	0	0	42	42	
Pineville	0	53	0	53	
Rockhill	25	0	4	29	
Statesville	0	27	11	38	
Total barrels of each site	65	80	88		
Max capacity	65	80	105		

Plan B: Use intermediate shipping points.

We have created two tables to illustrate two hops in the shipping route. Table 1 displays the number of waste barrels transported from all six plants to all locations, including plants and waste sites. Additionally, we have a corresponding table that lists the shipping cost per barrel for each route. Next, we present Table 2, which showcases the waste barrels transported from the previous waste-in locations to the three waste site destinations during Hop 2. The cost of each hop is computed in a similar manner, allowing us to obtain the total cost of Plan B.

This case introduces specific constraints, including the requirement that the number of waste barrels shipped from each location to the same dropping point must be equal to 0. Unfortunately, the total cost of implementing the intermediate shipping points plan amounts to $3656, which is significantly higher than that of Plan A. Consequently, we recommend Plan A for the shipping contract.

Figure 2: Plan B's optimal result

Hop 1		Waste in											Cost 1
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Total	Waste generated	
Waste out	Denver	0	44	0	0	0	0	0	0	1	45	45	1395
	Morganton	0	0	25	0	0	0	0	0	1	26	26	
	Morrisville	0	1	0	0	40	0	0	0	1	42	42	
	Pineville	0	0	29	0	1	0	1	21	1	53	53	
	Rockhill	0	0	27	0	0	0	1	0	1	29	29	
	Statesville	0	37	0	0	1	0	0	0	0	38	38	
	Total	0	82	81	0	42	0	2	21	5			
	Max in							65	80	105			

Hop 2		Waste in					
		Orangeburg	Florence	Macon	Total	from Hop 1	
Waste out = Hop 1's Waste in	Denver	0	0	0	0	0	Cost 2
	Morganton	0	77	5	82	82	2261
	Morrisville	0	0	81	81	81	Total Cost
	Pineville	0	0	0	0	0	3656
	Rockhill	41	0	1	42	42	
	Statesville	0	0	0	0	0	
	Orangeburg	0	0	2	2	2	
	Florence	19	0	2	21	21	
	Macon	5	0	0	5	5	
	Total	65	77	91			
	Max capacity	65	80	105			

Part 2: Investment Allocations

(i) Firstly, we manually complete the Covariance matrix of assets' returns table..

Covariance Matrix						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

Subsequently, we generate additional tables for optimized allocation and investment amounts, based on a total investment of $10,000. The expected investment return rate [e] is calculated as the [SUMPRODUCT] of the expected returns of each asset and its allocation percentage in the portfolio. Our objective is to achieve a minimum baseline expected return of 11% while minimizing the risk [r]. We once again utilize Solver to incorporate certain constraints, such as ensuring that the total allocation percentage of all assets sums up to 100% and that the minimum return rate is set at 11%. The tables and optimized results below indicate that the investor should allocate the largest investment amount to Foreign stocks (26.5%) and Put options (25.2%), while the smallest amount should be invested in Call options (4.6%).

	Expected Returns
Bonds	**7%**
High tech stocks	**12%**
Foreign stocks	**11%**
Call options	**14%**
Put options	**14%**
Gold	**9%**

Optimised Allocation	
Bonds	17.6%
High tech stocks	11.2%
Foreign stocks	26.5%
Call options	4.6%
Put options	25.2%
Gold	15.0%
Total	100%

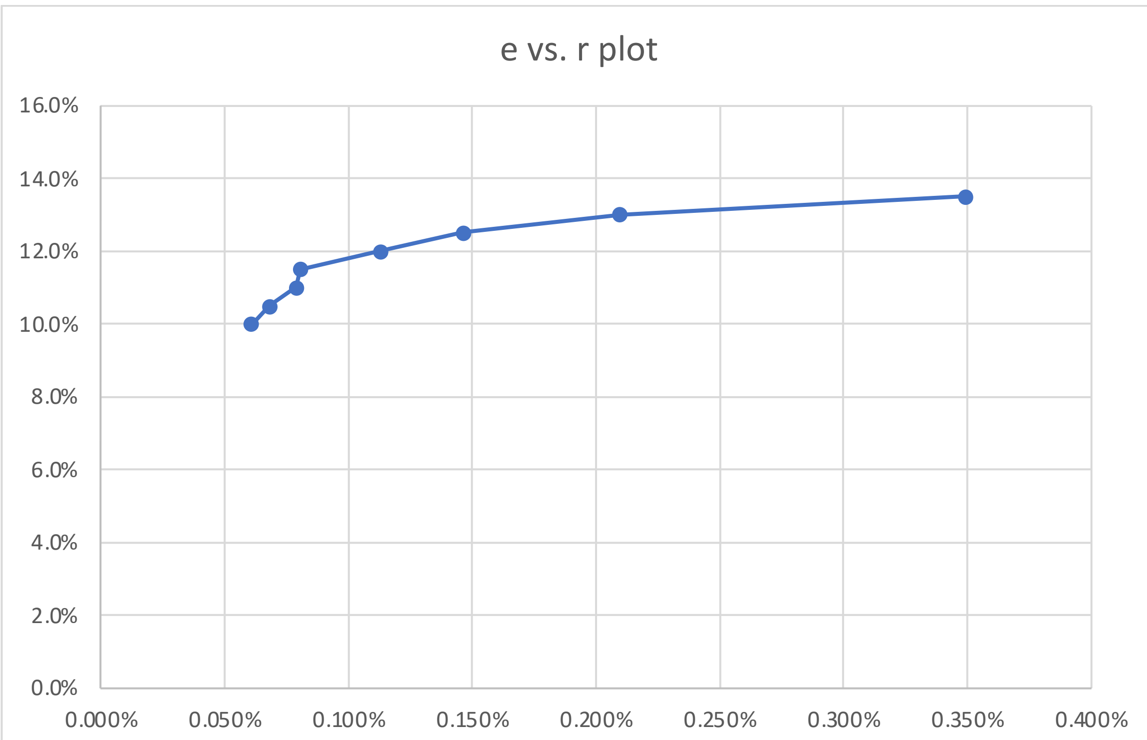
Optimised Return and Risk	
e	11.00%
r	0.079%

Investment Amount	
Bonds	$1,763.67
High tech stocks	$1,116.15
Foreign stocks	$2,646.77
Call options	$459.43
Put options	$2,517.65
Gold	$1,496.34
Total	$10,000.00

(ii) Use successive values of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13% and 13.5% as the baseline return values to obtain eight pairs of solutions (r, e)

We follow the same procedure, creating decision tables for each expected return rate. The results are summarized in the table and graph below. By examining the plot between e and r, I predict that the mathematical relationship between "r" and "e" may be logarithmic.

Full (r, e)	
r	e
0.061%	10.0%
0.068%	10.5%
0.079%	11.0%
0.081%	11.5%
0.113%	12.0%
0.146%	12.5%
0.210%	13.0%
0.350%	13.5%



Conclusion

By leveraging non-linear programming models, this report has focused on minimizing costs and investment risks while considering various constraints. The analysis has revealed the optimal transport solution for the logistics company as well as the portfolio management plan for investors. The more cost-effective strategy involves directly transferring the waste from the plants to the disposal destinations without making intermediate stops at other plants. Furthermore, we have demonstrated several investment scenarios, considering a range of baseline return values for the second optimization problem, while ensuring minimal risk for each scenario.

References

1. Evans, J. R. (2013). Statistics, data analysis, and decision modeling. Pearson Education.