**LAGUNA STATE POLYTECHNIC UNIVERSITY**

**LOS BAÑOS CAMPUS**

**COLLEGE OF COMPUTER STUDIES**

2ND SEMESTER, AY 2021-2022

**CMSC 101 – DISCRETE STRUCTURES 1**

PRACTICE EXERCISES ON MATHEMATICAL INDUCTION

Prove by mathematical Induction:

1. 2 + 4 + 6 + …. + 2n = n(n+1)

Step 1 Basis Step n = 1,2,3

|  |  |  |
| --- | --- | --- |
| **Counting** | **Left Side** | **Right Side** |
| n = 1 | 2 | 1(1+1) = 2 |
| n = 2 | 2+4 = 6 | 2(2+1) = 6 |
| n = 3 | 2+4+6 = 12 | 3(3+1) = 12 |

Step 2 Inductive Hypothesis n = k

2 + 4 + 6 + …. + 2k = k(k+1) is true

Step 3 Inductive Step n = k+1

2 + 4 + 6 + …. + 2k + 2(k+1)= (k+1)([k+1]+1)

k(k+1) + 2(k+1)= (k+1)([k+1]+1)

k2 + k + 2k + 2 = (k+1)(k+2)

k2 + 3k + 2 = k2 + 3k + 2

Step 4 Conclusion

We have shown that the 2 + 4 + 6 + …. + 2n = n(n+1) is true for values n = 1,2, 3, ...k. And we have proven that it is true for n = k+ 1. Thus, it is true for all counting numbers n. QED.

1. 2 + 6 + 10 + … + (4n - 2) = 2n2

Step 1 Basis Step n = 1,2,3

|  |  |  |
| --- | --- | --- |
| **Counting** | **Left Side** | **Right Side** |
| n = 1 | 2 | 2(1)2 = 2 |
| n = 2 | 2+6 = 8 | 2(2)2 = 8 |
| n = 3 | 2+6+10 = 18 | 2(3)2 = 18 |

Step 2 Inductive Hypothesis n = k

2 + 6 + 10 + … + (4k - 2) = 2k2 is true

Step 3 Inductive Step n = k+1

2 + 6 + 10 + … + (4k - 2) + (4(k+1) - 2) = 2(k+1)2

2k2 + (4(k+1) - 2) = 2(k+1)2

2k2 + 4k + 4 – 2 = 2(k2 + 2k + 1)

2k2 + 4k + 2 = 2k2 + 4k + 2

Step 4 Conclusion

We have shown that 2 + 6 + 10 + … + (4n - 2) = 2n2 is true for values n = 1,2, 3, ...k. And we have proven that it is true for n = k+ 1. Thus, it is true for all counting numbers n. QED.

1. 12 + 22 + 32 + … + n2 = (n (n+1)(2n+1))

Step 1 Basis Step n = 1,2,3

|  |  |  |
| --- | --- | --- |
| **Counting** | **Left Side** | **Right Side** |
| n = 1 | 12 = 1 | (1 (1+1)(2(1)+1))  = (2)(3)  x = 1 |
| n = 2 | 12 + 22= 5 | (2 (2+1)(2(2)+1)) = 8  = (6)(5)  x = 5 |
| n = 3 | 12 + 22 + 32 = 14 | (3 (3+1)(2(3)+1))= 18  = (12)(7)  x = 14 |

Step 2 Inductive Hypothesis n = k

12 + 22 + 32 + … + k2 = (k (k+1)(2k+1)) is true

Step 3 Inductive Step n = k+1

12 + 22 + 32 + … + k2 = (k (k+1)(2k+1))

12 + 22 + 32 + … + k2 + (k+1)2 = ((k+1) ((k+1)+1)(2(k+1)+1))

(k (k+1)(2k+1)) + (k+1)2 = ((k+1) ((k+1)+1)(2(k+1)+1))

((k2 + k)(2k+1)) + k2 + 2k + 1 = ((k+1) (k+2)(2k+3))

(2k3 + 3k2 + k) + k2 + 2k + 1 = (2k3 + 9k2 + 13k + 6)

1/3k3 + 1/2k2 + 1/6k + k2 + 2k + 1 = 1/3k3 + 3/2k2 + 13/6k + 1

1/3k3 + 3/2k2 + 1/6k+ 2k + 1 = 1/3k3 + 3/2k2 + 13/6k + 1

Step 4 Conclusion

We have shown that 12 + 22 + 32 + … + n2 = (n (n+1)(2n+1)) is true for values n = 1,2, 3, ...k. And we have proven that it is true for n = k+ 1. Thus, it is true for all counting numbers n. QED.

1. 1x2 + 3x4 + 5X6 + … + (2n-1)(2n) =

Step 1 Basis Step n = 1,2,3

|  |  |  |
| --- | --- | --- |
| **Counting** | **Left Side** | **Right Side** |
| n = 1 | 1x2 = 2 | =  = 2 |
| n = 2 | 1x2 + 3x4 = 14 | =  =  = 14 |
| n = 3 | 1x2 + 3x4 + 5x6 = 44 | =  =  = 44 |

Step 2 Inductive Hypothesis n = k

1x2 + 3x4 + 5X6 + … + (2k-1)(2k) = is true

Step 3 Inductive Step n = k+1

1x2 + 3x4 + 5X6 + … + (2k-1)(2k) + (2[k+1]-1)(2[k+1]) =

+ (2[k+1]-1)(2[k+1]) =

+ (2k+1)(2k+2) =

+ 4k2 + 6k + 2 =

4/3k3 + 5k2 + 17/3k + 2 = 4/3k3 + 5k2 + 17/3k + 2

Step 4 Conclusion

We have shown that 1x2 + 3x4 + 5X6 + … + (2n-1)(2n) = is true for values n = 1,2, 3, ...k. And we have proven that it is true for n = k+ 1. Thus, it is true for all counting numbers n. QED.

1. That 3n - 1 is divisible by 2 is true for all positive integers.

Or 1 + 3 + 5… + 3n -1 = n2

Step 1 Basis Step n = 1

3n – 1 3(1) – 1

= 2

2 / 2 = 1

Step 2 Inductive Hypothesis n = k

3k – 1 = 2x (x is for some integer)

3k = 2x + 1

Or

1 + 3 + 5… + 3k - 1 = k2

Step 3 Inductive Step n = k+1

3(k+1) – 1 = 3k + 3 – 1

= 3k + 2

= 2x + 1 + 2

= 2x + 3

Or

1 + 3 + 5 … +3k-1 + 3(k+1)-1 = (k+1)2

k2 + 3(k+1)-1 = (k+1)2

k2 + 3k+3-1 = k2 + 2k + 1

k2 + 3k+2 = k2 + 2k + 1

Step 4 Conclusion

Since 2x + 3 cannot be factored to have an equation that is divisible by 2, therefore the statement 3n-1 is not divisible by 2. By principle of mathematical induction, the statement is false for all positive integers.