Title:

Dimension optimisation and stress analysis of dome-shaped lunar bases

Course:

Mathematics: Analysis and Approaches Higher Level

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Introduction

I’ve been involved in space research projects for 3 years, and my recent passion became modeling interplanetary bases. As humanity is looking to space exploration, specifically the inhabitation of planets like the Moon, we need to create reliable structures. An especially promising material for creating such structures have been found to be lunar regolith base geopolymer, which is essentially structures made from geopolymer cement (GPC) based on lunar soil. More specifically,GPC is an amorphous, semi-crystalline cementitious material produced by the polymerization reaction of an aluminosilicate source with an alkaline reagent [1]. The space community is especially enthusiastic about this source as it provides superior qualities to traditional Portland cement, especially important when the utilization of in-situ materials becomes necessary due to the high costs of shipping building materials from Earth.

Figure 1.1.5-metric-ton (3,300 lb) block 3D-printed from simulated lunar dust, to demonstrate the feasibility of constructing a Moon base using local materials.

BODY

Background information and goal statement

Task 1: Optimizing spherical segment dimensions for using standard optimization technique and Lagrange multipliers

Material transfer between Earth and Moon remains one of the biggest logistical and financial obstacles to lunar construction. Currently, it costs NASA between USD 10,000 and USD 40,000 per kilogram to get materials to the Moon. It thus makes long haul transports of construction materials unsustainable for large lunar habitats.

An alternative has been in-situ resource utilization. The regolith on the Moon may be used to produce geopolymers for construction. However, this approach also has challenges and costs. Producing geopolymers requires regolith processing machinery - for example crushing and thermal activation systems. These machines need to be transported from Earth, significantly increasing upfront costs. Geopolymers also need activating agents and water, which are not available on the Moon. Water extraction from lunar ice deposits requires energy and is extremely costly compared to transporting water from Earth.

Given these constraints, lunar base surface area must be reduced while maintaining the same habitable volume. By optimizing the shape and dimensions of habitats resource expenditure can be minimized which will reduce transport and production costs. This approach makes lunar construction more feasible and fits within the larger goal of sustainable space exploration.

For lunar bases constructed from geopolymers, spherical segment structures are often proposed because they can distribute stress uniformly, retain pressure and provide high structural efficiency. Hemispheres are usually the best spherical segments for surface area minimization for a given volume.

We know that a hemisphere reduces the surface area for a given volume, so it is an efficient shape in many situations. However, when wall thickness is included in the calculations, the relationship between the dimensions may shift. **Specifically, I aim to determine whether the optimal shape remains a hemisphere or transitions to a different configuration once wall thickness is accounted for**. In doing so I try to answer the following key question::

1. Does the inclusion of wall thickness shift the optimal shape away from a hemisphere?

Task 2: Taylor series for analysis of angular deviations in dome structure supports

Geopolymers are now becoming an increasingly sustainable and versatile material for construction, when combined with advanced 3D printing technologies. That combination allows precise fabrication of complex structures with minimal material waste - an advantage for resource constrained environments like the Moon. However, small deviations in angular calculations may occur due to equipment limitations, environmental factors or execution errors during 3D printing. Those seemingly innocuous angular deviations can cause significant changes in stress distribution in the structure, which may affect the stability and integrity of critical components like support rings. Deep moonquakes range from 0.5 to 1.3 in magnitude on the Richter scale, while the strongest shallow moonquakes reach 4 to 5 [2]. Although moonquakes do not directly damage building structures in the low-gravity environment of the Moon, they can negatively affect the structural performance during the construction phase [3]. Thus, our first goal becomes ensuring that geopolymer -base structure can withstand such conditions.

So, I will use Taylor series expansion to mathematically analyze the effect of such angular deviations on the stress of supports of a dome structure. Expanding the stress formula around a reference angle I will define thresholds for acceptable angular errors that will keep the structure in safe stress limits.

Architectural design and data

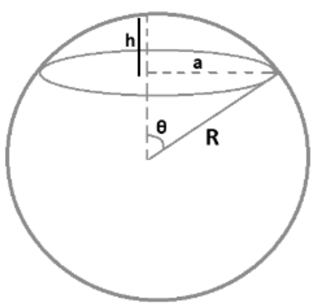

*Figure 2. **a** is the radius of the base of the cap, **h** is the height of the cap, and **r** is the radius of the sphere.*

For our examination, consider an inward-tapering, hollow, spherical segment for living purposes of a crew size of 6. It has been established that for each crewmate, there must be at least $120\ m^3$ of habitable space for long-term habituation. Thus, for a crew of 6 = this becomes $720\ m^3$. Moreover, there is a 20% of storage, which adds another $144\ m^3$, totalling to $864\ m^3$ of habitable space. Moreover, a recommended wall thickness is 1m [4]. The spherical segment subtends an angle of $120°$ at the support rings.

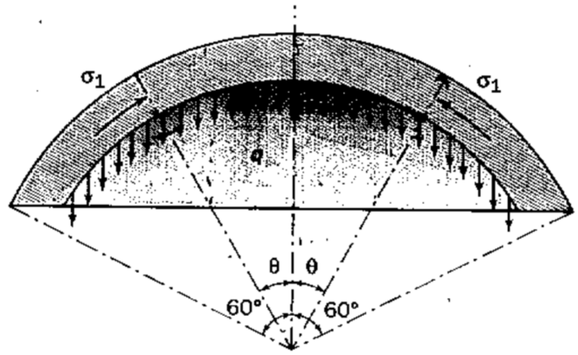

Figure 3. Concrete spherical segment diagram (adapted from Benham and Crawford, Mechanics of Engineering Materials).

Other data required include the gravitational acceleration of the Moon (1.625 m/s²) and the density of hard lunar regolith (3000 kg/m³)

Calculations

1.0 Task 1: Optimizing spherical segment dimensions for using standard optimization technique and Lagrange multipliers

First, I will optimize to minimize the surface area of a spherical cap, subject to the constraint that the volume of a spherical cap is $864\ m^3$.

Variables:

- h: Height of the spherical cap.
- a: Radius of the base of the spherical cap.
- T: Thickness of the wall.
- R: Radius of the sphere

If the wall has a thickness t, the external dimensions of the spherical cap are increased, so I need to adjust for this. Let h_{outer} and a_{outer} represent the height and base radius of the outer cap, respectively.

$$h_{outer} = h_{inner} + T$$

1.1 Finding the formula for the volume of a spherical cap

Let’s calculate the volume of a spherical cap using the concept of solids of revolution. I can visualize this as a sphere rotating around the y-axis.

A sphere of radius R, centered at the origin, has the equation

$$x^2 + y^2 = R^2$$

However, I am interested in the volume of a **spherical cap**, which is the portion of the sphere lying between $y = R - h$ and $y = R$ where h is the height of the cap.

The radius of the circular base of the cap, a, is obtained from the sphere's equation at $y = R - h$

$$a^2 + (R - h\,)^2 = R^2$$

$$a^2 = R^2 - (R - h\,)^2 = R^2 - R^2 + 2Rh - h^2$$

$$a = \sqrt{2Rh - h^2}$$

I calculate the volume of the spherical cap by rotating the segment of the sphere around the y-axis. The formula for the volume of a solid of revolution is:

$$V = \pi \int_{R-h}^{R} \left[\sqrt{R^2 - y^2}\right]^2 dy = \pi \int_{R-h}^{R} R^2 - y^2\, dy$$

Solving the integral:

$$\int R^2 - y^2 = R^2 y - \frac{y^3}{3}$$

Evaluating from R to R-h:

$$V = \pi \left[\left(R^2 R - \frac{R^3}{3}\right) - \left(R^2(R - h) - \frac{(R - h)^3}{3}\right)\right]$$

$$V = \pi \left(Rh^2 - \frac{h^3}{3}\right)$$

Plugging the value of $a = \sqrt{2Rh - h^2}$

Expressing R through a:

$$a^2 = 2Rh - h^2$$

$$R = \frac{h^2 + a^2}{2h}$$

Substituing

$$V = \pi\left(Rh^2 - \frac{h^3}{3}\right) = \pi\left(\left(\frac{h^2 + a^2}{2h}\right)h^2 - \frac{h^3}{3}\right) = \pi\left(\frac{h}{2}(h^2 + a^2) - \frac{h^3}{3}\right) = \frac{\pi h}{6}(3a^2 + h^2)$$

1.2 Finding the formula for the curved surface area of a spherical cap

I know the general formula for surface area about the y-axis, which is

$$S = 2\pi \int_{y_1}^{y_2} f(y)\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

As I am rotating the cap around the y-axis, I need to express x in terms of y:

$$x = f(y) = \sqrt{R^2 - y^2} = (R^2 - y^2)^{\frac{1}{2}}$$

Differentiating to plug into the formula:

$$\frac{dx}{dy} = \frac{1}{2}(R^2 - y^2)^{-\frac{1}{2}} \cdot -2y = \frac{-y}{\sqrt{R^2 - y^2}}$$

Squaring:

$$(\frac{dx}{dy})^2 = \frac{y^2}{R^2 - y^2}$$

Substituting $f(y)$ and $(\frac{dx}{dy})^2$back into the formula:

$$S = 2\pi \int_{R-h}^{R} \sqrt{R^2 - y^2}\sqrt{1 + \frac{y^2}{R^2 - y^2}}\,dy = 2\pi \int_{R-h}^{R} \sqrt{R^2 - y^2} \cdot \frac{R}{\sqrt{R^2 - y^2}}\,dy$$

$$S = 2\pi R \int_{R-h}^{R} 1 dy$$

Evaluating the integral:

$$\int_{R-h}^{R} 1 dy = R - (R - h) = h$$

Thus, the surface area is

$$S = 2\pi Rh$$

This way, I found the expressions for the volume and surface of a spherical cap.

1.3.0 Problem setup

I want to minimise:

$$A_{outer} = \pi\left({a_{out}}^2 + {h_{out}}^2\right)$$

The constraint is:

$$V_{in} = \frac{\pi h_{in}}{6}\left(3{a_{in}}^2 + {h_{in}}^2\right) = 864$$

Let's define other variables,

From section **1.1** (See above):

$${a_{out}}^2 = 2R_{out}h_{out} - {h_{out}}^2$$

$$R_{out} = R_{in} + T, \text{ where } R_{in} = \frac{{a_{in}}^2 + {h_{in}}^2}{2h_{in}}$$

1.3.1 Expressing the surface area function $\boldsymbol{A_{outer} = \pi\left({a_{out}}^2 + {h_{out}}^2\right)}$ in terms of $\boldsymbol{h_{in}}$, to get an objective function.

Writing out surface area in terms of h_{in} alone,

From section **1.1** (See above):

$${a_{out}}^2 = 2(R_{in} + T)(h_{in} + T) - (h_{in} + T)^2 = (h_{in} + T)[2(R_{in} + T) - (h_{in} + T)]$$

$$= (h_{in} + T)(2R_{in} + 2T - h_{in} - T) = (h_{in} + T)(2R_{in} + T - h_{in}).$$

Finding R_{in} in terms of h_{in},

From section **1.1** (See above):

$$R_{in} = \frac{{h_{in}}^2 + {a_{in}}^2}{2h_{in}}$$

Now, I need to express a_{in} in terms of h_{in} from the constraint equation to use the value in the expression for R above.

$$V_{in} = \frac{\pi h_{in}}{6}\left(3{a_{in}}^2 + {h_{in}}^2\right) = 864$$

$$h_{in}\left(3{a_{in}}^2 + {h_{in}}^2\right) = \frac{5184}{\pi}$$

Let $X = \frac{5184}{\pi}$, then

$$3{a_{in}}^2 + {h_{in}}^2 = \frac{X}{h_{in}}$$

$${a_{in}}^2 = \frac{1}{3}\left(\frac{X}{h_{in}} - {h_{in}}^2\right)$$

Then,

$$R_{in} = \frac{{h_{in}}^2 + \left[\frac{1}{3}\left(\frac{X}{h_{in}} - {h_{in}}^2\right)\right]}{2h_{in}} = \frac{\frac{3}{3}{h_{in}}^2 + \left[\frac{1}{3}\left(\frac{X}{h_{in}} - {h_{in}}^2\right)\right]}{2h_{in}} = \frac{\frac{1}{3}\left(\frac{X}{h_{in}} + 2{h_{in}}^2\right)}{2h_{in}}$$

$$= \frac{X}{6{h_{in}}^2} + \frac{1}{3}h_{in}$$

Now, substituting R into the ${a_{out}}^2$ expression,

$${a_{out}}^2 = (h_{in} + T)(2R_{in} + T - h_{in}) = (h_{in} + T)\left[2\left(\frac{X}{6{h_{in}}^2} + \frac{1}{3}h_{in}\right) + T - h_{in}\right]$$

$$= (h_{in} + T)(\frac{X}{3{h_{in}}^2} + T - \frac{1}{3}h_{in})$$

Plugging into the A_{out} function:

$$A_{out} = \pi[a_{out}{}^2 + (h_{in} + T)^2] = \pi\left[(h_{in} + T)\left(\frac{X}{3h_{in}{}^2} + T - \frac{1}{3}h_{in}\right) + (h_{in} + T)^2\right]$$

$$= \pi(h_{in} + T)\left[\left(\frac{X}{3h_{in}{}^2} + T - \frac{1}{3}h_{in}\right) + (h_{in} + T)\right]$$

$$= \pi(h_{in} + T)\left[\frac{X}{3h_{in}{}^2} + 2T + \frac{2}{3}h_{in}\right]$$

Now, we can differentiate this function.

1.3.2 Differentiation of the objective function

Using product rule, let $f(h_{in}) = \pi(h_{in} + T)$ and $g(h_{in}) = \left[\frac{X}{3h_{in}{}^2} + 2T + \frac{2}{3}h_{in}\right]$

$$f'(h_{in}) = \pi$$

Differentiation $g(h_{in})$ by terms:

$$\frac{d}{dh_{in}}\left(\frac{X}{3h_{in}{}^2}\right) = \frac{X}{3} \cdot \frac{d}{dh_{in}}\left(h_{in}{}^{-2}\right) = \frac{X}{3} \cdot (-2)h_{in}{}^{-3} = -\frac{2X}{3h_{in}{}^3}$$

$$\frac{d}{dh_{in}}(2T) = 0$$

$$\frac{d}{dh_{in}}\left(\frac{2}{3}h_{in}\right) = \frac{2}{3}$$

Thus,

$$g'(h_{in}) = -\frac{2X}{3h_{in}{}^3} + \frac{2}{3}$$

Combining:

$$\frac{d}{dh_{in}}[f(h_{in})g(h_{in})] = f'(h_{in})g(h_{in}) + g'(h_{in})f(h_{in})$$

$$= \pi\left(\frac{X}{3h_{in}{}^2} + 2T + \frac{2}{3}h_{in}\right) + \pi(h_{in} + T)\left(-\frac{2X}{3h_{in}{}^3} + \frac{2}{3}\right)$$

Substituing $X = \frac{5184}{\pi}$, the equation simplifies to:

$$\frac{d}{dh_{in}} = -1728h_{in} + \frac{4}{3}\pi {h_{in}}^4 - 3456T + \frac{8}{3}\pi T {h_{in}}^3$$

$$= -5184h_{in} + 4\pi {h_{in}}^4 - 10368T + 8\pi T {h_{in}}^3$$

Setting $\frac{d}{dh_{in}} = 0$:

$$-5184h_{in} + 4\pi {h_{in}}^4 - 10368T + 8\pi T {h_{in}}^3 = 0$$

Solving this polynomial graphically in the graphical calculator, I obtained:

$$h_{in\_1} = -2.00, h_{in\_2} = 7.44$$

Only the positive values are acceptance, as this is a physical height, so the height that minimizes the surface area of a spherical cap, while keep its volume 864 m^3 is is 7.44 metres (2 s.f).

1.3.3 Checking answers

Let's check our answer with our wall thickness T=1.

$$V_{in} = \frac{\pi h_{in}}{6}\left(3{a_{in}}^2 + {h_{in}}^2\right)$$

$$A_{out} = \pi(h_{in} + T)\left[\frac{X}{3{h_{in}}^2} + 2T + \frac{2}{3}h_{in}\right]$$

By calculating each value through the graphical calculator, I achieved these results:

h_{in}	V_{in}	A_{out}
5.00	864.00	515.2
6.00	864.00	467.94
7.00	864.00	449.67
7.44	864.00	448.02
8.00	864.00	450.34
9.00	864.00	464.66

10.00	864.00	489.57

So, indeed, we can observe that h=7.44 metres minimizes the surface while keeping the volume at 864 m3.

1.3.4 Lagrange method confirmation

Even though I obtained a correct answer in the first attempt, I believe that the inner and outer dimensions are linearly related due to the uniform wall thickness, and that it is unnecessary to explicitly consider the thickness when optimizing. To confirm this assumption and the validity of my solution, I want to redo the problem using the Lagrange method to confirm the results are consistent.

I will use different expressions for volume and surface area that account for R and h, instead of a and h, to ensure accuracy and consistency in my calculations. So, as the objective function, I will use

$$f(h, R) = 2\pi hR$$

And as the constraint becomes $\pi\left(-\frac{h^3}{3} + h^2\right) - 864 = 0$

$$L(h, R, \lambda) = 2\pi hR + \lambda\left(\pi\left(-\frac{h^3}{3} + h^2\right) - 864\right)$$

Finding all frist order partial derivatives:

$$\frac{\partial}{\partial h}\left(2\pi hR + \lambda\left(\pi\left(-\frac{h^3}{3} + h^2\right) - 864\right)\right) = \pi(-\lambda h(h - 2R) + 2R)$$

$$\frac{\partial}{\partial R}\left(2\pi hR + \lambda\left(\pi\left(-\frac{h^3}{3} + h^2\right) - 864\right)\right) = \pi h(\lambda h + 2)$$

$$\frac{\partial}{\partial\lambda}\left(2\pi hR + \lambda\left(\pi\left(-\frac{h^3}{3} + h^2\right) - 864\right)\right) = -\frac{\pi h^3}{3} + \pi h^2 R - 864$$

Creating the system:

$$\begin{cases} \dfrac{\partial}{\partial h} = 0 \\ \dfrac{\partial}{\partial R} = 0 \\ \dfrac{\partial}{\partial \lambda} = 0 \end{cases} \Rightarrow \begin{cases} \pi(-\lambda h(h - 2R) + 2R) = 0 \\ \pi h(\lambda h + 2) = 0 \\ -\dfrac{\pi h^3}{3} + \pi h^2 R - 864 = 0 \end{cases}$$

Solving the system through the graphical calculator: $(h, R) = \left(\frac{6\sqrt[3]{6}}{\sqrt[3]{\pi}}; \frac{6\sqrt[3]{6}}{\sqrt[3]{\pi}}\right) \approx (7.44; 7.44)$

Indeed, my hypothesis was correct. Now, let's check if R=a, to identify whether the wall thickness shift the optimal shape away from a hemisphere.

$$a_{in} = \sqrt{\frac{1}{3}\left(\frac{\frac{5184}{\pi}}{h_{in}} - {h_{in}}^2\right)} = 7.45$$

We can observe that the radius of the base of the spherical segment, a, is almost identical in value as the radius of the sphere, R. The small deviation from R is most probably due to the rounding.

Thus, I can answer to my exploration questions:

1. Does the inclusion of wall thickness shift the optimal shape away from a hemisphere? – No

1.4.0 Learning

I was expecting to receive a result where a and h would be different, as I was considering inner height and minimizing outer surface area, and I did not expect that it would turn out that a=h=R in the end of my work. This might have happened due to the fact that the hemispherical shape inherently provides optimal structural efficiency, even when a significant wall thickness of 1 meter is incorporated. In conclusion, the inclusion of wall thickness did not shift the optimal shape away from a hemisphere.

2.0. *Taylor series for analysis of angular deviations in dome structure supports*

From Breham and Crawford [5], the formulas for the stresses at the supports of a dome subjected to self-weight are:

$$\sigma_1 = -\frac{qr}{T}\left(\frac{1}{1+cos\theta}\right) \; and \; \sigma_2 = \frac{qr}{T}\left(\frac{1}{1+cos\theta} - cos\theta\right)$$

where q is the load per unit area due to self-weight, r is the dome's radius, T is the thickness, and θ is the angular position at the supports.

I want to use Taylor series to find out how the deviation in angle affects the stress at the supports of a dome subjected to self-weight because the original equations contain trigonometric and non-linear terms, and directly solving for deviations will be impractical. I will reduce the problem to a polynomial by transforming the stress function for one support point only (because the process will be identical for the second point) into a Taylor series about a reference angle. This will give a simple way to analyze small deviations from the reference angle to get a better idea of the stress variation with respect to angle. A few terms in the Taylor series also allow for approximating stress values with a very low computational effort while maintaining accuracy for small deviations.

2.1.1 Construction Taylor series

For our values, (r=7.44, T=1, $\rho = 3000\ kgm^{-3}$),

$$\sigma_1 = -\frac{qr}{T}\left(\frac{1}{1+\cos(\theta)}\right) = -\frac{(\rho\cdot g\cdot T)r}{t}\left(\frac{1}{1+\cos(\theta)}\right)$$

$$= -\frac{(3000\cdot 1.625\cdot 1)\cdot 7.44}{1}\left(\frac{1}{1+\cos(\theta)}\right) = -36270\left(\frac{1}{1+\cos(\theta)}\right)$$

Let’s do the Taylor’s series expansion around $\theta = \frac{\pi}{3}$

$$\sigma_1{}'(\theta) = -\frac{36270\cdot\sin(\theta)}{(1+\cos(\theta))^2}, f'\left(\frac{\pi}{3}\right) = -8060\sqrt{3}$$

$$\sigma_1{}''(\theta) = \frac{-36270\cos(\theta) + 18135\cos(2\theta) - 54405}{(cos(\theta)+1)^3}, f''\left(\frac{\pi}{3}\right) = -24180$$

$$\sigma_1{}'''(\theta) = \frac{18135\big(2sin(\theta)sin(2\theta)\big)(cos(\theta)+1) - 3(2cos(\theta) - \cos(2\theta) + 3)\sin(\theta))}{(cos(\theta)+1)^4},$$

$$f'''\left(\frac{\pi}{3}\right) = -24180\sqrt{3}$$

$$P(\theta) = \frac{-24180}{0!}\left(\theta - \left(\frac{\pi}{3}\right)\right)^0 + \frac{-8060\sqrt{3}}{1!}\left(\theta - \left(\frac{\pi}{3}\right)\right)^1 + \frac{-24180}{2!}\left(\theta - \left(\frac{\pi}{3}\right)\right)^2$$

$$+ \frac{-24180\sqrt{3}}{3!}\left(\theta - \left(\frac{\pi}{3}\right)\right)^3$$

$$\sigma(\theta) \approx P(\theta) = -24180 - 8060\sqrt{3}\left(\theta - \frac{\pi}{3}\right) - 12090\left(\theta - \frac{\pi}{3}\right)^2 - 4030\sqrt{3}\left(\theta - \frac{\pi}{3}\right)^3$$

Here I achieved the Taylor series in the 3rd power.

$\Delta\theta = \theta - \frac{\pi}{3}$ is the angle deviation from the reference angle of $\frac{\pi}{3}$.

The maximum allowable stress, σ_{max} of the support rings is 30000 Nm^{-2}.

Let’s now determine allowable angle deviations.

To ensure safety,

$$-24180 - 8060\sqrt{3}\Delta\theta - 12090\Delta\theta^2 - \cdots \geq -30000$$

$$-8060\sqrt{3}\Delta\theta - 12090\Delta\theta^2 - \cdots \geq -5820$$

$$8060\sqrt{3}\Delta\theta + 12090\Delta\theta^2 + \cdots \leq 5820$$

For small deviations, I can neglect higher order terms, therefore I approximate

$$8060\sqrt{3}\Delta\theta + 12090\Delta\theta^2 \leq 5820$$

Solving the quadractic inequality:

$$12090\Delta\theta^2 + 8060\sqrt{3}\Delta\theta - 5820 = 0$$

Using graphical calculator:

$$\Delta\theta_1 = 0.324\,rad(\approx 18.57°) \text{ and } \Delta\theta_2 = -1.480\,rad\ (\approx -84.78°)$$

Interpreting the results:

Positive deviation ($\Delta\theta \leq +18.57°$)	Negative deviation($\Delta\theta \geq -84.78°$)
• Stress remains within safe limits	• Not a practical concern, as angles can't realistically deviate this far downward in structural contexts.

2.2 Learning:

The result shows that the geopolymer is able to withstand large angular deviations safely. But such large deviations are unlikely to occur in practice as precision in 3D printing and construction is much higher. The material and structural design are well-engineered for the stresses expected in the system, indicating a large safety margin.

Conclusion

Through this exploration, I found that wall thickness does not change the optimal shape of spherical caps from hemisphere. I had initially reasoned that inner height and inner radius dim

ensions might impose optimization conditions different from those of
the outer surface - defined by outer height and outer radius. I hypothesized that this potential mismatch might have led to efficiencies in the structure's material usage.

Analyzing geometry and conducting optimization have revealed to me that the hemispherical design is still the most resource-efficient configuration, taking into account wall thickness. Its uniform stress distribution, along with its minimal surface area for a given volume, ensures that the hemispherical shape meets the efficiency goals regardless of wall thickness.

I also learned that geopolymers can be reliably printed 3D. A Taylor series expansion analysis indicated that the geopolymer material could withstand angular deviations of up to 18 degrees without exceeding allowable stress limits. Such a large margin of safety demonstrates the strength of the material and its suitability for lunar construction, where 3D printing may not always be perfect. This kind of tolerance to error is very important for the reliability of extraterrestrial habitats, given the difficulties of construction in remote and resource constrained locations.

This work validated the hemispherical design as optimal and demonstrated the practicality and resilience of geopolymers for lunar bases. Such findings complement the more inclusive view of sustainable space exploration through considerations of both theoretical design challenges and practical implementation challenges. Work could build upon these results by introducing dynamic factors like varying environmental loads or operational stresses to refine construction strategies for extraterrestrial environments.

Evaluation

Limitation: this work used one wall thickness value for the optimization analysis of spherical segments. I may have learned whether hemispherical shape remains resource-efficient under different thickness values. I could have tested several thickness values to see if the results - hemisphere being the optimal shape - are consistent or influenced by the thickness of the calculation.

A second limitation was that the Taylor series was truncated at the second order. This was sufficient to analyze small angular deviations, but may not have captured system behaviour under larger deviations. Enhanced computational resources might allow higher-order terms to join the expansion. This is especially relevant for identifying possible thresholds at which nonlinear effects dominate.

Future suggestions

Future work may optimize wall thickness to balance thermal insulation and radiation shielding. Thicker walls reduce heat loss and shield against cosmic rays better but increase material usage. An optimal thickness for minimizing heat transfer with maximum radiation protection is required for lunar habitats.

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