$$|Y - Z|_t \leq |Y|_t + |Z|_t.$$

Where for $0 = t_1 \leq \ldots \leq t_n = t$

$$|Y|_t = \sum_{i=1}^{n} |Y_{t_i} - Y_{t_{i-1}}|.$$

Lets say we can use the limit structure already

$$\begin{aligned}
|Y|_t &= \lim_{n\to\infty} \sum_{i=1}^{n} |Y_{t_i} - Y_{t_{i-1}}| = \lim_{n\to\infty} \sum_{i=1}^{n} \lim_{m\to\infty} \sum_{J\in\Pi_m} (\Delta_{J\cap[0,t_i]} X)^2 - \lim_{m\to\infty} \sum_{J\in\Pi_m} (\Delta_{J\cap[0,t_{i-1}]} X)^2 \\
&\leq \lim_{n\to\infty} \sum_{i=1}^{n} \lim_{m\to\infty} \sum_{J\in\Pi_m} (\Delta_{J\cap[0,t_i]} X)^2 \\
&= \lim_{n\to\infty} \lim_{m\to\infty} \sum_{i=1}^{n} \sum_{J\in\Pi_m} (\Delta_{J\cap[0,t_i]} X)^2
\end{aligned}$$

.

bzw.

$$\begin{aligned}
|Y|_t = \lim_{n\to\infty} \sum_{i=1}^{n} |Y_{t_i} - Y_{t_{i-1}}| &= \lim_{n\to\infty} \sum_{i=1}^{n} |\langle X\rangle_{t_i} - \langle X\rangle_{t_{i-1}}| \\
&\overset{\text{(i)}}{\leq} \lim_{n\to\infty} \sum_{i=1}^{n} |\langle X\rangle_{t_i}| \\
&= \sum_{i=1}^{k} \langle X\rangle_{t_i} + \lim_{n\to\infty} \sum_{k=1}^{n} \langle X\rangle_{t_i} \\
&= Y_t + 0.
\end{aligned}$$

If we know

$$\sum_{i=1}^{\infty} a_i < \infty.$$

Then

$$\lim_{k\to\infty} \sum_{i=k}^{\infty} a_i \to 0.$$

Exercise. Let $f(t,x) = t \cdot \frac{x^2}{2}$ show

$$f(t, B_t) = f(0,0) + \int_{\mathbb{R}} \frac{\partial f}{\partial s}(s, B_s) ds + \int_{\Omega} \frac{\partial f}{\partial x} + \frac{1}{2} \int \frac{\partial^2 f}{\partial x^2} ds \quad = 0 + \int_0^t \frac{B_s^2}{2} ds + \int_0^t s \cdot B_s dB_s + \frac{1}{2} \int_0^t s ds.$$