
$$|Y - Z|_t \leq |Y|_t + |Z|_t.$$

Where for $0 = t_1 \leq \dots \leq t_n = t$

$$|Y|_t = \sum_{i=1}^n |Y_{t_i} - Y_{t_{i-1}}|.$$

Lets say we can use the limit structure already

$$\begin{aligned} |Y|_t &= \lim_{n \rightarrow \infty} \sum_{i=1}^n |Y_{t_i} - Y_{t_{i-1}}| = \lim_{n \rightarrow \infty} \sum_{i=1}^n \lim_{m \rightarrow \infty} \sum_{J \in \Pi_m} (\Delta_{J \cap [0, t_i]} X)^2 - \lim_{m \rightarrow \infty} \sum_{J \in \Pi_m} (\Delta_{J \cap [0, t_{i-1}]} X)^2 \\ &\leq \lim_{n \rightarrow \infty} \sum_{i=1}^n \lim_{m \rightarrow \infty} \sum_{J \in \Pi_m} (\Delta_{J \cap [0, t_i]} X)^2 \\ &= \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} \sum_{i=1}^n \sum_{J \in \Pi_m} (\Delta_{J \cap [0, t_i]} X)^2 \end{aligned}$$

bzw.

$$\begin{aligned} |Y|_t &= \lim_{n \rightarrow \infty} \sum_{i=1}^n |Y_{t_i} - Y_{t_{i-1}}| = \lim_{n \rightarrow \infty} \sum_{i=1}^n |\langle X \rangle_{t_i} - \langle X \rangle_{t_{i-1}}| \\ &\stackrel{(i)}{\leq} \lim_{n \rightarrow \infty} \sum_{i=1}^n |\langle X \rangle_{t_i}| \\ &= \sum_{i=1}^k \langle X \rangle_{t_i} + \lim_{n \rightarrow \infty} \sum_{k=1}^n \langle X \rangle_{t_i} \\ &= Y_t + 0. \end{aligned}$$

If we know

$$\sum_{i=1}^{\infty} a_i < \infty.$$

Then

$$\lim_{k \rightarrow \infty} \sum_{i=k}^{\infty} a_i \rightarrow 0.$$