Exercise. Assume

$$\sup_y |\nabla_x K(x,y)| + \sup_x |\nabla_y K(x,y)| \le L.$$

Proof.

$$\begin{aligned}&\left|\int_{\mathbb{R}^d} K(v(x), v(y)) d\mu_0(y) - \int_{\mathbb{R}^d} K(w(x), w(y)) d\mu_0(y)\right| \\ &\le \int_{\mathbb{R}^d} |K(v(x), v(y))| + |K(w(x), w(y))| d\mu_0(y)\end{aligned}$$

.

and assume w.l.o.g $v(x) \le w(x)$ then by mean value theorem for some $c \in [v(x), w(x)]$

$$\frac{|K(v(x), v(y)) - K(w(x), w(y))|}{|v(x) - w(x)|} \le \sup_y \nabla_x K(c, y).$$

□