$\boxed{\text{2) SEGIDAK ETA SERIEAK}}$

$\boxed{1}$ Kalkulatu: $\lim\limits_{n\to\infty} \left(\sqrt{n^2+n+1} - \sqrt{n^2-n+1}\right)$

$$\lim_{n\to\infty} \left(\sqrt{n^2+n^2+1} - \sqrt{n^2-n+1}\right) = \lim_{n\to\infty} \frac{\left(\sqrt{n^2+n^2+1} - \sqrt{n^2-n+1}\right)\left(\sqrt{n^2+n+1} + \sqrt{n^2-n+1}\right)}{\sqrt{n^2+n+1} + \sqrt{n^2-n+1}} =$$

$$= \lim_{n\to\infty} \frac{(n^2+n+1) - (n^2-n+1)}{\sqrt{n^2+n+1} + \sqrt{n^2-n+1}} = \lim_{n\to\infty} \frac{2n}{\sqrt{n^2+n+1} + \sqrt{n^2-n+1}} = \lim_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}\left(\sqrt{1+\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}+\frac{1}{n^2}}\right)} =$$

$$= \frac{2}{\sqrt{1}+\sqrt{1}} = \frac{2}{2} = 1$$

$\boxed{2}$ Kalkulatu: $\lim\limits_{n\to\infty} \dfrac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}}$

$$\lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \lim_{n\to\infty} \frac{(n+1)^{n+1} n!}{n^n (n+1)!} = \lim_{n\to\infty} \frac{\cancel{(n+1)}(n+1)^n \cancel{n!}}{n^n \cancel{(n+1)!}} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^n = \overbrace{\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e}^{\text{Formula}}$$

$\boxed{3}$ Kalkulatu: $\lim\limits_{n\to\infty} (n+3) \ln\left(\dfrac{n^2+5n-1}{n^2-n+1}\right)$

$$\lim_{n\to\infty} (n+3) \ln\left(\frac{n^2+5n-1}{n^2-n+1}\right) = \left\| \ln(a_n) \sim a_n - 1 \right\| = \lim_{n\to\infty} (n+3)\left(\frac{n^2+5n-1}{n^2-n+1} - 1\right) = \lim_{n\to\infty} (n+3)\left(\frac{\cancel{n^2}+5n-1-\cancel{n^2}+n-1}{n^2-n+1}\right) =$$

$$= \lim_{n\to\infty} (n+3)\left(\frac{6n-2}{n^2-n+1}\right) = \left\| \begin{matrix} n+3 \sim n \\ 6n-2 \sim 6n \\ n^2-n+1 \sim n^2 \end{matrix} \right\| = \lim_{n\to\infty} n\left(\frac{6\cancel{n}}{\cancel{n^2}}\right) = 6$$

$\boxed{4}$ $\lim\limits_{n\to\infty} \left(\dfrac{n^2+3}{n^2+4n}\right)^{\frac{n^2-1}{n}}$

$\lim\limits_{n\to\infty} A = B \longrightarrow A = \ln B = e^B$

$$\lim_{n\to\infty} \ln\left(\frac{n^2+3}{n^2+4n}\right)^{\frac{n^2-1}{n}} = \lim_{n\to\infty} \left(\frac{n^2-1}{n}\right) \ln\left(\frac{n^2+3}{n^2+4n}\right) = \left\| \lim_{n\to\infty}(a_n) = 1 \to \ln(a_n) \sim a_n - 1 \right\| = \lim_{n\to\infty} \left(\frac{n^2-1}{n}\right)\left(\frac{n^2+3}{n^2+4n} - 1\right) =$$

$$= \lim_{n\to\infty} \left(\frac{n^2-1}{n}\right)\left(\frac{\cancel{n^2}+3-\cancel{n^2}-4n}{n^2+4n}\right) = \lim_{n\to\infty} \left(\frac{n^2-1}{n}\right)\left(\frac{3-4n}{n^2+4n}\right) = \lim_{n\to\infty} \left(\frac{3n^2-4n^3-3+4n}{n^3+4n^2}\right) \sim \lim_{n\to\infty} \frac{-4\cancel{n^3}}{\cancel{n^3}} = -4$$

$\boxed{5}$ $\lim\limits_{n\to\infty} \sqrt{n}\left(\sqrt{3+n} - \sqrt{n}\right) = \lim\limits_{n\to\infty} \sqrt{n} \dfrac{\left(\sqrt{3+n} - \sqrt{n}\right)\left(\sqrt{3+n} + \sqrt{n}\right)}{\sqrt{3+n} + \sqrt{n}} = \lim\limits_{n\to\infty} \sqrt{n} \dfrac{3+\cancel{n}-\cancel{n}}{\sqrt{3+n}+\sqrt{n}} =$

$$\lim_{n\to\infty} \frac{3\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\left(\sqrt{\frac{3}{n}+1} + 1\right)} = \frac{3}{2}$$

$\boxed{6}$ $\lim\limits_{n\to\infty} \dfrac{\ln(n)}{n} = \left\| n >> \ln(n) \right\| = 0$

$\boxed{7}$ $\lim_{n\to\infty} \left[\frac{\ln(n+a)}{\ln(n)}\right]^{n\ln(n)}$

$\lim_{n\to\infty} A = B \to A = \ln B = e^B$

$\lim_{n\to\infty} \ln\left[\frac{\ln(n+a)}{\ln(n)}\right]^{n\ln(n)} = \lim_{n\to\infty} n\ln(n)\ln\left[\frac{\ln(n+a)}{\ln(n)}\right] = \lim_{n\to\infty} n\ln(n)\left[\frac{\ln(n+a)}{\ln(n)} - 1\right] =$

$= \lim_{n\to\infty} n\cancel{\ln(n)}\left[\frac{\ln(n+a)-\ln(n)}{\cancel{\ln(n)}}\right] = \lim_{n\to\infty} n\ln\left(\frac{n+a}{n}\right) = \lim_{n\to\infty} n\left(\frac{n+a}{n} - 1\right) = \lim_{n\to\infty} \cancel{n}\left(\frac{\cancel{n}+a-\cancel{n}}{\cancel{n}}\right) = a$

$A = e^a$

$\boxed{8}$ $\lim_{n\to\infty} \frac{3n^4 \cdot \sin^2(1/n)\cdot\ln(1+1/n)}{(n+5)\cos\left(\frac{n\pi+5}{4n+1}\right)} \approx \left\| \begin{array}{l} 1/n \sim 0 \\ \sin(1/n) \sim 1/n \\ \ln(1+1/n) \sim 1/n \\ (n\pi+5)/(4n+1) \sim \pi/4 \\ \cos(\pi/4) = \sqrt{2}/2 \end{array} \right\| \sim \lim_{n\to\infty} \frac{3n^{\cancel{4}\,3}(1/n)^2(1/n)}{\cancel{n}\cdot(\sqrt{2}/2)} = \lim_{n\to\infty} \frac{3\cancel{n^3}\cancel{(1/n)^3}}{\sqrt{2}/2} = \frac{6}{\sqrt{2}}$

$\boxed{9}$ $\lim_{n\to\infty} \left(\frac{n+2}{3n^3-1}\right)^{\frac{1}{\ln(n^4-3)}}$

$\lim_{n\to\infty} A = B \to A = \ln B = e^B$

$\lim_{n\to\infty} \ln\left(\frac{n+2}{3n^3-1}\right)^{\frac{1}{\ln(n^4-3)}} = \lim_{n\to\infty}\left(\frac{1}{\ln(n^4-3)}\right)\ln\left(\frac{n+2}{3n^3-1}\right) \sim \lim_{n\to\infty}\left(\frac{1}{\ln(n^4)}\right)\left(\ln(n+2) - \ln(3n^3-1)\right) \sim$

$\sim \lim_{n\to\infty} \frac{\ln(n)}{\ln(n^4)} - \lim_{n\to\infty} \frac{\ln(n^3)}{\ln(n^4)} = \lim_{n\to\infty} \frac{\cancel{\ln(n)}}{4\cancel{\ln(n)}} - \lim_{n\to\infty} \frac{3\cancel{\ln(n)}}{4\cancel{\ln(n)}} = \frac{1}{4} - \frac{3}{4} = \frac{-2}{4} = \frac{-1}{2}$

$A = e^{-1/2}$

$\boxed{10}$ $\lim_{n\to\infty} \left[\cos\left(\frac{1}{\sqrt{n}}\right)\right]^n$

$\lim_{n\to\infty} A = B \to A = \ln B = e^B$

$\lim_{n\to\infty} \ln\left[\cos\left(\frac{1}{\sqrt{n}}\right)\right]^n = \lim_{n\to\infty} n\cdot\ln\left[\cos\left(\frac{1}{\sqrt{n}}\right)\right] \approx \left\| \ln\left[\cos\left(\frac{1}{\sqrt{n}}\right)\right] \sim \cos\left(\frac{1}{\sqrt{n}}\right) - 1 \right\| \sim \lim_{n\to\infty} n\cdot\left[\cos\left(\frac{1}{\sqrt{n}}\right) - 1\right] =$

$= \lim_{n\to\infty} -n\left[1 - \cos\left(\frac{1}{\sqrt{n}}\right)\right] \sim \left\| 1 - \cos(a_n) \sim \frac{a_n^2}{2} \right\| \sim \lim_{n\to\infty} -n\,\frac{(1/\sqrt{n})^2}{2} = \lim_{n\to\infty} -\cancel{n}\,\frac{1}{2\cancel{n}} = -\frac{1}{2}$

$A = e^{-1/2}$

$\boxed{11}$ $\lim_{n\to\infty} \left[1 + \tan^2\left(\frac{1}{n}\right)\right]^{\frac{1}{\sin^2(1/n)}}$

$\lim_{n\to\infty} A = B \to A = \ln B = e^B$

$\lim_{n\to\infty} \ln\left[1 + \tan^2\left(\frac{1}{n}\right)\right]^{\frac{1}{\sin^2(1/n)}} = \lim_{n\to\infty} \left(\frac{1}{\sin^2(1/n)}\right)\ln\left[1 + \tan^2\left(\frac{1}{n}\right)\right] \sim \left\| \begin{array}{l} \sin(1/n) \sim 1/n \\ \tan(1/n) \sim 1/n \\ \ln(a_n+1) \sim a_n \end{array} \right\| \sim$

$\sim \lim_{n\to\infty} \left(\frac{1}{(1/n)^2}\right)\left(\frac{1}{n}\right)^2 = \lim_{n\to\infty} \cancel{n^2}\,\frac{1}{\cancel{n^2}} = 1$

$A = e$

$\boxed{12}$ $\lim\limits_{n\to\infty} \frac{(-2)^n+3^n}{(-2)^{n+1}+3^{n+1}} = \lim\limits_{n\to\infty} \frac{\cancel{3^n}\left(\left(\frac{-2}{3}\right)^n+1\right)}{\cancel{3^{n+1}}\left(\left(\frac{-2}{3}\right)^{n+1}+1\right)} = \lim\limits_{n\to\infty} \frac{\left(\frac{-2}{3}\right)^n+1}{3\left(\left(\frac{-2}{3}\right)^{n+1}+1\right)} \sim \left\| \begin{matrix} -2/3<1 \\ (-2/3)^n \to 0 \\ (-2/3)^{n+1}\to 0 \end{matrix} \right\| \sim \frac{1}{3}$

$\boxed{13}$ Kalkulatu $\lim\limits_{n\to\infty}\left[\frac{n(n+1)(n+2)}{6\lambda(n+5)(n^2+1)}\right]^{5n-2}$ $\lambda\in\mathbb{R}e$ parametroaren arabera.

$$\lim_{n\to\infty}\left[\frac{n(n+1)(n+2)}{6\lambda(n+5)(n^2+1)}\right]^{5n-2} \sim \left\|\begin{matrix} n+1\sim n \\ n+2\sim n \\ n+5\sim n \\ n^2+1\sim n^2\end{matrix}\right\| \sim \lim_{n\to\infty}\left[\frac{\cancel{n\cdot n\cdot n}}{6\lambda\cdot \cancel{n\cdot n^2}}\right]^{5n-2} = \lim_{n\to\infty}\left(\frac{1}{6\lambda}\right)^{5n-2} = \left(\frac{1}{6\lambda}\right)^{\infty} = A$$

$\frac{1}{6\lambda}>1 \to A=\infty$ | $\lambda<\frac{1}{6}\to A=\infty$

$\frac{1}{6\lambda}<1\to A=0$ | $\lambda>\frac{1}{6}\to A=0$ | $\lambda=\frac{1}{6}\to A=1^{\infty}\to$ indeterminazioa

$\lambda=1/6$ denean:

$$\lim_{n\to\infty}\left[\frac{n(n+1)(n+2)}{(n+5)(n^2+1)}\right]^{5n-2}$$

$\lim\limits_{n\to\infty} A=B \quad\to A=\ln B=e^B$

$$\lim_{n\to\infty}\ln\left[\frac{n(n+1)(n+2)}{(n+5)(n^2+1)}\right]^{5n-2} = \lim_{n\to\infty}(5n-2)\ln\left[\frac{n(n+1)(n+2)}{(n+5)(n^2+1)}\right] \sim \left\|\begin{matrix}\ln(a_n)\sim a_n-1 \\ 5n-2\sim 5n\end{matrix}\right\| \sim \lim_{n\to\infty} 5n\left(\frac{n^3+3n^2+2n}{n^3+5n^2+n+5}-1\right)=$$

$$=\lim_{n\to\infty}5n\left(\frac{\cancel{n^3}+3n^2+2n-\cancel{n^3}-5n^2-n-5}{n^3+5n^2+n+5}\right) = \lim_{n\to\infty}5n\,\frac{-2n^2+n-5}{n^3+5n^2+n+5} \sim \left\|\frac{-2n^2+n-5}{n^3+5n^2+n+5}\sim\frac{-2n^2}{n^3}\right\| \sim \lim_{n\to\infty}5\cancel{n}\,\frac{-2}{\cancel{n}} =$$

$=-10$

$A=e^{-10}$

$\boxed{14}$ Kalkulatu $\lim\limits_{n\to\infty}\frac{(n+2)(an+1)^n}{(2+n)^{n+1}}$ $a\in\mathbb{R}$ parametroaren balioen arabera. $(a>0)$

$$\lim_{n\to\infty}\frac{(n+2)(an+1)^n}{(2+n)^{n+1}} = \lim_{n\to\infty}\frac{(an+1)^n}{(2+n)^n} = \lim_{n\to\infty}\left(\frac{an+1}{2+n}\right)^n \sim \left\|\begin{matrix}an+1\sim an \\ 2+n\sim n\end{matrix}\right\| \sim \lim_{n\to\infty}\left(\frac{a\cancel{n}}{\cancel{n}}\right)^n = a^n = A$$

$a>1\to A=\infty$

$a<1\to A=0$

$a=1\to A=1^{\infty}\to$ indeterminazioa

$a=1$ denean:

$$\lim_{n\to\infty}\frac{(n+2)(n+1)^n}{(2+n)^{n+1}} = \lim_{n\to\infty}\frac{(n+1)^n}{(n+2)^n} = \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n = A$$

$\lim\limits_{n\to\infty} A=B\to A=\ln B=e^B$

$$\lim_{n\to\infty}\ln\left(\frac{n+1}{n+2}\right)^n = \lim_{n\to\infty} n\cdot\ln\left(\frac{n+1}{n+2}\right) \sim \|\ln(a_n)\sim a_n-1\| \sim \lim_{n\to\infty} n\left(\frac{n+1}{n+2}-1\right) = \lim_{n\to\infty} n\left(\frac{\cancel{n}+1-\cancel{n}-2}{n+2}\right)=$$

$$=\lim_{n\to\infty} n\left(\frac{-1}{n+2}\right) = \lim_{n\to\infty}\frac{-n}{n+2} \sim \|n+2\sim n\| \sim \lim_{n\to\infty}\frac{-n}{n} = -1$$

$A=-1$