3) ALDAGAI ERREAL BAKARREKO FUNTZIO ERREALAK

1 Frogatu honako formula hauek:

A) $\cosh(ix) = \cos(x)$.

$$\cosh(ix) = \left\| \begin{array}{l}\text{Definizioa}\\ \text{erabiliz}\end{array} \right\| = \frac{e^{ix}+e^{-ix}}{2} = \frac{\cos(x)+\cancel{i\sin(x)}+\cos(x)-\cancel{i\sin(x)}}{2} = \frac{\cancel{2}\cos(x)}{\cancel{2}} = \cos(x)$$

B) $\tanh(ix) = i\tan(x)$

$$\tanh(ix) = \left\| \begin{array}{l}\text{Definizioa}\\ \text{erabiliz}\end{array} \right\| = \frac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}} = \frac{\cancel{\cos(x)}+i\sin(x)-\cancel{\cos(x)}+i\sin(x)}{\cos(x)+\cancel{i\sin(x)}+\cos(x)-\cancel{i\sin(x)}} = \frac{2i\sin(x)}{2\cos(x)} = i\tan(x)$$

c) $\sin(ix) = i\,sh(x)$

$$\sin(ix) = \left\| \begin{array}{l}\text{sinuaren era}\\ \text{esponentziala}\\ \text{erabiliz}\end{array} \right\| = \frac{e^{i(ix)}-e^{-i(ix)}}{2i} = \frac{e^{-x}-e^{x}}{2i} = \frac{-1}{i}\cdot\frac{e^{x}-e^{-x}}{2} = -(-i)\sinh(x) = i\,sh(x)$$

2 $sh(x) = \frac{1}{2\sqrt{2}}$ dela jakinda, $th(4x)$ kalkulatu

$$x = \text{argsh}\left(\frac{1}{2\sqrt{2}}\right) = \left\| \text{argsh}(x) = \ln(x+\sqrt{x^2+1}) \right\| = \ln\left(\frac{1}{2\sqrt{2}}+\sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2+1}\right) = \ln\left(\frac{1}{2\sqrt{2}}+\sqrt{\frac{9}{8}}\right) = \ln\left(\frac{1+3}{2\sqrt{2}}\right) =$$

$$= \ln\left(\frac{4}{2\sqrt{2}}\right) = \ln\left(\frac{2}{\sqrt{2}}\right) = \ln(\sqrt{2}) = \frac{1}{2}\ln(2)$$

$$th(4x) = th\left(4\frac{1}{2}\ln(2)\right) = th(2\ln(2)) = th(\ln(4)) = \frac{e^{\ln 4}-e^{-\ln 4}}{e^{\ln 4}+e^{-\ln 4}} = \frac{4-1/4}{4+1/4} = \frac{15/4}{17/4} = \frac{15}{17}$$

3 Kalkulatu $x \in \mathbb{R}$ non $sh^4(x) - 2ch^2(x) - 1 = 0$

$ch^2(x) - sh^2(x) = 1$ formula aplikatuz: $ch^2(x) = sh^2(x)+1$

$$sh^4(x) - 2(sh^2(x)+1) - 1 = 0 \rightarrow sh^4(x) - 2sh^2(x) - 3 = 0 \quad \| sh^2(x) = t \| \rightarrow t^2 - 2t - 3 = 0$$

$$t = \frac{2\pm\sqrt{2^2-4(-3)}}{2} = \frac{2\pm\sqrt{16}}{2} = \frac{2\pm 4}{2} < \begin{array}{l} 3 \rightarrow sh^2(x) = 3 \rightarrow sh(x) = \pm\sqrt{3} \\ \cancel{-1 \rightarrow sh^2(x) = -1} \end{array}$$

$$x = \text{argsh}(\pm\sqrt{3}) = \ln(\pm\sqrt{3}+\sqrt{3+1}) = \ln(\pm\sqrt{3}+\sqrt{4}) < \begin{array}{l} x = \ln(\sqrt{3}+2) \\ x = \ln(2-\sqrt{3}) \end{array}$$

4 Kalkulatu $x \in \mathbb{R}$ non $ch^4(x) - 2sh^2(x) - 10 = 0$

$ch^2(x) - sh^2(x) = 1 \longrightarrow ch^2(x) = 1 + sh^2(x)$

$$(sh^2(x)+1)^2 - 2sh^2(x) - 10 = 0 \longrightarrow sh^4(x) + \cancel{2sh^2(x)} + 1 - \cancel{2sh^2(x)} - 10 = 0 \rightarrow sh^4(x) - 9 = 0 \longrightarrow$$

$$sh^4(x) = 9 < \begin{array}{l} \cancel{sh^2(x) = -3} \\ sh^2(x) = 3 \rightarrow sh(x) = \pm\sqrt{3} \end{array}$$

$$x = \text{argsh}(\pm\sqrt{3}) = \ln(\pm\sqrt{3}+\sqrt{3+1}) = \ln(\pm\sqrt{3}+\sqrt{4}) < \begin{array}{l} x = \ln(\sqrt{3}+2) \\ x = \ln(2-\sqrt{3}) \end{array}$$

5 $sh(x) = \frac{1}{2\sqrt{6}}$ dela jakinda, $\coth(2x)$ kalkulatu.

$$x = \text{argsh}\left(\frac{1}{2\sqrt{6}}\right) = \ln\left(\frac{1}{2\sqrt{6}}+\sqrt{\left(\frac{1}{2\sqrt{6}}\right)^2+1}\right) = \ln\left(\frac{1}{2\sqrt{6}}+\sqrt{\frac{25}{(2\sqrt{6})^2}}\right) = \ln\left(\frac{1}{2\sqrt{6}}+\frac{5}{2\sqrt{6}}\right) = \ln\left(\frac{6}{2\sqrt{6}}\right) = \ln\left(\frac{3}{\sqrt{6}}\right) = \ln\left(\frac{\sqrt{6}}{2}\right)$$

$$\coth(2x) = \coth\left(2\ln\left(\frac{\sqrt{6}}{2}\right)\right) = \coth\left(\ln\left(\frac{6}{4}\right)\right) = \coth(\ln(3/2)) = \frac{e^{\ln(3/2)}+e^{-\ln(3/2)}}{e^{\ln(3/2)}-e^{-\ln(3/2)}} = \frac{3/2+2/3}{3/2-2/3} = \frac{9+4}{9-4} = \frac{13}{5}$$

$\boxed{6}$ Funtzio hiperbolikoen definizioak erabiliz, honako adierazpen hau sinplifikatu:

$$\frac{ch(\ln(x))+sh(\ln(x))}{ch(\ln(x))-sh(\ln(x))}=\frac{\frac{e^{\ln(x)}+e^{-\ln(x)}}{2}+\frac{e^{\ln(x)}-e^{-\ln(x)}}{2}}{\frac{e^{\ln(x)}+e^{-\ln(x)}}{2}-\frac{e^{\ln(x)}-e^{-\ln(x)}}{2}}=\frac{\frac{2e^{\ln(x)}}{\cancel{2}}}{\frac{2e^{-\ln(x)}}{\cancel{2}}}=\frac{e^{\ln(x)}}{e^{-\ln(x)}}=\frac{x}{1/x}=x^2$$

$\boxed{7}$ Zehaztu $x\in\mathbb{R}$ hurrengoa bete dadin: $\ln(sh^4(x)-4ch^2(x))=0$

Esponentzialak hartuz:

$$e^{\ln(sh^4(x)-4ch^2(x))}=0\longrightarrow sh^4(x)-4ch^2(x)=1\longrightarrow //ch^2(x)=sh^2(x)+1//\longrightarrow sh^4(x)-4(sh^2(x)+1)=1\longrightarrow$$

$$\longrightarrow sh^4(x)-4sh^2(x)-5=0\longrightarrow //t=sh^2(x)//\longrightarrow t^2-4t-5=0$$

$$t=\frac{4\pm\sqrt{4^2-4(-5)}}{2}=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm\sqrt{36}}{2}=\frac{4\pm6}{2}\begin{cases}\cancel{-1\rightarrow sh^2(x)\neq-1}\\5\rightarrow sh^2(x)=5\rightarrow sh(x)=\pm\sqrt{5}\end{cases}$$

$$x=\operatorname{argsh}(\pm\sqrt{5})=\ln(\pm\sqrt{5}+\sqrt{5+1})\begin{cases}\ln(\sqrt{5}+\sqrt{6})\\\ln(\sqrt{6}-\sqrt{5})\end{cases}$$

$\boxed{8}$ $$\lim_{x\to a}\frac{x^m-a^m}{x^p-a^p}=\frac{0}{0}=//L'Hopital//=\lim_{x\to a}\frac{mx^{m-1}}{px^{p-1}}=\lim_{x\to a}\frac{m}{p}x^{m-p}=\frac{m}{p}a^{m-p}$$

$\boxed{9}$ $$\lim_{x\to0}\frac{a-\sqrt{a^2-x^2}}{x^2}=\frac{0}{0}=//L'Hopital//=\lim_{x\to0}\frac{\frac{\cancel{x}}{\sqrt{a^2-x^2}}}{2\cancel{x}}=\lim_{x\to0}\frac{1}{2\sqrt{a^2-x^2}}=\frac{1}{2a}$$

$\boxed{10}$ $$\lim_{x\to0}\frac{tg(mx)-mx}{2x^2\cdot tg(mx)}=\frac{0}{0}\sim\lim_{x\to0}\frac{tg(mx)-mx}{2x^2(mx)}=\lim_{x\to0}\frac{tg(mx)-mx}{2x^3m}=\frac{0}{0}\overset{L'H}{=}\lim_{x\to0}\frac{\frac{m}{\cos^2(mx)}-m}{6x^2m}=$$

$$=\lim_{x\to0}\frac{\cancel{m}(1-\cos^2(mx))}{6x^2\cancel{m}\cos^2(mx)}=\lim_{x\to0}\frac{\sin^2(mx)}{6x^2\cos^2(mx)}\sim\lim_{x\to0}\frac{m^2\cancel{x^2}}{6\cancel{x^2}\cos^2(mx)}=\lim_{x\to0}\frac{m^2}{6\cos^2(mx)}=\frac{m^2}{6}$$

$\boxed{11}$ $$\lim_{x\to\pi/2}\frac{\ln(x-\pi/2)}{tg(x)}=//\nexists\lim_{x\to\pi/2^-}\ln(x-\pi/2)//=\lim_{x\to\pi/2^+}\frac{\ln(x-\pi/2)}{tg(x)}=\frac{\infty}{\infty}\overset{L'H}{=}\lim_{x\to\pi/2^+}\frac{\frac{1}{x-\pi/2}}{\frac{1}{\cos^2(x)}}=\lim_{x\to\pi/2^+}\frac{\cos^2(x)}{x-\pi/2}=\frac{0}{0}\overset{L'H}{=}$$

$$=\lim_{x\to\pi/2^+}\frac{-2\cos(x)\sin(x)}{1}=0$$

$\boxed{12}$ $$\lim_{x\to\pi/2}\frac{tg(x)}{tg(5x)}=\frac{\infty}{\infty}\overset{L'H}{=}\lim_{x\to\pi/2}\frac{\frac{1}{\cos^2(x)}}{\frac{5}{\cos^2(5x)}}=\lim_{x\to\pi/2}\frac{\cos^2(5x)}{5\cos^2(x)}=\frac{0}{0}\overset{L'H}{=}\lim_{x\to\pi/2}\frac{-10\cos(5x)\sin(5x)}{-10\cos(x)\sin(x)}=$$

$$=\lim_{x\to\pi/2}\frac{\cos(5x)\underbrace{\sin(5x)}_{1}}{\cos(x)\underbrace{\sin(x)}_{1}}=\lim_{x\to\pi/2}\frac{\cos(5x)}{\cos(x)}=\frac{0}{0}\overset{L'H}{=}\lim_{x\to\pi/2}\frac{-5\sin(5x)}{-\sin(x)}=5$$

$\boxed{13}$ $$\lim_{x\to0}\left[\left(\frac{1}{x}\right)^2-\frac{cotg(x)}{x}\right]=\lim_{x\to0}\left[\frac{1}{x^2}-\frac{\cos(x)}{x\sin(x)}\right]=\lim_{x\to0}\left[\frac{\sin(x)-x\cos(x)}{x^2\sin(x)}\right]\sim\lim_{x\to0}\frac{\sin(x)-x\cos(x)}{x^2(x)}=\frac{0}{0}$$

$$\overset{L'H}{=}\lim_{x\to0}\frac{\cancel{\cos(x)}-\cancel{\cos(x)}-x(-\sin(x))}{3x^2}=\lim_{x\to0}\frac{\sin(x)}{3x}\sim\lim_{x\to0}\frac{\cancel{x}}{3\cancel{x}}=\frac{1}{3}$$

$\boxed{14}$ $$\lim_{x\to\infty}x\left[\sqrt[3]{1+\frac{a}{x}}-1\right]\sim//a_n-1\sim\ln(a_n)//\sim\lim_{x\to\infty}x\ln\left(\sqrt[3]{1+\frac{a}{x}}\right)=\lim_{x\to\infty}x\frac{1}{3}\ln\left(1+\frac{a}{x}\right)\sim//\ln(a_n+1)\sim a_n//\sim$$

$$\sim\lim_{x\to\infty}\cancel{x}\frac{1}{3}\frac{a}{\cancel{x}}=\frac{a}{3}$$

15

$$\lim_{x\to 0} \left(\sqrt{1+x^2}\right)^{\frac{2}{\arcsin(x^2)}}$$

$$\lim_{x\to 0} A = B \to A = \ln B = e^{B}$$

$$\lim_{x\to 0} \ln\left(\sqrt{1+x^2}\right)^{\frac{2}{\arcsin(x^2)}} = \lim_{x\to 0}\left(\frac{2}{\arcsin(x^2)}\right)\ln\left(\sqrt{1+x^2}\right) = \lim_{x\to 0}\left(\frac{\cancel{2}}{\arcsin(x^2)}\right)\frac{1}{\cancel{2}}\ln(1+x^2) \sim$$

$$\sim \| \ln(a_n+1)\sim a_n \| \sim \lim_{x\to 0}\frac{x^2}{\arcsin(x^2)} \sim \lim_{x\to 0}\frac{x^2}{x^2} = 1$$

$$A = e$$

16

$$\lim_{x\to\infty}\frac{x\left(x^{1/x}-1\right)}{\ln(x)} = \frac{\infty}{\infty} \sim \left\| \lim_{x\to\infty}^{(*)} x^{1/x} = \underline{\underline{1}} \to a_n - 1 \sim \ln(a_n) \right\| \sim \lim_{x\to\infty}\frac{x\ln\left(x^{1/x}\right)}{\ln(x)} = \lim_{x\to\infty}\frac{\cancel{x}\frac{1}{\cancel{x}}\cancel{\ln(x)}}{\cancel{\ln(x)}} = \underline{1}$$

(*)

$$\lim_{x\to\infty} x^{1/x} = \left\| \begin{matrix} A = x^{1/x} \\ B = \ln A \end{matrix} \right\| = \lim_{x\to\infty}\ln\left(x^{1/x}\right) = \lim_{x\to\infty}\frac{1}{x}\ln(x) = \| \ln(x) << x \| = 0 \longrightarrow A = e^0 = \underline{\underline{1}}$$

17

$$\lim_{x\to 0}\frac{\ln(\sin(mx))}{\ln(\sin(x))} = \lim_{x\to 0^+}\frac{\ln(\sin(mx))}{\ln(\sin(x))} \sim \lim_{x\to 0^+}\frac{\ln(mx)}{\ln(x)} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0^+}\frac{\frac{m}{mx}}{\frac{1}{x}} =$$

$$= \lim_{x\to 0^+}\frac{mx}{mx} = 1$$

18

$$\lim_{x\to 0}\frac{a^x - b^x}{c^x - d^x} = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 0}\frac{\cancel{x}(a^{x-1}-b^{x-1})}{\cancel{x}(c^{x-1}-d^{x-1})} = \lim_{x\to 0}\frac{a^x\frac{1}{a} - b^x\frac{1}{b}}{c^x\frac{1}{c} - d^x\frac{1}{d}} = \lim_{x\to 0}\frac{\overbrace{a^x}^{1}\ln(a) - \overbrace{b^x}^{1}\ln(b)}{\underbrace{c^x}_{1}\ln(c) - \underbrace{d^x}_{1}\ln(d)} =$$

$$= \frac{\ln(a) - \ln(b)}{\ln(c) - \ln(d)} = \frac{\ln(a/b)}{\ln(c/d)}$$

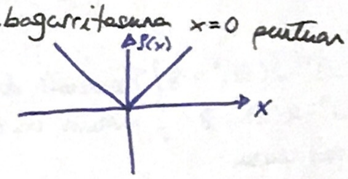
19

$$\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right) = \lim_{x\to 1}\left(\frac{x\ln(x) - x + 1}{(x-1)\ln(x)}\right) = \frac{0}{0} \sim \lim_{x\to 1}\left(\frac{x\ln(x) - x + 1}{(x-1)^2}\right) = \frac{0}{0} \overset{L'H}{=} \lim_{x\to 1}\frac{\ln(x) + 1 - 1}{2(x-1)} \sim$$

$$\sim \lim_{x\to 1}\frac{\cancel{x-1}}{2(\cancel{x-1})} = \frac{1}{2}$$

20 Aztertu honako funtzio honen jarraitasuna eta deribagarritasuna $x=0$ puntuan

$$f(x) = |x| \longrightarrow f(x)\begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



Jarraitasuna:

$$\left.\begin{matrix} \lim_{x\to 0^-} -x = 0 \\ \lim_{x\to 0^+} x = 0 \end{matrix}\right\}$$ Bi limiteak berdinak dira, beraz, jarraitua da.

Deribagarritasuna:

$$\left.\begin{matrix} f'(0^-) = \lim_{\Delta x\to 0^-}\frac{f(0+\Delta x) - f(0)}{\Delta x} = \frac{-\Delta x - 0}{\Delta x} = -1 \\ f'(0^+) = \lim_{\Delta x\to 0^+}\frac{f(0+\Delta x) - f(0)}{\Delta x} = \frac{\Delta x - 0}{\Delta x} = 1 \end{matrix}\right\}$$ Desberdinak dira, beraz, ez da deribagarria

21 Kalkulatu $\frac{dy}{dx}$ Katearen erregela erabiliz, non $y=\frac{u-1}{u+1}$ eta $u=x^2$.

$y \to u \to x$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\frac{(u+1)-(u-1)}{(u+1)^2}\cdot 2x=\frac{4x}{(u+1)^2}=\|u=x^2\|=\frac{4x}{(x^2+1)^2}$$

22 Alderantzizko funtzioaren deribatua: $y=\arcsin(x)$

$x=\sin(y)$

$$\frac{dx}{dy}=\cos(y)\longrightarrow\frac{dy}{dx}=\frac{1}{dx/dy}=\frac{1}{\cos(y)}=\|\cos^2(x)+\sin^2(x)=1\|=\frac{1}{\sqrt{1-\sin^2(y)}}=\|\sin(y)=x\|=\frac{1}{\sqrt{1-x^2}}$$

23 Kalkulatu $x^2(x+y)=a^2(x-y)$ Kurbaren ukitzailearen ekuazioa jatorrian $(0,0)$

Ekuazioa inplizituki deribatuz:

$$2x(x+y)+x^2(1+y')=a^2(1-y')$$

Jatorrian $(0,0)$:

$$2(0)(0+0)+(0)^2(1+y')=a^2(1-y')\to 0=a^2(1-y')\to y'=1$$ (Zuzen ukitzailearen malda.

Zuzen ukitzailearen ekuazioa:

$$y-y_0=y'(x-x_0)\to y-0=1(x-0)\to \boxed{y=x}$$

24 $y=x^{\sin(x)} \to y'$?

Deribazio logaritmikoa erabiliz:

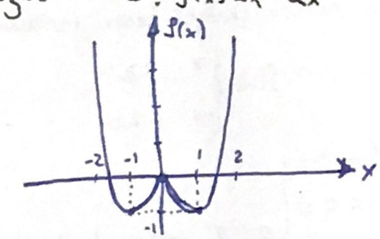
$$\ln(y)=\sin(x)\cdot\ln(x)$$

$$\frac{y'}{y}=\cos(x)\ln(x)+\sin(x)\frac{1}{x}\longrightarrow \boxed{y'=y\left(\cos(x)\ln(x)+\sin(x)\frac{1}{x}\right)}$$

25 Frogatu Rolle-ren teorema $[-2,2]$ tartean honako funtzio honako: $f(x)=x^4-2x^2$

Jarraitasuna:

$$\left.\begin{array}{l} f(-2)=(-2)^4-2(-2)^2=8 \\ f(2)=(2)^4-2(2)^2=8 \end{array}\right\}$$ Berdinak dira, beraz, jarraia da $[-2,2]$ tartean



Deribagarritasuna:

$$f'(x)=4x^3-4x=4x(x^2-1)=4x(x+1)(x-1)$$

$$f'(x)=0\begin{cases}x=0\\x=1\\x=-1\end{cases}$$

26 Kalkulatu $\frac{dy}{dx}$ eta $\frac{d^2y}{dx}$ non $y=x+\arctan(y)$ den

Inplizituki deribatuz:

$$y'=\frac{y'}{1+y^2}+1$$

y' askatuz:

$$y'-\frac{y'}{1+y^2}=1\longrightarrow y'\left(1-\frac{1}{1+y^2}\right)=1\longrightarrow y'\left(\frac{\cancel{1}+y^2-\cancel{1}}{1+y^2}\right)=1\longrightarrow y'=\frac{1+y^2}{y^2}$$

$$\boxed{y''}=\frac{d}{dx}[y']=\frac{d}{dx}\left[\frac{1+y^2}{y^2}\right]=\frac{2y\cdot y'\cdot y^2-(1+y^2)2y\cdot y'}{y^4}=\frac{\cancel{2y^3y'}-\cancel{2y^3y'}-2yy'}{y^4}=\frac{-2y'}{y^3}=\frac{-2}{y^3}\left(\frac{1+y^2}{y^2}\right)=\boxed{\frac{-2-2y^2}{y^5}}$$

27 $y=f(x)$ dela jakinda, aurkitu $x^3-3xy^2+y^3=1$ kurbaren zuzen ukitzailearen malda $(2,-1)$ puntuan.

Ekuazioa inplizituki deribatuz:

$3x^2-3y^2-6xyy'+3y^2y'=0 \longrightarrow x^2-y^2-2xyy'+y^2y'=0 \longrightarrow y'(-2xy+y^2)=y^2-x^2 \longrightarrow y'=\dfrac{y^2-x^2}{-2xy+y^2}$

$(2,-1)$ puntuan, $x=2$, $y=-1$:

$y'=\dfrac{(-1)^2-(2)^2}{-2(2)(-1)+(-1)^2}=\dfrac{1-4}{4+1}=\boxed{\dfrac{-3}{5}}$

28 Izan bedi $(xy)^{\sin(x)}=y^x$ funtzioa. Kalkulatu y' $(y=f(x))$

Logaritmoak hartuz:

$\ln((xy)^{\sin(x)})=\ln(y^x) \longrightarrow \sin(x)\ln(xy)=x\ln(y)$

Deribatuz:

$\cos(x)\ln(xy)+\sin(x)\dfrac{y+xy'}{xy}=\ln(y)+x\dfrac{y'}{y}$

$\cos(x)\ln(xy)+\dfrac{\sin(x)\cancel{y}}{x\cancel{y}}+\dfrac{\sin(x)\cancel{x}y'}{\cancel{x}y}=\ln(y)+\dfrac{xy'}{y}$

$\cos(x)\ln(xy)+\dfrac{\sin(x)y'-xy'}{y}+\dfrac{\sin(x)}{x}=\ln(y)$

$y'\left(\dfrac{\sin(x)-x}{y}\right)=\ln(y)-\cos(x)\ln(xy)-\dfrac{\sin(x)}{x}$

$$\boxed{y'=\left(\dfrac{y}{\sin(x)-x}\right)\left(\ln(y)-\cos(x)\ln(xy)-\dfrac{\sin(x)}{x}\right)}$$

29 Aurkitu $\dfrac{3x^2}{y}-\dfrac{y^2}{x^2}=xy^2-\dfrac{29}{4}$ kurbaren ukitzailearen malda $(1,2)$ puntuan.

Inplizituki deribatuz:

$\dfrac{6xy-3x^2y'}{y^2}-\dfrac{2yy'x^2-2xy^2}{x^4}=y^2+2xyy' \longrightarrow \dfrac{6x^5y-3x^6y'-2x^2y^3y'+2xy^4}{x^4y^2}=y^2+2xyy' \longrightarrow$

$6x^5y+2xy^4-x^4y^4=2x^5y^3y'+3x^6y'+2x^2y^3y' \longrightarrow y'=\dfrac{6x^5y+2xy^4-x^4y^4}{2x^5y^3+3x^6+2x^2y^3} \longrightarrow y'=\dfrac{6x^4y+2y^4-x^3y^4}{2x^4y^3+3x^5+2xy^3}$

$(1,2)$ puntuan, $x=1$ eta $y=2$

$y'=\dfrac{6(1)^4(2)+2(2)^4-(1)^3(2)^4}{2(1)^4(2)^3+3(1)^5+2(1)(2)^3}=\dfrac{12+32-16}{16+3+16}=\dfrac{28}{35} \longrightarrow \boxed{y'=\dfrac{4}{5}}$

30 Lortu $\dfrac{dy}{dx}$ non $y=(x^2+3)^{5x-1}$ den.

Logaritmoak hartuz:

$\ln(y)=\ln((x^2+3)^{5x-1}) \rightarrow \ln(y)=(5x-1)\ln(x^2+3)$

Deribatuz:

$\dfrac{y'}{y}=5\ln(x^2+3)+(5x-1)\dfrac{2x}{x^2+3}$

$y'=(x^2+3)^{5x-1}\left[5\ln(x^2+3)+\dfrac{2x(5x-1)}{x^2+3}\right]$

31 Kalkulatu $\frac{dy}{dx}$ non $x^4+x^2y^3-y^5=2x+1$

Inplizituki deribatuz:

$4x^3+2xy^3+3x^2y^2y'-5y^4y'=2 \rightarrow 5y^4y'+3x^2y^2y'=4x^3+2xy^3-2 \rightarrow y'(5y^4+3x^2y^2)=4x^3+2xy^3-2$

$$\boxed{y'=\frac{4x^3+2xy^3-2}{5y^4+3x^2y^2}}$$

32 Kalkulatu $\frac{dy}{dx}$ non $(y+5)^x=x^{\cos(y)}$ den.

Logaritmoak aplikatuz:

$\ln((y+5)^x)=\ln(x^{\cos(y)}) \rightarrow x\ln(y+5)=\cos(y)\ln(x)$

$\ln(y+5)+x\frac{y'}{y+5}=-y'\sin(y)\ln(x)+\cos(y)\frac{1}{x}$

$y'\left(\frac{x+(y+5)\sin(y)\ln(x)}{y+5}\right)=\frac{\cos(y)-x\ln(y+5)}{x}$

$$\boxed{y'=\frac{(y+5)\cos(y)-x(y+5)\ln(y+5)}{x^2+x(y+5)\sin(y)\ln(x)}}$$

33 Kalkulatu a eta b honako funtzio hau jarraia eta deribagarria izan dadin.

$$f(x)\begin{cases} x^2+2 & x\leq 0 \\ \sqrt{ax+b} & 0<x\leq 2 \\ \frac{-x}{2\sqrt{2}}+\frac{3}{\sqrt{2}} & x>2 \end{cases}$$

Alboko limiteak $x=0$ puntuan

$$\left.\begin{aligned} \lim_{x\to 0^+}(x^2+2)&=2 \\ \lim_{x\to 0^-}\sqrt{ax+b}&=\sqrt{b} \end{aligned}\right\} \text{Jarraia izan dadin, } \sqrt{b}=2 \text{ izan behar da, beraz, } \boxed{b=4}$$

Alboko limiteak $x=2$ puntuan

$$\left.\begin{aligned} \lim_{x\to 2^-}\sqrt{ax+b}&=\sqrt{2a+4} \\ \lim_{x\to 2^+}\left(\frac{-x}{2\sqrt{2}}+\frac{3}{\sqrt{2}}\right)&=\frac{-2+6}{2\sqrt{2}}=\frac{2}{\sqrt{2}} \end{aligned}\right\} \sqrt{2a+4}=\frac{2}{\sqrt{2}} \rightarrow 2a+4=\frac{4}{2} \rightarrow 2a=-2 \rightarrow \boxed{a=-1}$$

Beraz, $a=-1$ eta $b=4$ izanik:

$$f(x)\begin{cases} x^2+2 & x\leq 0 \\ \sqrt{4-x} & 0<x\leq 2 \\ \frac{-x}{2\sqrt{2}}+\frac{3}{\sqrt{2}} & 2<x \end{cases}$$

Deribatuz:

$$f'(x)\begin{cases} 2x & x\leq 0 \\ \frac{-1}{2\sqrt{4-x}} & 0<x\leq 2 \\ \frac{-1}{2\sqrt{2}} & 2<x \end{cases}$$

35 Aztertu $f(x)=\sqrt{x}$ funtzioaren deribagarritasuna $[0,\infty)$ tartean, $(0,\infty)$ tartean deribagarria da?

$$f'(x)=\frac{1}{2\sqrt{x}}$$

$f'(0^+)$?

$$f'(0^+)=\lim_{\Delta x\to 0^+}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0^+}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}=\frac{\sqrt{0+\Delta x}-\sqrt{0}}{\Delta x}=\frac{\sqrt{\Delta x}}{\Delta x}=\frac{1}{\Delta x}=\infty$$

Ez da deribagarria.

$[0,\infty)$ $\exists f'(0^+)$ baina infinitua da.

36 Aztertu honako funtzio horen deribagarritasuna $x=0$ puntuan:

$$f(x)\begin{cases}\sin(x) & x>0\\ 0 & x=0\\ x+x^3 & x<0\end{cases}\longrightarrow f'(x)\begin{cases}\cos(x) & x>0\\ 0 & x=0\\ 1+3x^2 & x<0\end{cases}$$

$x=0$ puntuan deribagarria?

$$f'(0^-)=\lim_{\Delta x\to 0^-}\frac{f(\Delta x+x)-f(x)}{\Delta x}=\frac{(x+\Delta x)+(x+\Delta x)^3-x-x^3}{\Delta x}=\frac{\Delta x+\Delta x^3}{\Delta x}=\Delta x^2=0$$

$$f'(0^+)=\lim_{\Delta x\to 0^+}\frac{f(\Delta x+x)-f(x)}{\Delta x}=\frac{\sin(x+\Delta x)-\sin(x)}{\Delta x}=\frac{\sin(\Delta x)-0}{\Delta x}=0$$

$f'(0^-)=f'(0^+)$ beraz, deribagarria da.

37 Aztertu honako funtzio horen jarraitasuna eta deribagarritasuna $x=0$ puntuan

$$f(x)\begin{cases}x^3\sin\left(\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$$

Jarraitasuna $x=0$ puntuan

$$\left.\begin{array}{l}\lim_{x\to 0^-} x^3\sin\left(\frac{1}{x}\right)=0\\ \lim_{x\to 0^+} x^3\sin\left(\frac{1}{x}\right)=0\end{array}\right\}$$ Limiteak berdinak dira, beraz, jarraia da.

Deribagarritasuna $x=0$ puntuan

$$f(0^-)=\lim_{\Delta x\to 0^-}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{(x+\Delta x)^3\sin(1/(x+\Delta x))-x^3\sin(1/x)}{\Delta x}=\frac{\Delta x^{\cancel{3}\,2}\sin(1/\Delta x)-0}{\cancel{\Delta x}}=0$$

Limiteak berdinak direnez, funtzioa deribagarria da $x=0$ puntuan

38 Aztertu honako funtzio honen jarraitasuna eta deribagarritasuna:

$$f(x)\begin{cases} e^x & x>0 \\ 1 & x=0 \\ 1+x+\frac{x^2}{2} & x<0 \end{cases}$$

Jarraitasuna $x=0$ puntuan

$$\left.\begin{aligned}\lim_{x\to 0^-}\left(1+x+\frac{x^2}{2}\right)=1\\ \lim_{x\to 0^+} e^x = 1\end{aligned}\right\}$$ Albo limiteak berdinak dira, beraz, jarraia da $x=0$ puntuan

Deribagarritasuna $x=0$ puntuan

$$f'(0^-)=\lim_{\Delta x\to 0^-}\frac{f(\Delta x+x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0^-}\frac{\cancel{1}+(x+\Delta x)+[(x+\Delta x)^2/2]-\cancel{1}-x-(x^2/2)}{\Delta x}=\lim_{\Delta x\to 0^-}\frac{\Delta x+\frac{\Delta x^2}{2}}{\Delta x}=\lim_{\Delta x\to 0^-}\frac{2\Delta x+\Delta x^2}{2\Delta x}=$$

$$=\lim_{\Delta x\to 0^-}\frac{2+\Delta x}{2}=\frac{2}{2}=1$$

$$f'(0^+)=\lim_{\Delta x\to 0^+}\frac{e^{x+\Delta x}-e^x}{\Delta x}=\lim_{\Delta x\to 0^+}\frac{e^{\Delta x}}{\Delta x}=0$$

$f'(0^-)\neq f'(0^+)$ beraz ez da deribagarria.

39 Izan bedi $y=(x+\sqrt{x^2+1})^5$ funtzioa.

A) Frogatu $y'=\frac{5y}{\sqrt{x^2+1}}$ dela.

$$y'=5(x+\sqrt{x^2+1})^4\cdot\left(1+\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x\right)=5(x+\sqrt{x^2+1})^4\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)=\frac{5(x+\sqrt{x^2+1})^5}{\sqrt{x^2+1}}=\frac{5y}{\sqrt{x^2+1}}$$

B) Aurreko berdintasuna erabiliz $\left(y'=\frac{5y}{\sqrt{x^2+1}}\right)$ frogatu y funtzioak honako ekuazio hau betetzen duela: $(1+x^2)y''+xy'-25y=0$

$$y''=\frac{5y'\sqrt{x^2+1}-5y\frac{x}{\sqrt{x^2+1}}}{x^2+1}=\frac{5y'(x^2+1)-5yx}{(x^2+1)\sqrt{x^2+1}}=\frac{5\frac{5y}{\sqrt{x^2+1}}(x^2+1)-5yx}{(x^2+1)\sqrt{x^2+1}}=\frac{25y(x^2+1)-5xy\sqrt{x^2+1}}{(x^2+1)^2}$$

Ekuazioan ordezkatuz:

$$\cancel{(1+x^2)}\frac{25y(x^2+1)-5xy\sqrt{x^2+1}}{(x^2+1)^{\cancel{2}}}+x\frac{5y}{\sqrt{x^2+1}}-25y=0$$

$$\frac{\cancel{25y(x^2+1)}-\cancel{5xy\sqrt{x^2+1}}}{(x^2+1)}+\frac{\cancel{5xy\sqrt{x^2+1}}}{(x^2+1)}-\frac{\cancel{25y(x^2+1)}}{(x^2+1)}=\frac{0}{x^2+1}=0$$

40 $f(x) = (x-1)\sqrt[3]{x^2} = (x-1)x^{2/3}$

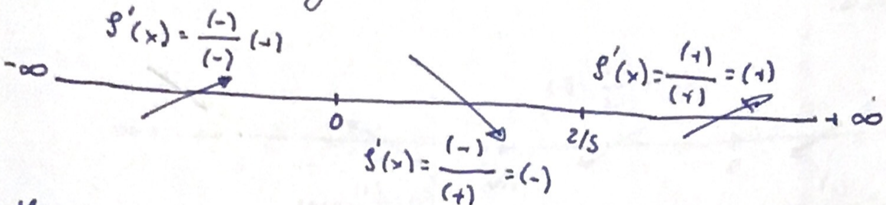
Puntu Kritikoak ($f'(x)=0$)

$$f'(x) = x^{2/3} + (x-1)\frac{2}{3}x^{-1/3} = \frac{2(x-1)+x^{2/3}\cdot 3x^{1/3}}{3x^{1/3}} = \frac{2x-2+3x}{3x^{1/3}} = \frac{5x-2}{3x^{1/3}}$$

$$f'(x)=0 \longrightarrow \frac{5x-2}{3x^{1/3}}=0 \longrightarrow 5x-2=0 \longrightarrow 5x=2 \longrightarrow \boxed{x=2/5}$$

$$\nexists f'(x) \longrightarrow 3x^{1/3}=0 \longrightarrow \underline{x=0}$$

Puntu Kritikoak aztertu:



Maximo erlatiboa $x=0$ puntuan $\longrightarrow (0,0)$

Minimo erlatiboa $x=2/5$ puntuan $\longrightarrow \left(2/5, -\frac{3}{5}\cdot\sqrt[3]{4/25}\right)$

41 $f(x)=\sqrt[3]{(x^2-4)^2} = (x^2-4)^{2/3}$

<u>Mutur erlatiboak:</u>

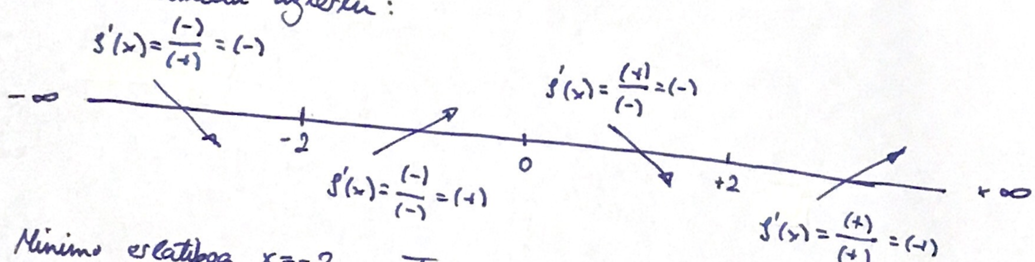
Puntu Kritikoak:

$$f'(x) = \frac{2\cdot 2x}{3\sqrt[3]{x^2-4}} = \frac{4x}{3\sqrt[3]{x^2-4}}$$

$$f'(x)=0 \longrightarrow \frac{4x}{3\sqrt[3]{x^2-4}}=0 \longrightarrow 4x=0 \longrightarrow \boxed{x=0}$$

$$\nexists f'(x) \longrightarrow 3\sqrt[3]{x^2-4}=0 \longrightarrow x^2-4=0 \longrightarrow x^2=4 \longrightarrow \underline{x=\pm 2}$$

Puntu Kritikoak aztertu:



Minimo erlatiboa $x=-2$ puntuan $\longrightarrow (-2,0)$

Maximo erlatiboa $x=0$ puntuan $\longrightarrow (0, 2\sqrt[3]{2})$

Minimo erlatiboa $x=2$ puntuan $\longrightarrow (2,0)$

<u>Mutur absolutuak $[0,3]$ tartean</u>

$f(0) = \sqrt[3]{16}$

$f(3) = \sqrt[3]{(9-4)^2} = \sqrt[3]{25} \longrightarrow$ Maximo absolutua $(3, \sqrt[3]{25})$

Minimo absolutua $(2,0)$

$\boxed{42}$ $f(x)=\frac{(x-a)(x-b)}{x}$

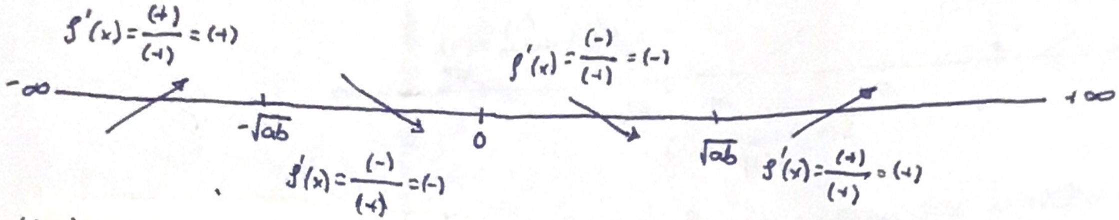
Puntu kritikoak:

$$f'(x)=\frac{((x-b)+(x-a))x-(x-a)(x-b)}{x^2}=\frac{\cancel{2}x^2\cancel{-ax}\cancel{-bx}\cancel{-x^2}\cancel{+ax}\cancel{+bx}-ab}{x^2}=\frac{x^2-ab}{x^2}$$

$$f'(x)=0 \longrightarrow \frac{x^2-ab}{x^2}=0 \longrightarrow x^2-ab=0 \longrightarrow \boxed{x=\pm\sqrt{ab}}$$

$$\nexists f'(x) \longrightarrow x^2=0 \longrightarrow \underline{x=0}$$

Aztertu puntu kritikoak:



Maximo erlatiboa: $(-\sqrt{ab}, \ldots$

Minimo erlatiboa: $(+\sqrt{ab}, \ldots$