BILBOKO INGENIARITZA ESKOLA
ESCUELA DE INGENIERÍA DE BILBAO

MATEMATIKA APLIKATUA

KUDEAKETAREN ETA INFORMAZIO SISTEMEN INFORMATIKAREN INGENIARITZAKO GRADUA

ANALISI MATEMATIKOA ***2019ko urtarrilaren 15a***

A) Ebatzi honako ekuazio hau eremu konplexuan:

$$z^2-(3-i)z+4=0$$

(puntu 1)

B) Kalkulatu honako segida honen limitea, a parametroaren balioen arabera ($a\in\mathbb{R}$):

$$\lim_{n\to\infty}\frac{1}{n^a}\sin\left(\frac{n^2+1}{n}\right)$$

(puntu 1)

C) Aztertu honako serie honen konbergentzia:

$$\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}$$

(puntu 1)

D) Kalkulatu $x\in\mathbb{R}$ non:

$$\ln\left(\mathrm{sh}^2x-5\mathrm{ch}x+8\right)=0$$

(puntu 1)

E) Kalkulatu $y(x)=x^{\sin x}$ funtzioaren Taylor-en garapena $a=\frac{\pi}{2}$ puntuan

← 2. gradoko garapena

(2 puntu)

F) Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua:

$$z(x,y)=\frac{\ln\left[\left(16-x^2-y^2\right)\left(x^2+y^2-4\right)\right]}{xy\sqrt{\frac{x^2}{9}+y^2-1}}$$

(2 puntu)

G) Kalkulatu $f(x,y)=x+y$ funtzioaren mutur erlatiboak $x^2+2y^2=1$ elipsean

(2 puntu)

2019 URTARRILA

A) Ebatzi honako ekuazio hau eremu Konplexuan:

$z^2-(3-i)z+4=0$

$z=\frac{3-i\pm\sqrt{(3-i)^2-4\cdot 4}}{2}=\frac{3-i\pm\sqrt{9-1-6i-16}}{2}=\frac{3-i\pm\sqrt{-8-6i}}{2}$ ⊛

$\sqrt{-8-6i}=a+bi \longrightarrow -8-6i=a^2-b^2+2abi$

$\begin{cases} -8=a^2-b^2 \longrightarrow -8=9/b^2-b^2 \rightarrow b^4-8b^2-9=0 \rightarrow b^2=\frac{8\pm\sqrt{64+36}}{2}=\frac{8\pm 10}{2} \begin{cases} b^2=9 \rightarrow \underline{b=\pm 3} \\ b^2=-1 \ \times \end{cases} \\ -6=2abi \rightarrow a=-3/b \rightarrow a=\pm 1 \end{cases}$

⊛

$z=\frac{3-i\pm(1-3i)}{2}\begin{cases}\boxed{z=2-2i}\\ \boxed{z=1+i}\end{cases}$

B) Kalkulatu honako segida honen limitea, a parametroaren arabera ($a\in\mathbb{R}$):

$\lim_{n\to\infty}\frac{1}{n^a}\sin\left(\frac{n^2+1}{n}\right)$

$\lim_{n\to\infty}\frac{1}{n^a}=\begin{cases}1, & a=0\\ 0, & a>0\\ \infty, & a<0\end{cases} \longrightarrow \begin{cases}\lim_{n\to\infty} 1\cdot\sin\left(\frac{n^2+1}{n}\right)\sim\lim_{n\to\infty}\frac{n^2+1}{n}\sim\lim_{n\to\infty}\frac{n^2}{n}=\infty \rightarrow \nexists \quad a=0\\ 0\cdot\lim_{n\to\infty}\sin\left(\frac{n^2+1}{n}\right)=0\rightarrow a>0\\ \infty\cdot\lim_{n\to\infty}\sin\left(\frac{n^2+1}{n}\right)=\infty\rightarrow\nexists \quad a<0\end{cases}$

$\lim_{n\to\infty}\frac{1}{n^a}\sin\left(\frac{n^2+1}{n}\right)=\begin{cases}0 \text{ baldin } a>0\\ \nexists \text{ baldin } a\leq 0\end{cases}$

C) Aztertu honako serie honen Konbergentzia:

$\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}=\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ Serie alternatua

Leibnitz-en irizpidea erabiliz:

- $\lim_{n\to\infty}a_n=0 \rightarrow \lim_{n\to\infty}\frac{(-1)^n}{n^2}=0$ ✓
- $|a_{n+1}|\leq|a_n| \rightarrow \frac{1}{(n+1)^2}\leq\frac{1}{n^2}$ ✓

Bi baldintzak betetzen dira, beraz, Konbergentea da

D) Kalkulatu $x\in\mathbb{R}$ non:

$\ln(sh^2(x)-5ch(x)+8)=0 \longrightarrow e^{\ln(sh^2(x)-5ch(x)+8)}=e^0 \rightarrow sh^2(x)-5ch(x)+8=1$

$ch^2(x)-sh^2(x)=1 \rightarrow sh^2(x)=ch^2(x)-1$

$ch^2(x)-1-5ch(x)+8-1=0 \rightarrow ch^2(x)-5ch(x)+6=0$

$ch(x)=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm 1}{2}\begin{cases}3\\2\end{cases}$

$x=\text{argch}(x)\begin{cases}\ln(3\pm\sqrt{3^2-1}) \rightarrow \boxed{x=\ln(3\pm 2\sqrt{2})}\\ \ln(2\pm\sqrt{2^2-1}) \longrightarrow \boxed{x=\ln(2\pm\sqrt{3})}\end{cases}$

E) Kalkulatu $y(x)=x^{\sin(x)}$ funtzioaren Taylor-en garapena, $a=\pi/2$ puntuan (2. gradeko garapena)

$$\ln(y)=\ln(x^{\sin(x)})=\sin(x)\ln(x)$$

$$\frac{y'}{y}=\cos(x)\ln(x)+\sin(x)\frac{1}{x}$$

$$y'=x^{\sin(x)}\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)\rightarrow y'\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\left(0+\frac{2}{\pi}\right)=1$$

$$y''=x^{\sin(x)}\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)^2+x^{\sin(x)}\left(-\sin(x)\ln(x)+\cos(x)\frac{1}{x}+\frac{\cos(x)x-\sin(x)}{x^2}\right)$$

$$y''=x^{\sin(x)}\left(\cos(x)\ln(x)+\frac{\sin(x)}{x}\right)^2+x^{\sin(x)}\left(\frac{2\cos(x)}{x}-\frac{\sin(x)}{x^2}-\sin(x)\ln(x)\right)$$

$$y''\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\left(0+\frac{2}{\pi}\right)^2+\frac{\pi}{2}\left(-\frac{4}{\pi^2}-\ln\left(\frac{\pi}{2}\right)\right)=\cancel{\frac{4\pi}{2\pi^2}}-\cancel{\frac{4\pi}{2\pi^3}}-\frac{\pi}{2}\ln\left(\frac{\pi}{2}\right)=-\frac{\pi}{2}\ln\left(\frac{\pi}{2}\right)$$

$$\boxed{f(x)=\frac{\pi}{2}+\left(x-\frac{\pi}{2}\right)-\frac{\pi}{4}\ln\left(\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)^2}$$

F) Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua:

$$z(x,y)=\frac{\ln[(16-x^2-y^2)(x^2+y^2-4)]}{xy\sqrt{\frac{x^2}{9}+y^2-1}}$$

1) Logaritmoaren argumentua $>0 \rightarrow (16-x^2-y^2)(x^2+y^2-4)>0$

1) $16-x^2-y^2>0 \rightarrow x^2+y^2<16 \rightarrow$ Zirkunferentzia $C(0,0)$, $r=4$

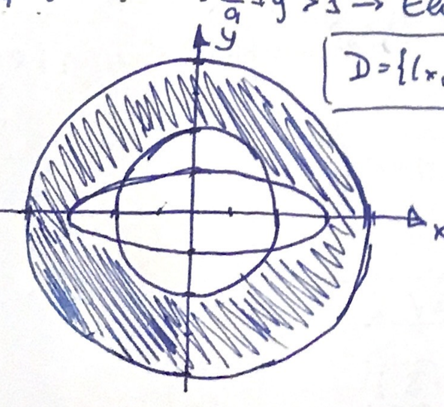
$x^2+y^2-4>0 \rightarrow x^2+y^2>4 \rightarrow$ Zirkunferentzia $C(0,0)$, $r=2$

2) $\left.\begin{array}{l}16-x^2-y^2<0 \rightarrow x^2+y^2>16\\ x^2+y^2-4<0 \rightarrow x^2+y^2<4\end{array}\right\}$ Ezinezkoa

2) $xy\sqrt{\frac{x^2}{9}+y^2-1}\neq 0$

- $x\neq 0$
- $y\neq 0$
- $\frac{x^2}{9}+y^2-1>0 \rightarrow \frac{x^2}{9}+y^2>1 \rightarrow$ Elipsea $C(0,0)$, $a=3$, $b=1$

$$\boxed{D=\left\{(x,y)\in\mathbb{R}^2/(x^2+y^2<16)\wedge(x^2+y^2>4)\wedge x\neq 0\wedge y\neq 0\wedge\left(\frac{x^2}{9}+y^2>1\right)\right\}}$$



Ardatzak eta kurbak ez daude domeinuaren barne.

9) Kalkulatu $f(x,y)=x+y$ funtzioaren mutur erlatiboak $x^2+2y^2=1$ elipsean.

$L(\lambda,(x,y)) = x+y+\lambda(x^2+2y^2-1)$

Puntu Kritikoak:

$$\begin{cases} L'x=0 \\ L'y=0 \\ \varphi(x,y)=0 \end{cases} \rightarrow \begin{cases} 1+2\lambda x=0 \rightarrow \lambda=-1/2x \\ 1+4\lambda y=0 \rightarrow \lambda=-1/4y \\ x^2+2y^2-1=0 \rightarrow 4y^2+2y^2=1 \rightarrow 6y^2=1 \rightarrow y=\pm 1/\sqrt{6} = \pm\sqrt{6}/6 \end{cases}$$

$\frac{-1}{2x}=\frac{-1}{4y} \rightarrow 2x=4y \rightarrow x=2y$

$x=\pm\sqrt{6}/3$

Puntu Kritikoak $\rightarrow \left(\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right), \lambda=-\frac{\sqrt{6}}{4}$; $\left(\frac{-\sqrt{6}}{3},\frac{-\sqrt{6}}{6}\right), \lambda=\frac{\sqrt{6}}{4}$

Matrizea hesiar orlatua:

$$H\left(\frac{-\sqrt{6}}{4},\left(\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right)\right)=\begin{vmatrix} 0 & 2x & 4y \\ 2x & 2\lambda & 0 \\ 4y & 0 & 4\lambda \end{vmatrix} = \begin{vmatrix} 0 & \frac{2\sqrt{6}}{3} & \frac{2\sqrt{6}}{3} \\ \frac{2\sqrt{6}}{3} & \frac{-\sqrt{6}}{2} & 0 \\ \frac{2\sqrt{6}}{3} & 0 & -\sqrt{6} \end{vmatrix} = \frac{24\sqrt{6}}{18}+\frac{24\sqrt{6}}{9} = \frac{4\sqrt{6}}{3}+\frac{8\sqrt{6}}{3} =$$

$$= \frac{12\sqrt{6}}{3} = 4\sqrt{6} > 0$$

$\rightarrow$ $\boxed{\text{Maximoa } \left(\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{2}\right) \text{ puntuan}}$

$$H\left(\frac{\sqrt{6}}{4},\left(\frac{-\sqrt{6}}{3},\frac{-\sqrt{6}}{6}\right)\right)=\begin{vmatrix} 0 & \frac{-2\sqrt{6}}{3} & \frac{-2\sqrt{6}}{3} \\ \frac{-2\sqrt{6}}{3} & \frac{\sqrt{6}}{2} & 0 \\ \frac{-2\sqrt{6}}{3} & 0 & \sqrt{6} \end{vmatrix} = -\frac{24\sqrt{6}}{18}-\frac{24\sqrt{6}}{9} = \frac{-4\sqrt{6}}{3}-\frac{8\sqrt{6}}{3} = \frac{-12\sqrt{6}}{3} =$$

$= -4\sqrt{6} < 0 \longrightarrow$ $\boxed{\text{Minimoa } \left(\frac{-\sqrt{6}}{3},\frac{-\sqrt{6}}{6},\frac{-\sqrt{6}}{2}\right) \text{ puntuan}}$