# Integral mugatua:

1) $\int 3^x e^x dx = \int (3e)^x dx = \dfrac{(3e)^x}{\ln(3e)} + K = \dfrac{3^x \cdot e^x}{\ln(3) + \ln(e)} + K = \dfrac{3^x \cdot e^x}{\ln(3) + 1} + K$

2) $\int e^x \cdot \sin(x)\,dx = \left\| \begin{matrix} u = \sin(x) \to du = \cos(x)dx \\ dv = e^x dx \to v = e^x \end{matrix} \right\| = e^x \sin(x) - \int e^x \cdot \cos(x)\,dx =$

$= \left\| \begin{matrix} u = \cos(x) \to du = -\sin(x)dx \\ dv = e^x dx \to v = e^x \end{matrix} \right\| = e^x \sin(x) - \left(e^x \cdot \cos(x) - \int e^x (-\sin(x))dx\right) \longrightarrow$

$\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x)dx \to 2\int e^x \sin(x)dx = e^x(\sin(x) - \cos(x)) + K$

$\int e^x \sin(x)\,dx = \frac{1}{2} e^x (\sin(x) - \cos(x)) + K$

3) $\int x^2 \cdot \ln(1+x^2)\,dx = \left\| \begin{matrix} u = \ln(1+x^2) \to du = \frac{2x\,dx}{1+x^2} \\ dv = x^2 dx \to v = x^3/3 \end{matrix} \right\| = \dfrac{x^3}{3} \ln(1+x^2) - \int \dfrac{x^3}{3} \cdot \dfrac{2x}{1+x^2} dx =$

$= \dfrac{x^3}{3} \ln(1+x^2) - \dfrac{2}{3} \int \dfrac{x^4}{1+x^2} dx = \left\| \begin{array}{l|l} x^4 & x^2+1 \\ \hline -x^4 + x^2 & x^2 - 1 \\ \quad / \; -x^2 & \\ \quad - \; -x^2 - 1 & \\ \hline \qquad / \quad 1 & \end{array} \right\| = \dfrac{x^3}{3} \ln(1+x^2) - \dfrac{2}{3} \int \left(x^2 - 1 + \dfrac{1}{1+x^2}\right) dx =$

$= \dfrac{x^3}{3} \ln(1+x^2) - \dfrac{2}{3}\left(\dfrac{x^3}{3} - x + \operatorname{arctg}(x)\right) + K = \dfrac{x^3}{3} \ln(1+x^2) - \dfrac{2x^3}{9} + \dfrac{2x}{3} - \dfrac{2}{3} \operatorname{arctg}(x) + K$

4) $\int \dfrac{3x+1}{(x^2+3x+4)^2} dx = \dfrac{Ax+B}{x^2+3x+4} + \int \dfrac{Mx+N}{x^2+3x+4} dx$

$$\frac{3x+1}{(x^2+3x+4)^2} = \frac{A(x^2+3x+4) - (Ax+B)(2x+3)}{(x^2+3x+4)^2} + \frac{Mx+N}{x^2+3x+4}$$

$$3x+1 = Ax^2 + \cancel{3Ax} + 4A - 2Ax^2 - \cancel{3Ax} - 2Bx - 3B + Mx^3 + 3Mx^2 + 4Mx + Nx^2 + 3Nx + 4N$$

$x^3$: $\boxed{0 = M}$

$x^2$: $0 = -A + \cancel{3M} + N \to A = N \xrightarrow{1} \boxed{A = -1}$

$$x:\ 3 = -2B + \cancel{4M} + 3N \rightarrow B = \frac{3N-3}{2} \;②\; \longrightarrow \boxed{B=-3}$$

$$x^0:\ 1 = 4A - 3B + 4N \xrightarrow{①} 1 = 4N - 3B + 4N \xrightarrow{②} 1 = 8N - 3\,\frac{3N-3}{2}$$

$$2 = 16N - 9N + 9 \rightarrow -7 = 7N \rightarrow \boxed{N=-1}$$

$$\frac{-x-3}{x^2+3x+4} + \int \frac{-1}{x^2+3x+4}\,dx = \frac{-x-3}{x^2+3x+4} - \int \frac{dx}{(x+3/2)^2+7/4} =$$

$$= \frac{-x-3}{x^2+3x+4} - \frac{1}{\sqrt{7}/2}\,\mathrm{arctg}\left(\frac{x+3/2}{\sqrt{7}/2}\right) + K = \frac{-x+3}{x^2+3x+4} - \frac{\sqrt{7}}{2}\,\mathrm{arctg}\left(\frac{2x+3}{\sqrt{7}}\right) + K$$

5) $$\int \frac{3x+2}{x(x+1)^3}\,dx = \frac{Ax+B}{(x+1)^2} + \int \frac{Mx+N}{x(x+1)}\,dx$$

$$\frac{3x+2}{x(x+1)^3} = \frac{A(x+1)^{\cancel{2}} - (Ax+B)2\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}} + \frac{Mx+N}{x(x+1)}$$

$$3x+2 = Ax^2 + Ax - 2Ax^2 - 2Bx + Mx^3 + 2Mx^2 + Mx + Nx^2 + 2Nx + N$$

$$x^3:\ \boxed{0 = M}$$

$$x^2:\ 0 = -A + \cancel{2M} + N \longrightarrow A = N \;①\; \longrightarrow \boxed{A=2}$$

$$x:\ 3 = A - 2B + \cancel{M} + 2N \xrightarrow{①} 3 = N - 2B + 2N \rightarrow B = \frac{3N-3}{2} \rightarrow \boxed{B=3/2}$$

$$x^0:\ \boxed{2 = N}$$

$$\frac{2x+3/2}{(x+1)^2} + \int \frac{2}{x(x+1)}\,dx = \frac{4x+3}{2(x+1)^2} + 2\int\left(\frac{A}{x} + \frac{B}{x+1}\right)dx$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1} = \frac{Ax+A+Bx}{x(x+1)}$$

$x: \quad 0 = A + B \rightarrow \boxed{B = -1}$

$x^0: \quad \boxed{1 = A}$

$$\frac{4x+3}{(x+1)^2} + 2\int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \frac{4x+3}{(x+1)^2} + 2\ln|x| - 2\ln|x+1| + K =$$

$$= \frac{4x+3}{(x+1)^2} + 2\ln\left|\frac{x}{x+1}\right| + K = \frac{4x+3}{(x+1)^2} + \ln\left|\frac{x^2}{(x+1)^2}\right| + K$$

6) $$\int\frac{x}{a^4+x^4}dx = \left\| \begin{matrix} t = x^2 \\ dt = 2x\,dx \end{matrix} \right\| = \frac{1}{2}\int\frac{dt}{(a^2)^2+t^2} = \frac{1}{2}\cdot\frac{1}{a^2}\operatorname{arctg}\left(\frac{t}{a^2}\right) + K =$$

$$= \frac{1}{2a^2}\operatorname{arctg}\left(\frac{x^2}{a^2}\right) + K$$

7) $$\int x\cdot\ln\frac{1+x}{1-x}dx = \int x\left(\ln(1+x) - \ln(1-x)\right)dx = \left\| \begin{matrix} u = \ln\frac{1+x}{1-x} \rightarrow du = \frac{1}{1+x} - \frac{-1}{1-x}\,dx \\ dv = x\,dx \rightarrow v = x^2/2 \end{matrix} \right\| =$$

$$= \frac{x^2}{2}\ln\frac{1+x}{1-x} - \int\frac{x^2}{2}\left(\frac{1}{1+x} - \frac{-1}{1-x}\right)dx = \frac{x^2}{2}\ln\frac{1+x}{1-x} - \frac{1}{2}\int\left(x - 1 + \frac{1}{1+x} - x - 1 - \frac{1}{1-x}\right)dx =$$

$$= \frac{x^2}{2}\ln\frac{1+x}{1-x} - \frac{1}{2}\left(\cancel{\frac{x^2}{2}} - x + \ln|1+x| - \cancel{\frac{x^2}{2}} - x - \ln|1-x|\right) + K =$$

$$= \frac{x^2}{2}\ln\frac{1+x}{1-x} + x - \frac{1}{2}\ln\frac{1+x}{1-x} + K = \frac{x^2-1}{2}\ln\frac{1+x}{1-x} + x + K$$

8) $$\int\frac{x^3\cdot\sqrt{1+x^4}}{\sqrt{1+x^4}+1}dx = \left\| \begin{matrix} t = \sqrt{1+x^4} \\ dt = \frac{2x^3dx}{\sqrt{1+x^4}} \end{matrix} \right\| = \int\frac{t}{t+1}\,\frac{t}{2}dt = \frac{1}{2}\int\frac{t^2}{t+1}dt = \frac{1}{2}\int\left(t - 1 + \frac{1}{t+1}\right)dt =$$

$$= \frac{1}{2}\left(\frac{t^2}{2} - t + \ln|t+1|\right) + K = \frac{t^2}{4} - \frac{t}{2} + \frac{1}{2}\ln(t+1) + K =$$

$$= \frac{1+x^4}{4} - \frac{\sqrt{1+x^4}}{2} + \frac{1}{2}\ln\left(\sqrt{1+x^4}+1\right) + K$$

9) $$\int\frac{x\,dx}{\sqrt{1-x^4}} = \left\| \begin{matrix} t = x^2 \\ dt = 2x\,dx \end{matrix} \right\| = \int\frac{1}{\sqrt{1-t^2}}\,\frac{dt}{2} = \frac{1}{2}\arcsin(t) + K = \frac{1}{2}\arcsin(x^2) + K$$

10) $$\int \frac{\sqrt{x}}{\sqrt[4]{x^3}+1}\,dx = \int x^{1/2}\cdot\left(x^{3/4}+1\right)^{-1}dx = \left\| \begin{array}{l} p=-1 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = \frac{1/2+1}{3/4} = 2 \in \mathbb{Z}\ \checkmark \\ t = x^{3/4} \to x = t^{4/3} \\ dx = \frac{4}{8}t\,dt \end{array} \right\| = \int t^{2/3}\cdot(t+1)^{-1}\frac{4}{3}t^{1/3}dt =$$

$$= \frac{4}{3}\int t(t+1)^{-1}dt = \frac{4}{3}\int \frac{t}{t+1}dt = \frac{4}{3}\int\left(1-\frac{1}{t+1}\right)dt = \frac{4}{3}t - \frac{4}{3}\ln|t+1| + K =$$

$$= \frac{4}{3}x^{3/4} - \ln\left|x^{3/4}+1\right| + K = \frac{4}{3}\left(\sqrt[4]{x^3} - \ln\left(\sqrt[4]{x^3}+1\right)\right) + K$$

11) $$\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx = \int x^{-2/3}\cdot\left(1+x^{1/3}\right)^{1/2}dx = \left\| \begin{array}{l} p=1/2 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = \frac{-2/3+1}{1/3} = 1 \in \mathbb{Z}\ \checkmark \\ t = x^{1/3} \to x = t^3 \\ dx = 3t^2dt \end{array} \right\| = \int \cancel{t^{-2}}\cdot(1+t)^{1/2}\cdot 3\cancel{t^2}dt =$$

$$= 3\int (1+t)^{1/2}dt = 2(1+t)^{3/2} + K = 2\left(1+x^{1/3}\right)^{3/2} + K = 2\sqrt{\left(1+\sqrt[3]{x}\right)^3} + K$$

12) $$\int \frac{dx}{x^2\sqrt{4-x^2}} = \int x^{-2}\left(4-x^2\right)^{-1/2}dx = \left\| \begin{array}{l} p=-1/2 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = \frac{-2+1}{2} = \frac{-1}{2} \notin \mathbb{Z} \\ \frac{m+1}{n}+p = \frac{-1}{2}-\frac{1}{2} = -1 \in \mathbb{Z}\ \checkmark \\ 3.\ \text{mota} \end{array} \right| \begin{array}{l} t = x^2 \to x = t^{1/2} \\ dx = \frac{1}{2}t^{-1/2}dt \end{array} \Bigg\| = \int t^{-1}(4-t)^{-1/2}\cdot\frac{1}{2}t^{-1/2}dt =$$

$$= \frac{1}{2}\int t^{-3/2}(4-t)^{-1/2}dt = \frac{1}{2}\int t^{-3/2}(4-t)^{-1/2}\frac{t^{-1/2}}{t^{-1/2}}dt = \frac{1}{2}\int t^{-2}\left(\frac{4-t}{t}\right)^{-1/2}dt = \left\| \begin{array}{l} z^2 = \frac{4-t}{t} \to t = \frac{4}{z^2+1} \\ dt = -8\left(z^2+1\right)^{-2}dz \end{array} \right\| =$$

$$= \frac{1}{2}\int \frac{4^{-2}}{\cancel{\left(z^2+1\right)^{-2}}}\cdot \cancel{z}^{-1}\cdot(-8)\cancel{z}\,\cancel{\left(z^2+1\right)^{-2}}dz = \frac{-1}{4}\int dz = \frac{-1}{4}z + K = \frac{-1}{4}\left(\frac{4-t}{t}\right)^{1/2} + K =$$

$$= \frac{-1}{4}\left(\frac{4-x^2}{x^2}\right)^{1/2} + K = \frac{-1}{4}\left(\frac{4}{x^2}-1\right)^{1/2} + K$$

13) $$\int \frac{dx}{x\sqrt{x}\cdot\sqrt[3]{1+\sqrt[4]{x^3}}} = \int \frac{dx}{\sqrt{x^3}\cdot\sqrt[3]{1+\sqrt[4]{x^3}}} = \int x^{-3/2}\left(1+x^{3/4}\right)^{-1/3}dx = \left\| \begin{array}{l} p=-1/3 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = \frac{-3/2+1}{3/4} = \frac{-2}{3} \notin \mathbb{Z} \\ \frac{m+1}{n}+p = \frac{-2}{3}+\frac{-1}{3} = -1 \in \mathbb{Z}\ \checkmark \end{array} \right| \begin{array}{l} t = x^{3/4} \\ x = t^{4/3} \\ dx = \frac{4}{3}t^{1/3}dt \end{array} \Bigg\| =$$

$$= \int t^{-2}(1+t)^{-1/3}\frac{4}{3}t^{1/3}dt = \frac{4}{3}\int t^{-5/3}(1+t)^{-1/3}dt = \frac{4}{3}\int t^{-5/3}(1+t)^{-1/3}\frac{t^{-1/3}}{t^{-1/3}}dt =$$

$$= \frac{4}{3}\int t^{-2}\left(\frac{1+t}{t}\right)^{-1/3}dt = \left\| \begin{array}{l} z^3 = \frac{1+t}{t} \to t = \left(z^3-1\right)^{-1} \\ dt = -3z^2\left(z^3-1\right)^{-2}dz \end{array} \right\| = \frac{4}{3}\int \frac{1}{\cancel{\left(z^2-1\right)^{-2}}}\cdot \cancel{z}^{-1}\cdot(-3)z^{\cancel{2}}\,\cancel{\left(z^3-1\right)^{-2}}dz =$$

$$= -4\int z\,dz = -4\frac{z^2}{2} + K = -2z^2 + K = -2\left(\frac{1+t}{t}\right)^{2/3} + K = -2\left(\frac{1+x^{3/4}}{x^{3/4}}\right)^{2/3} + K = -2\left(1+x^{-3/4}\right)^{2/3} + K$$

14) $$\int \sqrt{\frac{\sqrt[3]{x}+1}{x^2}}\,dx = \int \frac{\sqrt{\sqrt[3]{x}+1}}{x}dx = \int x^{-1}\left(x^{1/3}+1\right)^{1/2}dx = \left\| \begin{array}{l} p=1/2 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = \frac{-1+1}{1/3} = 0 \in \mathbb{Z}\ \checkmark \\ 2.\ \text{mota} \end{array} \right| \begin{array}{l} t = x^{1/3} \to x = t^3 \\ dx = 3t^2dt \end{array} \Bigg\| =$$

$$= \int t^{-3}(t+1)^{1/2}3t^2dt = 3\int t^{-1}(t+1)^{1/2}dt = \left\| \begin{array}{l} z^2 = t+1 \to t = z^2-1 \\ dt = 2z\,dz \end{array} \right\| = 3\int \frac{1}{z^2-1}z\cdot 2z\,dz =$$

$$= 6\int \frac{z^2}{z^2-1}\,dz = 6\int\left(1+\frac{1}{z^2-1}\right)dz = 6z + \int\left(\frac{A}{z+1}+\frac{B}{z-1}\right)dz$$

$$\frac{1}{z^2-1} = \frac{A}{z+1} + \frac{B}{z-1} = \frac{Az - A + Bz + B}{z^2-1}$$

$$z: \quad 0 = A+B \rightarrow A = -B \rightarrow \boxed{A = -1/2}$$

$$z^0: \quad 1 = B - A \rightarrow B + B = 1 \rightarrow \boxed{B = 1/2}$$

$$= 6z - 3\int\frac{1}{z+1}\,dz + 3\int\frac{1}{z-1}\,dz = 6z - 3\ln|z+1| + 3\ln|z-1| + K =$$

$$= 6z + 3\ln\left|\frac{z-1}{z+1}\right| + K = 6\sqrt{t+1} + 3\ln\left|\frac{\sqrt{t+1}-1}{\sqrt{t+1}+1}\right| + K = 6\sqrt{\sqrt[3]{x}+1} + 3\ln\left|\frac{\sqrt{\sqrt[3]{x}+1}-1}{\sqrt{\sqrt[3]{x}+1}+1}\right| + K$$

15) $$\int\frac{dx}{x\sqrt{x^2+x+1}} = \left\|\begin{matrix} t = 1/x \rightarrow x = 1/t \\ dx = -1/t^2\,dt \end{matrix}\right\| = \int\frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{1}{t}+1}}\cdot\frac{-1}{t^2}\,dt = \int\frac{1}{\frac{1}{\cancel{t^2}}\sqrt{1+t+t^2}}\cdot\frac{-1}{\cancel{t^2}}\,dt =$$

$$= \int\frac{-dt}{\sqrt{1+t+t^2}} = \int\frac{-dt}{\sqrt{(t+1/2)^2+3/4}} = -\ln\left|t+\frac{1}{2}+\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}\right| + K =$$

$$= -\ln\left|t+\frac{1}{2}+\sqrt{t^2+t+1}\right| + K = -\ln\left|\frac{1}{x}+\frac{1}{2}+\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\right| + K =$$

$$= -\ln\left|\frac{1}{x}+\frac{1}{2}+\frac{\sqrt{x^2+x+1}}{x}\right| + K = -\ln\left|\frac{2+x+2\sqrt{x^2+x+1}}{2x}\right| + K =$$

$$= \ln\left|\frac{2x}{x+2+2\sqrt{x^2+x+1}}\right| + K$$

16) $$\int\frac{x-2}{\sqrt{x^2+x+1}} = A\sqrt{x^2+x+1} + M\int\frac{dx}{\sqrt{x^2+x+1}}$$

$$\frac{x-2}{\cancel{\sqrt{x^2+x+1}}} = \frac{A(2x+1)}{2\cancel{\sqrt{x^2+x+1}}} + \frac{M}{\cancel{\sqrt{x^2+x+1}}} \rightarrow x - 2 = Ax + \frac{1}{2}A + M$$

$$x: \quad \boxed{1 = A} \qquad\qquad x^0: \quad -2 = \frac{1}{2}A + M \rightarrow \boxed{M = -5/2}$$

$$= \sqrt{x^2+x+1} - \frac{5}{2}\int \frac{dx}{\sqrt{(x+1/2)^2+3/4}} = \sqrt{x^2+x+1} - \frac{5}{2}\,\mathrm{argsh}\left(\frac{x+1/2}{\sqrt{3}/2}\right)+K =$$

$$= \sqrt{x^2+x+1} - \frac{5}{2}\,\mathrm{argsh}\left(\frac{2x+1}{\sqrt{3}}\right)+K$$

17) $$\int\sqrt{2x^2+3x-1}\,dx = \int \sqrt{2x^2+3x-1}\cdot\frac{\sqrt{2x^2+3x-1}}{\sqrt{2x^2+3x-1}}\,dx = \int\frac{2x^2+3x-1}{\sqrt{x^2+3x-1}}\,dx =$$

$$=(Ax+B)\sqrt{2x^2+3x-1}+M\int\frac{dx}{\sqrt{2x^2+3x-1}}$$

$$\frac{2x^2+3x-1}{\sqrt{2x^2+3x-1}} = A\sqrt{2x^2+3x-1}+\frac{(Ax+B)(4x+3)}{2\sqrt{2x^2+3x-1}}+\frac{M}{\sqrt{2x^2+3x-1}}$$

$$2x^2+3x-1 = 2Ax^2+3Ax-A+2Ax^2+\frac{3}{2}Ax+2Bx+\frac{3}{2}B+M$$

$$x^2:\ 2=4A \to \boxed{A=1/2}$$

$$x:\ 3=\frac{9}{2}A+2B \to 2B=3-\frac{9}{4}=\frac{3}{4} \to \boxed{B=3/8}$$

$$x^0:\ -1=-A+\frac{3}{2}B+M \to -1=-\frac{1}{2}+\frac{9}{16}+M \to \boxed{M=-17/16}$$

$$=\left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17}{16}\int\frac{dx}{\sqrt{2x^2+3x-1}} = \left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17}{16}\int\frac{dx}{\sqrt{2(x+3/4)^2-17/8}}=$$

$$=\left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17}{16}\int\frac{dx}{\sqrt{2\frac{(4x+3)^2}{16}-\frac{17}{8}}} = \left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17}{16}\int\frac{\sqrt{8}\,dx}{\sqrt{(4x+3)^2-17}}=$$

$$=\left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17\sqrt{8}}{16}\int\frac{dx}{\sqrt{(4(x+\frac{3}{4}))^2-17}} = \left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17\sqrt{8}}{16}\int\frac{dx}{\sqrt{16(x+\frac{3}{4})^2-17}}=$$

$$=\left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17\sqrt{8}}{64}\,\mathrm{arcch}\left(\frac{x+3/4}{\sqrt{17}/4}\right)+K = \left(\frac{x}{2}+\frac{3}{8}\right)\sqrt{2x^2+3x-1}-\frac{17\sqrt{8}}{64}\,\mathrm{arcch}\left(\frac{4x+3}{\sqrt{17}}\right)+K$$

18) $$\int\frac{dx}{\sin^3(x)\cdot\cos(x)} = \int\frac{dx}{\sin^3(x)\cdot\cos(x)\cdot\frac{\cos^3(x)}{\cos^3(x)}} = \int\frac{dx}{tg^3(x)\cdot\cos^4(x)} = \Big/\!\!\Big/\ t=tg(t)\to x=argtg(t);\ dx=\frac{1}{1+t^2}dt;\ \cos^2(x)=\frac{1}{1+t^2}\ \Big/\!\!\Big/ =$$

$$=\int\frac{1}{t^3\cdot\left(\frac{1}{1+t^2}\right)^2}\cdot\frac{dt}{1+t^2} = \int\frac{(1+t^2)^{\cancel{2}}}{t^3\cancel{(1+t^2)}}\,dt = \int\frac{1+t^2}{t^3}\,dt = \int\frac{1}{t^3}\,dt+\int\frac{\cancel{t^2}}{\cancel{t^3}}\,dt = \frac{-1}{2t^2}+\ln|t|+K=$$

$$=\frac{-1}{2tg^2(x)}+\ln|tg(x)|+K$$

19) $$\int\frac{dx}{1+8\cos^2(x)}=\left\|\begin{array}{l}t=tg(x)\to x=arctg(t)\\ dx=\frac{1}{1+t^2}dt\\ \cos^2(x)=\frac{1}{1+t^2}\end{array}\right\|=\int\frac{1}{1+8\frac{1}{1+t^2}}\cdot\frac{dt}{1+t^2}=\int\frac{\cancel{1+t^2}}{1+t^2+8}\cdot\frac{dt}{\cancel{1+t^2}}=\int\frac{dt}{9+t^2}=$$
$$=\frac{1}{3}arctg\left(\frac{t}{3}\right)+K=\frac{1}{3}arctg\left(\frac{tg(x)}{3}\right)+K$$

20) $$\int\frac{\sec^2(x)}{\sqrt{\sec^2(x)-1}}dx=\int\frac{\frac{1}{\cos^2(x)}}{\sqrt{\frac{1}{\cos^2(x)}-1}}dx=\int\frac{dx}{\cos^2(x)\sqrt{\frac{1-\cos^2(x)}{\cos^2(x)}}}=\int\frac{dx}{\cos(x)\sqrt{1-\cos^2(x)}}=$$
$$=\int\frac{dx}{\cos(x)\cdot\sqrt{\sin^2(x)}}=\int\frac{1}{\sin(x)}\cdot\frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\sin(x)}dx=\int\frac{\sin(x)}{\cos(x)}\frac{1}{\sin^2(x)}dx=$$
$$=\int tg(x)\cdot\frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)}dx=\int tg(x)\left(1+\frac{\cos^2(x)}{\sin^2(x)}\right)dx=\int\left(tg(x)+\frac{\cancel{\sin(x)}}{\cancel{\cos(x)}}\cdot\frac{\cos^{\cancel{2}}(x)}{\sin^{\cancel{2}}(x)}\right)dx=$$
$$=\int\left(tg(x)+cotg(x)\right)dx=-\ln|\cos(x)|+\ln|\sin(x)|+K=\ln\left|\frac{\sin(x)}{\cos(x)}\right|+K=\ln|tg(x)|+K$$

21) $$\int\frac{dx}{\sin(x)}=\left\|\begin{array}{l}t=tg(x/2)\to x=2arctg(t)\\ dx=\frac{2dt}{1+t^2}\\ \sin(x)=\frac{2t}{1+t^2}\end{array}\right\|=\int\frac{\cancel{1+t^2}}{\cancel{2}t}\cdot\frac{\cancel{2}dt}{\cancel{1+t^2}}=\int\frac{dt}{t}=\ln|t|+K=\ln|tg(x/2)|+K$$

22) $$\int sh^3(x)\cdot ch(x)dx=\left\|\begin{array}{l}t=sh(x)\\ dt=ch(x)\end{array}\right\|=\int t^3dt=\frac{t^4}{4}+K=\frac{sh^4(x)}{4}+K$$

23) $$\int sh(x)\cdot ch(x)dx=\left\|\begin{array}{l}t=ch(x)\\ dt=sh(x)\end{array}\right\|=\int tdt=\frac{t^2}{2}+K=\frac{ch^2(x)}{2}+K=\frac{sh^2(x)+1}{2}+K=\frac{ch(2x)}{2}+K$$

24) $$\int\frac{cosech(1/x)\cdot coth(1/x)}{x^2}dx=\int\frac{\frac{1}{sh(1/x)}\cdot\frac{ch(1/x)}{sh(1/x)}}{x^2}dx=\int\frac{ch(1/x)}{x^2\cdot sh^2(1/x)}dx=\left\|\begin{array}{l}t=sh^{-1}(1/x)\\ dt=sh^{-2}(1/x)\cdot ch(1/x)\cdot x^{-2}dx\end{array}\right\|=$$
$$=\int dt=t+K=sh^{-1}(1/x)+K=cosech(1/x)+K$$

25) $$\int\ln\frac{x-2}{x+2}dx=\int\left(\ln(x-2)-\ln(x+2)\right)dx=\left\|\begin{array}{l}u=\ln(x-2)-\ln(x+2)\to du=\frac{1}{x-2}-\frac{1}{x+2}dx\\ dv=dx\to v=x\end{array}\right\|=$$
$$=x\ln\frac{x-2}{x+2}-\int x\left(\frac{1}{x-2}-\frac{1}{x+2}\right)dx=x\ln\frac{x-2}{x+2}-\int\left(1+\frac{2}{x-2}\right)dx+\int\left(1-\frac{2}{x+2}\right)dx=$$
$$=x\ln\frac{x-2}{x+2}-\cancel{x}-2\ln|x-2|+\cancel{x}-2\ln|x+2|+K=x\ln\frac{x-2}{x+2}-2\ln|x^2-4|+K$$

26) $\int x^2 \ln\sqrt{1-x}\,dx = \left\| \begin{array}{l} u = \frac{1}{2}\ln(1-x) \rightarrow du = \frac{-dx}{2(1-x)} \\ dv = x^2dx \rightarrow v = \frac{x^3}{3} \end{array} \right\| = \frac{x^3}{3}\ln\sqrt{1-x} - \int \frac{-x^3}{6(1-x)}dx =$

$= \frac{x^3}{3}\ln\sqrt{1-x} - \frac{1}{6}\int\left(x^2+x+1-\frac{1}{1-x}\right)dx = \frac{x^3}{3}\ln\sqrt{1-x} - \frac{1}{6}\left(\frac{x^3}{3}+\frac{x^2}{2}+x+\ln|1-x|\right)+K =$

$= \frac{x^3}{3}\ln\sqrt{1-x} - \frac{1}{6}\ln|1-x| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + K$

27) $\int x\cdot \operatorname{arctg}\left(\frac{1}{x+1}\right)dx = \left\| \begin{array}{l} u = \operatorname{arctg}\left(\frac{1}{x+1}\right) \rightarrow du = \frac{\frac{-1}{(x+1)^2}dx}{1+\left(\frac{1}{x+1}\right)^2} \\ dv = x\,dx \rightarrow v = \frac{x^2}{2} \end{array} \right\| = \frac{x^2}{2}\operatorname{arctg}\left(\frac{1}{x+1}\right) - \int \frac{x^2}{2}\,\frac{-\cancel{(x+1)^2}\,dx}{(x+1)^2\,\cancel{(x+1)^2}}$

$= \frac{x^2}{2}\operatorname{arctg}\left(\frac{1}{x+1}\right) + \frac{1}{2}\int\frac{x^2dx}{x^2+2x+1} = \frac{x^2}{2}\operatorname{arctg}\left(\frac{1}{1+x}\right) + \frac{1}{2}\int\left(1-\frac{2x+2}{x^2+2x+1}\right)dx =$

$= \frac{x^2}{2}\operatorname{arctg}\left(\frac{1}{1+x}\right) + \frac{x}{2} - \ln\left|\sqrt{x^2+2x+1}\right| + K$

28) $\int\frac{dx}{x^4+5x^2+4} = \int\frac{dx}{x^4+4x^2+x^2+4} = \int\frac{dx}{x^2(x^2+4)+(x^2+4)} - \int\frac{dx}{(x^2+1)(x^2+4)} =$

$= \int\left(\frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+4}\right)dx$

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1}+\frac{Cx+D}{x^2+4} = \frac{Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx^2+D}{(x^2+1)(x^2+4)}$$

$x^3:\ 0 = A+C \rightarrow A=-C \rightarrow \boxed{C=0}$

$x^2:\ 0 = B+D \rightarrow B=-D \rightarrow \boxed{D=-1/3}$

$x:\ 0 = 4A+C \rightarrow 3A=0 \rightarrow \boxed{A=0}$

$x^0:\ 1 = 4B+D \rightarrow 5B=1 \rightarrow \boxed{B=1/3}$

$= \int\left(\frac{1/3}{x^2+1}-\frac{1/3}{x^2+4}\right)dx = \frac{1}{3}\operatorname{arctg}(x) - \frac{1}{3}\cdot\frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+K =$

$= \frac{1}{6}\left(2\operatorname{arctg}(x)-\operatorname{arctg}\left(\frac{x}{2}\right)\right)+K$

29) $\displaystyle\int \frac{x^4-x^3-x}{(x^3-1)^2}dx = \frac{Ax^2+Bx+C}{x^3-1} + \int \frac{Mx^2+Nx+P}{x^3-1}dx$

$$\frac{x^4-x^3-x}{(x^3-1)^2} = \frac{(2Ax+B)(x^3-1)-(Ax^2+Bx+C)\,3x^2}{(x^3-1)^2} + \frac{Mx^2+Nx+P}{x^3-1}$$

$$x^4-x^3-x = 2Ax^4-2Ax+\cancel{Bx^3}-B-3Ax^4-\cancel{Bx^3}-3Cx^2-Mx^5-Mx^2+Nx^4-Nx+Px^3-P$$

$x^5$: $\boxed{0=-M}$

$x^4$: $1=-A+N \longrightarrow N=A+1 \longrightarrow \boxed{N=1}$

$x^3$: $\boxed{-1=P}$

$x^2$: $0=-3C-M \longrightarrow \boxed{C=0}$

$x$: $-1=-2A-N \to -1=-2A-A-1 \to \boxed{A=0}$

$x^0$: $0=-B-P \longrightarrow \boxed{B=1}$

$$= \frac{x}{x^3-1} + \int \frac{x-1}{x^3-1}dx = \frac{x}{x^3-1} + \int \frac{\cancel{x-1}}{(x^2+x+1)\cancel{(x-1)}}dx = \frac{x}{x^3-1} + \int \frac{dx}{(x+1/2)^2+3/4} =$$

$$= \frac{x}{x^3-1} + \frac{1}{\sqrt{3}/2}\,\mathrm{arctg}\left(\frac{x+1/2}{\sqrt{3}/2}\right) + K = \frac{x}{x^3-1} + \frac{2}{\sqrt{3}}\,\mathrm{arctg}\left(\frac{2x+1}{\sqrt{3}}\right) + K$$

30) $\displaystyle\int x^5 \cdot \sqrt[3]{(1+x^3)^2}\,dx = \int x^5\cdot(1+x^3)^{2/3}dx = \left\| \begin{array}{l} p=2/3 \in \mathbb{Z} \\ \frac{m+1}{n}=\frac{6}{3}=2\in\mathbb{Z}\ \checkmark \\ t=x^3 \to x=t^{1/3} \\ dx=(3t^{2/3})^{-1}dt \end{array} \right\| = \int t^{5/3}\cdot(1+t)^{2/3}\cdot\frac{dt}{3t^{2/3}} =$

$$= \frac{1}{3}\int t(1+t)^{2/3}dt = \left\| \begin{array}{l} z^3=1+t \to t=z^3-1 \\ dt=3z^2dz \end{array} \right\| = \frac{1}{\cancel{3}}\int (z^3-1)z^2\cdot\cancel{3}z^2dz = \int (z^7-z^4)dz =$$

$$= \frac{z^8}{8} - \frac{z^5}{5} + K = \frac{(1+t)^{8/3}}{8} - \frac{(1+t)^{5/3}}{5} + K = \frac{\sqrt[3]{(1+x^3)^8}}{8} - \frac{\sqrt[3]{(1+x^3)^5}}{5} + K$$

31) $\displaystyle\int \frac{x^2}{\sqrt{x^2-a^2}}dx = Ax\sqrt{x^2-a^2} + M\int\frac{dx}{\sqrt{x^2-a^2}}$

$$\frac{x^2}{\sqrt{x^2-a^2}} = A\sqrt{x^2-a^2} + \frac{Ax^2}{\sqrt{x^2-a^2}} + \frac{M}{\sqrt{x^2-a^2}} = \frac{Ax^2 - Aa^2 + Ax^2 + M}{\sqrt{x^2-a^2}}$$

$$x^2: \quad 1 = 2A \rightarrow \boxed{A = 1/2}$$

$$x^0: \quad 0 = M - Aa^2 \rightarrow \boxed{M = a^2/2}$$

$$= \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2-a^2}\right| + K$$

32) $$\int \frac{dx}{x\sqrt{x^2 - x + 3}} = \left\| \begin{matrix} t = 1/x \rightarrow x = 1/t \\ dx = \frac{-dt}{t^2} \end{matrix} \right\| = \int \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2} - \frac{1}{t} + 3}} \frac{-dt}{t^2} = \int \frac{1}{\frac{1}{t}\sqrt{\frac{1-t+3t^2}{t^2}}} \frac{-dt}{t^2} =$$

$$= \int \frac{-\cancel{t^2}}{\sqrt{1-t+3t^2}} \frac{dt}{\cancel{t^2}} = \int \frac{-dt}{\sqrt{(\sqrt{3}t - 1/2\sqrt{3})^2 + 11/12}} = \frac{-1}{\sqrt{3}} \int \frac{dt}{\sqrt{(t - 1/6)^2 + 11/12 \cdot 3}} = \frac{-1}{\sqrt{3}} \ln\left| t - \frac{1}{6} + \sqrt{\left(t - \frac{1}{6}\right)^2 + \frac{11}{36}} \right| + K =$$

$$= \frac{-1}{\sqrt{3}} \ln\left| t - \frac{1}{6} + \sqrt{t^2 - \frac{t}{3} + \frac{12}{36}} \right| + K = \frac{-1}{\sqrt{3}} \ln\left| \frac{1}{x} - \frac{1}{6} + \frac{\sqrt{1-x+x^2}}{\sqrt{3}x} \right| + K = \frac{-1}{\sqrt{3}} \ln\left| \frac{\sqrt{3} + \sqrt{1-x+x^2}}{\sqrt{3}x} - \frac{1}{6} \right| + K$$

$$= \frac{-1}{\sqrt{3}} \ln\left| \frac{1}{\sqrt{3}} \left( \frac{\sqrt{3} + \sqrt{1-x+x^2}}{x} - \frac{1}{2\sqrt{3}} \right) \right| + K = \frac{-1}{\sqrt{3}} \left( \ln\left|\frac{1}{\sqrt{3}}\right| + \ln\left| \frac{\sqrt{3} + \sqrt{1-x+x^2}}{x} - \frac{1}{2\sqrt{3}} \right| \right) + K =$$

$$= \frac{-1}{\sqrt{3}} \ln\left| \frac{\sqrt{3} + \sqrt{1-x+x^2}}{x} - \frac{1}{2\sqrt{3}} \right| + K$$

33) $$\int \sqrt{2ax - x^2}\, dx = \int \frac{2ax - x^2}{\sqrt{2ax - x^2}} dx = (Ax + B)\sqrt{2ax - x^2} + M \int \frac{dx}{\sqrt{2ax - x^2}}$$

$$\frac{2ax - x^2}{\sqrt{2ax - x^2}} = A\sqrt{2ax - x^2} + \frac{(Ax+B)(\cancel{2}a - \cancel{2}x)}{\cancel{2}\sqrt{2ax - x^2}} + \frac{M}{\sqrt{2ax - x^2}}$$

$$2ax - x^2 = 2aAx - Ax^2 + aAx - Ax^2 + aB - Bx + M$$

$$x^2: \quad -1 = -2A \rightarrow \boxed{A = 1/2}$$

$$x: \quad 2a = 3aA - B \rightarrow 4a = 3a - 2B \rightarrow a = -2B \rightarrow \boxed{B = -a/2}$$

$$x^0: \quad 0 = aB + M \rightarrow \boxed{M = -a^2/2}$$

$$= \left(\frac{x}{2} - \frac{a}{2}\right)\sqrt{2ax - x^2} - \frac{a^2}{2}\int \frac{dx}{\sqrt{2ax-x^2}} = \left(\frac{x-a}{2}\right)\sqrt{2ax-x^2} - \frac{a^2}{2}\int\frac{dx}{\sqrt{a^2-(x-a)^2}} =$$

$$= \left(\frac{x-a}{2}\right)\sqrt{2ax-x^2} - \frac{a^2}{2}\arcsin\left(\frac{x-a}{a}\right) + K$$

34) $$\int \frac{\cos^2(x)}{(\sin^2(x) - 4\cos^2(x))^2}dx = \left\| \begin{array}{l} t = tg(x) \to x = arctg(t) \\ dx = \frac{dt}{1+t^2} \\ \sin^2(x) = \frac{t^2}{1+t^2} \,;\, \cos^2(x) = \frac{1}{1+t^2} \end{array} \right\| = \int \frac{\frac{1}{1+t^2}}{\left(\frac{t^2}{1+t^2} + \frac{4}{1+t^2}\right)^2} \cdot \frac{dt}{1+t^2} =$$

$$= \int \frac{\cancel{(1+t^2)^2}}{\cancel{(1+t^2)^3}(t^2+4)^2}dt = \frac{At+B}{t^2+4} + \int \frac{Mt+N}{t^2+4}dx$$

$$\frac{1}{(t^2+4)^2} = \frac{A(t^2+4) - 2t(At+B)}{(t^2+4)^2} + \frac{M\ \ +N}{t^2-4} = \frac{At^2+4A-2At^2-2Bt+Mt^3+4Mt+Nt^2+4N}{(t^2+4)^2}$$

$t^3$: $\boxed{0 = M}$

$t^2$: $0 = -A + N \to A = N \to \boxed{N = 1/8}$

$t$: $0 = -2B + \cancel{4M}^{0} \to \boxed{B = 0}$

$t^0$: $1 = 4A + 4N \to 1 = 8A \to \boxed{A = 1/8}$

$$= \frac{1/8}{t^2-4} + \int\frac{1/8}{t^2+4}dx = \frac{1}{8(t^2-4)} - \frac{1}{8}\cdot\frac{1}{2}\,arctg\left(\frac{t}{2}\right) + K =$$

$$= \frac{-1}{8(tg^2(x)+4)} + \frac{1}{16}\,arctg\left(\frac{tg(x)}{2}\right) + K$$

35) $$\int\frac{\sin(x)}{1+\sin(x)}dx = \int\frac{\sin(x)+1-1}{1+\sin(x)}dx = \int dx - \int\frac{dx}{1+\sin(x)} = x - \overbrace{\int\frac{\sec^2(x/2)}{(tg(x/2)+1)^2}dx}^{I}$$

$$I = \left\| \begin{array}{l} t = tg(x/2)+1 \\ dt = \frac{\sec^2(x/2)}{2}dx \end{array} \right\| = 2\int\frac{1}{t^2}dt = -\frac{2}{t} + K = \frac{-2}{tg(x/2)+1} + K$$

$$= x + \frac{2}{tg(x/2)+1} + C$$

36) $\displaystyle\int \frac{dx}{sh(x)} = \left\| \begin{array}{l} t = th(x/2) \to x = 2arcth(t) \\ dx = \frac{2dt}{1-t^2}\ ;\ sh(x) = \frac{2t}{1-t^2} \end{array} \right\| = \int \frac{1}{\frac{2t}{1-t^2}} \frac{2dt}{1-t^2} = \int \frac{\cancel{1-t^2}}{\cancel{2}t} \cdot \frac{\cancel{2}dt}{\cancel{1-t^2}} = \int \frac{dt}{t} =$

$= \ln|t| + K = \ln\left|tg\left(x/2\right)\right| + K$

37) $\displaystyle\int \frac{\cos(2x)+1}{2+16\sin^2(x)}dx = \left\| \begin{array}{l} \cos^2(x) = \frac{\cos(2x)+1}{2} \\ 2\cos^2(x) = \cos(2x)+1 \end{array} \right\| = \int \frac{2\cos^2(x)}{2+16\sin^2(x)}dx = \int \frac{\cos^2(x)}{1+8\sin^2(x)}dx =$

$= \left\| \begin{array}{l} t = tg(x) \to x = arctg(t) \\ dx = \frac{dt}{1+t^2} \\ \sin^2(x) = \frac{t^2}{1+t^2}\ ;\ \cos^2(x) = \frac{1}{1+t^2} \end{array} \right\| = \displaystyle\int \frac{\frac{1}{1+t^2}}{1+8\frac{t^2}{1+t^2}} \cdot \frac{dt}{1+t^2} = \int \frac{\frac{1}{\cancel{1+t^2}}}{\frac{1+t^2+8t^2}{\cancel{1+t^2}}} \cdot \frac{dt}{1+t^2} = \int \frac{dt}{(1+9t^2)(1+t^2)} =$

$= \displaystyle\int \left( \frac{At+B}{1+9t^2} + \frac{Ct+D}{1+t^2} \right) dt$

$$\frac{1}{(1+9t^2)(1+t^2)} = \frac{(At+B)(1+t^2)+(Ct+D)(1+}{(1+9t^2)(1+t^2)}$$

$t^3$: $0 = A+9C \to 0 = -C+9C \to 0 = 8C \to \boxed{C=0}$

$t^2$: $0 = B+9D \to 0 = 1-D+9D \to \boxed{D=-1/8}$

$t$: $0 = A+C \to A = -C \to \boxed{A=0}$

$t^0$: $1 = B+D \to B = 1-D \to \boxed{B = 9/8}$

$= \displaystyle\int \left( \frac{9/8}{1+9t^2} - \frac{1/8}{1+t^2} \right) dt = \frac{9}{8}\int \frac{dt}{1+(3t)^2} - \frac{1}{8}\int \frac{dt}{1+t^2} = \frac{3}{8}arctg(3t) - \frac{1}{8}arctg(t) + K =$

$= \frac{3}{8}arctg(3tg(x)) - \frac{1}{8}arctg(tg(x)) + K = \frac{3}{8}arctg(3tg(x)) - \frac{x}{8} + K$

38) $\displaystyle\int \frac{dx}{x^3\sqrt{(2+3/x^2)^3}} = \int x^{-3}(2+3x^{-2})^{-3/2}dx = \left\| \begin{array}{l} p = -3 \notin \mathbb{Z}^+ \\ \frac{m+1}{n} = -1 \in \mathbb{Z} \checkmark \\ t = x^{-2} \to x = t^{-1/2} \\ dx = \frac{-dt}{2t^{3/2}} \end{array} \right\| = \int \cancel{t^{3/2}}(2+3t)^{-3/2}\frac{-dt}{2\cancel{t^{3/2}}} =$

$= \left\| \begin{array}{l} z^2 = 2+3t \\ t = (z^2-2)/3 \\ dt = \frac{2z}{3}dz \end{array} \right\| = \displaystyle\frac{-1}{\cancel{2}}\int z^{-3}\cdot\frac{\cancel{2}z}{3}dz = \frac{-1}{3}\int z^{-2}dz = \frac{-1}{3}\cdot\frac{-1}{z} + K = \frac{1}{3\sqrt{2+3t}} + K = \frac{1}{3\sqrt{2+\frac{3}{x^2}}} + K$

39) $$\int x\cdot \operatorname{arctg}(2x+3)\,dx = \left\|\begin{array}{l} u=\operatorname{arctg}(2x+3) \to du=\frac{2dt}{1+(2x+3)^2} \\ dv = x\,dx \longrightarrow v=x^2/2 \end{array}\right\| = \frac{x^2}{2}\operatorname{arctg}(2x+3) - \int \frac{x^2}{\cancel{2}}\,\frac{\cancel{2}dx}{1+(2x+3)^2} =$$

$$= \frac{x^2}{2}\operatorname{arctg}(2x+3) - \int\left(\frac{1}{4} - \frac{6x+5}{2(4x^2+6x+10)}\right)dx = \frac{x^2}{2}\operatorname{arctg}(2x+3) - \frac{x}{4} + \frac{1}{2}\int\frac{3(2x+3)-4}{1+(2x+3)^2}dx =$$

$$= \frac{x^2}{2}\operatorname{arctg}(2x+3) - \frac{x}{4} + \frac{3}{8}\ln|4x^2+12x+10| - 2\underbrace{\int\frac{dx}{1+(2x+3)^2}}_{I} =$$

$$\int\frac{dx}{1+(2x+3)^2} = \left\|\begin{array}{l} t=2x+3 \\ x=\frac{t-3}{2} \\ dx=\frac{dt}{2}\end{array}\right\| = \frac{1}{2}\int\frac{dt}{1+t^2} = \frac{1}{2}\operatorname{arctg}(t)+K = \frac{1}{2}\operatorname{arctg}(2x+3)+K$$

$$= \frac{x^2}{2}\operatorname{arctg}(2x+3) - \frac{x}{4} + \frac{3}{8}\ln\left|4\left(x^2+3x+\frac{5}{2}\right)\right| - \operatorname{arctg}(2x+3) + K =$$

$$= \left(\frac{x^2}{2}-1\right)\operatorname{arctg}(2x+3) - \frac{x}{4} + \ln(4) + \ln\left|x^2+3x+\frac{5}{2}\right| + K =$$

$$= \left(\frac{x^2}{2}-1\right)\operatorname{arctg}(2x+3) - \frac{x}{4} + \ln\left|x^2+3x+\frac{5}{2}\right| + C$$

40) $$\int\frac{dx}{1+\sin(x)-\cos(x)} = \left\|\begin{array}{l} t=\operatorname{tg}(x/2) \to x=2\operatorname{arctg}(t) \\ dx=\frac{2dt}{1+t^2} \\ \sin(x)=\frac{2t}{1+t^2}\ ;\ \cos(x)=\frac{1-t^2}{1+t^2}\end{array}\right\| = \int\frac{1}{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2} =$$

$$= 2\int\frac{\cancel{1+t^2}}{\cancel{1}+t^2+2t-\cancel{1}+t^2}\,\frac{dt}{\cancel{1+t^2}} = \int\frac{dt}{t^2+2t} = \int\left(\frac{A}{t}+\frac{B}{t+1}\right)dt$$

$$\frac{1}{t^2+t} = \frac{A(t+1)+Bt}{t(t+1)}$$

$$t:\ 0=A+B \to \boxed{B=-1}$$

$$t^0:\ \boxed{1=A}$$

$$= \int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt = \ln|t| - \ln|t+1| + K = \ln\left|\frac{t}{t+1}\right| + K = \ln\left|\frac{\operatorname{tg}(x/2)}{\operatorname{tg}(x/2)+1}\right| + K$$