1) Ebatzi problema Simplex algoritmoa erabiliz

$$\text{Max} \quad z = 2x_1 + 4x_2 + \frac{5}{2}x_3$$

non
$$3x_1 + 4x_2 + 2x_3 \leq 600$$
$$2x_1 + x_2 + 2x_3 \leq 400$$
$$x_1 + 3x_2 + 3x_3 \leq 300$$
$$x_1, x_2, x_3 \geq 0$$

$\longrightarrow$

Lasaiera aldagaiak:

$$\max \quad z = 2x_1 + 4x_2 + \frac{5}{2}x_3$$
$$3x_1 + 4x_2 + 2x_3 + x_4 = 600$$
$$2x_1 + x_2 + 2x_3 + x_5 = 400$$
$$x_1 + 3x_2 + 3x_3 + x_6 = 300$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

$$A = \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \left(\begin{array}{ccc|ccc} 3 & 4 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 1 & 3 & 3 & 0 & 0 & 1 \end{array}\right) \\ \qquad\qquad\qquad B \end{array}$$

$$x_B = (x_4, x_5, x_6) = B^{-1} b \overset{B=I}{=} b = (600, 400, 300)$$
$$x_N = (x_1, x_2, x_3) = (0, 0, 0)$$

Hasierako soluzio bideragarria:

$$x_B = (x_4, x_5, x_6) = (600, 400, 300)$$
$$x_N = (x_1, x_2, x_3) = (0, 0, 0)$$

Simplex taula:

Coin	Aoin	$B^{-1}b$	2	4	5/2	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	600	3	4	2	1	0	0
0	x_5	400	2	1	2	0	1	0
0	x_6	300	1	$\boxed{3}$ PIBOTA	3	0	0	1
$z = 0$		z_j	0	0	0	0	0	0
		$z_j - c_j$	-2	-4	-5/2	0	0	0

- $\exists\, w_j < 0 \rightarrow$ Jarraitu
- Sartze irizpidea: $w_j = \min\, z_k - c_k = \min\{-2, -4, -5/2\} = -4 \rightarrow x_2$ sartzen da
- Irtetze irizpidea: $\min\left\{\frac{x_{Bk}}{y_{jk}} \,/\, y_{jk} > 0\right\} = \min\left\{\frac{600}{4}, \frac{400}{1}, \frac{300}{3}\right\} =$
  $= \min\{150, 400, 100\} = 100 \rightarrow x_6$ irtetzen da

Simplex taula berria:

Coin	Aoin	$B^{-1}b$	2	4	5/2	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
0	x_4	200	$\boxed{5/3}$ PIBOTA	0	-2	1	0	-4/3
0	x_5	300	5/3	0	1	0	1	-1/3
4	x_2	100	1/3	1	1	0	0	1/3
$z = 400$		z_j	4/3	4	4	0	0	4/3
		$z_j - c_j$	-2/3	0	3/2	0	0	4/3

Aldaketak:

$$\begin{cases} e_{3b} \leftarrow e_3 \cdot 1/3 \\ e_{1b} \leftarrow e_1 - 4e_{3b} \\ e_{2b} \leftarrow e_2 - e_{3b} \end{cases}$$

- $\exists w_j < 0 \rightarrow$ Jarraitu
- Sartze irizpidea: $w_j = \min z_k - c_k = \min\left\{-\frac{2}{3}, \frac{3}{2}, \frac{4}{3}\right\} = -\frac{2}{3} \rightarrow x_1$ sartzen da
- Irtetze irizpidea: $\min\left\{\frac{x_{Bk}}{y_{jk}} / y_{jk} > 0\right\} = \min\left\{\frac{200}{5/3}, \frac{300}{5/3}, \frac{100}{1/3}\right\} =$

$= \min\{120, 180, 300\} = 120 \rightarrow x_4$ irtetzen da

Simplex taula berria:

C_{oin}	A_{oin}	$B^{-1}\cdot b$	2	4	5/2	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
2	x_1	120	1	0	-6/5	3/5	0	-4/5
0	x_5	100	0	0	3	-1	1	1
4	x_2	60	0	1	7/5	-1/5	0	3/5
$z = 480$		z_j	2	4	16/5	2/5	0	4/5
		$z_j - c_j$	0	0	7/10	2/5	0	4/5

Aldaketak:

$$\begin{cases} e_{1b} \leftarrow e_1 \cdot 3/5 \\ e_{2b} \leftarrow e_2 - e_{1b} \cdot 5/3 \\ e_{3b} \leftarrow e_3 - e_{1b} \cdot 1/3 \end{cases}$$

- $\forall w_j \geq 0 \rightarrow$ Gelditu, optimoa aurkitu dugu. Gainera, soluzio optimoa bakarra da, oinarrizkoak ez diren aldagaien kostu minimoak $\neq 0$ direlako.

$x_1^* = 120$, $x_2^* = 60$, $x_3^* = 0$, $x_4^* = 0$, $x_5^* = 100$, $x_6^* = 0$, $z^* -$

2) Problema primalaren taula optimoa erabiliz, problema dualaren soluzioa lortu.

Problema primalaren azken taula:

Coin	Aoin	$B^{-1}\cdot b$	2	4	5/2	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6
2	x_1	120	1	0	-6/5	3/5	0	-4/5
0	x_5	100	0	0	3	-1	1	1
4	x_2	60	0	1	7/5	-1/5	0	3/5
$z = 480$		z_j	2	4	16/5	2/5	0	4/5
		$z_j - c_j$	0	0	7/10	2/5	0	4/5

Problema primalaren soluzio optimoa

$x_1^* = 120$, $x_2^* = 60$, $x_3^* = 0$, $x_4^* = 0$, $x_5^* = 100$, $x_6^* = 0$, $z^* = 480$

Problema duala:

$$\begin{aligned} \min \quad & z = 600u_1 + 400u_2 + 300u_3 \\ & 3u_1 + 2u_2 + u_3 \geq 2 \\ & 4u_1 + u_2 + 3u_3 \geq 4 \\ & 2u_1 + 2u_2 + 3u_3 \geq 5/2 \\ & u_1, u_2, u_3 \geq 0 \end{aligned}$$

Problema dualaren soluzioa:

$u_1^* = 2/5$, $u_2^* = 0$, $u_3^* = 4/5$, $z^* = 480$

3) Problema duala idatzi eta osagarriizko lasaitasuna erabiliz ebatzi

Problema primala:

$$\max \quad z = 2x_1 + 4x_2 + \frac{5}{2}x_3$$

$$\begin{aligned} 3x_1 + 4x_2 + 2x_3 &\leq 600 \\ 2x_1 + x_2 + 2x_3 &\leq 400 \\ x_1 + 3x_2 + 3x_3 &\leq 300 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 2 & 2 & 3 \end{pmatrix}$$

Problema duala:

$$\min \quad z = 600u_1 + 400u_2 + 300u_3$$

$$\begin{aligned} 3u_1 + 2u_2 + u_3 &\geq 2 \\ 4u_1 + u_2 + 3u_3 &\geq 4 \\ 2u_1 + 2u_2 + 3u_3 &\geq 5/2 \\ u_1, u_2, u_3 &\geq 0 \end{aligned}$$

$\rightarrow$

Problema duala lasaiera aldagaiekin:

$$\min \quad z = 600u_1 + 400u_2 + 300u_3$$

$$\begin{aligned} 3u_1 + 2u_2 + u_3 - u_4 &= 2 \\ 4u_1 + u_2 + 3u_3 - u_5 &= 4 \\ 2u_1 + 2u_2 + 3u_3 - u_6 &= 5/2 \\ u_1, u_2, u_3, u_4, u_5 &\geq 0 \end{aligned}$$

Problema primalaren soluzio osoa:

$$x_1^* = 120,\quad x_2^* = 60,\quad x_3^* = 0,\quad x_4^* = 0,\quad x_5^* = 100,\quad x_6^* = 0,\quad z^* = 480$$

Soluzio duala:

$$x^T = (x_1, x_2, x_3) \rightarrow (u^h)^T = (u_4, u_5, u_6)$$

$$(x^h)^T = (x_4, x_5, x_6) \rightarrow u^T = (u_1, u_2, u_3)$$

$$x_1 \cdot u_4 = 0 \rightarrow 120 \cdot u_4 = 0 \rightarrow \boxed{u_4 = 0}$$

$$x_2 \cdot u_5 = 0 \rightarrow 60 \cdot u_5 = 0 \rightarrow \boxed{u_5 = 0}$$

$$x_3 \cdot u_6 = 0$$

$$x_4 \cdot u_1 = 0$$

$$x_5 \cdot u_2 = 0 \rightarrow 100 \cdot u_2 = 0 \rightarrow \boxed{u_2 = 0}$$

$$x_6 \cdot u_3 = 0$$

Ekuazioak:

$$z = 600u_1 + 300u_3 \xrightarrow{1\ 2} z = 600 \cdot (2/5) + 300 \cdot (4/5) \rightarrow \boxed{z = 480}$$

$$3u_1 + u_3 = 2 \rightarrow u_3 = 2 - 3u_1 \ (1) \xrightarrow{2} u_3 = 2 - 3(2/5) \rightarrow \boxed{u_3 = 4/5}$$

$$4u_1 + 3u_3 = 4 \xrightarrow{1} 4u_1 + 3(2 - 3u_1) = 4 \rightarrow -5u_1 = -2 \rightarrow \boxed{u_1 = 2/5}\ (2)$$

$$2u_1 + 3u_3 - u_6 = 5/2 \xrightarrow{1\ 2} 2(2/5) + 3(4/5) - u_6 = 5/2 \rightarrow 4/5 + 12/5 - 5/2 = u_6 \rightarrow \boxed{u_6 = 7/10}$$

Soluzio optimoa:

$$u_1^* = 2/5,\quad u_2^* = 0,\quad u_3^* = 4/5,\quad u_4^* = 0,\quad u_5^* = 0,\quad u_6^* = 7/10,\quad z^* = 480$$

4) Posiblea al da Simplex metodoa erabiliz problema duala ebaztea?

Ez da posible problema duala Simplex metodoaren bidez ebaztea, problemako murrizketak "$\geq$" motakoak direlako. Lasaiera aldagaiak sartzean, honakoa lortzen dugu:

$$\begin{aligned}
\min \ z &= 600u_1 + 400u_2 + 300u_3 \\
& 3u_1 + 2u_2 + u_3 - u_4 = 2 \\
& 4u_1 + u_2 + 3u_3 - u_5 = 4 \\
& 2u_1 + 2u_2 + 3u_3 - u_6 = 5/2 \\
& u_1, u_2, u_3, u_4, u_5 \geq 0
\end{aligned}
\quad \longrightarrow \quad A = \begin{pmatrix} 3 & 2 & 1 & -1 & 0 & 0 \\ 4 & 1 & 3 & 0 & -1 & 0 \\ 2 & 2 & 3 & 0 & 0 & -1 \end{pmatrix}$$

Ikus daitekeenez, ezinezkoa da modu honetan oinarri kanonikoa lortzea, lasaiera aldagaiak kenduz sartzen direlako. Hortaz, problema ebazteko beste metodo bat erabili behar da.