5) ALDAGAI BEKTORIALEKO FUNTZIO ERREALA

1 Kalkulatu honako funtzio honen definizio eremua

$$f(x,y)=\frac{\ln(x^2+y^2-1)}{\exp\left(\frac{1}{x^2-y^2}\right)}$$

1) Logaritmoaren argumentua >0

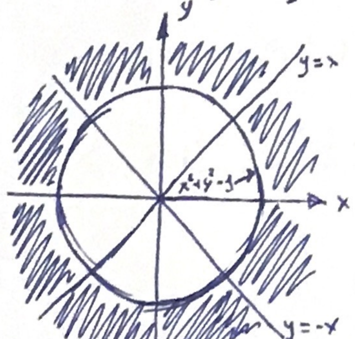
$x^2+y^2-1>0$

2) Izendatzailea $\neq 0$

$x^2-y^2\neq 0$

1) $x^2+y^2-1=0 \longrightarrow x^2+y^2=1 \begin{cases} C=(x_0,y_0)=(0,0) \\ r=\text{erradioa}=\sqrt{1}=1 \end{cases}$

2) $x^2-y^2=0 \rightarrow (x-y)(x+y)=0 \left.\begin{matrix} x-y\neq 0 \longrightarrow x\neq y \\ x+y\neq 0 \longrightarrow x\neq -y \end{matrix}\right\}$ Zuzenak



1) $x^2+y^2>1$

2) $x^2-y^2\neq 0 < \begin{matrix} x\neq y \\ x\neq -y \end{matrix}$

$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / x^2+y^2>1 \wedge x\neq y \wedge y\neq -x\}}$$

2 Kalkulatu $z(x,y)=x^2y+xy^2+xy$ funtzioaren deribatu partzialak

$z'x=2xy+y^2+y$

$z'y=x^2+2xy+x$

3 Kalkulatu $z(x,y)=2x^2-3y^2$ ekuazioaren plano ukitzailearen ekuazioa $P(-2,1,5)$ puntuan

Plano ukitzailearen ekuazioa:

$z-c=z'x]_{(a,b)}(x-a)+z'y]_{(a,b)}(y-b)$

Deribatu partzialak:

$z'x=4x]_{(-2,1)}=-8$

$z'y=-6y]_{(-2,1)}=-6$

Ekuazioan ordezkatuz:

$z-5=-8(x+2)-6(y-1)$

$z-5=-8x-16-6y+6$

$\boxed{z+8x+6y+5=0}$

[4] Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua:

$f(x,y) = \ln[(x^2+4y^2-4)(-x^2-y^2+2x+3)]$

Logaritmoaren argumentua > 0

1) $x^2+4y^2-4=0 \rightarrow$ Elipsea

2) $-x^2-y^2+2x+3=0 \rightarrow x^2+y^2-2x-3=0 \rightarrow$ Zirkunferentzia

1) $x^2+4y^2-4=0$

$C(0,0)$

$\frac{x^2}{4}+y^2=1 \quad a=2, \; b=1$

2) $(x-x_0)^2+(y-y_0)^2=r^2$

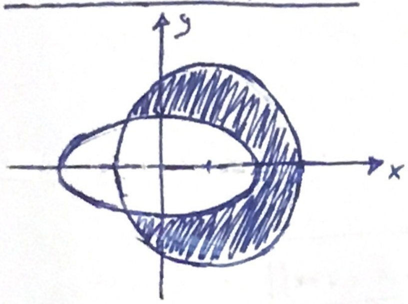
$(x-1)^2+y^2=4$

$x^2+1-2x+y^2=4$

$y_0=0, \; x_0=1$

$C=(1,0), \; r=2$

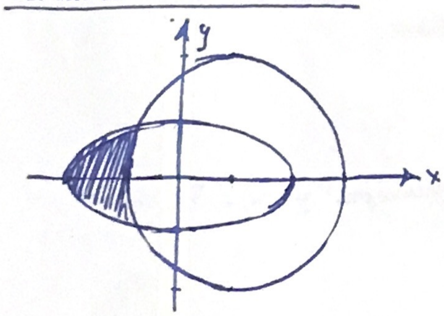
1. aukera: BIAK POSITIBO



$D=\{(x,y)\in\mathbb{R}^2 / x^2+4y^2-4>0 \wedge x^2+y^2-2x-3<0\}$

$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / \frac{x^2}{4}+y^2>1 \wedge (x-1)^2+y^2<4\}}$$

2. aukera: BIAK NEGATIBO



$D=\{(x,y)\in\mathbb{R}^2 / x^2+4y^2-4<0 \wedge x^2+y^2-2x-3>0\}$

$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / \frac{x^2}{4}+y^2<1 \wedge (x-1)^2+y^2>4\}}$$

[5] Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua:

$f(x,y) = \frac{\arccos(\ln(x))}{\sqrt{36-4x^2-9y^2}}$

1) $-1 \leq \ln(x) \leq 1 \rightarrow e^{-1} \leq x \leq e \rightarrow x \in [e^{-1}, e]$ (arcosinuaren argumentua $\in [-1,1]$)

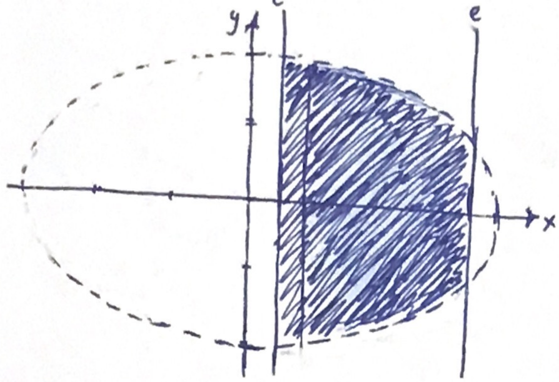
2) $\sqrt{36-4x^2-9y^2} \neq 0 \rightarrow 36-4x^2-9y^2>0 \rightarrow 4x^2+9y^2<36$

3) $\ln(x) \rightarrow x>0 \rightarrow$ 1. eta 4. koadranteak

2) $36-4x^2-9y^2=0 \rightarrow$ elipsea

$$4x^2+9y^2=36 \rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1 \begin{cases} C(0,0) \\ a=3 \\ b=2 \end{cases}$$

1) $x \neq 0$



$$\boxed{D=\left\{(x,y)\in\mathbb{R}^2 / (e^{-1}\leq x\leq e)\wedge\left(\frac{x^2}{9}+\frac{y^2}{4}<1\right)\right\}}$$

$\boxed{6}$ Lortu $z(x,y)=2x^2+2y^2-xy+1$ gainazalaren plano ukitzailearen ekuazioa $P(1,1,4)$ puntuan

Plano ukitzailearen ekuazioa:

$$z-c = z'_x\Big]_{(a,b)}(x-a)+z'_y\Big]_{(a,b)}(y-b)$$

Deribatu partzialak:

$$z'_x=4x-y\Big]_{(1,1)}=3$$

$$z'_y=4y-x\Big]_{(1,1)}=3$$

Ekuazioan ordezkatuz:

$$z-4=3(x-1)+3(y-1) \rightarrow z-4=3x-3+3y-3 \rightarrow z-3x-3y+2=0$$

$\boxed{7}$ Izan bedi $f(x,y)=\sqrt{x^2+2y^2}$ funtzioa, kalkulatu gradiente bektorea $(1,1)$ puntuan eta norabide bektorearen balio maximoa puntu horretan.

Gradiente bektorea:

$$\nabla f(1,1)=\left(\frac{\partial f(1,1)}{\partial x}, \frac{\partial f(1,1)}{\partial y}\right)$$

Deribatu partzialak:

$$\frac{\partial f}{\partial x}=\frac{x}{\sqrt{x^2+2y^2}}\Bigg]_{(1,1)}=\frac{1}{\sqrt{3}}$$

$$\frac{\partial f}{\partial y}=\frac{2y}{\sqrt{x^2+2y^2}}\Bigg]_{(1,1)}=\frac{2}{\sqrt{3}}$$

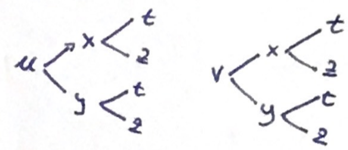
Beraz:

$$\nabla f(1,1) = \left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$$

Norabide-deribatuaren balio maximoa gradiente bektorearen modulua da.

$$|\nabla f(1,1)| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2} = \sqrt{\frac{5}{3}}$$

$\boxed{8}$ Izan bitez $\begin{cases} u(x,y) = x^2+y^2 \\ v(x,y) = 4x-y \end{cases}$ eta $\begin{cases} x(t,z) = 2t^2-5z^2 \\ y(t,z) = t^2-z \end{cases}$



Kalkulatu $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial z}$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t} = 2x\cdot 4t + 2y\cdot 2t = 8xt + 4yt$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial z} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial z} = 2x(-10z) + 2y(-1) = -20xz - 2y$$

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial t} = 4\cdot 4t + (-1)2t = 16t - 2t = 14t$$

$$\frac{\partial v}{\partial z} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial z} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial z} = 4(-10z) + (-1)(-1) = 1 - 40z$$

$\boxed{9}$ Kalkulatu $\frac{\partial^3 u}{\partial x \partial y \partial z}$ non $u(x,y,z) = e^{xyz}$ den

$$\frac{\partial u}{\partial x} = yz\cdot e^{xyz}$$

$$\frac{\partial u^2}{\partial y} = \frac{\partial}{\partial y}\left[yz\cdot e^{xyz}\right] = z\cdot e^{xyz} + yz\cdot xy\cdot e^{xyz} = e^{xyz}(z + xyz^2)$$

$$\frac{\partial u^3}{\partial z} = \frac{\partial}{\partial z}\left[e^{xyz}(z + xyz^2)\right] = xye^{xyz}(z+xyz^2) + e^{xyz}(1+2xyz) = e^{xyz}(xyz + x^2y^2z^2 + 1 + 2xyz) =$$
$$= e^{xyz}(x^2y^2z^2 + 3xyz + 1)$$

$\boxed{10}$ $x\sin(y-z) = \sin(z)$ ekuazioak $z = f(x,y)$ funtzioa definitzen du. Kalkulatu $\frac{\partial z}{\partial x}$ eta $\frac{\partial z}{\partial y}$

Ekuazioa inplizituki deribatuz (x-rekiko)

$$\sin(y-z) + x\cos(y-z)\left(-\frac{\partial z}{\partial x}\right) = \cos(z)\left(\frac{\partial z}{\partial x}\right)$$

$$\left(\frac{\partial z}{\partial x}\right)\left(\cos(z) + x\cos(y-z)\right) = \sin(y-z)$$

$$\frac{\partial z}{\partial x} = \frac{\sin(y-z)}{\cos(z) + x\cos(y-z)}$$

Ekuazioa inplizituki deribatuz: (y-rekiko)

$$x\cos(y-z)\cdot\left(1-\frac{\partial z}{\partial y}\right)=\cos(z)\left(\frac{\partial z}{\partial y}\right)$$

$$x\cos(y-z)=\cos(z)\left(\frac{\partial z}{\partial y}\right)+x\cos(y-z)\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial z}{\partial y}=\frac{x\cos(y-z)}{\cos(z)+x\cos(y-z)}$$

$\boxed{11}$ Izan bedi $z(x,y)=x\,tg(xy)$ funtzioa, Schwarz-en Teorema egiaztatu.

Teorema: $\dfrac{\partial^2 z}{\partial x\partial y}=\dfrac{\partial^2 z}{\partial y\partial x}$

$$\frac{\partial z}{\partial x}=tg(xy)+x\frac{y}{\cos^2(xy)}$$

$$\frac{\partial}{\partial y}\left[\frac{\partial z}{\partial x}\right]=\frac{\partial}{\partial y}\left[tg(xy)+\frac{xy}{\cos^2(xy)}\right]=\frac{x}{\cos^2(xy)}+\frac{x\cos^2(xy)-xy\,2\cancel{\cos(xy)}(-\sin(xy))x}{\cos^4(xy)}=$$

$$=\frac{x\cos(xy)+2x^2y\sin(xy)}{\cos^3(xy)}+\frac{x\cos(xy)}{\cos^3(xy)}=\boxed{\frac{2x\cos(xy)+2x^2y\sin(xy)}{\cos^3(xy)}}$$

$=$ ✓

$$\frac{\partial z}{\partial y}=x\frac{x}{\cos^2(xy)}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial z}{\partial y}\right]=\frac{\partial}{\partial x}\left[\frac{x^2}{\cos^2(xy)}\right]=\frac{2x\cos^2(xy)-x^2\,2\cancel{\cos(xy)}(-\sin(xy))\cdot y}{\cos^4(xy)}=\boxed{\frac{2x\cos(xy)+2x^2y\sin(xy)}{\cos^3(xy)}}$$

$\boxed{12}$ $z^3+2y^3-3xyz-2y+3=0$ ekuazioak $z(x,y)$ funtzioa definitzen du. Kalkulatu $\dfrac{\partial z}{\partial x}$ eta $\dfrac{\partial z}{\partial y}$

Inplizituki deribatuz x-rekiko

$$3z^2\frac{\partial z}{\partial x}-3yz-3xy\frac{\partial z}{\partial x}=0\longrightarrow\left(\frac{\partial z}{\partial x}\right)(3z^2-3xy)=3yz\longrightarrow\frac{\partial z}{\partial x}=\frac{3yz}{3z^2-3xy}$$

$$\boxed{\frac{\partial z}{\partial x}=\frac{yz}{z^2-xy}}$$

y-rekiko deribatuz

$$3z^2\frac{\partial z}{\partial y}+6y^2-3xz-3xy\frac{\partial z}{\partial y}-2=0\longrightarrow\left(\frac{\partial z}{\partial y}\right)(3z^2-3xy)=2+3xz-6y^2$$

$$\boxed{\frac{\partial z}{\partial y}=\frac{2+3xz-6y^2}{3z^2-3xy}}$$

$\boxed{13}$ Kalkulatu $\frac{\partial^6 u}{\partial x^3 \partial y^3}$ non $u(x,y) = x^3 \sin(y) + y^3 \sin(x)$ den

$$\frac{\partial u}{\partial x} = 3x^2 \sin(y) + y^3 \cos(x)$$

$$\frac{\partial^2 u}{\partial^2 x} = 6x \sin(y) - y^3 \sin(x)$$

$$\frac{\partial^3 u}{\partial^3 x} = 6 \sin(y) - y^3 \cos(x)$$

$$\frac{\partial^4 u}{\partial y} = 6\cos(y) - 3y^2 \cos(x)$$

$$\frac{\partial^5 u}{\partial^2 y} = -6\sin(y) - 6y\cos(x)$$

$$\frac{\partial^6 u}{\partial^3 y} = -6\cos(y) - 6\cos(x)$$

$\boxed{14}$ $z^3 = xz + y$ ekuazioak $z = f(x,y)$ funtzioak definitzen du. Kalkulatu $\frac{\partial^2 z}{\partial x \partial y}$

Inplizituki deribatuz (x-rekiko)

$$3z^2 \frac{\partial z}{\partial x} = z + x\frac{\partial z}{\partial x} \longrightarrow \frac{\partial z}{\partial x} = \frac{z}{3z^2 - x}$$

$$\frac{\partial}{\partial y}\left[\frac{z}{3z^2 - x}\right] = \frac{\frac{\partial z}{\partial y}(3z^2 - x) - \cancel{2}6z^2\frac{\partial z}{\partial y}}{(3z^2-x)^2} = \frac{\frac{\partial z}{\partial y}(3z^2 - x - 6z^2)}{(3z^2-x)^2} = \frac{\frac{\partial z}{\partial y}(-3z^2 - x)}{(3z^2-x)^2}$$

Inplizituki deribatuz (y-rekiko)

$$3z^2 \frac{\partial z}{\partial y} = x\frac{\partial z}{\partial y} + 1 \longrightarrow \frac{\partial z}{\partial y} = \frac{1}{3z^2 - x}$$

Ordezkatuz:

$$\frac{\frac{\partial z}{\partial y}(-3z^2 - x)}{(3z^2-x)^2} = \frac{\frac{(-3z^2-x)}{(3z^2-x)}}{(3z^2-x)^2} = \frac{-3z^2 - x}{(3z^2-x)^2(3z^2-x)} = \frac{-3z^2-x}{(3z^2-x)^3}$$

$\boxed{15}$ Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua

$$f(x,y) = \frac{\arccos(x^2+y^2-5)}{\ln(e^x - y)}$$

$$\left.\begin{array}{l} 1)\ \arccos(x^2+y^2-5) \rightarrow -1 \leq (x^2+y^2-5) < 1 \\ 2)\ \ln(e^x - y) \rightarrow e^x - y > 0 \\ 3)\ \text{Izendatzailea} \neq 0 \rightarrow \ln(e^x-y) \neq 0 \rightarrow e^x - y \neq 1 \end{array}\right\}$$

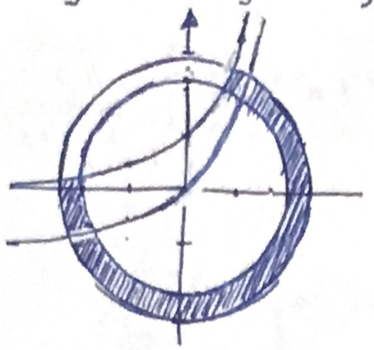
1) $x^2+y^2 = 5 \rightarrow$ Zirkunferentzia

$$x^2+y^2-5 = -1 \rightarrow x^2+y^2 = 4 \begin{cases} C(0,0) \\ r = 2 \end{cases}$$

$$x^2+y^2-5 = 1 \rightarrow x^2+y^2 = 6 \begin{cases} C(0,0) \\ r = \sqrt{6} \end{cases}$$

2) $e^x - y > 0 \to e^x > y$ (Kurbaren azpitik)

3) $e^x - y \neq 1 \to e^x - y = 1 \to y = e^x - 1$ (Kurba domeinu kanpotik)



$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / (-1\le x^2+y^2-5\le 1) \wedge (y<e^x) \wedge (y\neq e^x-1)\}}$$

$\boxed{16}$ Kalkulatu eta grafikoki adierazi hurrengo funtzioaren definizio eremua

$f(x,y)=\sqrt{(x^2+y^2-1)(2x-x^2-y^2)}$

$$(x^2+y^2-1)(2x-x^2-y^2)\geq 0 \begin{cases} (x^2+y^2-1\geq 0) \wedge (2x-x^2-y^2\geq 0) \\ (x^2+y^2-1\leq 0) \wedge (2x-x^2-y^2\leq 0) \end{cases}$$

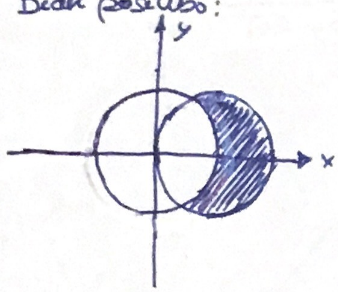
1) $x^2+y^2-1 \to$ Zirkunferentzia

$x^2+y^2-1=0 \to x^2+y^2=1 \begin{cases} C(0,0) \\ r=1 \end{cases}$

2) $2x-x^2-y^2 \to$ Zirkunferentzia

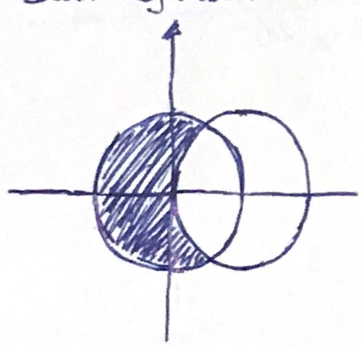
$2x-y^2-x^2=0 \to x^2+y^2-2x=0 \to (x-1)^2+y^2=1 \begin{cases} C(1,0) \\ r=1 \end{cases}$

Biak positibo:



$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / (x^2+y^2-1\geq 0) \wedge (x^2+y^2-2x\leq 0)\}}$$

Biak negatibo:



$$\boxed{D=\{(x,y)\in\mathbb{R}^2 / (x^2+y^2-1\leq 0) \wedge (x^2+y^2-2x\geq 0)\}}$$

Beraz, $D=\{(x,y)\in\mathbb{R}^2 / [(x^2+y^2\geq 1) \wedge ((x-1)^2+y^2\leq 1)] \vee [(x^2+y^2\leq 1) \wedge ((x-1)^2+y^2\geq 1)]\}$

$\boxed{17}$ Izan bedi $z(u,v)=e^{uv+u-v}$ funtzioa, non $\begin{cases} u(x,y)=x^2+y^2 \\ v(x,y)=\dfrac{1}{e^{x-y}} \end{cases}$ diren, Kalkulatu $\frac{\partial z}{\partial x}$ eta $\frac{\partial z}{\partial y}$ $(x,y)=(0,0)$ puntuan.

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}=(v+1)e^{uv+u-v}\cdot 2x+(u-1)e^{uv+u-v}\cdot(-e^{y-x}) = \left.\frac{\partial z}{\partial x}\right]_{(0,0)} = \left\| \begin{matrix} x=0 \\ y=0 \\ u=0 \\ v=1 \end{matrix} \right\| =$$

$$=0+e^{-1}=\frac{1}{e}$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}=(v+1)e^{uv+u-v}\cdot 2y+(u-1)e^{uv+u-v}\cdot e^{y-x} = \left.\frac{\partial z}{\partial y}\right]_{(0,0)} = \left\| \begin{matrix} x=0 \\ y=0 \\ u=0 \\ v=1 \end{matrix} \right\| =$$

$$=0-e^{-1}=-\frac{1}{e}$$

$\boxed{18}$ $z^2-5xyz+x^3+5x^2y=0$ ekuazioak $z=f(x,y)$ funtzioa definitzen du. Kalkulatu $\frac{\partial^2 z}{\partial x \partial y}$ $(1,1)$, $z(1,1)=2$ izanik.

Ekuazioa inplizituki deribatuz (x-rekiko)

$$2z\frac{\partial z}{\partial x}-5yz-5xy\frac{\partial z}{\partial x}+3x^2+10y=0 \longrightarrow \frac{\partial z}{\partial x}=\frac{3x^2+10y-5yz}{5xy-2z}$$

$$\frac{\partial}{\partial y}\left[\frac{3x^2+10xy-5yz}{5xy-2z}\right]=\frac{\left(10x-5z-5y\frac{\partial z}{\partial y}\right)(5xy-2z)-(3x^2+10xy-5yz)\left(5x-2\frac{\partial z}{\partial y}\right)}{(5xy-2z)^2}=$$

$$=\frac{\cancel{50x^2y}-20xz-\cancel{25xyz}+10z^2-25xy^2\left(\frac{\partial z}{\partial y}\right)+\cancel{10yz\left(\frac{\partial z}{\partial y}\right)}-15x^3+6x^2\left(\frac{\partial z}{\partial y}\right)-\cancel{50x^2y}+20xy\left(\frac{\partial z}{\partial y}\right)+\cancel{25xyz}-\cancel{10yz\left(\frac{\partial z}{\partial y}\right)}}{(5xy-2z)^2}=$$

$$=\frac{10z^2-20xz-15x^3+\left(\frac{\partial z}{\partial y}\right)(6x^2-25xy^2+20xy)}{(5xy-2z)^2}$$

Ekuazioa inplizituki deribatuz (y-rekiko)

$$2z\frac{\partial z}{\partial y}-5xz-5xy\frac{\partial z}{\partial y}+5x^2=0 \longrightarrow \frac{\partial z}{\partial y}=\frac{5x^2-5xz}{5xy-2z}$$

Aurreko ekuazioan ordezkatuz:

$$\frac{\partial}{\partial y}\left[\frac{\partial z}{\partial x}\right]=\frac{10z^2-20xz-15x^3+\left(\frac{\partial z}{\partial y}\right)(6x^2-25xy^2+20xy)}{(5xy-2z)^2}=\frac{10z^2-20xz-15x^3+\left(\frac{5x^2-5xz}{5xy-2z}\right)(6x^2-25xy^2+20xy)}{(5xy-2z)^2}=$$

$$=\frac{10z^2-20xz-15x^3+\dfrac{\quad}{5xy-2z}}{\quad}$$

$\boxed{18}$ $z^2-5xyz+x^3+5x^2y=0$ ekuazioak $z=f(x,y)$ funtzioa definitzen du. Kalkulatu $\frac{\partial^2 z}{\partial x \partial y}(1,1)$, $z(1,1)=2$ izanik.

Ekuazioa x-rekiko inplizituki deribatuz:

$$2z\frac{\partial z}{\partial x}-5yz-5xy\frac{\partial z}{\partial x}+10xy+3x^2=0 \rightarrow \frac{\partial z}{\partial x}=\frac{3x^2+10xy-5yz}{5xy-2z}$$

$$\frac{\partial}{\partial y}\left[\frac{\partial z}{\partial x}\right]=\frac{\left(10x-5z-5y\frac{\partial z}{\partial y}\right)(5xy-2z)-(3x^2+10xy-5z)\left(5x-2\frac{\partial z}{\partial y}\right)}{(5xy-2z)^2}$$

Ekuazioa y-rekiko inplizituki deribatuz:

$$2z\frac{\partial z}{\partial y}-5xz-5xy\frac{\partial z}{\partial y}+5x^2=0 \rightarrow \frac{\partial z}{\partial y}=\frac{5x^2-5xz}{5xy-2z}$$

$$\left.\frac{\partial z}{\partial y}\right]_{(1,1)}=\frac{5(1)^2-5(1)(2)}{5(1)(1)-2(2)}=\frac{5-10}{5-4}=-5$$

Aurreko ekuazioan ordezkatuz:

$$\left.\frac{\partial^2 z}{\partial x \partial y}\right]_{(1,1)}=\frac{(10(1)-5(2)-5(1)(-5))(5(1)(1)-2(2))-(3(1)^2+10(1)(1)-5(2))(5(1)-2(-5))}{(5(1)(1)-2(2))^2}=$$

$$=\frac{(10-10+25)(5-4)-(3+10-10)(5+10)}{(5-4)^2}=\frac{25-3\cdot 15}{1}=25-45=-20$$

$\boxed{19}$ Izan bedi $\omega(x,y,z)=\cos(xz)+f(u,v)$ eta $\begin{cases} u(x,y)=x^2+2xy \\ v(x,y)=y^2\cos(2yz) \end{cases}$, Kalkulatu ω'_x, ω'_y eta ω'_z

$\omega'_x=-z\sin(xz)+f'_u\cdot(2x+2y)$

$\omega'_y=f'_u(2x)+f'_v\cdot(2y\cos(2yz)-2y^2z\sin(2yz))$

$\omega'_z=-x\sin(xz)+f'_v\cdot(y^2(-\sin(2yz))2y)=-x\sin(xz)-f'_v(2y^3\sin(2yz))$

$\boxed{20}$ Izan bedi $f(x,y)=-2x^2+2xy-2y^2+100$ funtzioa. Kalkulatu funtzioaren mutur erlatiboak.

$f'_x=-4x+2y$

$f'_y=2x-4y$

Puntu kritikoak: Bi deribatuak polinomioak direnez, beti existitzen dira.

$$\begin{cases} f'_x=0 \\ f'_y=0 \end{cases} \rightarrow \begin{cases} -4x+2y=0 \\ 2x-4y=0 \end{cases} \rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \rightarrow (0,0)$$

Matrize hesiarra:

$$Hf(x,y)=\begin{bmatrix} \frac{\partial^2 f(x,y)}{\partial x^2} & \frac{\partial^2 f(x,y)}{\partial x \partial y} \\ \frac{\partial^2 f(x,y)}{\partial y \partial x} & \frac{\partial^2 f(x,y)}{\partial y^2} \end{bmatrix}=\begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix}$$

$$Hf(0,0)=\begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix}=16-4=12$$

$\rightarrow \frac{\partial^2 f}{\partial x^2}<0 \rightarrow$ Maximo erlatiboa $(0,0,100)$ puntuan

21 Kalkulatu honako funtzio honen mutur erlatiboak: $z(x,y) = x^3 + y^3 - 9xy + 27$

$f'_x = 3x^2 - 9y$

$f'_y = 3y^2 - 9x$

Puntu Kritikoak: Bi deribatuak polinomioak direnez, beti existitzen dira.

$$\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases} \longrightarrow \begin{cases} 3x^2 - 9y = 0 \\ 3y^2 - 9x = 0 \end{cases} \left\langle \begin{array}{l} \to (0,0) \\ \to (3,3) \end{array} \right.$$

Matrize hesiarra:

$$Hf(x,y) = \begin{bmatrix} \dfrac{\partial^2 f(x,y)}{\partial x^2} & \dfrac{\partial f(x,y)}{\partial x \partial y} \\ \dfrac{\partial^2 f(x,y)}{\partial y \partial x} & \dfrac{\partial^2 f(x,y)}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6x & -9 \\ -9 & 6y \end{bmatrix}$$

$$Hf(0,0) = \begin{bmatrix} 0 & -9 \\ -9 & 0 \end{bmatrix} = -81 < 0 \longrightarrow \underline{\text{Zela puntua}}$$

$$Hf(3,3) = \begin{bmatrix} 18 & -9 \\ -9 & 18 \end{bmatrix} = 324 - 81 = 243 > 0$$

$$\to \frac{\partial^2 f}{\partial x^2} > 0 \to \underline{\text{Minimo erlatiboa } (3,3,0) \text{ puntuan}}$$

22 $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ ekuazioak $z = z(x,y)$ funtzioa inplizituki definitzen du. Kalkulatu z funtzioaren mutur erlatiboak.

x-rekiko inplizituki deribatuz:

$$2x + 2z \cdot z'_x - 2 - 6z'_x = 0 \longrightarrow z'_x(2z-6) = 2 - 2x \longrightarrow z'_x = \frac{2-2x}{2z-6} \longrightarrow z'_x = \frac{1-x}{z-3}$$

y-rekiko inplizituki deribatuz:

$$2y + 2z \cdot z'_y + 4 - 6z'_y = 0 \longrightarrow z'_y(2z-6) = -2y - 4 \longrightarrow z'_y = \frac{-2y-4}{2z-6} \longrightarrow z'_y = \frac{-y-2}{z-3}$$

Puntu Kritikoak:

$$\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases} \longrightarrow \begin{cases} 1 - x = 0 \\ -y - 2 = 0 \end{cases} \longrightarrow \begin{cases} x = 1 \\ y = -2 \end{cases} \longrightarrow (1,-2)$$

Funtzioa ez dago definituta $z = 3$ puntuan

Matrize hesiarra:

$$Hf(x,y) = \begin{bmatrix} \dfrac{\partial^2 f(x,y)}{\partial x^2} & \dfrac{\partial^2 f(x,y)}{\partial x \partial y} \\ \dfrac{\partial^2 f(x,y)}{\partial y \partial x} & \dfrac{\partial^2 f(x,y)}{\partial y^2} \end{bmatrix} = \begin{bmatrix} \dfrac{-(z-3)^2 - (1-x)^2}{(z-3)^3} & \dfrac{(1-x)(-y-2)}{(z-3)^3} \\ \dfrac{(-y-2)(1-x)}{(z-3)^3} & \dfrac{-(z-3)^2 - (-y-2)^2}{(z-3)^3} \end{bmatrix}$$

$$z(1,-2) \longrightarrow 1 + 2^2 + z^2 - 2 + 4 \cdot 2 - 6z = 11 \longrightarrow 1 + 4 + z^2 - 2 - 8 - 6z = 11 \longrightarrow z^2 - 6z - 16 = 0 \left\langle \begin{array}{l} z_1 = 8 \\ z_2 = -2 \end{array} \right.$$

$$\text{Puntu Kritikoak} \left\langle \begin{array}{l} (1,-2,8) \\ (1,-2,-2) \end{array} \right.$$

$$Hf(1,-2,8)=\begin{bmatrix} \frac{-(8-3)^2-(1-1)^2}{(8-3)^3} & \frac{(1-1)(-(-2)-2)}{(8-3)^3} \\ \frac{(-(-2)-2)(1-1)}{(8-3)^3} & \frac{-(8-3)^2-(-(-2)-2)^2}{(8-3)^3} \end{bmatrix} = \begin{bmatrix} \frac{-5^2}{5^3} & 0 \\ 0 & \frac{-5^2}{5^3} \end{bmatrix} = \frac{-1}{5}\cdot\frac{-1}{5} = \frac{1}{25} > 0$$

$\to -\frac{1}{5} < 0 \to$ Maximo erlatiboa $(1,-2,8)$ puntuan

$$Hf(1,-2,-2)=\begin{bmatrix} \frac{-(-2-3)^2-(1-1)^2}{(-2-3)^3} & \frac{(1-1)(2-2)}{(-2-3)^3} \\ \frac{(2-2)(1-1)}{(-2-3)^3} & \frac{-(-2-3)^2-(2-2)^2}{(-2-3)^3} \end{bmatrix} = \begin{bmatrix} \frac{-25}{(-5)25} & 0 \\ 0 & \frac{-25}{(-5)25} \end{bmatrix} = \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25} > 0$$

$\to \frac{1}{5} > 0 \to$ Minimo erlatiboa $(1,-2,-2)$ puntua

$\boxed{23}$ Izan bedi $f(x,y)=xy$ funtzioa. Kalkulatu f funtzioaren muturrak $x^2+y^2=1$ kurban.

Funtzio Lagrangiarra idatzi:

$$L(\lambda,(x,y)) = xy + \lambda(x^2+y^2-1)$$

Puntu kritikoak kalkulatu:

$$\begin{cases} L'x=0 \\ L'y=0 \\ \varphi(x,y)=0 \end{cases} \to \begin{cases} y+2\lambda x=0 \to \lambda=-y/2x \\ x+2\lambda y=0 \to \lambda=-x/2y \\ x^2+y^2-1=0 \to x^2+x^2-1=0 \to 2x^2=1 \end{cases} \frac{-y}{2x}=\frac{-x}{2y} \to 2x^2=2y^2 \to \underline{x^2=y^2}$$

$2x^2=1 \to x=\pm 1/\sqrt{2}$, $y=\pm 1/\sqrt{2}$

$(1/\sqrt{2}, 1/\sqrt{2})$ $\to \lambda=-1/2$; $(1/\sqrt{2}, -1/\sqrt{2})$ $\to \lambda=1/2$; $(-1/\sqrt{2}, 1/\sqrt{2})$ $\to \lambda=1/2$; $(-1/\sqrt{2}, -1/\sqrt{2})$ $\to \lambda=-1/2$

Matrize hesiar orlatua

$$HL(\lambda,(x,y)) = \begin{bmatrix} 0 & \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} \\ \frac{\partial\varphi}{\partial x} & L'x^2 & L'xy \\ \frac{\partial\varphi}{\partial y} & L'yx & L'y^2 \end{bmatrix} = \begin{bmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 1 \\ 2y & 1 & 2\lambda \end{bmatrix}$$

Matrizearen determinantea puntu bakoitzean.

$$|HL(-1/2,(1/\sqrt{2},1/\sqrt{2}))| = \begin{vmatrix} 0 & 2/\sqrt{2} & 2/\sqrt{2} \\ 2/\sqrt{2} & -1 & 1 \\ 2/\sqrt{2} & 1 & -1 \end{vmatrix} = \frac{4}{2}+\frac{4}{2}+\frac{4}{2}+\frac{4}{2} = 8 > 0 \to \text{Maximo lokala.}$$

$$|HL(1/2,(1/\sqrt{2},-1/\sqrt{2}))| = \begin{vmatrix} 0 & 2/\sqrt{2} & -2/\sqrt{2} \\ 2/\sqrt{2} & 1 & 1 \\ -2/\sqrt{2} & 1 & 1 \end{vmatrix} = \frac{-4}{2}-\frac{4}{2}-\frac{4}{2}-\frac{4}{2} = -8 < 0 \to \text{Minimo lokala}$$

$$|H\mathcal{L}(1/2,(-1/\sqrt{2},1/\sqrt{2}))| = \begin{vmatrix} 0 & -2/\sqrt{2} & 2/\sqrt{2} \\ -2/\sqrt{2} & 1 & 1 \\ 2/\sqrt{2} & 1 & 1 \end{vmatrix} = -2-2-2-2 = -8 < 0 \rightarrow \text{Minimo lokala}$$

$$|H\mathcal{L}(-1/2,(-1/\sqrt{2},-1/\sqrt{2})| = \begin{vmatrix} 0 & -2/\sqrt{2} & -2/\sqrt{2} \\ -2/\sqrt{2} & -1 & 1 \\ -2/\sqrt{2} & 1 & -1 \end{vmatrix} = 2+2+2+2 = 8 > 0 \rightarrow \text{Maximo lokala}$$

$\boxed{24}$ Izan bedi $f(x,y)=x^2+y^2$ funtzioa, Kalkulatu f funtzioaren muturrak $x+y=1$ kurban

Funtzio Lagrangiarra idatzi

$\mathcal{L}(\lambda,(x,y)) = x^2+y^2+\lambda(x+y-1)$

Puntu kritikoak:

$$\begin{cases} \mathcal{L}'x=0 \\ \mathcal{L}'y=0 \\ \varphi(x,y)=0 \end{cases} \rightarrow \begin{cases} 2x+\lambda=0 \rightarrow \lambda=-2x \\ 2y+\lambda=0 \rightarrow \lambda=-2y \\ x+y-1=0 \rightarrow x+x=1 \rightarrow x=1/2=y \end{cases} \quad -2x=-2y \rightarrow x=y$$

$(1/2,1/2) \rightarrow \lambda=-1$

Matrize hessiar orlatua:

$$H\mathcal{L}(\lambda,(x,y)) = \begin{bmatrix} 0 & \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} \\ \frac{\partial\varphi}{\partial x} & \mathcal{L}'x^2 & \mathcal{L}'xy \\ \frac{\partial\varphi}{\partial y} & \mathcal{L}'yx & \mathcal{L}'y^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$|H\mathcal{L}(-1,(1/2,1/2))| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = -2-2 = -4 < 0 \rightarrow \text{Minimo lokala}$$

$\boxed{25}$ Izan bedi $z(x,y)=x\ln(x)+y\ln(y)$ funtzioa Kalkulatu z funtzioaren muturrak $x+y=2$ kurban.

Funtzio Lagrangiarra:

$\mathcal{L}(\lambda,(x,y)) = x\ln(x)+y\ln(y)+\lambda(x+y-2)$

Puntu kritikoak

$$\begin{cases} \mathcal{L}'x=0 \\ \mathcal{L}'y=0 \\ \varphi(x,y)=0 \end{cases} \rightarrow \begin{cases} \ln(x)+1+\lambda=0 \rightarrow \lambda=-\ln(x)-1 \\ \ln(y)+1+\lambda=0 \rightarrow \lambda=-\ln(y)-1 \\ x+y-2=0 \rightarrow x+x=2 \rightarrow x=1=y \end{cases} \quad -\ln(x)-1=-\ln(y)-1 \rightarrow \ln(x)=\ln(y) \rightarrow x=y$$

$(1,1) \rightarrow \lambda=-1$

Matrize hesiar orlatua

$$HL(\lambda,(x,y)) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1/x & 0 \\ 1 & 0 & 1/y \end{bmatrix}$$

$$|HL(-1,(1,1))| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -1-1 = -2 < 0 \rightarrow \text{Minimo lokala } (1,1,0)$$

$\boxed{26}$ Izan bedi $f(x,y) = 49 - x^2 - y^2$ funtzioa, Kalkulatu f funtzioaren muturrak $x+3y-10=0$ kurban.

$$L(\lambda,(x,y)) = 49 - x^2 - y^2 + \lambda(x+3y-10)$$

Puntu Kritikoak:

$$\begin{cases} L'_x = 0 \\ L'_y = 0 \\ \varphi(x,y) = 0 \end{cases} \rightarrow \begin{cases} -2x+\lambda=0 \rightarrow \lambda = 2x \\ -2y+3\lambda=0 \rightarrow \lambda = 2y/3 \\ x+3y-10=0 \rightarrow x+9x=10 \rightarrow x=1 \rightarrow y=3 \end{cases}$$

$2x = 2y/3 \rightarrow 6x = 2y \rightarrow 3x = y$

$(1,3) \rightarrow \lambda = 2$

Matrize hesiar orlatua

$$H(\lambda,(x,y)) = \begin{bmatrix} 0 & 1 & 3 \\ 1 & -2 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

$$|HL(2,(1,3))| = \begin{vmatrix} 0 & 1 & 3 \\ 1 & -2 & 0 \\ 3 & 0 & 3 \end{vmatrix} = 18 - 3 = 15 > 0 \rightarrow \text{Maximoa } (1,3,39)$$

$\boxed{27}$ Izan bedi $f(x,y) = x+2y$ funtzioa Kalkulatu f funtzioaren muturrak $x^2+y^2=5$ Kurban.

Funtzio Lagrangiarra idatzi:

$$L(\lambda,(x,y)) = x+2y+\lambda(x^2+y^2-5)$$

Puntu Kritikoak:

$$\begin{cases} L'_x = 0 \\ L'_y = 0 \\ \varphi(x,y) = 0 \end{cases} \rightarrow \begin{cases} 1+2\lambda x = 0 \rightarrow \lambda = -1/2x \\ 2+2\lambda y = 0 \rightarrow \lambda = -1/y \\ x^2+y^2- \;=0 \rightarrow x^2+(2x)^2- \;=0 \rightarrow 5x^2=5 \rightarrow x=\pm 1 \end{cases} \quad , y = \pm 2$$

$\frac{-1}{2x} = \frac{-1}{y} \rightarrow 2x = y$

$(1,2)$, $(-1,-2)$

$(1,2) \downarrow \lambda = -1/2$, $(-1,-2) \downarrow \lambda = 1/2$

Matrize Hesiar orlatua:

$$HL(\lambda,(x,y)) = \begin{bmatrix} 0 & \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} \\ \frac{\partial\varphi}{\partial x} & L'_{x^2} & L'_{xy} \\ \frac{\partial\varphi}{\partial y} & L'_{yx} & L'_{y^2} \end{bmatrix} = \begin{bmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{bmatrix}$$

$$|HL(1/2,(1,2))| = \begin{vmatrix} 0 & 2 & 4 \\ 2 & -1 & 0 \\ 4 & 0 & -1 \end{vmatrix} = 16+4 = +20>0 \rightarrow$$ <u>Maximoa $(1,2,5)$ puntuan</u>

$$|HL(-1/2,(-1,-2))| = \begin{vmatrix} 0 & -2 & -4 \\ -2 & 1 & 0 \\ -4 & 0 & 1 \end{vmatrix} = -16-4 = -20<0 \rightarrow$$ <u>Minimoa $(-1,-2,-5)$ puntuan</u>

$\boxed{28}$ Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua

$$f(x,y) = e^{\left(\frac{1}{\pi^2-y^2-x^2}\right)} + \frac{\ln(x+\pi)\ln(-y)}{\sqrt{|\sin(\frac{x}{2})|}}$$

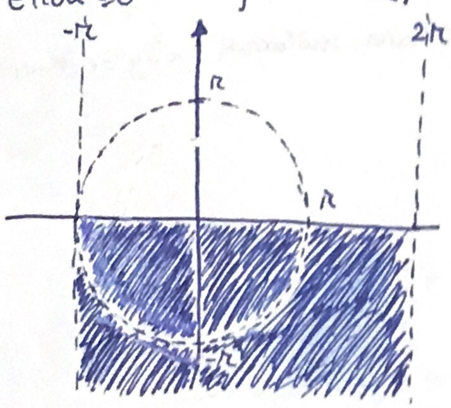
1) e beti positiboa $\rightarrow \pi^2-y^2-x^2=0 \rightarrow$ Zirkunferentzia

$x^2+y^2=\pi^2 \rightarrow C(0,0),\ r=\pi$

2) ln-ren argumentua $>0 \rightarrow \ln(x+\pi) \rightarrow x>-\pi$

$\ln(-y) \rightarrow y<0$

3) Zatitzailea $\neq 0$, Erroa ≥ 0 $\Big\} \rightarrow \sin(\frac{x}{2}) \neq 0 \begin{cases} x\neq 0 \\ x\neq 2K\pi \end{cases}$



$D = \{(x,y)\in\mathbb{R}^2 / (x^2+y^2-\pi^2\neq 0) \wedge (x>-\pi) \wedge (y<0) \wedge (x\neq 2K\pi)\}$

29 Kalkulatu eta grafikoki adierazi honako funtzio honen definizio eremua

$f(x,y) = \ln\left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right) + \sqrt{y^2 - x^2}$

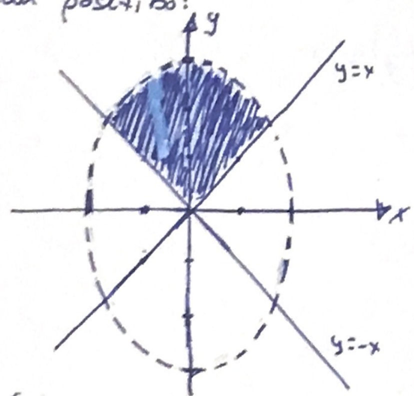
1) ln-ren argumentua $>0 \longrightarrow \ln\left(1 - \frac{x^2}{4} - \frac{y^2}{9}\right) \longrightarrow 1 - \frac{x^2}{4} - \frac{y^2}{9} > 0$

$\frac{x^2}{4} + \frac{y^2}{9} = 1 \longrightarrow$ Elipsea $\longrightarrow C(0,0)$, $a=2$, $b=3$

2) Erroa $\geq 0 \longrightarrow y^2 - x^2 \geq 0 \longrightarrow (y+x)(y-x) \geq 0$

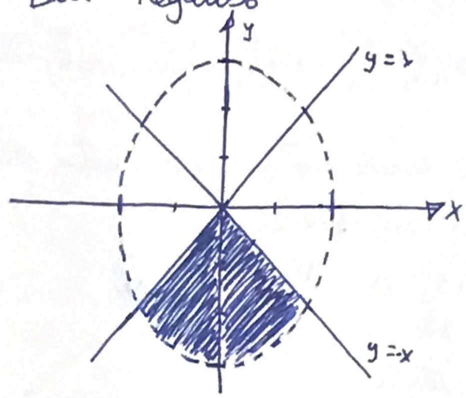
1. $(y+x) \geq 0 \longrightarrow y \geq -x \longrightarrow$ Zuzena
   $(y-x) \geq 0 \longrightarrow y \geq x \longrightarrow$ Zuzena
2. $(y+x) \leq 0 \longrightarrow y \leq -x \longrightarrow$ Zuzena
   $(y-x) \leq 0 \longrightarrow y \leq x \longrightarrow$ Zuzena

Biak positibo:



$D = \{(x,y) \in \mathbb{R}^2 / \frac{x^2}{9} + \frac{y^2}{9} < 1 \wedge y \geq x \wedge y \geq -x\}$

Biak negatibo



$D = \{(x,y) \in \mathbb{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} < 1 \wedge y \leq x \wedge y \leq -x\}$

$$\boxed{D = \left\{(x,y) \in \mathbb{R}^2 / \frac{x^2}{4} + \frac{y^2}{9} < 1 \wedge \left((y \geq x \wedge y \geq -x) \vee (y \leq x \wedge y \leq -x)\right)\right\}}$$

30 Izan bitez $\omega = f(u,v)$; $u(x,y,z) = x^2 + 2yz$; $v(x,y,z) = y^2 + 2xz$. Frogatu honako berdintza hau:

$(y^2 - xz)\omega'x + (x^2 - yz)\omega'y + (z^2 - xy)\omega'z = 0$

$\omega'x = \omega'u \cdot 2x + \omega'v \cdot 2z$

$\omega'y = \omega'u \cdot 2z + \omega'v \cdot 2y$

$\omega'z = \omega'u \cdot 2y + \omega'v \cdot 2x$

Formulan ordezkatuz:

$(y^2 - xz)(2x\omega'u + 2z\omega'v) + (x^2 - yz)(2z\omega'u + 2y\omega'v) + (z^2 - xy)(2y\omega'u + 2x\omega'v) =$

$= 2xy^2\omega'u - 2x^2z\omega'u + 2y^2z\omega'v - 2xz^2\omega'v + 2x^2z\omega'u - 2yz^2\omega'u + 2x^2y\omega'v - 2y^2z\omega'v + 2yz^2\omega'u - 2xy^2\omega'u + 2xz^2\omega'v - 2x^2y\omega'v =$

$= \omega'u(\cancel{2xy^2} - \cancel{2x^2z} + \cancel{2x^2z} - \cancel{2yz^2} + \cancel{2yz^2} - \cancel{2xy^2}) + \omega'v(\cancel{2y^2z} - \cancel{2xz^2} + \cancel{2x^2y} - \cancel{2y^2z} + \cancel{2xz^2} - \cancel{2x^2y}) =$

$= \omega'u(0) + \omega'v(0) = 0$

$\boxed{31}$ Izan bedi $z(x,y)=e^{x/y}\cdot\sin(x/y)+e^{y/x}\cdot\sin(y/x)$ funtzioa, frogatu honako berdintza hau:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$$

$$\frac{\partial z}{\partial x}=\frac{1}{y}e^{x/y}\cdot\sin\left(\frac{x}{y}\right)+e^{x/y}\cos\left(\frac{x}{y}\right)\cdot\frac{1}{y}+\frac{-y}{x^2}e^{y/x}\cdot\sin\left(\frac{y}{x}\right)+e^{y/x}\cos\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)=$$

$$=\frac{e^{x/y}}{y}\sin\left(\frac{x}{y}\right)+\frac{e^{x/y}}{y}\cos\left(\frac{x}{y}\right)-\frac{ye^{y/x}}{x^2}\sin\left(\frac{y}{x}\right)-\frac{ye^{y/x}}{x^2}\cos\left(\frac{y}{x}\right)$$

$$\frac{\partial z}{\partial y}=\frac{-x}{y^2}e^{x/y}\cdot\sin\left(\frac{x}{y}\right)+e^{x/y}\cos\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right)+\frac{1}{x}e^{y/x}\sin\left(\frac{y}{x}\right)+e^{y/x}\cos\left(\frac{y}{x}\right)\left(\frac{1}{x}\right)=$$

$$=\frac{e^{y/x}}{x}\sin\left(\frac{y}{x}\right)+\frac{e^{y/x}}{x}\cos\left(\frac{y}{x}\right)-\frac{xe^{x/y}}{y^2}\sin\left(\frac{x}{y}\right)-\frac{xe^{x/y}}{y^2}\cos\left(\frac{x}{y}\right)$$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\cancel{\frac{xe^{x/y}}{y}\sin\left(\frac{x}{y}\right)}+\cancel{\frac{xe^{x/y}}{y}\cos\left(\frac{x}{y}\right)}-\cancel{\frac{xye^{y/x}}{x^2}\sin\left(\frac{y}{x}\right)}-\cancel{\frac{xye^{y/x}}{x^2}}+$$

$$+\cancel{\frac{ye^{y/x}}{x}\sin\left(\frac{y}{x}\right)}+\cancel{\frac{ye^{y/x}}{x}\cos\left(\frac{y}{x}\right)}-\cancel{\frac{xye^{x/y}}{y^2}\sin\left(\frac{x}{y}\right)}-\cancel{\frac{xye^{x/y}}{y^2}\cos\left(\frac{x}{y}\right)}=0$$

$\boxed{32}$ Kalkulatu honako funtzio honen mutur erlatiboak:

$z(x,y)=x^3+3xy^2-15x-12y$

$z'x=3x^2+3y^2-15$

$z'y=6xy-12$

Puntu kritikoak:

$$\begin{cases}z'x=0\\ z'y=0\end{cases}\rightarrow\begin{cases}3x^2+3y^2=15\\ 6xy=12\end{cases}\rightarrow\begin{cases}x^2+y^2=5\\ xy=2\end{cases}\longrightarrow\begin{cases}x_1=\pm1 & =\pm2\\ y_1=\pm2 & y_2=\pm1\end{cases}$$

$(1,2),\ (-1,-2),\ (2,1),\ (-2,-1)$

Matrize hesiarra:

$$Hf(x,y)=\begin{bmatrix}\frac{\partial^2 f(x,y)}{\partial x^2} & \frac{\partial^2 f(x,y)}{\partial x\partial y}\\ \frac{\partial^2 f(x,y)}{\partial y\partial x} & \frac{\partial^2 f(x,y)}{\partial y^2}\end{bmatrix}=\begin{bmatrix}6x & 6y\\ 6y & 6x\end{bmatrix}$$

$$|Hf(1,2)|=\begin{vmatrix}6 & 12\\ 12 & 6\end{vmatrix}=36-144=-108<0\rightarrow \text{Zela puntua}$$

$$|Hf(-1,-2)|=\begin{vmatrix}-6 & -12\\ -12 & -6\end{vmatrix}=36-144=-108<0\rightarrow \text{Zela puntua}$$

$$|Hf(2,1)|=\begin{vmatrix}12 & 6\\ 6 & 12\end{vmatrix}=144-36=108>0\rightarrow$$

$\rightarrow 12>0\rightarrow$ Minimo erlatiboa $(2,1,-28)$

$$|Hf(-2,-1)|=\begin{vmatrix}-12 & -6\\ -6 & -12\end{vmatrix}=144-36=108>0$$

$\rightarrow 12>0\rightarrow$ Minimo erlatiboa $(2,1,28)$