4) ZENBAKIZKO METODOAK

1 Izan bedi $\frac{1}{x} - \ln(x) = 0$ ekuazioa:

A) Froga ezazu ekuazioak $\hat{x}$ soluzio bakarra duela $[1,2]$ tartean.

1) $g(x)$ deribagarria $[1,2]$ tartean ✓

2) Lehenengo deribatuak zeinua mantentzen du? ✓

$$g'(x) = -\frac{1}{x^2} - \frac{1}{x} = \frac{-1-x}{x^2} = -\frac{(x+1)}{x^2}$$

3) $g(a)g(b) < 0$?

$$\left.\begin{array}{l} g(a)g(b)<0? \\ g(1) = 1 - 0 = 1 > 0 \end{array}\right\} g(1)g(2) < 0$$

B) Dikotomia metodoa erabiliz, aurkitu $\hat{x}$-ren balio hurbildu bat errorea $\leq 1/8$ izanik

$[a,b] = [1,2] \longrightarrow x_1 = \frac{1+2}{2} = \frac{3}{2} \rightarrow$ Egindako errorea $\leq \frac{b-a}{2} \leq \frac{1}{2} \rightarrow$ 1. Iterazioa

$g(1) = 1 > 0$

$g(2) = -0'1931 < 0$

$g(x_1) = g(3/2) = \frac{2}{3} - \ln\left(\frac{3}{2}\right) = 0'2612 > 0$

2. Iterazioa $\cancel{[a, x_1]} \vee [x_1, b]$

$[x_1, b] = [3/2, 2]$

$x_2 = \frac{3/2 + 2}{2} = \frac{7}{4} \rightarrow$ Egindako errorea $\leq \frac{b-a}{2^2} = \frac{1}{4}$

$g(3/2) = 0'2612 > 0$

$g(2) = -0'1931 < 0$

$g(7/4) = \frac{4}{7} - \ln\left(\frac{7}{4}\right) = 0'0318 > 0$

3. Iterazioa $[7/4, 2]$

$x_3 = \frac{7/4 + 2}{2} = \frac{15}{8} = 1'875$

Egindako errorea $\leq \frac{b-a}{2^3} = \frac{1}{8}$

2 Ondoz ondoko hurbilketen metodoa erabiliz, kalkulatu $x - \cos(x) = 0$ ekuazioaren soluzio hurbildua $[0,1]$ tartean (10 iterazio egin).

- $x = g(x) \longrightarrow x = \cos(x) \longrightarrow g(x) = \cos(x)$

Baldintzak konprobatuko ditugu

1) $g(x)$ C^1 klasekoa da. ✓

2) $|g'(x)| < K$ $(K \geq 1)$ ✓

$g'(x) = -\sin(x) \rightarrow |g'(x)| = \underbrace{|-\sin(x)| < 1}_{[0,1] \text{ tartean}}$

3) $g([0,1]) \subset [0,1]$

$\cos([0,1]) \subset [0,1]$ ✓

• $x_0 = \frac{1}{2}$ ($x_0 \in [0,1]$ hasierako balio bat aukeratu)

$x_1 = g(x_0) = \cos(1/2) = 0'8775$

$x_2 = g(x_1) = \cos(0'8775) = 0'639$

$x_3 = g(x_2) = \cos(0'639) = 0'8027$

$x_4 = \cos(0'8027) = 0'6947$

$x_5 = \cos(0'6947) = 0'7682$

$x_6 = \cos(0'7682) = 0'7191$

$x_7 = \cos(0'7191) = 0'7523$

$x_8 = \cos(0'7523) = 0'73008$

$x_9 = \cos(0'73008) = 0'7451$

$x_{10} = \cos(0'7451) = 0'735$

$\boxed{3}$ Izan bedi $xe^x - 2 = 0$ ekuazioa.

A) Frogatu soluzio bakarra duela $[0,2]$ tartean

$g(x) = xe^x - 2$

1) C^2 klasekoa $[0,2]$ tartean ✓

2) $\left.\begin{aligned} g'(x) &= e^x + xe^x = e^x(x+1) \\ g''(x) &= e^x + e^x + xe^x = e^x(x+2) \end{aligned}\right\}$ zeinua mantentzen dute $[0,2]$ tartean

B) Newton-Raphson-en metodoa erabiliz, kalkulatu:

$x_0 = 1'5$ ($x_0 \in [0,2] / g'(x_0) \neq 0$)

1. iterazioa

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = 1'5 - \frac{g(1'5)}{g'(1'5)} = \frac{1'5e^{1'5} - 2}{2'5(e^{1'5})} = 1'078$$

2. iterazioa

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = \frac{1'078\,e^{1'078} - 2}{2'078\,e^{1'078}} = 0'887$$

3. iterazioa

$$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = \frac{0'887e^{0'887} - 2}{1'887e^{0'887}} \approx 0'854$$

$\boxed{4}$ A) Kalkulatu $f(x)=\sqrt{x}$ funtzioaren Taylor-en garapeneko 7 gaiak $a=1$ puntuan

Taylor-en garapena:

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\dots+\frac{f^{(n)}(a)}{n!}(x-a)^{(n)}$$

$\underline{a=1}$

$f(1)=\sqrt{1}=1$

$f'(1)=\frac{1}{2\sqrt{x}}=\frac{1}{2}$

$f^{II}(1)=\frac{-1}{4\sqrt{x^3}}=\frac{-1}{4}$

$f^{III}(1)=\frac{3}{8\sqrt{x^5}}=\frac{3}{8}$

$f^{IV}(1)=\frac{-15}{16\sqrt{x^7}}=\frac{-15}{16}$

$f^{V}(1)=\frac{105}{32\sqrt{x^9}}=\frac{105}{32}$

$f^{VI}(1)=\frac{-945}{64\sqrt{x^{11}}}=\frac{-945}{64}$

Beraz,

$$f(x)=1+\frac{1}{2}(x-1)-\frac{1}{4}\frac{(x-1)^2}{2!}+\frac{3}{8}\frac{(x-1)^3}{3!}-\frac{15}{16}\frac{(x-1)^4}{4!}+\frac{105}{32}\frac{(x-1)^5}{5!}-\frac{945}{64}\frac{(x-1)^6}{6!}$$

B) Lehenengo 7 gaiak erabiliz, $\sqrt{2}$-ren balio hurbildua kalkulatu

$\sqrt{2}$ kalkulatzeko $\longrightarrow f(2)$

$$f(2)=1+\frac{1}{2}(2-1)-\frac{1}{4}\frac{(2-1)^2}{2!}+\frac{3}{8}\frac{(2-1)^3}{3!}-\frac{15}{16}\frac{(2-1)^4}{4!}+\frac{105}{32}\frac{(2-1)^5}{5!}-\frac{945}{64}\frac{(2-1)^6}{6!}=1'40527$$

C) Aurreko atalean egindako erroreak kalkulatu:

$$R_{n+1}\leq\left|\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}\right| \quad \xi\in(a,x)$$

$n=6$

$$R_7\leq\left|\frac{f^{(7)}(\xi)}{7!}(2-1)^7\right| \quad \xi\in(1,2)$$

$$f^{(7)}(x)=\frac{-945}{64}\left(\frac{-11}{2}\right)x^{-13/2}=\frac{10395}{128}x^{-13/2}$$

$$R_7\leq\left|\frac{\frac{10395}{128}(\xi)^{-13/2}}{7!}(1)^7\right|=\left\| \xi=1, \text{ errorearen balio handiena kalkulatzeko } =0'0161 \right\| \quad \xi\in(1,2)$$

$\boxed{5}$ Izan bedi $x=e^{-x/2}$ ekuazioa

1) Froga ezazu ekuazioak $\hat{x}$ soluzio bakarra duela $[0,1]$ tartean

$x=g(x) \rightarrow x=e^{-x/2} \rightarrow g(x)=e^{-x/2}$

Baldintzak konprobatuko ditugu:

1) $g(x)$ C^1 klasekoa da $[0,1]$ tartean ✓

2) $|g'(x)|<K$ $(K\leq 1)$

$g'(x)=-e^{-x/2} \rightarrow |g'(x)|=|-e^{-x/2}|<1$ $[0,1]$ ✓

3) $g([0,1])\subset[0,1]$

$(-e^{[0,1]/2})\subset[0,1]$ ✓

B) Ondoz ondoko hurbilketa metodoa erabiliz, aurkitu $\hat{x}$-ren soluzio hurbildu bat. Hartu $x_0=1/2$ eta 10 iterazio erabili

$x_0=\frac{1}{2}$ ($x_0\in[0,1]$ hasierako balio bat aukeratu)

$x_1=g(x_0)=e^{-1/2}=0'77880$

$x_2=g(x_1)=0'677462$

$x_3=g(x_2)=0'712673$

$x_4=g(x_3)=0'700236$

$x_5=g(x_4)=0'704604$

$x_6=g(x_5)=0'703067$

$x_7=g(x_6)=0'703608$

$x_8=g(x_7)=0'703417$

$x_9=g(x_8)=0'703484$

$x_{10}=g(x_9)=0'703461$

$\boxed{6}$ Izan bedi $x+\cos(x)=0$ ekuazioa

1) Froga ezazu ekuazioak soluzio bakarra duela $[-1,0]$ tartean

$g(x)=x+\cos(x)$

1) $g(x)$ C^2 klasekoa da $[-1,0]$ tartean ✓

2) $\left.\begin{array}{l} g'(x)=1-\sin(x)=0 \\ g''(x)=-\cos(x)=0 \end{array}\right\}$ zeinua mantentzen da $[-1,0]$ tartean ✓

3) $g(-1)g(0)<0 \rightarrow (-0'459)(1)<0$ ✓

B) Newton-Raphson-en metodoa erabiliz, aurkitu $\hat{x}$-ren balio hurbildu bat (Hartu $x_0=-1$ eta 3 iterazio erabili).

1. Iterazioa

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = -1 - \frac{1+\cos(-1)}{1-\sin(-1)} = -0'750364$$

2. Iterazioa

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = -0'739113$$

3. Iterazioa

$$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = -0'739085$$

$\boxed{7}$ A) Lortu $f(x)=\sin(x)$ funtzioaren McLaurin-en garapena

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n$$

$f(0) = \sin(0) = 0$

$f'(0) = \cos(0) = 1$

$f''(0) = -\sin(0) = 0$

$f'''(0) = -\cos(0) = -1$

$f^{IV}(0) = \sin(0) = 0$

B) Kalkulatu $\sin(\pi/4)$-ren balio hurbildua garapeneko lehenengo 3 gai ez-nuluak erabiliz

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5} + \dots + (-1)^{2k+1}\frac{x^{2k+1}}{(2k+1)!}$$

$$\sin(\pi/4) \approx f(\pi/4) = (\pi/4) - \frac{(\pi/4)^3}{3!} + \frac{(\pi/4)^5}{5!} = 0'707143$$

c) Aurreko atalean egindako errorea kalkulatu.

$n=6$

$$R_{n+1} = \left| \frac{f^{(n+1)}(\theta_x)}{(n+1)!} x^{n+1} \right| =$$

$$R_7 \leq \left| \frac{f^{(7)}(\theta_x)}{7!} x^7 \right| = 0'000637$$