DUALTASUNA ETA SENTIKORTASUN ANALISIA

- (p. 9)

$\min \ z = 3x_1 - 3x_2 - x_3$

$x_1 + 2x_2 - 3x_3 \leq 5$

$-2x_1 - 2x_2 + 5x_3 \geq -12$ $\Rightarrow$

$x_1 - 2x_2 - 7x_3 \leq 16$

$x_1, x_2, x_3 \geq 0$

$\min \ z = 3x_1 - 3x_2 - x_3$

$-x_1 - 2x_2 + 3x_3 \geq 5$

$-2x_1 - 2x_2 + 5x_3 \geq -12$ $\Rightarrow$ $A = \begin{pmatrix} -1 & -2 & 3 \\ -2 & -2 & 5 \\ -1 & 2 & 7 \end{pmatrix} \Rightarrow$

$-x_1 + 2x_2 + 7x_3 \geq 16$

$x_1, x_2, x_3 \geq 0$

$\Rightarrow A^T = \begin{pmatrix} -1 & -2 & -1 \\ -2 & -2 & 2 \\ 3 & 5 & 7 \end{pmatrix} \Rightarrow$

$\max \ z = 5u_1 - 12u_2 + 16u_3$

$-u_1 - 2u_2 - u_3 \leq 3$

$-2u_1 - 2u_2 + 2u_3 \leq -3$

$3u_1 + 5u_2 + 7u_3 \leq -1$

$u_1, u_2, u_3 \geq 0$

- (p. 10)

$\min \ z = 3x_1 + 5x_2 - 7x_3$

$x_1 + x_2 - 3x_3 \leq 4$

$2x_2 + 5x_3 = 12$ $\Rightarrow A = \begin{pmatrix} 1 & 1 & -3 \\ 0 & 2 & 5 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ -3 & 5 \end{pmatrix} \Rightarrow$

$x_1, x_2 \geq 0$

x_3 ez-murriztua

$\max \ z = 4u_1 + 12u_2$

$u_1 \leq 3$

$u_1 + 2u_2 \leq 5$

$-3u_1 + 5u_2 = -7$

$u_1, u_2 \leq 0$

- (p. 14)

$\max \ z = 4x_1 + 3x_2$

$-x_1 + 2x_2 \leq 4$

$2x_1 + 3x_2 \leq 13$

$x_1 - x_2 \leq 4$

$x_1, x_2 \geq 0$

Simplex metodoaren azken taula $\Longrightarrow$

C_{oin}	A_{oin}	$B^{-1}\cdot b$	4	3	0	0	0
			x_1	x_2	x_3	x_4	x_5
0	x_3	7	0	0	1	-1/5	7/5
3	x_2	1	0	1	0	1/5	-2/5
4	x_1	5	1	0	0	1/5	3/5
$z = 23$		z_j	4	3	0	7/5	6/5
		$z_j - c_j$	0	0	0	7/5	6/5

Problema primalaren soluzio optimoa: $x_1^* = 5$, $x_2^* = 1$, $x_3^* = 7$, $z^* = 23$

Problema duala:

$\min \ z = 4u_1 + 13u_2 + 4u_3$

$-u_1 + 2u_2 + u_3 \geq 4$ $\Rightarrow$ $u_1^* = 0$, $u_2^* = 7/5$, $u_3^* = 6/5$, $z = 23$

$2u_1 + 3u_2 - u_3 \geq 3$

$u_1, u_2, u_3 \geq 0$

OSAGARRIZKO LASAITASUNA (p. 16)

- $$\begin{aligned} \max\ z &= 2x_1 + 4x_2 + 3x_3 + x_4 \\ 3x_1 + x_2 + x_3 + 4x_4 &\leq 12 \\ x_1 - 3x_2 + 2x_3 + 3x_4 &\leq 7 \\ 2x_1 + x_2 + 3x_3 - x_4 &\leq 10 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

$$\Rightarrow A = \begin{pmatrix} 3 & 1 & 1 & 4 \\ 1 & -3 & 2 & 3 \\ 2 & 1 & 3 & -1 \end{pmatrix} \Rightarrow$$

$$\begin{aligned} \max\ z &= 2x_1 + 4x_2 + 3x_3 + x_4 \\ 3x_1 + x_2 + x_3 + 4x_4 + x_5 &\leq 12 \\ x_1 - 3x_2 + 2x_3 + 3x_4 + x_6 &\leq 7 \\ 2x_1 + x_2 + 3x_3 - x_4 + x_7 &\leq 10 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7 &\geq 0 \end{aligned}$$

$x_1^* = x_3^* = 0,\ x_2^* = 10'4,\ x_4^* = 0'4$ $\xrightarrow{\text{Ordezkatuz lortzen dira}}$ $x_5^* = 0,\ x_6^* = 37,\ x_7^* = 0$

$$A^T = \begin{pmatrix} 3 & 1 & 2 \\ 1 & -3 & 1 \\ 1 & 2 & 3 \\ 4 & 3 & -1 \end{pmatrix} \Rightarrow$$

$$\begin{aligned} \min\ z &= 12u_1 + 7u_2 + 10u_3 \\ 3u_1 + u_2 + 2u_3 &\geq 2 \\ u_1 - 3u_2 + u_3 &\geq 4 \\ u_1 + 2u_2 + 3u_3 &\geq 3 \\ 4u_1 + 3u_2 - u_3 &\geq 1 \\ u_1, u_2, u_3 &\geq 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} \min\ z &= 12u_1 + 7u_2 + 10u_3 \\ 3u_1 + u_2 + 2u_3 - u_4 &= 2 \\ u_1 - 3u_2 + u_3 - u_5 &= 4 \\ u_1 + 2u_2 + 3u_3 - u_6 &= 3 \\ 4u_1 + 3u_2 - u_3 - u_7 &= 1 \\ u_1, u_2, u_3, u_4, u_5, u_6, u_7 &\geq 0 \end{aligned}$$

$x^T = (x_1, x_2, x_3, x_4)^T$ $\qquad$ $u^T = (u_1, u_2, u_3)^T$

$(x^h)^T = (x_5, x_6, x_7)^T$ $\qquad$ $(u^h)^T = (u_4, u_5, u_6, u_7)^T$

$x_1 \cdot u_4 = 0 \rightarrow u_4 = 7$

$x_2 \cdot u_5 = 0 \rightarrow u_5 = 0$

$x_3 \cdot u_6 = 0 \rightarrow u_6 = 7$

$x_4 \cdot u_7 = 0 \rightarrow u_7 = 0$

$x_5 \cdot u_1 = 0 \rightarrow u_1 = 1$

$x_6 \cdot u_2 = 0 \rightarrow u_2 = 0$

$x_7 \cdot u_3 = 0 \rightarrow u_3 = 3$

<u>SIMPLEX DUAL METODOA</u> (p.20)

- min $z = x_1 + x_2 + x_3 + x_4$ $\Rightarrow$ min $z = x_1 + x_2 + x_3 + x_4$ $\Rightarrow$ min $z = x_1 + x_2 + x_3 + x_4$

$2x_1 + x_4 \geq 250$	$-2x_1 - x_4 \leq -250$	$-2x_1 - x_4 + x_5 = -250$
$3x_2 \geq 1000$	$-3x_2 \leq -1000$	$-3x_2 + x_6 = -1000$
$3x_2 + 10x_3 + 6x_4 \geq 750$	$-3x_2 - 10x_3 - 6x_4 \leq -750$	$-3x_2 - 10x_3 - 6x_4 + x_7 = -750$
$x_1, x_2, x_3, x_4 \geq 0$	$x_1, x_2, x_3, x_4 \geq 0$	$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

$$A = \left(\begin{array}{cccc|ccc} -2 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 1 & 0 \\ 0 & -3 & -10 & -6 & 0 & 0 & 1 \end{array}\right) \quad \underbrace{\hphantom{000}}_{B \,(\text{last three columns})}$$

Hasierako oinarrizko soluzio ez-bideragarria: $x_D = (x_5, x_6, x_7) = (-250, -1000, -750)$

C_{oin}	A_{oin}	$B^{-1} \cdot b$	1	1	1	1	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6	x_7
0	x_5	-250	-2	0	0	-1	1	0	0
0	x_6	-1000	0	$\boxed{-3}$	0	0	0	1	0
0	x_7	-750	0	-3	-10	-6	0	0	1
$z = 0$		z_j	0	0	0	0	0	0	0
		$z_j - c_j$	-1	-1	-1	-1	0	0	0

$\exists k: x_{Dk} < 0 \rightarrow$ Jarraitu

Irtetze irizpidea: $\max\{|x_D| / x_{Dk} < 0\} = \max\{|-250|, |-1000|, |-750|\} = 1000 \rightarrow x_6$ irteten da

Sartze irizpidea: $\min_{k, a_{ik} < 0} \left\{ \frac{|z_k - c_k|}{|a_{ik}|} \right\} = \min\left\{\frac{1}{3}\right\} = \frac{1}{3} \rightarrow x_2$ sartzen da

C_{oin}	A_{oin}	$B^{-1} \cdot b$	1	1	1	1	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6	x_7
0	x_5	-250	$\boxed{-2}$	0	0	-1	1	0	0
1	x_2	1000/3	0	1	0	0	0	-1/3	0
0	x_7	250	0	0	-10	-6	0	-1	1
$z = \frac{1000}{3}$		z_j	0	1	0	0	0	-1/3	0
		$z_j - c_j$	-1	0	-1	-1	0	-1/3	0

$$\begin{cases} e_{2b} \leftarrow e_2 / -3 \\ e_{1b} \leftarrow e_1 \\ e_{3b} \leftarrow e_3 + 3e_{2b} \end{cases}$$

$\exists k: x_{Dk} < 0 \rightarrow$ Jarraitu

Irtetze irizpidea: $\max\{|x_D| / x_{Dk} < 0\} = \max\{|-250|\} = 250 \rightarrow x_5$ irteten da

Sartze irizpidea: $\min_{k, a_{ik} < 0} \left\{ \frac{|z_k - c_k|}{|a_{ik}|} \right\} = \min\left\{\frac{1}{2}, 1\right\} = \frac{1}{2} \rightarrow x_1$ sartzen da

C_{oin}	A_{oin}	$B^{-1}\cdot b$	1	1	1	1	0	0	0
			x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	x_1	125	1	0	0	1/2	-1/2	0	0
1	x_2	1000/3	0	1	0	0	0	-1/3	0
0	x_7	250	0	0	-10	-6	0	-1	1
$z=\frac{1750}{3}$		z_j	1	1	0	1/2	-1/2	-1/3	0
		z_j-c_j	0	0	-1	-1/2	-1/2	-1/3	0

$$\begin{cases} e_{1b} \leftarrow e_1 \mid -2 \\ e_{2b} \leftarrow e_2 \\ e_{3b} \leftarrow e_3 \end{cases}$$

$\forall x_B \geq 0 \rightarrow$ Soluzioa bideragarria da $\rightarrow$ Gelditu $\rightarrow$ Optimoa aurkitu dugu

$x_1^* = 125$, $x_2^* = 1000/3$, $x_3^* = 0$, $x_4^* = 0$, $x_5^* = 0$, $x_6^* = 0$, $x_7^* = 250$, $z^* = 1750/3$

SENTIKORTASUN ANALISIA

- ALDAKETAK b BEKTOREAN

$$\max\ z = 3x_1 + x_2 + 4x_3$$
$$6x_1 + 3x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

$\Rightarrow$

$$\max\ z = 3x_1 + x_2 + 4x_3$$
$$6x_1 + 3x_2 + 5x_3 + x_4 = 25$$
$$3x_1 + 4x_2 + 5x_3 + x_5 = 20$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

$x_B = (x_4, x_5) = (25, 20)$

Simplex problemaren azken taula:

C_{oin}	A_{oin}	$B^{-1}\cdot b$	3	1	-4	0	0
			x_1	x_2	x_3	x_4	x_5
3	x_1	1'667	1	-1/3	0	1/3	-1/3
4	x_3	3	0	1	1	-0'2	0'4
$z=17$		z_j	3	3	4	0'2	0'4
		z_j-c_j	0	2	0	0'2	0'4

a) $b^T = (40, 30)$

$$\hat{x}_B = B^{-1}\cdot b$$

$$B^{-1} = \begin{pmatrix} 1/3 & -1/3 \\ -0'2 & 0'4 \end{pmatrix}\begin{pmatrix} 40 \\ 30 \end{pmatrix} = \begin{pmatrix} 10/3 \\ 4 \end{pmatrix} \geq 0$$

$\hat{x}_B \geq 0$ denez, bideragarritasuna mantentzen da

Soluzio optimoaren (optimo bideragarriaren) soluzioa:

$x_1^* = 10/3$, $x_2^* = 0$, $x_3^* = 4$, $x_4^* = 0$, $x_5^* = 0$, $z^* = 25'99$

b) $b^T = (50, 20)$

$\hat{x}_B = B^{-1} \cdot b$

$$B^{-1} = \begin{pmatrix} 1/3 & -1/3 \\ -0'2 & 0'4 \end{pmatrix} \begin{pmatrix} 50 \\ 20 \end{pmatrix} = \begin{pmatrix} 9'99 \\ -2 \end{pmatrix} \not\geq 0$$

$\hat{x}_B \not\geq 0$ denez, bideragarritasuna galdu egiten da → 2 ereduko lasaierako ~~~ ?

simplex dual metodoa aplikatu behar da

C_{oin}	A_{oin}	$B^{-1} \cdot b$	3	1	-4	0	0
			x_1	x_2	x_3	x_4	x_5
3	x_1	9'99	1	-1/3	0	1/3	-1/3
4	x_3	-2	0	1	1	-0'2	0'4
$z = 21'97$		z_j	3	3	4	0'2	0'6
		$z_j - c_j$	0	2	0	0'2	0'6

$\exists k : x_{Bk} < 0 \rightarrow$ Jarraitu

Irtetze irizpidea: $\max\{|x_D| / x_{DK} < 0\} = 2 \rightarrow x_3$ irtetzen da

Sartze irizpidea: $\min_{k, a_{ik} < 0} \left\{ \frac{|z_k - c_k|}{|a_{ik}|} \right\} = \min \left\{ \frac{0'2}{0'2} \right\} = 1 \rightarrow x_4$ sartzen da

C_{oin}	A_{oin}	$B^{-1} \cdot b$	3	1	-4	0	0
			x_1	x_2	x_3	x_4	x_5
3	x_1	6'66	1	1'33	1'66	0	0'333
0	x_4	10	0	-5	-5	1	-2
$z = 19'98$		z_j	3	4	5	0	1
		$z_j - c_j$	0	3	1	0	1

$\{ e_2 \leftarrow e_2 / (-0'2)$

$x_D \geq 0 \rightarrow$ Soluzio bideragarria da → Gelditu, optimoa aurkitu dugu

$x_1^* = 6'66$, $x_2^* = 0$, $x_3^* = 0$, $x_4^* = 10$, $x_5^* = 0$, $z^* = 19'98$

c) $\hat{x}_B \geq 0$

$$\hat{x}_B = B^{-1} \cdot \hat{b} = \begin{pmatrix} 1/3 & -1/3 \\ -0'2 & 0'4 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ 20 \end{pmatrix} \geq 0$$

$$\hat{x}_B = B^{-1} \cdot \hat{b} = \begin{pmatrix} b_1/3 - 20/3 \\ -0'2b_1 + 8 \end{pmatrix} \geq 0 \rightarrow \begin{cases} b_1/3 - 20/3 \geq 0 \rightarrow b_1 \geq 20 \\ -0'2b_1 + 8 \geq 0 \rightarrow b_1 \leq 40 \end{cases} \rightarrow 20 \leq b_1 \leq 40$$

Ondorioz, $20 \leq b_1 \leq 40$ betetzen bada, soluzioak bideragarria izaten jarraituko du.

- Adibidea (d. 30) (Aurreko adibideari murrizketa bat gehitu)

$$\max\ z = 3x_1 + x_2 + 4x_3$$

$$6x_1 + 3x_2 + 5x_3 \leq 25$$

$$3x_1 + 4x_2 + 5x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

a)

• Algo ? (34)

a)

$$\max\ z = 4x_1 + 2x_2 + 5x_3$$

$$6x_1 + 3x_2 + 5x_3 \leq 25$$

$$3x_1 + 4x_2 + 5x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

Coin	Δoin	$B^{-1} \cdot b$	4	2	5	0	0
			x_1	x_2	x_3	x_4	x_5
4	x_1	1'667	1	-1/3	0	1/3	-1/3
5	x_3	3	0	1	1	-0'2	0'4
$z = 21'668$	z_j		4	1/3	5	1/3	2/3
	w_j		0	5/3	0	1/3	2/3

Kasu honetan $w_j \geq 0\ \forall j$ → soluzio bideragarriak optimoa izaten jarraitzen du.

$x_1^* = 1'667,\ x_3^* = 3,\ x_2^* = x_4^* = x_5^* = 0\ ,\ z^* = 21'668$

b) $\max\ z = x_1 + x_2 + 2x_3$

$$6x_1 + 3x_2 + 5x_3 \leq 25$$

$$3x_1 + 4x_2 + 5x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

Coin	Δoin	$B^{-1} \cdot b$	1	1	2	0	0
			x_1	x_2	x_3	x_4	x_5
1	x_1	1'667	1	-1/3	0	1/3	-1/3
2	x_3	3	0	1	1	-0'2	0'4
$z = 7'667$	z_j		1	5/3	2	-1/15	7/15
	w_j		0	2/3	0	-1/15	7/15

$\exists j : w_j < 0$ → Optimaltasuna galdu da ⟶ Simplex aplikatu

Sartze irizpidea: $w_j = \min\{w_j / w_j < 0\} = -1/15$ → x_4 sartu

Irtetze irizpidea: $\min\left\{\frac{1'667}{1/3}\right\} = 3 \cdot 1'667$ → x_1 irten.

Coin	Δoin	$B^{-1} \cdot b$	1	1	2	0	0
			x_1	x_2	x_3	x_4	x_5
0	x_4	5	3	-1	0	1	-1
2	x_3	4	3/5	4/5	1	0	1/5
$z = 8$	z_j		6/5	8/5	2	0	2/5
	w_j		1/5	3/5	0	0	2/5

$e_{1b} \leftarrow e_1 \cdot 3$

$e_{2b} \leftarrow e_2 + 0'2 \cdot e_{1b}$

$w_j \geq 0\ \forall j$ → Optimoa lortu dugu

$x_1^* = x_2^* = 0,\ x_3^* = 4\ ,\ x_4^* = 5,\ x_5^* = 0,\ z^* = 8$ izanik