1) ZENBAKI KONPLEXUAK

1 z_1 eta $z_2 \in \mathbb{C}$ (ez-nuluak) kalkulatu, hurrengoa jakinda:

A) $z_1 + z_2 = -3 + 5i$

$$\begin{cases} z_1 = a + bi \\ z_2 = c + di \end{cases}$$

$$z_1 + z_2 = (a+bi) + (c+di) = (a+c) + (b+d)i = -3 + 5i$$

$$\begin{cases} a + c = -3 \\ b + d = 5 \end{cases}$$

B) z_1/z_2 irudikari purua dela

$$\Re\left(\frac{z_1}{z_2}\right) = 0 \Big/ \Im\left(\frac{z_1}{z_2}\right) = xi$$

$$\frac{z_1}{z_2} = \frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac - adi + bci - bdi^{2 \to -1}}{c^2 + \ldots} = \frac{(ac+bd)^{\to 0} + (bc-ad)i}{c^2+d^2}$$

Irudikari purua denez, $\Re = 0 \to \frac{ac+bd}{c^2+d^2} = 0 \to ac + bd = 0$

C) $\Re(z_2) = -3$

$$\Re(z_2) = c \to \boxed{c = -3}$$

$$\begin{cases} a + c = -3 \to \boxed{a = 0} \\ b + d = 5 \to \boxed{b = 5} \\ ac + bd = 0 \to bd = 0 \begin{cases} \to \cancel{b = 0} \quad z_1 \neq 0 \text{ izan behar delako} \\ \to \boxed{d = 0} \end{cases} \end{cases}$$

Beraz, $\boxed{z_1 = 5i \ \wedge \ z_2 = -3}$

2 i^{1127} kalkulatu:

$$i^{1127} = \left\| \begin{array}{r|l} 1127 & 4 \\ 32 & 281 \\ 07 & \\ 3 & \end{array} \right\| = i^{4 \cdot 281 + 3} = i^3 = -i$$

3 $z_1 = 1 + \frac{1}{\sqrt{3}} i$ eta $z_2 = 1_{\pi/3}$ zenbaki konplexuak emanda, hurrengo biderkadura kalkulatu: $z_1 \cdot z_2$

<u>ERA POLARREAN:</u>

$$z_1 = 1 + \frac{1}{\sqrt{3}} i \begin{cases} \rho = \sqrt{1^2 + (1/\sqrt{3})^2} = \sqrt{1 + (1/3)} = \sqrt{4/3} = 2/\sqrt{3} \\ \alpha = \text{argtan}[(1/\sqrt{3})/1] = \text{arg}t \ (1/\sqrt{3}) \begin{cases} \pi/6 \to 1.\text{ Koad.} \\ \cancel{7\pi/6} \to 4.\text{ Koad} \end{cases} \end{cases}$$

$$\boxed{z_1 \cdot z_2 = \frac{2}{\sqrt{3}}_{\pi/6} \cdot 1_{\pi/3} = \left(\frac{2}{\sqrt{3}} \cdot 1\right)_{\frac{\pi}{6} + \frac{\pi}{3}} = \frac{2}{\sqrt{3}}_{\frac{\pi}{2}}}$$

<u>ERA BINOMIKOAN:</u>

$$z_2 = 1_{\frac{\pi}{3}} = 1[\cos(\pi/3) + i \sin(\pi/3)] = 1\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$\boxed{z_1 \cdot z_2 = \left(1 + \frac{1}{\sqrt{3}} i\right) \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = \frac{1}{2} + \frac{\sqrt{3}}{2} i + \frac{1}{2\sqrt{3}} i + \frac{\sqrt{3}}{2\sqrt{3}} i^{2 \to -1} = \left(\cancel{\frac{1}{2} - \frac{1}{2}}\right) + \left(\frac{1 + (\sqrt{3})^2}{2\sqrt{3}}\right) i = \frac{2}{\sqrt{3}} i}$$

4 Hurrengo zenbaki konplexuak era binomikoan jarri:

A) $z_1 = (2_{\frac{\pi}{3}})^{15} \cdot \left[\left(\frac{1}{2}\right)_{\frac{\pi}{8}}\right]^{12} = (2^{15})_{\frac{15\pi}{3}} \cdot (2^{-12})_{\frac{12\pi}{8}} = (2^3)_{5\pi + \frac{3\pi}{2}} = (2^3)_{\frac{13\pi}{2}} = 8_{6\pi + \frac{\pi}{2}} = 8[\overset{0}{\cancel{\cos(\pi/2)}} + i\overset{1}{\cancel{\sin(\pi/2)}}] = 8i$

B) $z_2 = \left(\frac{3+i}{1+2i}\right)^7 = \left(\frac{(3+i)(1-2i)}{(1+2i)(1-2i)}\right)^7 = \left(\frac{3-6i+i-2i^2}{1+2^2}\right)^7 = \left(\frac{(3+2)+(1-6)i}{1+4}\right)^7 = \left(\frac{5-5i}{5}\right)^7 = (1-i)^7 =$

$= \left\| \begin{matrix} \rho = \sqrt{1^2+1^2} = \sqrt{2} \\ \alpha = \arg\tan(-1/1) < \begin{matrix} \cancel{3\pi/4} \\ 7\pi/4 \end{matrix} \end{matrix} \right\| = (\sqrt{2}_{\frac{7\pi}{4}})^7 = (2^{1/2}_{\frac{-\pi}{4}})^7 = (2^{7/2})_{\frac{-7\pi}{4}} = (2^{7/2})_{\frac{\pi}{4}} = 2^3\sqrt{2}[\cos(\pi/4) + i\sin(\pi/4)] =$

$= 2^3\sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2^3 + 2^3 i = 8 + 8i$

5 Lortu bi zenbaki konplexu, halako moldez non haien kendura erreala den, haien batura eta zatidura irudikari puruak diren, eta haien biderkadura -2 den.

$z_1 = a + bi$

$z_2 = c + di$

- $z_1 - z_2 = \mathbb{R}e$ $\quad\quad \mathbb{I}m = 0$

  $z_1 - z_2 = (a+bi) - (c+di) = (a-c) + \cancel{(b-d)}i = a - c$

- $z_1 + z_2 = \mathbb{I}m$ $\quad\quad \mathbb{R}e = 0$

  $z_1 + z_2 = (a+bi) + (c+di) = \cancel{(a+c)} + (b+d)i = (b+d)i$

- $\frac{z_1}{z_2} = \mathbb{I}m$

  $\frac{z_1}{z_2} = \frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{\overset{0}{\cancel{(ac+bd)}} + (bc-ad)i}{c^2+d^2}$

  $ac + bd = 0$

- $z_1 \cdot z_2 = -2$

  $z_1 \cdot z_2 = (a+bi)(c+di) = ac + adi + bci + bdi^2 = (ac-bd) + (ad+bc)i = -2$

$\begin{cases} ac - bd = -2 \\ ad + bc = 0 \end{cases}$

$b - d = 0 \rightarrow b = d \rightarrow d = \pm 1$

$a + c = 0 \rightarrow a = -c \rightarrow c = \pm 1$

$ac + bd = 0 \rightarrow -a^2 + b^2 = 0 \rightarrow a^2 = b^2 \rightarrow a = b \rightarrow b = \pm 1$

$ac - bd = -2 \rightarrow -a^2 - b^2 = -2 \rightarrow 2a^2 = 2 \rightarrow a = \pm 1$

$\boxed{z_1 = 1 + i \ \wedge \ z_2 = -1 + i} \ \vee \ \boxed{z_1 = -1 - i \ \wedge \ z_2 = 1 - i}$

6 $\sqrt{3+4i}$ era binomikoan adierazi

$\sqrt{3+4i} = a + bi \rightarrow 3 + 4i = (a+bi)^2 \rightarrow 3 + 4i = a^2 + 2abi - b^2 \rightarrow 3 + 4i = (a^2 - b^2) + 2abi$

$\begin{cases} a^2 - b^2 = 3 \rightarrow \frac{4}{b^2} - b^2 = 3 \rightarrow 4 - b^4 = 3b^2 \rightarrow b^4 + 3b^2 - 4 = 0 \\ 2ab = 4 \rightarrow ab = 2 \rightarrow a = 2/b \rightarrow a = \pm 2 \end{cases}$

$b^2 = \frac{-3 \pm \sqrt{3^2 - 4(-4)}}{2} = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3 \pm 5}{2} < \begin{matrix} b^2 = 1 \rightarrow b = \pm 1 \\ \cancel{b^2 = -4} \notin \mathbb{R}e \end{matrix}$

$\boxed{z = 2 + i} \ \vee \ \boxed{z = -2 - i}$

[7] Honako erro hau Kalkulatu: $\sqrt[4]{-3+3\sqrt{3}\,i}$

$$z\begin{cases}\rho=\sqrt{3^2+(3\sqrt{3})^2}=\sqrt{9+27}=\sqrt{36}=6\\ \alpha=\arctan\left(3\sqrt{3}/-3\right)=\arctan(-\sqrt{3})\begin{smallmatrix}2\pi/3\\5\pi/3\end{smallmatrix}\end{cases}\Bigg\}\ z=6_{\frac{2\pi}{3}}$$

$$\sqrt[4]{6_{\frac{2\pi}{3}}}=\sqrt[4]{6}_{\frac{\frac{2\pi}{3}+2k\pi}{4}}\quad k=0,1,2,3$$

[8] Idatzi honako zenbaki hauek era binomikoan

A) $(1-i)^{12}$

$$\begin{cases}\rho=\sqrt{1^2+1^2}=\sqrt{2}\\ \alpha=\arctan(-1/1)=\arctan(-1)=3\pi/4\end{cases}$$

$$(1-i)^{12}=\left(\sqrt{2}_{\frac{3\pi}{4}}\right)^{12}=\left(2^{12/2}\right)_{\frac{12\cdot3\pi}{4}}=2^6_{9\pi}=2^6_{8\pi+\pi}=2^6[\cancel{\cos(\pi)}^{-1}+i\cancel{\sin(\pi)}^{0}]=-2^6=-64$$

B) $(1-4i)^3$

Neaitonen binomioa

$$\frac{3!}{0!\,3!}1^3(-4i)^0+\frac{3!}{1!\,2!}1^2(-4i)^1+\frac{3!}{2!\,1!}1^1(-4i)^2+\frac{3!}{3!\,0!}1^0(-4i)^3=1-12i-48+64i=-47+52i$$

[9] Kalkulatu ezazu (Kalkulagailurik gabe) $\cos(3x)$ non x, $z=2+i$ zenbaki konplexuaren argumentua den.

$$\begin{cases}\rho=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}\\ \alpha=x\end{cases}$$

$$2+i=\sqrt{5}\,[\cos(x)+i\sin(x)]$$

$$\begin{cases}2=\sqrt{5}\cos(x)\longrightarrow\cos(x)=2/\sqrt{5}\\ 1=\sqrt{5}\sin(x)\longrightarrow\sin(x)=1/\sqrt{5}\end{cases}$$

Moivre-ren formula: $(\cos(x)+i\sin(x))^n=\cos(nx)+i\sin(nx)$

$$(\cos(x)+i\sin(x))^3=\cos(3x)+i\sin(3x)$$

$$\cos^3(x)+3i\cos^2(x)\sin(x)-3\cos(x)\sin^2(x)-i\sin^3(x)=\cos(3x)+i\sin(3x)$$

Parte errealak konparatuz:

$$\cos^3(x)-3\cos(x)\sin^2(x)=\cos(3x)$$

$$\left(\frac{2}{\sqrt{5}}\right)^3-3\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)^2=\cos(3x)$$

$$\frac{8}{5\sqrt{5}}-\frac{6}{5\sqrt{5}}=\cos(3x)$$

$$\boxed{\frac{2}{5\sqrt{5}}=\cos(3x)}$$

[10] Ebatzi $\cos(z)=2$ ekuazioa eremu konplexuan

$$\cos(z)=\frac{e^{iz}+e^{-iz}}{2}=2\longrightarrow e^{iz}+e^{-iz}=4\longrightarrow e^{2iz}+1=4e^{iz}\longrightarrow e^{2iz}-4e^{iz}+1=0$$

$$e^{iz}=\frac{4\pm\sqrt{4^2-4\cdot1\cdot1}}{2}=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

Logaritmoak erabiliz:

$$\ln(e^{iz})=\ln(2\pm\sqrt{3})\rightarrow iz\cdot\ln(e)=\ln(2\pm\sqrt{3})\rightarrow z=\frac{\ln(2\pm\sqrt{3})}{i}\rightarrow\boxed{z=-i\ln(2\pm\sqrt{3})}$$

11 $\ln(2+2i)$ Kalkulatu:

Logaritmo

$\ln(z)=\ln|z|+i(\alpha+2k\pi)$

z era polarrera pasatu:

$z=(2+2i)\begin{cases}\rho=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}\\ \alpha=\arg\tan(1)=\pi/4\end{cases}\rightarrow z=2\sqrt{2}_{\frac{\pi}{4}}$

$\ln(2+2i)=\ln|\sqrt{8}|+i\left(\frac{\pi}{4}+2k\pi\right)\rightarrow \boxed{\ln(2+2i)=\frac{1}{2}\ln|8|+i\left(\frac{\pi}{4}+2k\pi\right)}$

12 $(1-i)^i$ Kalkulatu:

Era esponentzialean idatzi

$z=e^{\ln|z|+i(\alpha+2k\pi)}$

$z=(1-i)\begin{cases}\rho=\sqrt{1^2+1^2}=\sqrt{2}\\ \alpha=\arg\tan(-1)=7\pi/4\end{cases}$

$z=(1-i)^i=\left[e^{\ln|\sqrt{2}|+i(-\pi/4+2k\pi)}\right]^i=\left[e^{\frac{1}{2}\ln(2)+i(-\frac{\pi}{4}+2k\pi)}\right]^i=$

$\boxed{(1-i)^i=e^{i\frac{1}{2}\ln(2)+(\frac{\pi}{4}+2k\pi)}}$

13 $z^3+4z^2+8z=0$ ekuazioa ebatzi

$z(z^2+4z+8)=0 <\begin{matrix}\boxed{z=0}\\ z^2+4z+8=0\end{matrix}$

$z=\frac{-4\pm\sqrt{4^2-4\cdot 8}}{2}=\frac{-4\pm\sqrt{16-32}}{2}=\frac{-4\pm\sqrt{-16}}{2}=\frac{-4\pm 4i}{2}=\boxed{-2\pm 2i=z}$

14 Ebatzi $tg\,\theta=2i$ era konplexuan (Euler-en formulak erabiliz)

$tg\,\theta=\frac{\sin\theta}{\cos\theta}=2i\longrightarrow \sin\theta=2i\cos\theta\longrightarrow\frac{e^{i\theta}-e^{-i\theta}}{2i}=2i\frac{e^{i\theta}+e^{-i\theta}}{2}\longrightarrow e^{i\theta}-e^{-i\theta}=-2(e^{i\theta}+e^{-i\theta})$

$3e^{i\theta}+e^{-i\theta}=0\longrightarrow 3e^{2i\theta}+1=0\rightarrow e^{2i\theta}=-1/3$

Logaritmoak erabiliz:

$\ln(e^{2i\theta})=\ln(-1/3)=\ln(1/3_{\pi})\rightarrow 2i\theta\ln(e)=\ln(1/3_{\pi})\rightarrow 2i\theta=\ln(1/3)+i(\pi+2k\pi)$

15 Ebatzi $z^4-2z^2+2=0$ ekuazioa

$z^2=t\rightarrow t^2-2t+2=0\rightarrow t=\frac{2\pm\sqrt{2^2-4\cdot 2}}{2}=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm 2i}{2}<\begin{matrix}1+i=t_1\\ 1-i=t_2\end{matrix}$

$(1+i)\begin{cases}\rho=\sqrt{1^2+1^2}=\sqrt{2}\\ \alpha=\arg\tan(1)=\pi/4\end{cases}\rightarrow z_1^2=\sqrt{2}_{\pi/4}$

$z_1=\sqrt{\sqrt{2}_{\pi/4}}=((2)^{1/2})^{1/2}_{\frac{(\pi/4)+2k\pi}{2}}=\sqrt[4]{2}_{\frac{\pi}{8}+k\pi}\quad k=0,1$

$(1-i)\begin{cases}\rho=\sqrt{1^2+1^2}=\sqrt{2}\\ \alpha=\arg\tan(-1)=7\pi/4=-\pi/4\end{cases}\rightarrow z_2^2=\sqrt{2}_{-\frac{\pi}{4}}$

$z_2=\sqrt{\sqrt{2}_{-\frac{\pi}{4}}}=((2)^{1/2})^{1/2}_{\frac{(-\pi/4)+2k\pi}{2}}\quad k=0,1=\sqrt[4]{2}_{k\pi-\frac{\pi}{8}}\quad k=0,1$

16 Ebatzi $\sin(\theta)=i$ ekuazioa eremu konplexuan.

$$\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i}=i\longrightarrow e^{i\theta}-e^{-i\theta}=-2\longrightarrow e^{2i\theta}-1=-2e^{i\theta}\longrightarrow e^{2i\theta}+2e^{i\theta}-1=0$$

$$e^{i\theta}=\frac{-2\pm\sqrt{2^2-4(-1)}}{2}=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm2\sqrt{2}}{2}\begin{cases}-1+\sqrt{2}=e^{i\theta_1}\\-1-\sqrt{2}=e^{i\theta_2}\end{cases}$$

① $\ln(e^{i\theta})=\ln(-1+\sqrt{2})\longrightarrow i\theta=\ln(-1+\sqrt{2})\longrightarrow\boxed{\theta=-i\ln(-1+\sqrt{2})}$

② $\ln(e^{i\theta})=\ln(-1-\sqrt{2})\longrightarrow i\theta=\ln(-1-\sqrt{2})\longrightarrow i\theta=\ln(\sqrt{3}_\pi)\longrightarrow i\theta=\ln(\sqrt{3})+i(\pi+2k\pi)\longrightarrow$

$\boxed{\theta=-i[\ln(\sqrt{3})+i(\pi+2k\pi)]}$

17 Kalkula itzazu a eta b, non $a+bi=\ln\sqrt{w}$ den, $|w|=1$ den eta $\frac{w}{1+i\sqrt{3}}$ erreala den $(w\in\mathbb{C})$

$w=x+iy \qquad |w|=\sqrt{x^2+y^2}=1$

$$\frac{w}{1+i\sqrt{3}}=\frac{x+iy}{1+i\sqrt{3}}=\frac{(x+iy)(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}=\frac{x-xi\sqrt{3}+iy+y\sqrt{3}}{1+3}=\frac{(x+y\sqrt{3})+\cancel{(y-x\sqrt{3})i}^{\to 0}}{4}$$

$y-x\sqrt{3}=0\longrightarrow y=x\sqrt{3}$

$$\begin{cases}\sqrt{x^2+y^2}=\sqrt{x^2+3x^2}=\pm2x=1\longrightarrow\pm2x=1\longrightarrow x=\pm1/2\\ y=x\sqrt{3}\longrightarrow y=\pm\sqrt{3}/2\end{cases}\longrightarrow ①\ w=\frac{1}{2}+i\frac{\sqrt{3}}{2}\ \Big/\ ②\ w=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$$

① $a+bi=\ln\sqrt{\frac{1}{2}+i\frac{\sqrt{3}}{2}}\longrightarrow a+bi=\frac{1}{2}\ln\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\longrightarrow a+bi=\frac{1}{2}\ln(1_{\frac{\pi}{3}})\longrightarrow$

$a+bi=\frac{1}{2}\left[\ln(1)+i\left(\frac{\pi}{3}+2k\pi\right)\right]$

$$\begin{cases}a=\frac{1}{2}\ln(1)\longrightarrow\boxed{a=0}\\ b=\frac{1}{2}\left(\frac{\pi}{3}+2k\pi\right)\longrightarrow\boxed{b=\frac{\pi}{6}+k\pi}\end{cases}$$

② $a+bi=\ln\sqrt{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}\longrightarrow a+bi=\frac{1}{2}\ln\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\longrightarrow a+bi=\frac{1}{2}\left[\ln(1)+i\left(\frac{4\pi}{3}+2k\pi\right)\right]\longrightarrow$

$a=\frac{1}{2}\ln(1)\longrightarrow\boxed{a=0}$

$b=\frac{1}{2}\left(\frac{4\pi}{3}+2k\pi\right)\longrightarrow\boxed{b=\frac{2\pi}{3}+k\pi}$

18 Frogatu honako adierazpen hau:

1) $\frac{Ai}{B+Ci}\in\mathbb{R}e\iff A\cdot B=0$

$$\frac{Ai}{B+Ci}=\frac{Ai(B-Ci)}{(B+Ci)(B-Ci)}=\frac{\cancel{ABi}^{\to 0}+AC}{B^2+C^2}\longrightarrow\boxed{AB=0}$$

B) Aurreko atala kontuan hartuta, kalkula itzazu a eta λ balioak, halako moldez non $\lambda = \log_{a+i}(i)$ zenbaki erreal bat den.

$$\lambda = \log_{a+i}(i) = \frac{\ln(i)}{\ln(a+i)} = \frac{\cancel{\ln(1)}^{\,0} + i(\pi/2 + 2k_1\pi)}{\ln(\sqrt{a^2+1^2}) + i(\operatorname{atan}(1/a) + 2k_2\pi)}$$

$$\frac{Ai}{B+Ci} = \frac{i(\pi/2 + 2k_1\pi)}{\ln(\sqrt{a^2+1}) + i(\operatorname{atan}(1/a) + 2k_2\pi)}$$

$$A \cdot B = 0$$

$$\left(\frac{\pi}{2} + 2k_1\pi\right) \cdot \left(\ln(\sqrt{a^2+1^2})\right) = 0 \longrightarrow \ln(\sqrt{a^2+1^2}) = 0 \longrightarrow \sqrt{a^2+1^2} = 1 \longrightarrow a^2+1 = 1$$

$$a^2 = 0 \longrightarrow \boxed{a=0}$$

$$\lambda = \frac{\ln(i)}{\ln(a+i)} = \frac{\cancel{i}(\pi/2 + 2k_1\pi)}{\cancel{\ln(1)}^{\,0} + \cancel{i}(\pi/2 + 2k_2\pi)} = \frac{\pi + 4k_1\pi}{\pi + 4k_2\pi} = \boxed{\frac{1+4k_1}{1+4k_2} = \lambda}$$

19 $z + \frac{1}{z} = 2\cos(t)$ dela jakinda, kalkulatu honako adierazpen hau: $z^2 - (5+i)z + 4 + 4i = 0$ eta soluzioaren logaritmo nepertarrak kalkulatu (balio nagusia $k=0$)

$$z + \frac{1}{z} = 2\cos(t) \longrightarrow z^2 + 1 = 2z\cos(t) \longrightarrow z^2 - 2z\cos(t) + 1 = 0 \longrightarrow z = \frac{2\cos(t) \pm \sqrt{4\cos^2(t) - 4}}{2} =$$

$$= \frac{2\cos(t) \pm \sqrt{4(\cos^2(t) - 1)}}{2} = \frac{2\cos(t) \pm \sqrt{-4\sin^2(t)}}{2} = \frac{2\cos(t) \pm i2\sin(t)}{2} \begin{cases} \cos(t) + i\sin(t) & ① \\ \cos(t) - i\sin(t) & ② \end{cases}$$

Moivre-ren formula aplikatuz: // $(\cos(\alpha) + i\sin(\alpha))^n = \cos(n\alpha) + i\sin(n\alpha)$

① $(\cos(t) + i\sin(t))^n = \cos(nt) + i\sin(nt)$

$$\frac{1}{\cos(t) + i\sin(t)} \cdot \frac{\cos(t) - i\sin(t)}{\cos(t) - i\sin(t)} = \frac{\cos(t) - i\sin(t)}{\underbrace{\cancel{\cos^2(t) + \sin^2(t)}}_{1}} = \cos(t) - i\sin(t)$$

$$z^n + \frac{1}{z^n} = \cos(nt) + \cancel{i\sin(nt)} + \cos(nt) - \cancel{i\sin(t)} = 2\cos(nt)$$

②

$$z^n + \frac{1}{z^n} = \cos(nt) - \cancel{i\sin(nt)} + \cos(nt) + \cancel{i\sin(nt)} = 2\cos(nt)$$

$$z^2 - (5+i)z + 4 + 4i = 0 \longrightarrow z = \frac{(5+i) \pm \sqrt{(5+i)^2 - 4(4+4i)}}{2} = \frac{(5+i) \pm \sqrt{25 + 10i - 1 - 16 - 16i}}{2} =$$

$$= \frac{(5+i) \pm \sqrt{8-6i}}{2} \quad ⊛'$$

$$\sqrt{8-6i} = a + bi \longrightarrow 8 - 6i = a^2 + 2abi - b^2 \longrightarrow \begin{cases} a^2 - b^2 = 8 \rightarrow \frac{9}{b^2} - b^2 = 8 \rightarrow b^4 + 8b^2 - 9 = 0 \quad ⊛ \\ -2ab = 6 \rightarrow \underline{a = -3/b} \rightarrow \underline{\underline{a = \pm 3}} \end{cases}$$

⊛
$$b^2 = \frac{-8 \pm \sqrt{8^2 - 4(-9)}}{2} = \frac{-8 \pm \sqrt{100}}{2} = \frac{-8 \pm 10}{2} \begin{cases} b^2 = 1 \rightarrow \underline{\underline{b = \pm 1}} \\ \cancel{b^2 = -9} \end{cases}$$

⊛'
$$z = \frac{5 + i \pm (3 - i)}{2} \begin{cases} \underline{z = 4} \\ \underline{z = 1 + i} \end{cases}$$

$$\boxed{\ln(z) = \ln(4)}$$

$$\ln(z) = \ln(1+i) = \ln\sqrt{2} + i\left(\frac{\pi}{4} + \cancel{2k\pi}^{\,k=0}\right) \longrightarrow \boxed{\ln(z) = \frac{1}{2}\ln(2) + i\frac{\pi}{4}}$$