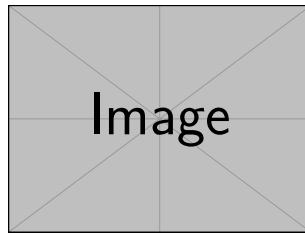


# Mathematics Class Report

Assignment Title



Your Name

Course: MVA – Time Series Analysis

Instructor: Dr. \_ \_ \_ \_ \_

Deadline: \_ \_ \_ \_ \_

ACADEMIC YEAR: 2023–2024

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Theory</b>	<b>2</b>
<b>3</b>	<b>Methodology</b>	<b>3</b>
<b>4</b>	<b>Results</b>	<b>3</b>
<b>5</b>	<b>Discussion</b>	<b>3</b>
<b>6</b>	<b>Conclusion</b>	<b>3</b>

# 1 Introduction

- Briefly describe the objective of your report.
- Provide context and motivation.

# 2 Theory

## Definition

Here, you can state and define important mathematical concepts.

## Theorem

Place for theorems, lemmas, properties, etc.

### Question 1

Consider the following Lasso regression:

$$\min_{\beta \in \mathbb{R}^p} \frac{1}{2} \|y - X\beta\|_2^2 + \lambda \|\beta\|_1 \quad (1)$$

where  $y \in \mathbb{R}^n$  is the response vector,  $X \in \mathbb{R}^{n \times p}$  the design matrix,  $\beta \in \mathbb{R}^p$  the vector of regressors and  $\lambda > 0$  the smoothing parameter.

Show that there exists  $\lambda_{\max}$  such that the minimizer of (1) is  $\mathbf{0}_p$  (a  $p$ -dimensional vector of zeros) for any  $\lambda > \lambda_{\max}$ .

### Answer 1

$$\lambda_{\max} = \max_{j \in \{1, \dots, p\}} \left| \sum_{i=1}^n X_{ij} y_i \right| \quad (2)$$

Proof: Let  $f(\beta) = \frac{1}{2} \|y - X\beta\|_2^2$ . The function is twice differentiable everywhere, and the Hessian is  $\nabla^2 f(\beta) = X^T X \succeq 0$ . Therefore  $f$  is convex. We know the function satisfies, for every pair of points  $(x, y)$ :  $f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle$ , and so by letting  $y = \beta$ ,  $x = 0$ , and computing the gradient of the function at 0 which is  $\nabla f(0) = -X^T y$  we finally get the identity:

$$f(\beta) \geq f(0) - \langle X^T y, \beta \rangle \quad (3)$$

Now, if we let  $\lambda > \lambda_{\max}$ , we have that  $\lambda |\beta_i|$  is strictly greater than the absolute value of the  $i$ 'th component of the dot product (this is true by definition of  $\lambda_{\max}$ ), and therefore by adding all the terms we get

$$f(\beta) + \lambda \|\beta\|_1 > f(0) \quad \forall \beta \in \mathbb{R}^p$$

This proves what we wanted.

### 3 Methodology

- Explain the approaches, algorithms, or tools used.
- Illustrate with formulas:

$$x(t) = \sum_{i=1}^n a_i \cdot \phi_i(t)$$

### 4 Results

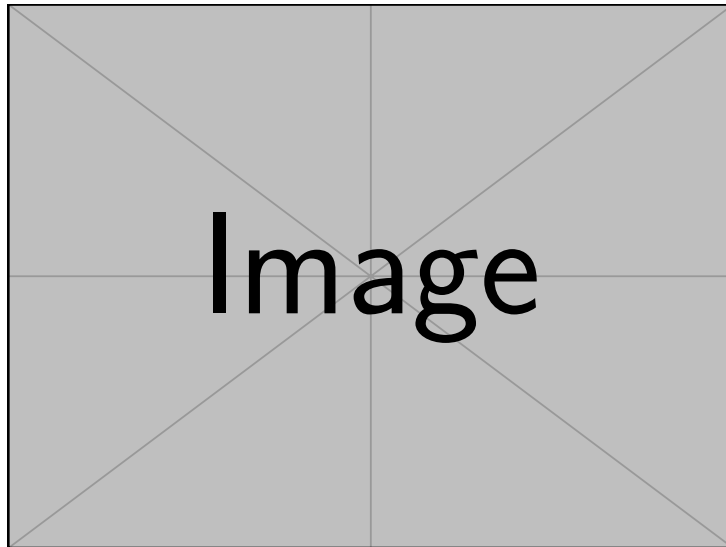


Figure 1: Sample Plot – describe your results here.

Parameter	Value	Description
$a$	2	Slope coefficient
$b$	3	Intercept

Table 1: An example table

### 5 Discussion

- Interpret and critique your results.
- Discuss any surprising findings.

### 6 Conclusion

- Summarize what you learned.
- Propose future directions or applications.

## References

1. Author, “Title of Book or Article,” Journal/Publisher, Year.
2. ...