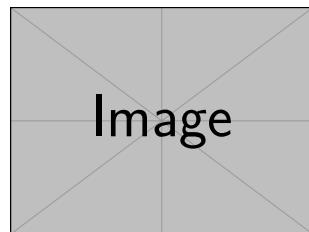


Mathematics Class Report

Assignment Title



Your Name

Course: MVA – Time Series Analysis

Instructor: Dr. _____

Deadline: _____

ACADEMIC YEAR: 2023–2024

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1 Introduction

- Briefly describe the objective of your report.
- Provide context and motivation.

2 Theory

Definition

Here, you can state and define important mathematical concepts.

Theorem

Place for theorems, lemmas, properties, etc.

Question 1

Consider the following Lasso regression:

$$\min_{\beta \in \mathbb{R}^p} \frac{1}{2} \|y - X\beta\|_2^2 + \lambda \|\beta\|_1 \quad (1)$$

where $y \in \mathbb{R}^n$ is the response vector, $X \in \mathbb{R}^{n \times p}$ the design matrix, $\beta \in \mathbb{R}^p$ the vector of regressors and $\lambda > 0$ the smoothing parameter.

Show that there exists λ_{\max} such that the minimizer of (1) is $\mathbf{0}_p$ (a p -dimensional vector of zeros) for any $\lambda > \lambda_{\max}$.

Answer 1

$$\lambda_{\max} = \max_{j \in \{1, \dots, p\}} \left| \sum_{i=1}^n X_{ij} y_i \right| \quad (2)$$

Proof: Let $f(\beta) = \frac{1}{2} \|y - X\beta\|_2^2$. The function is twice differentiable everywhere, and the Hessian is $\nabla^2 f(\beta) = X^T X \succeq 0$. Therefore f is convex. We know the function satisfies, for every pair of points (x, y) : $f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle$, and so by letting $y = \beta$, $x = 0$, and computing the gradient of the function at 0 which is $\nabla f(0) = -X^T y$ we finally get the identity:

$$f(\beta) \geq f(0) - \langle X^T y, \beta \rangle \quad (3)$$

Now, if we let $\lambda > \lambda_{\max}$, we have that $\lambda |\beta_i|$ is strictly greater than the absolute value of the i 'th component of the dot product (this is true by definition of λ_{\max}), and therefore by adding all the terms we get

$$f(\beta) + \lambda \|\beta\|_1 > f(0) \quad \forall \beta \in \mathbb{R}^p$$

This proves what we wanted.

3 Methodology

- Explain the approaches, algorithms, or tools used.
- Illustrate with formulas:

$$x(t) = \sum_{i=1}^n a_i \cdot \phi_i(t)$$

4 Results

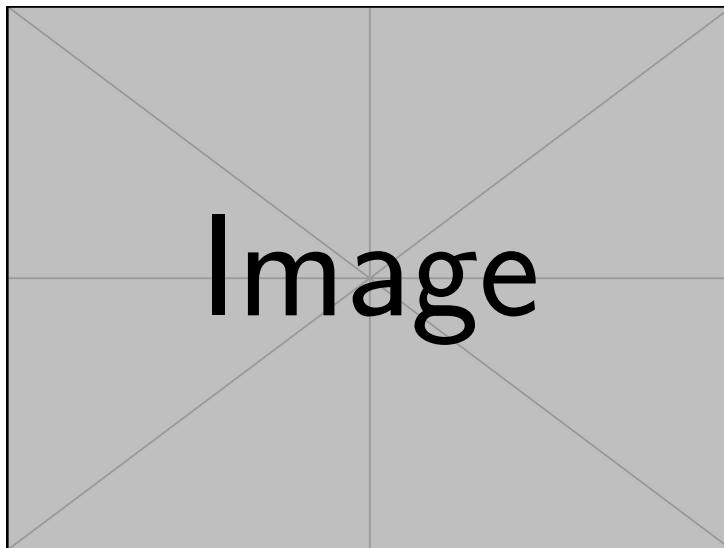


Figure 1: Sample Plot – describe your results here.

Parameter	Value	Description
a	2	Slope coefficient
b	3	Intercept

Table 1: An example table

5 Discussion

- Interpret and critique your results.
- Discuss any surprising findings.

6 Conclusion

- Summarize what you learned.
- Propose future directions or applications.

References

1. Author, "Title of Book or Article," Journal/Publisher, Year.
2. ...