Lesson:

Problems based on Recursion - 2

PW SKILLS

Pre-Requisites

- Functions
- Recursion basics

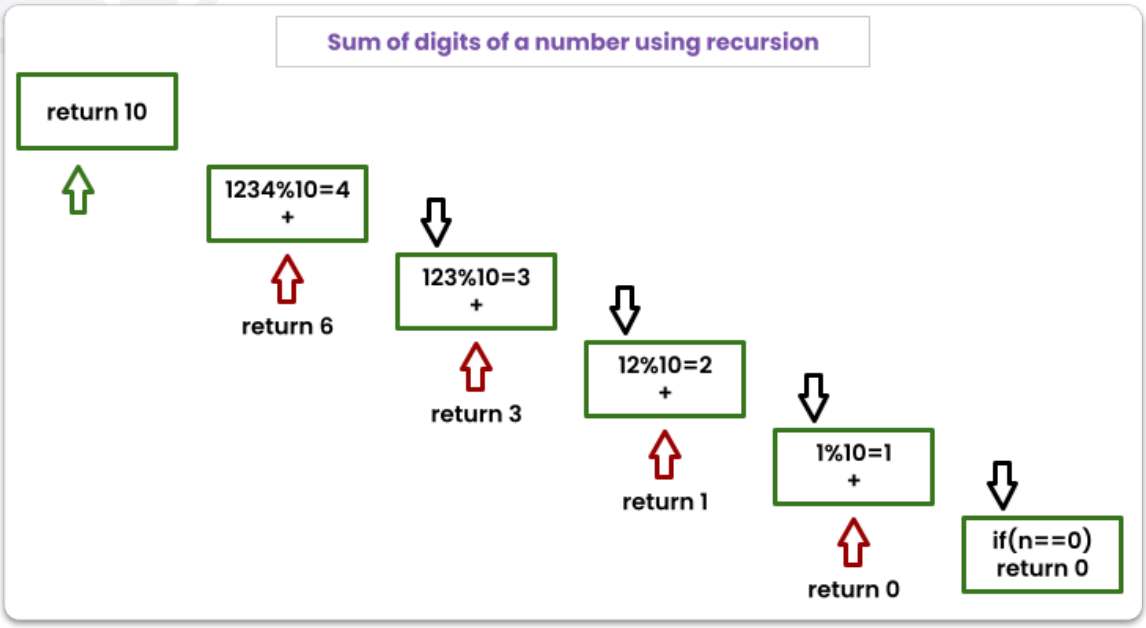
List of Concepts Involved :

- Given an integer, find out the sum of its digits using recursion.
- Given two numbers p & q, find the value p^q using a recursive function.

Problem 1 : Given an integer, find out the sum of its digits using recursion.
Input n= 1234
Output 10
Explanation 1+2+3+4=10

Approach
Let's define a function func which will take a number as input and will return the sum of all of its digits. now this function can we written in this way first digit + sum of remaining digit (the same way human do sum in left to right manner) but as extracting first digit is relatively tough thus we will start with last digit so the function func(num) can be written as func(num) = last_digit + func(num after removing last digit). Now to get the last digit we can use % operator and to remove the last digit we can use / operator.

Let's see the step-by-step approach for a better understanding of how the algorithm works.
Let the number be 1234.
Step 1-> 1234 % 10 which is equal-to 4 + (send 1234/10 to next step)
Step 2-> 123 % 10 which is equal-to 3 + (send 123/10 to next step)
Step 3-> 12 % 10 which is equal-to 2 + (send 12/10 to next step)
Step 4-> 1 % 10 which is equal-to 1 + (send 1/10 to next step)
Step 5-> 0 algorithm stops



Code link https://pastebin.com/hfS1SKb5

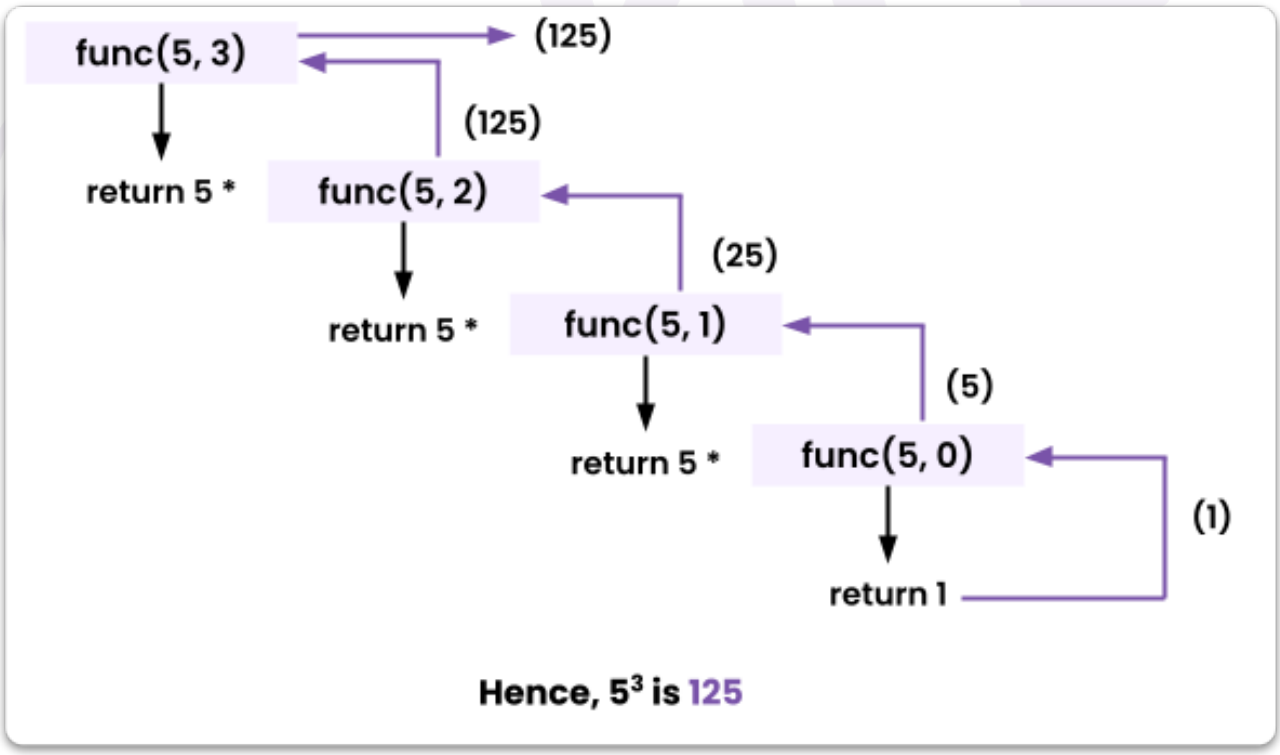
```
Enter the number
1234
Sum of digits in 1234 is 10
```

Problem 2 Given two numbers a & b, find the value a^b using a recursive function.
Input a =5 , b=3
Output 125
Explanation (5*5*5)=125

Approach

- Create a recursive function say func(int a, int b)
- Base condition : if(b==0) return 1.
- Otherwise, return (a* func(a, b-1))

Code link https://pastebin.com/BYLridJC



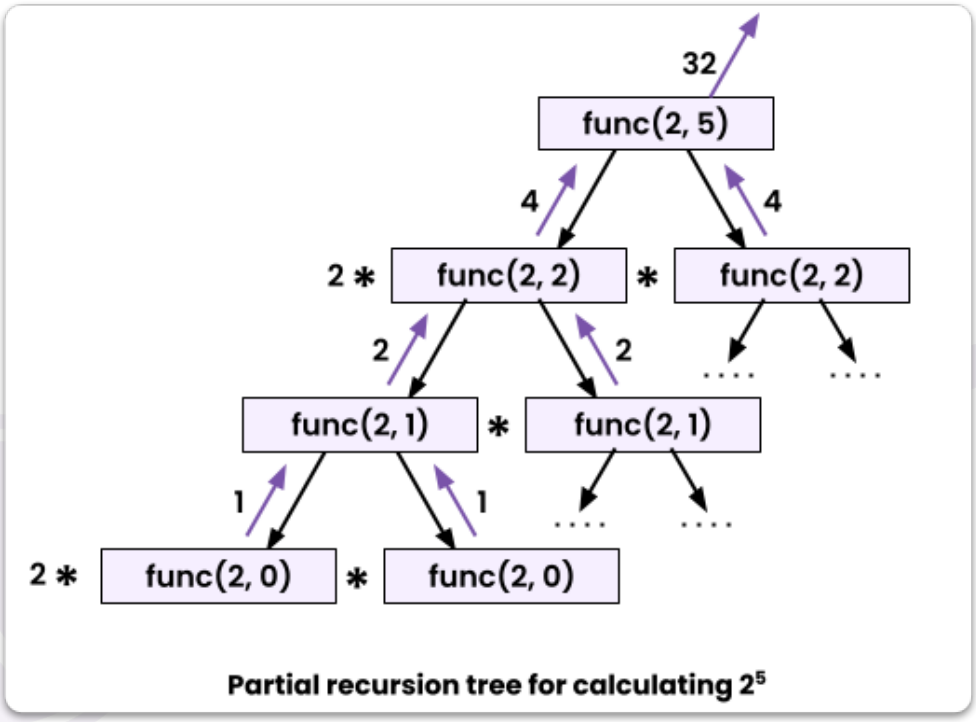
In the above program, the function func() is a recursive function. If the power is zero, then the function returns 1 because any number raised to power 0 is 1. If the power is not 0, then the function recursively calls itself.

- Time Complexity : O(b), b denotes the power.
- Space Complexity : O(b), b is the size of the recursion stack.

Efficient approach:
Lets see a better approach on how we can improve the time complexity of our solution, lets define a function func which will take two numbers a and b as input and will return a^b. Now this function can we written in this way func(a, b) =func(a, b / 2) ×func(a, b / 2); Like humans do in real life for example a=2, b=4, so for calculating 2^4 = we can find (2^2), take square of it in case b is even. In case b is odd, for example a=2, b=5, so for calculating 2^5 = we can find (2^2), take square of it and multiply it with 2 like (2^5)= (2) * (2^2) * (2^2).

func(a, b) =func(a, b / 2) ×func(a, b / 2); // if b is even
func(a, b) = a ×func(a, (b-1) / 2) ×func(a, (b-1) / 2); // if b is odd

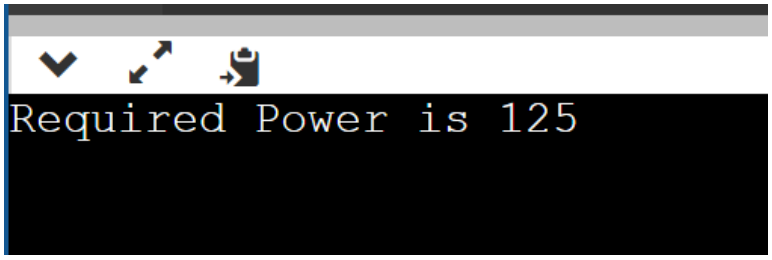
The generated recursion tree when above recurrence relation is implemented to calculate power(2,5) is shown below.



Partial recursion tree for calculating 2^5

As you can observe, func(2,2,), func(2,1) are being computed multiple times. To avoid these redundant computations, we store the result in a result variable while making the recursive call and use this variable value subsequently.

Code
https://pastebin.com/LX9wwNt8



- This function has a base case: b = 0.
- This function has two recursive calls. Only one of them is made in any given call.
- Let's first consider the case when b is even: In that case, the recursive call is made with b / 2. I will store the value return by func(a,b/2) in result variable and will return square of result variable i.e (result *result).
- If all calls would use this case, then you half b in each call down until you reach 1. This is indeed log(b) calls (plus 1 for the base case).
- In that case when b is odd, the recursive call is made with (b-1) / 2. I will store the value return by func(a, (b-1)/2) in result variable and will return square of result variable multiplied by a i.e (a*result *result).

Time Complexity

- To more easily observe that the algorithm is O(logb), you can rewrite the function to always reduce the power by half at each iteration, so that in the worst case, there are only logb call.
- The reason why the algorithm remains O(logb) even with the extra calls for the odd number case is because the number of extra calls is bounded by a constant. In the worst case, b/2 is odd at each iteration, but this would only double the number of extra calls (the constant is 2). That is, at worst, there will be 2logb calls to complete the algorithm.
- The time complexity of this algorithm is O(log(b)) while computing power(a,b). This is because at every level in the recursion sub-tree, we are doing only one computation(and using that value subsequently) and there are log(b) levels overall and Space Complexity is O(1).

Upcoming Class Teasers

- Recursion on arrays
- Recursion on strings