Optimal Control of Multiroom HVAC System: An Event-Based Approach

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***Abstract*— Building energy saving is of great practical interest due to the increasing energy consumption in buildings. The optimal control of the heating, ventilation, and air-conditioning (HVAC) systems leads to great energy saving potential. However, this problem is challenging due to the exponentially increasing state space and policy space. In this brief, we consider this important problem and make the following major contributions. First, we formulate the multiroom HVAC control problem as an event-based optimization, where decisions are made only when certain events occur. The size of the event space is significantly smaller than that of the state space. Second, to further simplify the calculation process, we develop an approximate solution method which focuses on local-event-based policies. These policies control the terminal devices in a room using solely the information in that room. Third, we demonstrate the performance of this method through two sets of numerical examples. In the small-scale two-room example, it is shown that our method can achieve a near-optimal solution. In the large-scale example, it is shown that the local-event-based approach can achieve a policy which is better than the threshold-based control method, hysteresis control method, and predictive control method.**

***Index Terms*— Building energy saving, event-based optimization (EBO), Markov decision process (MDP).**

I. Introduction

BUILDINGS account for 70% of electricity consumption in the United States [1], 35% of primary energy consumption in China [2], and about 40% of the energy in buildings is consumed by heating, ventilation, and air-conditioning (HVAC) systems [3]. Since improving the HVAC systems control can result in significant energy savings, the optimal control of the HVAC systems is becoming more and more attractive.

As people spend more than 90% of their time indoors on average [4], thermal comfort is an important factor in HVAC control. The temperature and humidity in a room are affected by HVACs, lights, natural ventilation, number of occupants, plug-in load, and the outdoor environment, including outdoor temperature, direct and diffuse radiation, and so on. The HVAC system needs to provide the required comfortable environment to the occupants with the minimal energy cost. There is a tradeoff between the energy cost and the thermal comfort due to the contradiction between them.

In a multiroom building, a room is coupled with the others in that the occupants may move from one room to another. Therefore, the optimal control of the HVAC system of an individual room should consider not only the states of itself but also the states of the other rooms [3]. However, the optimal control of the HVAC systems is challenging due to the exponentially increasing state space. A policy determines whether to turn ON or OFF the HVACs in all the rooms of a building at each decision stage. The set of the policies is called the policy space. But the size of the state space and policy space increases exponentially with the number of rooms in the building, and could become extremely large for practical problems. It is difficult to solve the problem directly by Markov decision processes (MDPs). Many efforts have been done to overcome the large scale difficulty in MDP problem, such as neurodynamic programming [15], state aggregation [10].

Event-based optimization (EBO) provides an alternative approach to solve large-scale MDPs. One unique feature of EBO is that by appropriately defining the events, the number of the events may only increase linearly or stay as a constant when the problem scale increases. Note that events are defined as a set of state transitions that share some common properties. Therefore, there is a tradeoff between how the details of the system dynamics are captured by the events and the number of the events. In particular, events that describe the state transitions in all rooms in the building are called global events, and the set of the global events is called the global event space; events that only describe the state transitions in an individual room are called local events, and the set of the local events are is called the local event space. Global events provide more information, but the number of global events explodes exponentially. Local events provide limited information, but the number of local events stays as a constant. Therefore, an important problem is to analyze whether global events or local events should be used in a specific EBO problem.

In this brief, we consider this important problem and make the following major contributions. First, we formulate the multiroom HVAC control problem as an EBO, where decisions are made only when certain events occur. Second, to further simplify the calculation process, we develop an approximate solution method which focuses on local-event-based policies. These policies control the terminal devices in a room using

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solely the information in that room. Third, we demonstrate the performance of this method through two sets of numerical examples. In the small-scale two-room example, it is shown that our method can achieve a near-optimal solution. In the large-scale example, it is shown that the local-event-based approach can achieve a policy which is better than the threshold-based control method, hysteresis control method, and predictive control method.

The rest of this brief is organized as follows. We briefly review the related literature in Section II, mathematically formulate the HVAC control problem in Section III, provide the global and local event-based solution methodologies in Section IV, provide the numerical results in Section V, and briefly conclude in Section VI.

II. Literature Review

There have been many efforts focusing on obtaining effective strategies for the control of the HVAC systems to reduce the energy cost. There are studies focusing on small-scale HVAC control problems [5]–[7]. A personalized occupancy profile-based HVAC schedules method was used in [8]. The start/stop of the variable air volume systems was controlled by the occupancy profile instead of predetermined HVAC schedules. But this was also a rule-based control method, not optimal control.

MDP [9] provides a general framework for many control, decision making, and optimization problems. A major difficulty of MDP is that the state and action space increase exponentially with the scale of the problem. A great amount of efforts have been done to overcome this difficulty. Some studies focus on simplifying the structure of the optimal policies based on the problem information, such as state aggregation [10], time aggregation [11], [12], and action elimination [13], [14]. Some studies focus on solving the MDP problem approximately, such as neurodynamic programming [15], reinforcement learning [16], and EBO approach [17], [18].

EBO approach [17], [18] is a general framework for the optimization problems in discrete event dynamic systems. By exploiting the special feature of the problem, the policies can respond to certain events instead of all the state transitions. But [19]–[21] did not consider how to define the events in the large-scale problems. The performance of the EBO approach depends on the definition of the events. In this brief, we show that the global-event-based method cannot solve the large scale problem due to the large event space. As a result, we use a local-event-based method instead of global-event-based method to approximately solve the HVAC control problem.

III. Problem Formulation

We formulate the HVAC control problem as an EBO in this section. We introduce the device models that are required for control implementation in Section III-A, introduce the thermal comfort model in Section III-B, present the system dynamics in Section III-C, define the events in Section III-D, and present the EBO problem in Section III-E.

A. Device Models

Consider a building of I rooms [34]. The room index i ranges from 1 to I. The decision stage k ranges from 1 to K with equal duration Δt. Each room has a glass-curtain wall with blinds for shading, and is also equipped with HVAC system. In summer days, indoor air is supplied to the fan coil unit (FCU) system by a fan, cooled and dehumidified by chilled water, then supplied to the room again. To simplify the discussion, we do not consider the control of the fresh air unit, and only control the FCU.

The air in rooms is cooled by FCUs. The energy consumption of the FCU includes the energy consumption of the FCU fan and the FCU cooling energy consumption. The power of the FCU fan $P_{i,\text{fan},\text{FCU},k}$ is a nonlinear function of the air flow rate as [22]

$$P_{i,\text{fan},\text{FCU},k} = P_{i,\text{fan},\text{FCU},\text{Rated},k} \cdot (G_{i,a,\text{FCU},k}/G_{i,a,\text{FCU},\text{Rated},k})^3 \tag{1}$$

where $P_{i,\text{fan},\text{FCU},\text{Rated},k}$ and $G_{i,a,\text{FCU},\text{Rated},k}$ are the rated FCU fan power and the rated FCU air flow rate in room i at stage k, respectively; $G_{i,a,\text{FCU},k}$ is the air flow rate in room i at stage k.

The FCU cooling power $P_{i,\text{FCU},k}$ equals the difference between the inlet air enthalpy $Q_{i,\text{FCU},\text{inlet},k}$ and the outlet air enthalpy $Q_{i,\text{FCU},\text{outlet},k}$. The enthalpy indicates the energy that is contained in the air and the water vapor in the air. The $P_{i,\text{FCU},k}$ is given as [22]

$$\begin{aligned} P_{i,\text{FCU},k} &= Q_{i,\text{FCU},\text{inlet},k} - Q_{i,\text{FCU},\text{outlet},k} \\ &= G_{i,a,\text{FCU},k}[C_p T_{i,a,k} + H_{i,k}(2500 + 1.84T_{i,a,k})] \\ &\quad - G_{i,a,\text{FCU},k}[C_p T_{i,\text{FCU},k} \\ &\qquad + H_{i,\text{FCU},k}(2500 + 1.84T_{i,\text{FCU},k})] \end{aligned} \tag{2}$$

where C_p is the air specific heat; $T_{i,a,k}$ and $H_{i,k}$ are the air temperature and humidity in room i at stage k, respectively; $T_{i,\text{FCU},k}$ and $H_{i,\text{FCU},k}$ are the outlet air temperature and humidity of the FCU in room i, respectively.

B. Thermal Comfort Model

The human thermal comfort requirement is described by the predicted mean vote (PMV) index. Fanger [24] related PMV to the imbalance between the actual heat flow from the body in a given environment and the heat flow that is required for optimum comfort at the specified activity. The PMV value F can be described by the following equation [24]:

$$F = [0.303\exp(-0.036M) + 0.028]L \tag{3}$$

where M is the metabolic activity level and L is the thermal load on the body, which is defined as the difference between the internal heat production and the heat loss to the actual environment for a person.

C. System Dynamics

The building energy optimization problem can be modeled as a discrete-time MDP problem.

For room i at stage k, the state vector is

$$s_{i,k} = \{T_{i,a,k}, T_{i,w,k}, H_{i,k}, x_{i,l,k}, n_{i,k}, P_{i,d,k}, T_{o,k}, L_{i,o,k}\} \quad (4)$$

where

$T_{i,a,k}$ air temperature of room i;
$T_{i,w,k}$ wall temperature of room i;
$H_{i,k}$ humidity of room i;
$x_{i,l,k}$ ON/OFF state of the lights in room i;
$n_{i,k}$ number of the occupants in room i;
$P_{i,d,k}$ power of the plug-in device in room i;
$T_{o,k}$ outdoor temperature;
$L_{i,o,k}$ outdoor radiant heat gains of room i.

Therefore, the state space of the whole building at stage k is

$$\mathcal{S}_k = \left\{ s_k | s_k \in \bigcup_{i=1}^{I} s_{i,k} \right\}. \quad (5)$$

The state space of the system is defined as

$$\mathcal{S} = \left\{ s | s \in \bigcup_{k=1}^{K} \mathcal{S}_k \right\}. \quad (6)$$

The state transition is described by dynamic equations, which are given in detail in the following [3].

The indoor air temperature at stage $k+1$ is affected by 1) heat that is generated by occupants, lights, and other plug-in devices; 2) heat that is transferred through the interior wall; 3) heat that is transferred through the glass curtain wall; 4) heat that is provided by the HVAC system; and 5) heat that is contained in the remaining indoor air. The dynamic of the air temperature of room i is described by

$$\begin{aligned} m_{i,a} T_{i,a,k+1} &= m_{i,a} T_{i,a,k} + \Delta t \cdot [n_{i,k} L_g + x_{i,l,k} \cdot P_{i,l,k} + P_{i,d,k} \\ &\quad + h_{gs} A_{i,gs} (T_{o,k} - T_{i,a,k}) + h_w A_{i,w} (T_{i,w,k} - T_{i,a,k})]/C_p \\ &\quad + \Delta t \cdot G_{i,a,FCU,k} \cdot (T_{i,FCU,k} - T_{i,a,k}) \end{aligned} \quad (7)$$

where $m_{i,a}$ is the mass of air in room i; $P_{i,l,k}$ is the power of the lights; L_g is the heat generation rate per person; h_w is the heat convection coefficient between the interior walls and the indoor air; h_{gs} is the heat transfer coefficient between outdoor and indoor air through the glass curtain wall; $A_{i,w}$ is the area of the interior walls; and $A_{i,gs}$ is the area of the glass curtain.

The interior wall temperature is affected by 1) heat convection between the wall and the indoor air and 2) radiant heat gains. The dynamic of the wall temperature of room i is described by

$$\begin{aligned} \frac{m_{i,w}}{2} C_w T_{i,w,k+1} &= \frac{m_{i,w}}{2} C_w T_{i,w,k} \\ &\quad + \Delta t [h_w A_{i,w} (T_{i,a,k} - T_{i,w,k}) + L_{i,o,k}] \end{aligned} \quad (8)$$

where C_w is the wall capacitance; $m_{i,w}$ is the mass of the wall; and $L_{i,o,k}$ is related to the orientations of the window. The mass of the wall is divided by two because an interior wall is shared by two adjacent rooms. In our problem, each two adjacent rooms are separated by concrete walls. The temperature of each side of a wall may be different, and is affected by the room that it belongs to.

The indoor air humidity is affected by 1) humidity that is generated by occupants; 2) humidity that is provided by the HVAC system; and 3) humidity that is contained in the remaining indoor air. The dynamic of the humidity of room i is described by

$$\begin{aligned} m_{i,a} H_{i,k+1} &= m_{i,a} H_{i,k} + \Delta t \cdot n_{i,k} H_g \\ &\quad + \Delta t \cdot G_{i,a,\text{FCU},k} \cdot (H_{i,\text{FCU},k} - H_{i,k}) \end{aligned} \quad (9)$$

where H_g is the humidity generation rate per person.

The load of the room includes heat that is generated by occupants, lights, plug-in device, and heat that is transferred into the room from outside. The number of occupants in room i is described by a Markov chain. The one-step transition matrix is [23]

$$P\{n_{i,k+1} = b | n_{i,k} = c\} = \pi_{bc,i}, \quad i = 1, \ldots, I. \quad (10)$$

The state of the lights in room i is ON during the working hours, which is $x_{i,l,k} = 1$, and OFF during the other hours, which is $x_{i,l,k} = 0$

$$x_{i,l,k} = \begin{cases} 1, & k \in K_w \\ 0, & k \notin K_w \end{cases} \quad (11)$$

where K_w is the working hours of each day.

The power of the plug-in device in each room is modeled as a constant level plus a normally distributed noise [23], which is

$$P_{i,d,k} = C_{i,d} + \omega_{i,d,k} \quad (12)$$

where $C_{i,d}$ is a constant and $\omega_{i,d,k}$ is a normally distributed variable. The mean and variance of $\omega_{i,d,k}$ come from the statistical data of room i.

The computational formula of $T_{o,k}$ and $L_{i,o,k}$ can be obtained in [25] and [26]. The full-state information can be measured by sensors [3], [26].

The action space of room i is defined as

$$\mathcal{A}_i = \{a_{i,j} | i = 1, 2, \ldots, I, \quad j = 1, 2\} \quad (13)$$

where $a_{i,1} = \{S_{i,\text{HVAC}} = \text{OFF}\}$, $a_{i,2} = \{S_{i,\text{HVAC}} = \text{ON}\}$; $S_{i,\text{HVAC}}$ is the state of the HVAC system in room i. Therefore, the action space of the system is defined as

$$\mathcal{A} = \left\{ a(k_a) | a(k_a) \in \prod_{i=1}^{I} \mathcal{A}_i \right\} \quad (14)$$

where $\prod \mathcal{A}_i$ is the Cartesian product of $\mathcal{A}_i$; k_a is the index of the elements in $\mathcal{A}$.

The one-step cost will contain the two aspects, which is

$$f_k(s, a(k_a)) = \sum_{i=1}^{I} (\alpha \cdot E_{i,k} + (1-\alpha) \cdot |F_{i,k}|) \quad (15)$$

$$\begin{aligned} E_{i,k} &= (P_{i,\text{fan},\text{FCU},k} + x_{i,l,k} \cdot P_{i,l,k} + P_{i,d,k} \\ &\quad + P_{i,\text{FCU},k}/\rho) \cdot \Delta t \end{aligned} \quad (16)$$

where $F_{i,k}$ is the PMV value in room i at stage k; α is a weighting factor; $E_{i,k}$ is the energy consumption in room i at stage k; and ρ is the coefficient of performance.

The state-based policy can be deterministic or randomized. A state-based randomized policy d_s specifies a probability distribution $\gamma_{d_s}(\cdot)$ over the action space, which means that d_s is defined as a mapping from $\mathcal{S}$ to the set of the probability distribution over $\mathcal{A}$, that is $d_s : \mathcal{S} \to \mathcal{P}(\mathcal{A})$. A deterministic policy can be regarded as a special case of a randomized policy in that the probability distribution over the action space is degenerate. The policy space is denoted as $\mathcal{D}$. Finally, the optimization problem is

$$\eta = \min_{d_s \in \mathcal{D}} \left\{ \lim_{K\to\infty} \frac{1}{K} E\left(\sum_{k=1}^{K} f_k\left(s, d_s\left(s\right)\right) \right) \right\} \tag{17}$$

the expectation is over all the sample paths, each of which is uncertain with the outside temperature, the number of occupants in each room, and the uncertain of the power of the plug-in device.

D. Definition of Events

An event is defined as a set of state transitions with certain common properties. The HVAC in each room is ON when the room temperature is $T_{i,a,k} \geq T_h$, and OFF when $T_{i,a,k} < T_l$, where T_h and T_l are the upper and lower bounds of the thermostats, respectively. The EBO approach will be used to determine whether the HVAC is ON or OFF when the room temperature is between T_l and T_h. For room i, the event space is defined as

$$\mathcal{E}_i = \{e_i(k_1)|k_1 = 1, 2, \quad i = 1, \ldots, I\} \tag{18}$$

where

$$e_i(1) = \{\langle s_{i,k}, s_{i,k+1}\rangle | T_m \leq T_{i,a,k} < T_h\}$$
$$e_i(2) = \{\langle s_{i,k}, s_{i,k+1}\rangle | T_l \leq T_{i,a,k} < T_m\}$$

where $e_i(1)$ means that the air temperature of room i at stage k is between T_m and T_h, and $e_i(2)$ means that the air temperature of room i at stage k is between T_l and T_m. Note that the event space $\mathcal{E}_i$ can consist of any number of events, and T_m can be any number between T_l and T_h. We use two events for a proof of concept here. Therefore, the event space of the system can be defined as

$$\mathcal{E}_{\text{global}} = \left\{ e(k_e) | e(k_e) \in \prod_{i=1}^{I} \mathcal{E}_i \right\} \tag{19}$$

where k_e is the index of the elements in $\mathcal{E}_{\text{global}}$. From (19), we can see that there are 2^I global events. An global event $e(k_e) \in \mathcal{E}_{\text{global}}$ represents that the air temperature of all rooms at stage k is $T_m \leq T_{i,a,k} < T_h$ or $T_l \leq T_{i,a,k} < T_m$, where $i = 1, 2, \ldots, I$. The input state space of $e(k_e)$ is defined as

$$\mathcal{I}(e(k_e)) = \{\text{all } s_k \in \mathcal{S} | \langle s_k, s_{k+1}\rangle \in e(k_e)\} \tag{20}$$

and the output state space of an input state s_k of the event $e(k_e)$ is defined as

$$\mathcal{O}_{s_k}(e(k_e)) = \{\text{all } s_{k+1} \in \mathcal{S} | \langle s_k, s_{k+1}\rangle \in e(k_e)\}. \tag{21}$$

The policy that is based on the global events can be deterministic or randomized, too. A deterministic policy can be regarded as a special case of a randomized policy. From [20], we know that the EBO problem can be converted to a partially observable MDP (POMDP) problem. In the POMDP problem, the performance of the randomized policy may be better than that of the deterministic policy [27]. Furthermore, it is easier to obtain a randomized policy since the randomized policy is continuous. Therefore, we will find a randomized optimal policy instead of a deterministic policy in this brief. The randomized global-event-based policy is defined as

$$d : \mathcal{E}_{\text{global}} \to \mathcal{P}(\mathcal{A}). \tag{22}$$

From (22), a global-event-based policy d specifies a probability distribution γ over the action space $\mathcal{A}$, that is

$$\gamma \in \mathcal{P}(\mathcal{A}), \quad \gamma = [\gamma_{k_e,k_a}], \quad \gamma_{k_e,k_a} = P(a(k_a)|e(k_e)) \tag{23}$$

which means that when the event $e(k_e)$ happens, the system will take the action $a(k_a)$ with probability γ_{k_e,k_a}. It can be seen that γ is a k_e by k_a matrix and $\sum_{k_a=1}^{N(\mathcal{A})} \gamma_{k_e,k_a} = 1$, where $N(\cdot)$ is the number of elements in the set. In practical application, the control computer can generate a uniform distributed random number δ with $0 < \delta \leq 1$. Denote $\gamma_{k_e,0} = 0$. If $\sum_{t=0}^{k_a-1} \gamma_{k_e,t} < \delta \leq \sum_{t=0}^{k_a} \gamma_{k_e,t}$, the action $a(k_a)$ is taken. The selected action, which is a pseudorandom sequence that determines the ON/OFF of the HVAC system of each room, is sent to the controller by BACnet protocols, e.g., Honeywell HT961 [33] controller. The controller can control the ON/OFF of the HVAC system according to the action.

E. EBO Problem

For our EBO problem, the event space is defined in (19); the action space is defined in (14). The system dynamics are described in (7)–(12). The steady-state probability of the event $e(k_e)$ under policy d is denoted as $\pi^d(e(k_e))$. The steady-state probability that the system state is s under policy d when the event $e(k_e)$ happens is denoted as $\pi^d(s|e(k_e))$. The probability of an action $a(k_a)$ is taken under policy d when an event $e(k_e)$ happens is denoted as $P^d(k_a|k_e)$. The objective function of the optimization problem is (17).

IV. Solution Methodologies

In this section, we provide the solution methodologies of the event-based approach, including global-event-based approach and local-event-based approach.

A. Global-Event-Based Approach

In global-event-based approach, the actions of the HVAC system in one room are affected by the other rooms. To obtain the EBO algorithms, the performance difference formula is needed.

1) Difference Formula of Global-Event-Based Policies: The global event space is defined in (19). For any two different policies h and d, let $\pi^h(e(k_e))$ denote the steady-state probability of the event $e(k_e)$ under policy h; $\pi^h(s|e(k_e))$ denote the steady-state probability that the system state is s under policy h when the event $e(k_e)$ happens; $P^h(k_a|k_e)$ denote the probability of the action $a(k_a)$ is taken under policy h when the event $e(k_e)$ happens. Following [28, eq. (8.19)], the difference formula with global-event-based policies can be derived as:

$$\eta^h - \eta^d = \sum_{k_e=1}^{N(\mathcal{E}_{\text{global}})} \pi^h(e(k_e)) \cdot \sum_{a(k_a)\in\mathcal{A}} \{P^h(k_a|k_e) - P^d(k_a|k_e)\} q^{d,h}(k_e,k_a) \quad (24)$$

where

$$q^{d,h}(k_e,k_a) = \sum_{s\in\mathcal{I}(e(k_e))} \sum_{s'\in\mathcal{O}_s(e(k_e))} \times \{\pi^h(s|e(k_e)) \cdot P(s'|k_e,k_a) g^d(s')\} \quad (25)$$

where $P(s'|k_e,k_a)$ is the probability of that when the event $e(k_e)$ happens, and the action $a(k_a)$ is taken, the system state is s' at the next stage, which is determined by the system dynamic equations (7)–(12); $q^{d,h}(k_e,k_a)$ is the aggregated potential that is depending on both the policies h and d and $g^d(s')$ is the potential of the state s' under policy d. Equation (24) provides the performance difference of the system between the two different policies h and d.

2) Derivative Formula of Global-Event-Based Policies: The event space is defined in (19), the action space is defined in (14), and the randomized policy is defined in (22).

Consider that there is another policy d', similarly as in (24), the difference formula can be derived as

$$\eta' - \eta = \sum_{k_e=1}^{N(\mathcal{E}_{\text{global}})} \pi'(e(k_e)) \cdot \sum_{a(k_a)\in\mathcal{A}} (\gamma'_{k_e,k_a} - \gamma_{k_e,k_a}) q'(k_e,k_a) \quad (26)$$

where

$$q'(k_e,k_a) = \sum_{s\in\mathcal{I}(e(k_e))} \sum_{s'\in\mathcal{O}_s(e(k_e))} \{\pi'(s|e(k_e)) \cdot P(s'|k_e,k_a) \cdot g(s')\}. \quad (27)$$

In (26), when γ' approaches γ, from (26), the sensitivity formula can be derived as

$$\frac{\partial \eta}{\partial \gamma_{k_e,k_a}} = \pi(e(k_e)) q(k_e,k_a) \quad (28)$$

where

$$q(k_e,k_a) = \sum_{s\in\mathcal{I}(e(k_e))} \sum_{s'\in\mathcal{O}_s(e(k_e))} \{\pi(s|e(k_e)) \cdot P(s'|k_e,k_a) \cdot g(s')\}. \quad (29)$$

Equation (28) provides the performance derivatives with respect to the probability of which action is taken when the event $e(k_e)$ happens. In (28), it can be seen that the sensitivity formula only depends on the sample path under the policy d. The aggregated potential $q(k_e,k_a)$ can be estimated based on the sample path under the original policy d, and the policy d' does not need to be considered.

3) Estimate the Steady-State Probability of the Event and the Aggregated Potential: The steady-state probability of the event and the aggregated potential can be estimated based on the sample path under the policy d. As an example, the detailed estimation method for $\pi(e(k_e))$ and $q(k_e,k_a)$ is explained here.

Consider a sample path $\{s_0, s_1, \ldots, s_N\}$ under policy d with $N \gg 1$. Denote the set of the time instants at which the event $e(k_e)$ happens on the sample path as

$$\mathcal{T}(k_e) = \{l_j, j = 1, 2, \ldots, N(\mathcal{T}(k_e))\}. \quad (30)$$

The steady-state probability of the event $e(k_e)$ can be estimated as

$$\pi(e(k_e)) = \frac{N(\mathcal{T}(k_e))}{N}. \quad (31)$$

Denote the set of the time instants at which the event $e(k_e)$ happens on the sample path, and then the action $a(k_a)$ is taken as

$$\mathcal{T}(k_e,k_a) = \{l_j, j = 1, 2, \ldots, N(\mathcal{T}(k_e,k_a))\}. \quad (32)$$

Denote the set of the time instants at which the system is at the state s when the event $e(k_e)$ happens and action $a(k_a)$ is taken as

$$\mathcal{T}(k_e,s,k_a) = \{l_j, j = 1, 2, \ldots, N(\mathcal{T}(k_e,s,k_a))\}. \quad (33)$$

It is obvious that

$$\mathcal{T}(k_e,k_a) = \bigcup_{s\in\mathcal{I}(e(k_e))} \mathcal{T}(k_e,s,k_a) \quad (34)$$

$$N(\mathcal{T}(k_e,k_a)) = \sum_{s\in\mathcal{I}(e(k_e))} N(\mathcal{T}(k_e,s,k_a)). \quad (35)$$

Choose a large integer N_l, set

$$g_{l_j} = \sum_{i=l_j}^{l_j+N_l} f_i(s, a(k_a)), \quad l_j \in \mathcal{T}(k_e,k_a). \quad (36)$$

From the above definitions, we have

$$\frac{1}{N(\mathcal{T}(k_e,k_a))} \sum_{j=1}^{N(\mathcal{T}(k_e,k_a))} g_{l_j} = \sum_{s\in\mathcal{I}(e(k_e))} \frac{N(\mathcal{T}(k_e,s,k_a))}{N(\mathcal{T}(k_e,k_a))} \cdot \left(\frac{1}{N(\mathcal{T}(k_e,s,k_a))} \sum_{n=1}^{N(\mathcal{T}(k_e,s,k_a))} g_{l_j} \right). \quad (37)$$

And by definition, we have

$$\lim_{N(\mathcal{T}(k_e,k_a))\to\infty} \frac{N(\mathcal{T}(k_e,s,k_a))}{N(\mathcal{T}(k_e,k_a))} = \pi(s|e(k_e)) \quad (38)$$

$$\lim_{N_l\to\infty} \lim_{N(\mathcal{T}(k_e,s,k_a))\to\infty} \frac{1}{N(\mathcal{T}(k_e,s,k_a))} \sum_{n=1}^{N(\mathcal{T}(k_e,s,k_a))} g_{l_j} = \sum_{s'\in\mathcal{O}_s(e(k_e))} (P(s'|k_e,k_a) \cdot g(s')). \quad (39)$$

Algorithm 1 Gradient-Based Algorithm

- Step 1: initialization. Randomly pick an initial policy d^0 from the event-based policy space, set $j = 0$.
- Step 2: policy evaluation. Estimate the aggregated potential based on the sample path of the system under policy d^j with the method which is provided in (30)-(40).
- Step 3: policy improvement. Obtain the performance derivatives defined in (28). Denote $\gamma^j_{k_e}$ the k_eth row of γ^j. For $k_e = 1, 2, \cdots, N(\mathcal{E}_{global})$, update the policy to d^{j+1} by
$$\gamma^{j+1}_{k_e} = \gamma^j_{k_e} + \beta \cdot \left(\tilde{\gamma}^j_{k_e} - \gamma^j_{k_e}\right),$$
where β is a constant stepsize,
$$\tilde{\gamma}^j_{k_e} = \arg\min_{\gamma_{k_e} \in \Gamma} \left\{ \left[\frac{\partial \eta}{\partial \gamma_{k_e,1}}, \frac{\partial \eta}{\partial \gamma_{k_e,2}}, \cdots, \frac{\partial \eta}{\partial \gamma_{k_e,N(\mathcal{A})}}\right] \cdot \left(\gamma_{k_e} - \gamma^j_{k_e}\right)^T \right\}$$
$$\Gamma = \left\{ \gamma_{k_e} | 0 \le \gamma_{k_e,k_a} \le 1, \sum_{k_a}^{N(\mathcal{A})} \gamma_{k_e,k_a} = 1 \right\}$$
- Step 4: if $\left|\eta^{\gamma^{j+1}} - \eta^{\gamma^j}\right| > \varepsilon$, let $j = j + 1$ and go to step 2; else, stop.

Therefore, the aggregated potential $q(k_e, k_a)$ can be estimated as

$$q(k_e, k_a) = \lim_{N(\mathcal{T}(k_e,k_a)) \to \infty} \frac{1}{N(\mathcal{T}(k_e, k_a))} \sum_{j=1}^{N(\mathcal{T}(k_e,k_a))} g_{l_j}. \quad (40)$$

The gradient-based algorithm can be derived, as shown in Algorithm 1.

From the definitions of the events in (18) and (19), we can see that the size of the event space $N(\mathcal{E}_{\text{global}})$ explodes exponentially when the number of the rooms I increases, and could be extremely large for practical problem. To solve the problem, the local-event-based approach is provided below.

B. Local-Event-Based Approach

In this section, we will give a local-event-based approach to solve the HVAC control problem.

1) Local Events: The local-event-based approach is used to approximately solve the HVAC control problem. In the local-event-based approach, the HVAC in one room is controlled only by the information of that room itself. The local event space is defined as

$$\mathcal{E}_{\text{local}}(i) = \{e(k_e, i) | k_e = 1, 2; \quad i = 1, \ldots, I\} \quad (41)$$

where

$$e(1, i) = \{\langle s_{i,k}, s_{i,k+1}\rangle | T_m \le T_{i,a,k} < T_h\}$$
$$e(2, i) = \{\langle s_{i,k}, s_{i,k+1}\rangle | T_l \le T_{i,a,k} < T_m\}.$$

The input state space of the event $e(k_e, i)$ is defined as

$$\mathcal{I}(e(k_e, i)) = \{s_{i,k} | \langle s_{i,k}, s_{i,k+1}\rangle \in e(k_e, i)\} \quad (42)$$

and the output state space of an input state $s_{i,k}$ of the event $e(k_e, i)$ is defined as

$$\mathcal{O}_{s_{i,k}}(e(k_e, i)) = \{s_{i,k+1} | \langle s_{i,k}, s_{i,k+1}\rangle \in e(k_e, i)\}. \quad (43)$$

The action space of room i is defined in (13). The randomized policy is used to determine the probability of which action is taken when an event happens. Therefore, the randomized local-event-based policy $d_{l,i}$ is defined as

$$d_{l,i} : \mathcal{E}_{\text{local}}(i) \to \mathcal{P}(\mathcal{A}_i). \quad (44)$$

From (44), a local-event-based policy $d_{l,i}$ specifies a probability distribution $\gamma_{l,i}$ over the action space $\mathcal{A}_i$, that is

$$\gamma_{l,i} \in \mathcal{P}(\mathcal{A}_i), \quad \gamma_{l,i} = [\gamma_{l,i,k_e}], \quad i = 1, \ldots, I; \quad k_e = 1, 2 \quad (45)$$

which means that when the event $e(k_e, i)$ happens in room i, the system will take the action $a_{i,1}$ with probability γ_{l,i,k_e}, and will take the action $a_{i,2}$ with probability $1 - \gamma_{l,i,k_e}$. To obtain the local-event-based algorithm, the performance difference formula is provided below.

2) Difference Formula of Local-Event-Based Policies: For any two different local-event-based policies $h_{l,i}$ and $d_{l,i}$, let $\pi^{h_{l,i}}(e(k_e, i))$ denote the steady-state probability of the event $e(k_e, i)$ under policy $h_{l,i}$; $P^{h_{l,i}}[a_{i,j} | e(k_e, i)]$ and $P^{d_{l,i}}[a_{i,j} | e(k_e, i)]$ denote the probability of an action $a_{i,j}$ is taken under policy $h_{l,i}$ and $d_{l,i}$ when an event $e(k_e, i)$ happens, respectively. Following [28, eq. (8.19)], the difference formula with event-based policies can be derived as:

$$\eta^{h_{l,i}} - \eta^{d_{l,i}} = \sum_{i=1}^{I} \sum_{k_e=1}^{2} \pi^{h_{l,i}}(e(k_e, i)) \cdot \sum_{j=1}^{2} \{P^{h_{l,i}}[a_{i,j} | e(k_e, i)] - P^{d_{l,i}}[a_{i,j} | e(k_e, i)]\} q(k_e, j, i) \quad (46)$$

$$q(k_e, j, i) = \sum_{s \in \mathcal{I}(e(k_e,i))} \sum_{s' \in \mathcal{O}_s(e(k_e,i))} \times \{\pi^{h_{l,i}}(s | e(k_e, i)) P[s' | a_{i,j}, e(k_e, i)] g^{d_{l,i}}(s')\} \quad (47)$$

where $q(k_e, j, i)$ is the aggregated potential that is depending on both policy $h_{l,i}$ and $d_{l,i}$.

3) Derivative Formula of Local-Event-Based Policies: The action space and local event space here are defined the same as in (13) and (41), respectively. With the definition in (44), for another policy $d'_{l,i}$, the difference formula can be derived as

$$\eta' - \eta = \sum_{i=1}^{I} \sum_{k_e=1}^{2} \pi'(e(k_e, i)) \cdot \{({\gamma'}_{l,i,k_e} - \gamma_{l,i,k_e}) \cdot [q'(e(k_e, i), a_{i,1}) - q'(e(k_e, i), a_{i,2})]\} \quad (48)$$

where $q'(e(k_e, i), a_{i,j})$ is the aggregated potential in room i. When $\gamma'_{l,i}$ approaches $\gamma_{l,i}$, the sensitivity formula is

$$\frac{\partial \eta}{\partial \gamma_{l,i,k_e}} = \pi(e(k_e, i)) \cdot [q(e(k_e, i), a_{i,1}) - q(e(k_e, i), a_{i,2})]. \quad (49)$$

The gradient-based algorithm can be derived similarly, as shown in Algorithm 1, and use the local-event-based policy $d_{l,i}$ instead of the global-event-based policy d.

V. Numerical Results

In this section, we will give the numerical results of the EBO approach. Since the policy iteration algorithm [18] cannot be used to solve our problem [34], we use the gradient algorithm to solve our problem. First, a two-room example is given to show that the local-event-based method can converge to a near-optimal solution to the problem. Second, a large-scale example is given to show that the global-event-based approach cannot be used to solve the large-scale problem, and the local-event-based approach can achieve a policy which is better than the threshold-based control method, hysteresis control method, and predictive control method.

A. Two-Room Example

In this section, we propose a two-room example to show that the HVAC control problem can be approximately solved by the local-event-based approach. The two-room system is depicted in [34].

1) Global-Event-Based Approach: In global-event-based approach, the event space is defined as in (19). Here, $T_l = 22$ °C, $T_m = 25$ °C, $T_h = 28$ °C, and $I = 2$. Therefore, the event space is

$$\mathcal{E}_{\text{global}} = \{e(k_e) | k_e = 1, 2, 3, 4\}. \tag{50}$$

The action space is defined as in (14), which is

$$\mathcal{A}_{\text{global}} = \{a(k_a) | k_a = 1, \ldots, 4\}. \tag{51}$$

The policy is defined as in (22) and (23), which is

$$d : \gamma = [\gamma_{k_e,k_a}], \quad k_e = 1, \ldots, 4, \quad k_a = 1, \ldots, 4. \tag{52}$$

From (28), the sensitivity formula can be derived as

$$\frac{\partial \eta}{\partial \gamma_{k_e,k_a}} = \pi(e(k_e)) q(k_e, k_a). \tag{53}$$

We simulate the global-event-based approach for 11 times from different initial values, and the results are in Table I, where STD and IT are standard deviation and iteration times of Algorithm 1, respectively. In each iteration, we simulate 30 sample paths, and $\bar{\eta}_{\text{global}}$ is the mean value of η_{global} over the 30 sample paths.

2) Local-Event-Based Approach: In the local-event-based approach, the HVAC in each room is controlled by the information of itself. The event space is defined as in (41), here $I = 2$. Therefore, the event space is defined as

$$\mathcal{E}_{\text{local}}(i) = \{e(k_e, i) | k_e = 1, 2; \quad i = 1, 2\}. \tag{54}$$

The action space is defined as in (13), which is

$$\mathcal{A}_i = \{a_{i,j} | i = 1, 2, \quad j = 1, 2\}. \tag{55}$$

The policy is defined as in (44) and (45), which is

$$d_{l,i} : \gamma_{l,i} = [\gamma_{l,i,k_e}], \quad i = 1, 2; \quad k_e = 1, 2. \tag{56}$$

From (49), the sensitivity formula can be derived as

$$\frac{\partial \eta}{\partial \gamma_{l,i,k_e}} = \pi(e(k_e, i)) \cdot [q(e(k_e, i), a_{i,1}) - q(e(k_e, i), a_{i,2})]. \tag{57}$$

TABLE I
NUMERICAL RESULTS OF THE GLOBAL AND LOCAL EVENT-BASED METHOD

Index	Global-event-based method			Local-event-based method		
	$\bar{\eta}_{global}$	STD	IT	$\bar{\eta}_{local}$	STD	IT
1	3,667.39	10.49	3	3,669.34	12.82	3
2	3,350.73	71.99	20	3,667.79	10.13	2
3	3,230.80	146.52	46	3,214.19	87.72	13
4	3,176.95	81.94	13	3,135.58	86.71	15
5	3,355.70	57.05	5	3,329.09	73.49	13
6	3,142.28	83.34	12	3,156.96	54.10	7
7	3,153.99	56.83	4	3,164.18	65.55	7
8	3,153.23	117.66	7	3,166.10	115.48	18
9	3,190.18	120.42	13	3,166.08	122.70	17
10	3,138.64	181.40	28	3,139.77	109.53	10
11	3,102.67	144.39	15	3,280.36	164.40	36

We simulate the local-event-based approach for 11 times from different initial values as well, and the results are in Table I, where $\bar{\eta}_{\text{local}}$ is the mean value of η_{local} over 30 sample paths as well. The differences among the values of the STD are caused by the stochastic error. By the central limit theorem [29] and Student's t-test theory [30], we can see that the local-event-based approach can achieve a near-optimal solution to the HVAC control problem with probability $\zeta > 0.95$. But the local-event-based approach can control the HVAC system without the events of the other rooms, so the size of the event space stays as a constant in the large-scale problem.

B. Large-Scale Example

In this section, we consider a practical problem based on a real building. We consider the Future Internet Technology (FIT) building with 211 rooms in Tsinghua University [34]. If we use the global-event-based approach to solve the problem, the size of the event space is $N(\mathcal{E}_{\text{global}}) = 2^{211}$, which is extremely large. Therefore, the local-event-based approach is used to solve the problem.

The action space and local event action space are defined as in (13) and (41), where $I = 211$. Since each room in the system is controlled separately, the size of the event space for each room is $N(\mathcal{E}_{\text{local}}(i)) = 2$, which does not increase with the number of rooms I. The performance of the EBO approach is compared with that of the other three methods here, which are the following.

1) *Threshold Control Method:* If $T_{i,a,k} \geq T_m$, the HVAC is ON; if $T_{i,a,k} < T_m$, the HVAC is OFF.
2) *Hysteresis Control Method:* The HVAC is ON if $T_{i,a,k} \geq T_h$, and is not OFF until $T_{i,a,k} < T_l$. The HVAC is OFF if $T_{i,a,k} < T_l$, and is not ON until $T_{i,a,k} \geq T_h$.
3) *Predictive Control Method:* A predictive control strategy where weather forecasts and building thermal mass are included [3], [31], [32].

The parameters in the four methods are: $T_l = 22$ °C, $T_m = 25$ °C, and $T_h = 28$ °C. The numerical results of the four policies are in Table II [34], where $\bar{\eta}$ is the mean value of η over 30 sample paths.

TABLE II
NUMERICAL RESULTS OF THE FOUR STRATEGIES

Method	$\overline{\eta}$	STD
A (Threshold control)	5.9537×10^6	2.1364×10^3
B (Hysteresis control)	6.0277×10^6	9.4981×10^3
C (Predictive control)	5.2680×10^6	2.1514×10^4
D (EBO)	4.8798×10^6	1.1133×10^4

From Table II, it can be seen that although the global-event-based approach cannot be used here, the local-event-based approach can give a solution that is better than the other three methods.

VI. CONCLUSION

The performance of the EBO approach depends on the definition of the events. In our HVAC control problem, we develop a local-event-based approach to simplify the calculation process. We formulate the HVAC control problem as an EBO, and show that the local-event-based method can achieve a near-optimal solution. Then, we solve a large-scale problem with the local-event-based method. Numerical results show that the local-event-based approach can achieve a near-optimal policy that is better than the commonly used policies in acceptable time and memory space.

In the future work, the local-event-based approach will be tested on the experiment platform of the Center for Intelligent and Networked Systems in the FIT building of Tsinghua University. The performance and the economic implications of the local-event-based approach will be analyzed.

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