(9) Value Assignment

- fuzzification
- Mem value assign for relations

value Assignment

Say A and B are two fuzzy sets

R : $A \rightarrow B$ is a fuzzy relation

then mem. value of each elem in relation

give by

$$\boxed{\mu_{R_{ij}} = \alpha} \quad , \quad \alpha \in [0, 1]$$

let's use r_{ij} to rep mem. value.

$\mu_{R_{ij}}$ කියන එක ආදේශ කිරීම

[illegible] වලට

How to find α — 7 methods

1. Cartesian product → prev videos
2. Closed form expression
3. look up tables
4. linguistic rules of language
5. Classification
6. Automated method from I/O data
7. Similarity method in data manipulation

(2) closed form expression

two number sets X & Y

our relation $R : X \to Y$ such that

$$y_j = x_i^2$$

create mem. fn

$$r_{ij} = 1 - \left| \frac{x_i^2 - y_j}{x_i^2} \right|$$

let's take $x_i = 3$ & $y_j = 7$

$$r_{3,7} = 1 - \left| \frac{3^2 - 7}{3^2} \right|$$

$$= 0.77$$

$3^2 = 9$

7, 9ට ළඟයි

$\therefore$ x_i හා y_j අතර relation

එකට 1 ආසන්න (0.77)

අගයක් ඇත.

③ look-up tables

already is give by someone

steel strength vs weight

$$\begin{array}{c|c}
 & \begin{array}{cccc} 100 & 200 & 300 & \text{MPa} \end{array} \\
\begin{array}{c} 10 \\ 15 \\ 20 \\ \text{kg} \end{array} & \begin{bmatrix} 0.7 & 0.1 & 0.4 \\ 0.2 & 0.35 & 0.8 \\ 0.9 & 0.8 & 0.3 \end{bmatrix}
\end{array}$$

④ linguistic rules

expressed as if-else rules

rules created base on experience & expert knowledge

⑤ classification

⑥ automation

$\Big\}$ later

③ Similarity

by Zadeh in 1907

rep similarity btwn two data, expressed as

- Data x_1 is rather similar to data X_2
- Data x_1 is very similar to data X_2

Similarity methods

- ① Cosine Amplitude
- ② Max-Min method

① Cosine Amplitude method

Data set array X of size n

$$X = \{ X_1 \; X_2 \; X_3 \; X_4 \; \cdots \; \underset{\substack{\uparrow \\ \text{each universe}}}{X_n} \}$$

each X_i is a vector of size m

$$X_i = \{ x_{i1}, x_{i2}, \cdots \underset{\substack{\uparrow \\ \text{each element}}}{x_{im}} \}$$

for example cities

$$X = \{ col, kan, Gmp, keg \} \quad n = 4$$

each city has properties

$$X_i = \{ lat, long, temp, area, population \}$$

$$m = 5$$

$$X_{col} = \{ 76^\circ E, 9^\circ N, 25^\circ C, 94\ km^2, 100k \}$$

$$X_{kan} = \{ 77^\circ E, 12^\circ N, 20^\circ C, 741\ km^2, 900k \}$$

$$X_{Gmp} = \{$$

$$X_{keg} =$$

we've 4 cities, each city describe by a 1×5 vector now relation btwn (X_i & X_j) btwn two cities given by

$$r_{ij} = \frac{\left| \sum_{k=1}^{m} (x_{ik}\, x_{jk}) \right|}{\sqrt{\left(\sum_{k=1}^{m} x_{ik}^2 \right) \cdot \left(\sum_{k=1}^{m} x_{jk}^2 \right)}}$$

$i, j = 1, 2, \ldots\ldots n$

$= \mu_R (X_i, X_j)$

<u>Ex</u> try to find weather pattern btwn 5 cities based on rain data from time periods

x_1 Jan - Apr, x_2 May - Aug, x_3 Set - Dec

findings are,

cities

time period	A	B	C	D	E
x_1	0.3	0.2	0.1	0.7	0.4
x_2	0.8	0.4	0.6	0.2	0.6
x_3	0.1	0.4	0.3	0.1	0.0

$n = 5$

$m = 3$

find similarity btwn rain patterns of 5 cities ?

using cosine method

$$r_{ij} = \frac{\left| \sum_{k=1}^{3} x_{ik} x_{jk} \right|}{\left(\sum_{k=1}^{3} x_{ik}^2 \right) \left(\sum_{k=1}^{3} x_{jk}^2 \right)}$$

$i, j = 1, 2, \ldots n$

for ex $i=1$, $j=2$ we get

1st prop in 2nd city

$$r_{12} = \frac{\left| (x_{11} \cdot x_{21}) + (x_{12} \cdot x_{22}) + (x_{13} \cdot x_{23}) \right|}{(x_{11}^2 + x_{12}^2 + x_{13}^2) \cdot (x_{21}^2 + x_{22}^2 + x_{23}^2)}$$

given matrix in $m \times n$ for our example $n \times m$ only

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.6 & 0.3 \\ 0.7 & 0.2 & 0.1 \\ 0.4 & 0.6 & 0.0 \end{bmatrix}_{n \times m} \end{array}$$

now calc all the r_{ij} values & we get

$$R = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 1 & 0.836 & 0.914 & 0.682 & 0.982 \\ B & 0.836 & 1 & 0.934 & 0.6 & 0.74 \\ C & 0.914 & 0.934 & 1 & 0.441 & 0.818 \\ D & 0.682 & 0.6 & 0.441 & 1 & 0.774 \\ E & 0.982 & 0.74 & 0.818 & 0.774 & 1 \end{array}$$

city C is more similar to A (0.914) than D (0.441) in the rain fall perspective

all diagonal elem = 1

bcuz city A = city A similar

all non diagonal elem mirror

bcuz $A \to C = C \to A$

R is tolerance relation

② Max - Min Similarity method

composition එකේ max - min ගන්නවා.

Consider data array

$$X = \{X_1, X_2, X_3 \dots X_n\}$$

and each set has elem

$$X_i = \{x_{i1}, x_{i2}, \dots x_{im}\}.$$

then mem value for each elem

$$r_{ij} = \frac{\sum_{k=1}^{m} \min(x_{ik}, x_{jk})}{\sum_{k=1}^{m} \max(x_{ik}, x_{jk})}$$

$$i, j = 1, 2, \dots n$$

Ex

කලින් example එක

$$
R = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\left[\begin{array}{ccccc}
1 & & & & \\
0.538 & 1 & & \text{sym} & \\
0.667 & 0.667 & 1 & & \\
0.429 & 0.333 & 0.250 & 1 & \\
0.818 & 0.429 & 0.538 & 0.429 & 1
\end{array}\right]
\end{array}
$$

values diff from කලින් method

but idea is same //