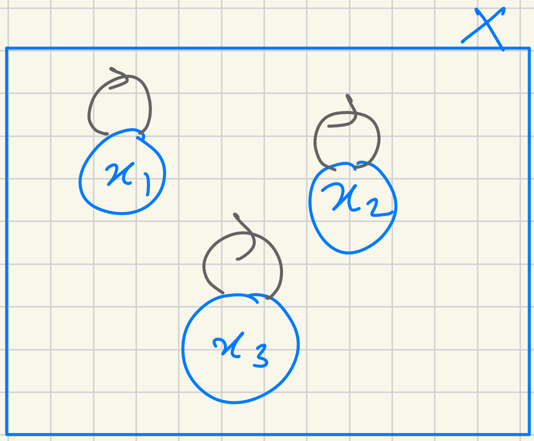
[8] Equivalance & Tolerance relation

Reflexivity for crisp

universe X with elem x_1, x_2, x_3



say all elem in X related to itself only

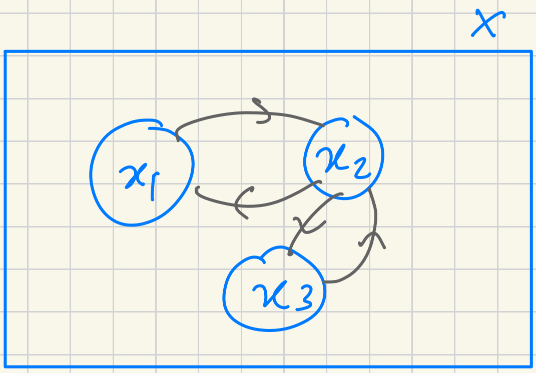
↳ Reflexivity

relation R is reflexive if,

$$(x_i, x_i) \in R$$

$$\chi_R(x_i, x_i) = 1$$

Symmetry for crisp



for each edge from $x_i \to x_j$ there should be edge from $x_j \to x_i$

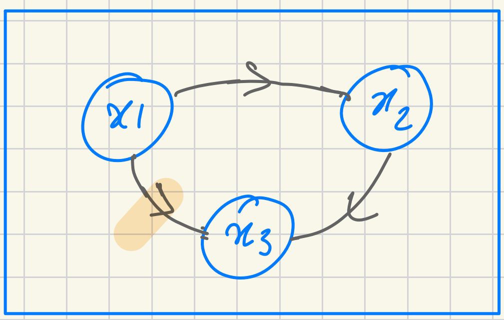
Relation R is symmetric

$$(x_i, x_j) \in R \to (x_j, x_i) \in R$$

$$\chi_R(x_i, x_j) = 1$$

$$\chi_R(x_j, x_i) = 1$$

Transitivity for crisp



$x_1 \to x_2 \to x_3$ rel

હોય તો

$x_1 \to x_3$ પણ

હોય.

$$(x_i, x_j) \in R \ \& \ (x_j, x_k) \in R \longrightarrow (x_i, x_k) \in R$$

$$\chi_R(x_i\, x_j) = 1 \ \& \ \chi_R(x_j\, x_k) = 1 \to \chi_R(x_i\, x_k) = 1$$

Crisp equivalance Relation

relation R on universe X is equivalance if it hold reflexivity, symmetry, transitivity

Crisp tolerance Relation

relation R on universe X is tolerance if it have only reflexivity & symmetry

universe X with cardinality n

R be a tolerance relation in X

Then R can → equivalance Relation

$$R_e = R \cdot R \cdot \ldots\ldots R$$

$$= R^{\leq (n-1)}$$

— or CTR

Example

consider universe $X = \{x_1, x_2, x_3, x_4, x_5\}$

& Relation R

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

χ values

- all diagonal elem $= 1$

  $\therefore$ R is reflexive

- all diagonally opposite elem are equal

  $\therefore$ R is symmetric

- $\chi(x_1, x_2) = 1$ $\qquad$ $\chi(x_1, x_5) \neq 1$

  $\chi(x_2, x_5) = 1$ $\qquad$ $\therefore$ R is NOT transitive

$\therefore$ R is a tolerance Relation //

Now,

R cardinality = 5 elem 5 $\xi l \sigma$.

$$R^1 = R \cdot R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

for R^1 check reflexitivity ✓

symmetry ✓

transitivity ✓

$\therefore R^1$ equivalance //

if R^1 not equivalance

calc $R^2 = R^1 \circ R$

or

calc $R^3 = R^2 \circ R$

upto R^4 because $n=5$ $\rightarrow eq^n$ CTR

for fuzzy

reflexivity

$$\mu_R(x_i, x_i) = 1$$

Symmetric

$$\mu_R(x_i, x_j) = 1 \rightarrow \mu_R(x_j, x_i) = 1$$

Transitive

$$\mu_R(x_i, x_j) = \lambda_1 \quad \& \quad \mu_R(x_j, x_k) = \lambda_2$$

then

$$\mu_R(x_i, x_k) = \lambda \quad \text{where } \lambda \geq \min(\lambda_1, \lambda_2)$$

fuzzy equivalance
fuzzy tolerance $\Bigg\}$ crisp වල def එක
තමයි.

Ex

$$R = \begin{bmatrix} 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ & & \vdots & & \end{bmatrix}_{5\times 5}$$