Classical sets

collection of distinct obj

has crisp values

ex/ set of marks of students mark above 75

$$A = \{ 76, 80, 90, 75, \ldots \}$$

member / element

classical set is defined in a way such that it has two groups;

1. members
2. non-members

no partial memberships exists

Let S is a given set, the membership fn use to define set S is given by

$$\chi(x) = \begin{cases} 1 & ; x \in S \\ 0 & ; x \notin S \end{cases}$$

Cardinality

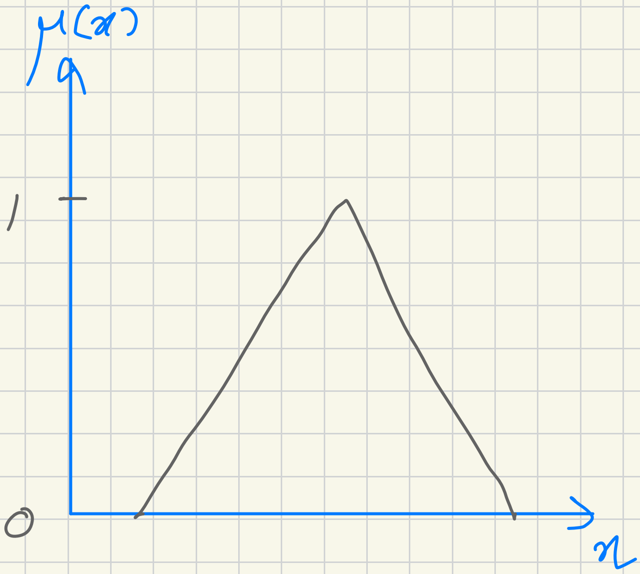
denoted by $|S|$: number of elements in the set S

Cardinal number

$B = \{a, b, c, d\}$ $\quad |B| = 4$ //

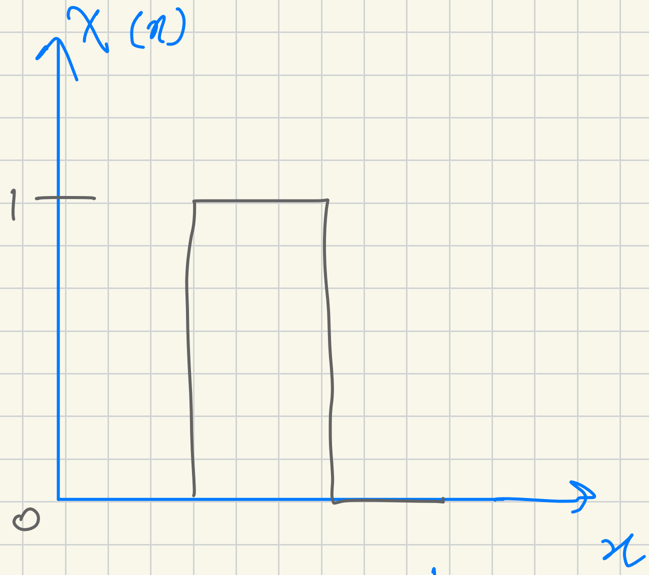
fuzzy sets

elements has a partial membership



fuzzy

for given x we can have any value btwn 1 or 0



classical

for given x (input) we only have either 1 or 0 values

Def

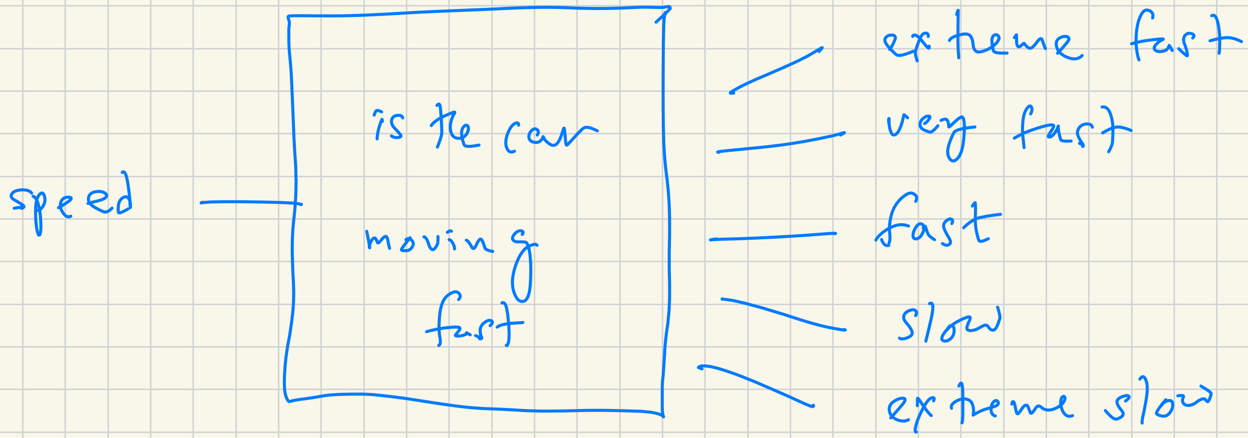
fuzzy set $\tilde{A}$ in the universe U can be defined as set of ordered pairs

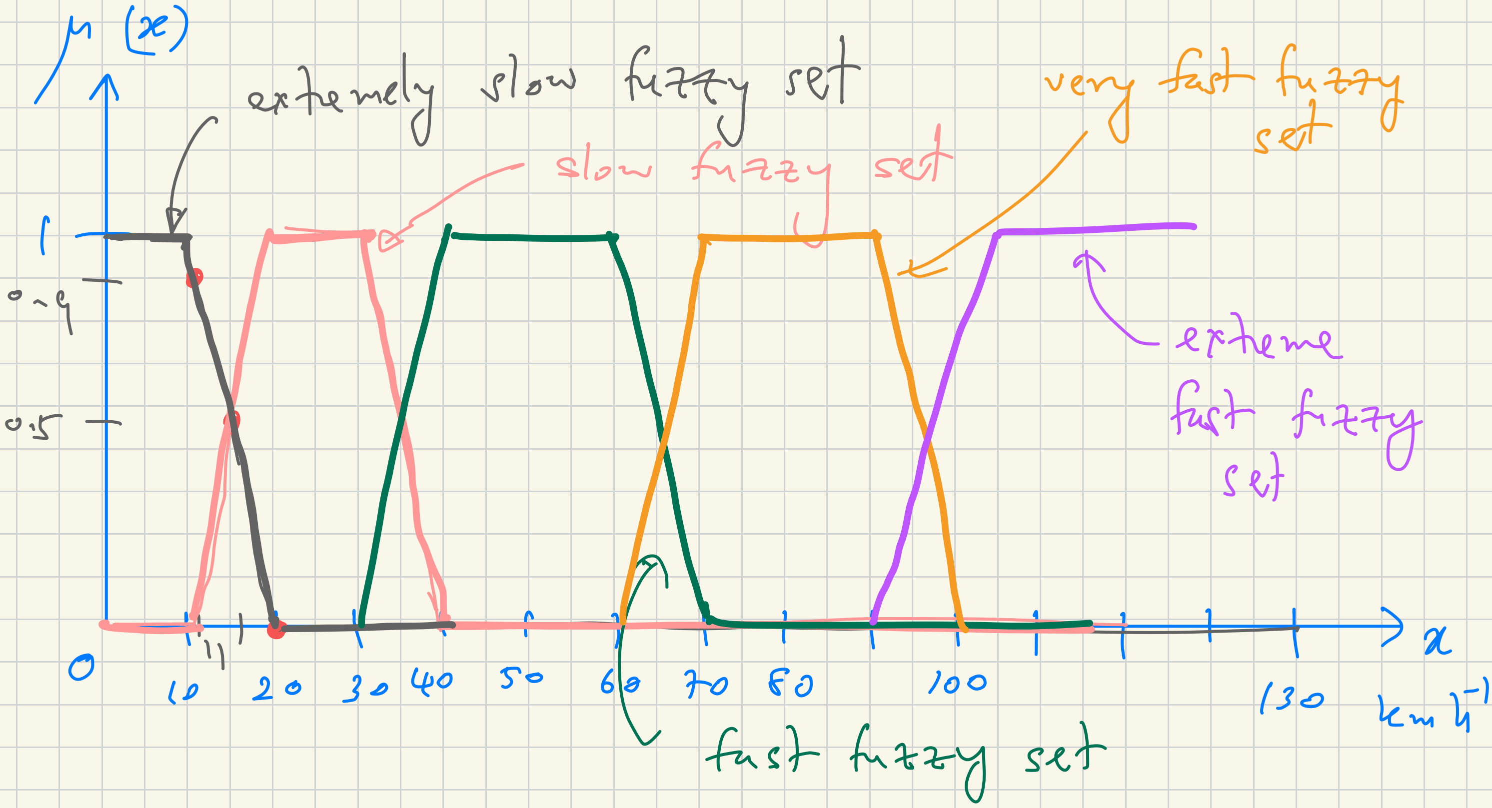
$$\tilde{A} = \{ y, \mu_A(y) \mid y \in U \}$$

(value) element

mem. f^n $\mu_A \in [0,1]$

EX





we all agree 0 – 10 km/h is extreme slow

now what about 11 km/h,

say 90% extremely slow

now what about 15 km/h,

say 50% extremely slow

what about 20 km/h,

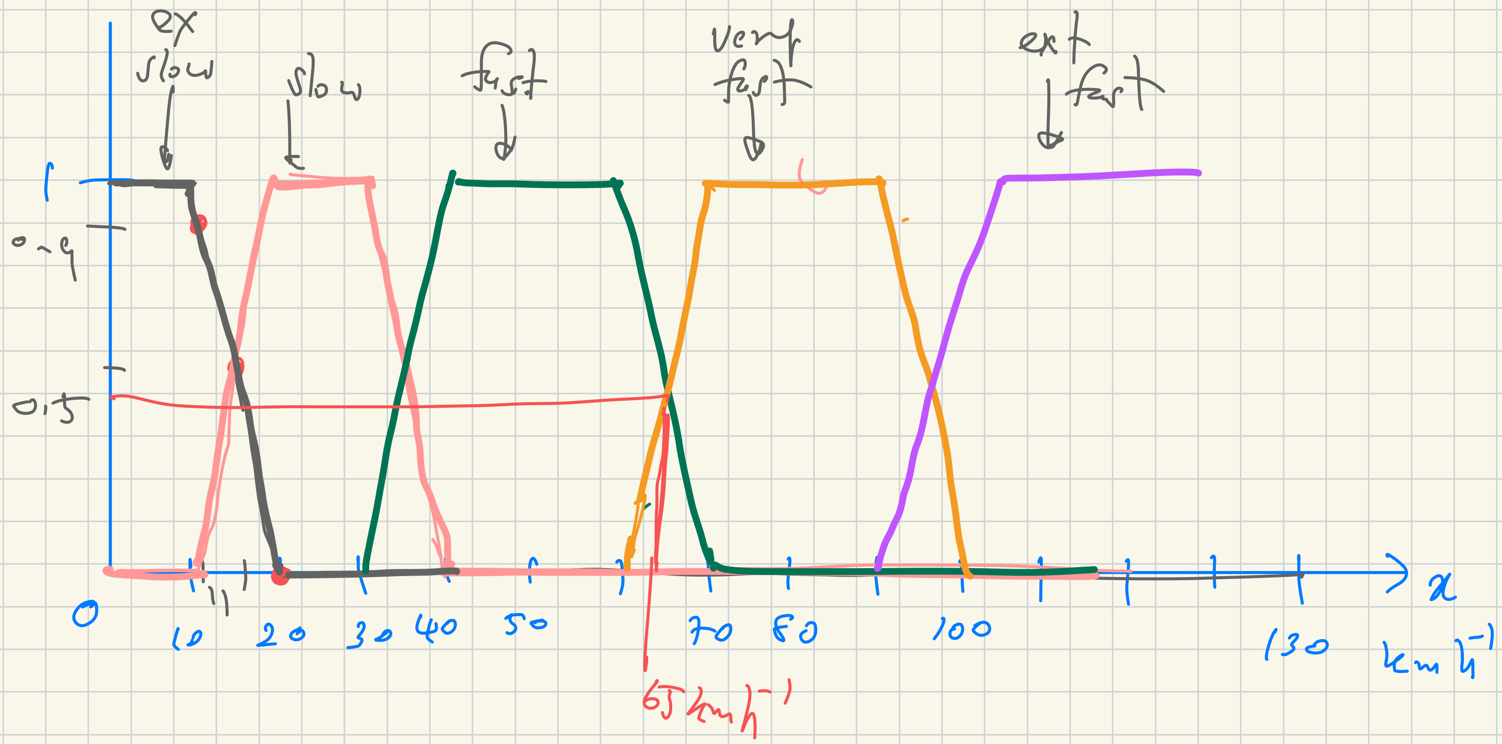
not a extremely slow at all 0%

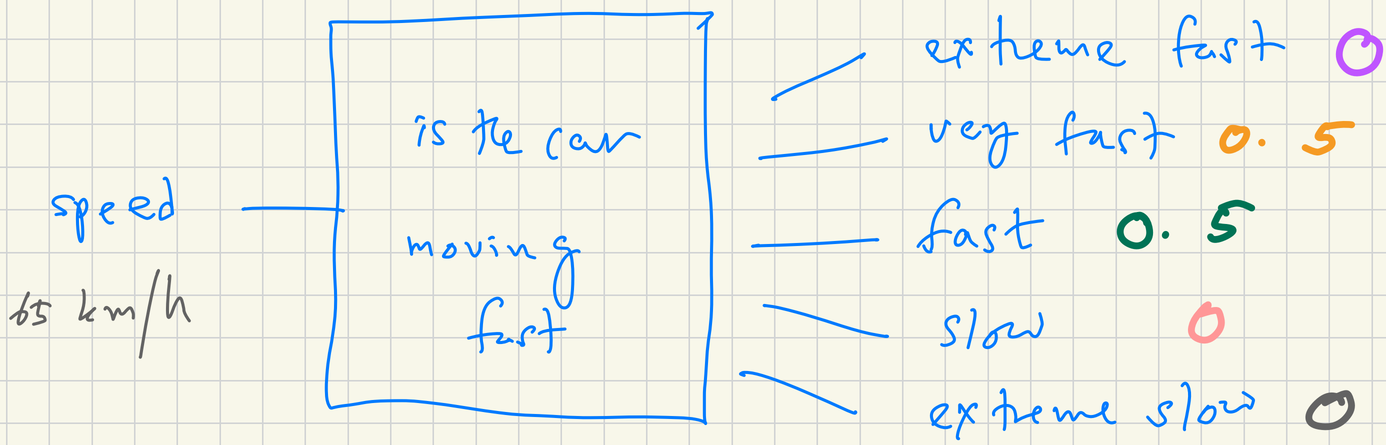
Let's select 20-30 km/h is slow.

then 15 km/h is 50% slow

32 km/h is 80% slow

say our speed input is 65 km/h





Representing fuzzy sets

Case 01: universe U is discrete & finite

$$\tilde{A} = \left\{ \frac{\mu_A(y_1)}{y_1} + \frac{\mu_A(y_2)}{y_2} + \cdots + \frac{\mu_A(y_n)}{y_n} \right\}$$

$$= \sum_{i=1}^{n} \frac{\mu_A(y_i)}{y_i}$$

not actual addition or divisions. just a notation only

$$\tilde{A} = \{ (y_1, \mu_A(y_1)), (y_2, \mu_A(y_2)), \ldots, (y_3, \mu_A(y_3)) \}$$

Ex

element

	y	1	2	3	4	km/h
$\tilde{A}$	$\mu_A(y)$	0.1	0.3	0.6	0.9	

fuzzy set

extreme slow [illegible]

$$\tilde{A} = \left\{ \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.6}{3} + \frac{0.9}{4} \right\}$$

$$\tilde{A} = \{ (1, 0.1), (2, 0.3), (3, 0.6), (4, 0.9) \}$$

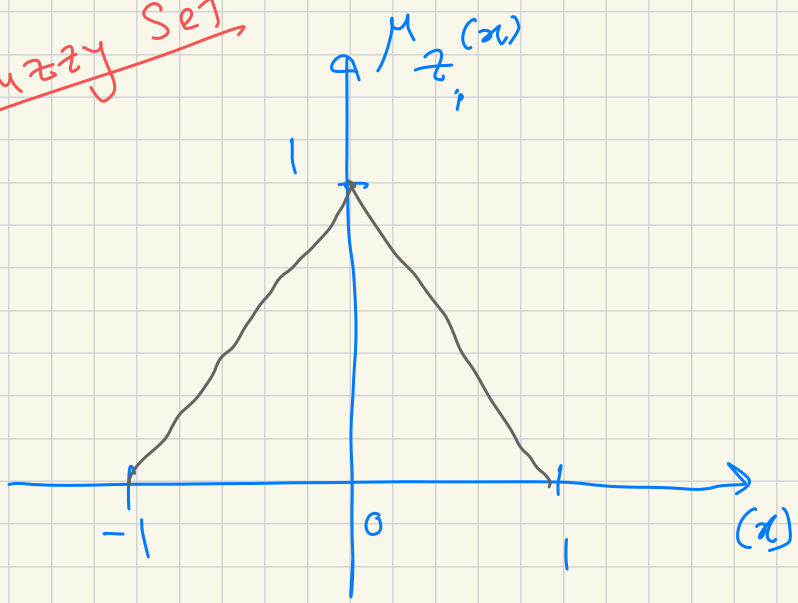
Case 02: universe u is continous & finite

$$\tilde{A} = \int_{i=1}^{n} \frac{\mu_A(y_i)}{y_i}$$

$\int$ & — are just notation no real math meaning

EX Let Z be a set named "Numbers close to zero". Define fuzzy & crisp sets for Z & show membership curve in a graph.

fuzzy set

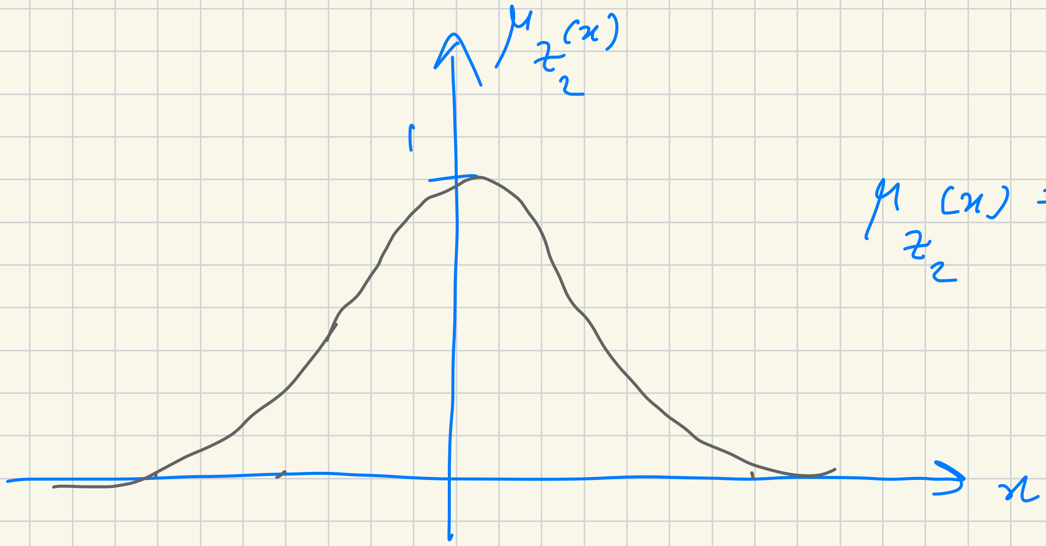


as numbers (x) close to zero

membership fn close to 1

$$\mu_{Z_1}(x) = \begin{cases} 0 & ; \; x < -1 \\ x+1 & ; \; -1 \leq x < 0 \\ x-1 & ; \; 0 \leq x < 1 \\ 0 & ; \; x > 1 \end{cases}$$

we can define something else, both are correct



$$\mu_{z_2}(x) = \left\{ e^{-x^2} ; x \in \mathbb{R} \right.$$

fuzzy sets are used to "defuzzify" the real world.

i.e. characterizing a fuzzy description with a membership fn

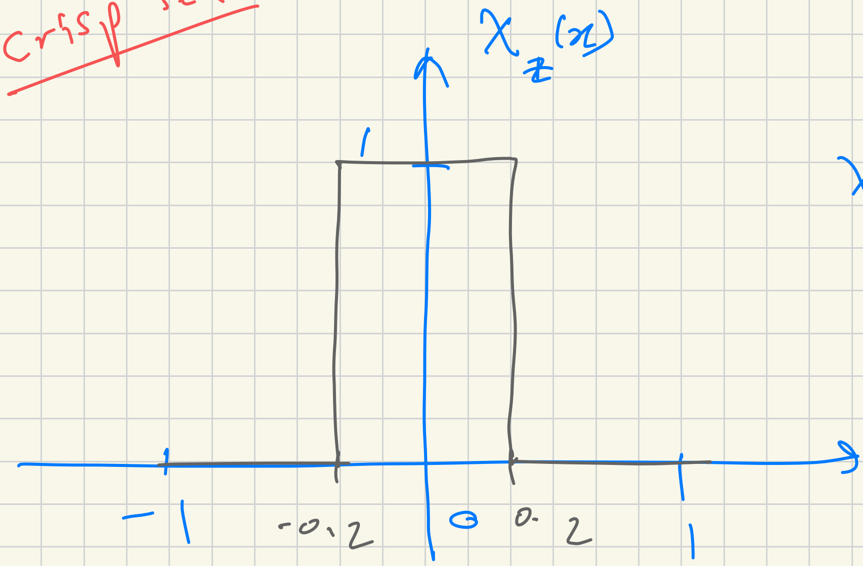
how to select a correct membership fn

different applications have diff mem. fns

use experts experiences
collected big datasets

no hard & fast rule.

crisp set



$$\chi_z(x) = \begin{cases} 1 & -0.2 \le x < 0.2 \\ 0 & \text{otherwise} \end{cases}$$

any number not within -0.2 — 0.2 is not closer to zero.

(not a common sense correct statement