(6) fuzzy relations & operations

- fuzzy relations
- operations on fuzzy relations
- Cartesian products

fuzzy relations

various degree of relationship from "completely related" to "not related" btwn two or more sets

fuzzy relation R;

mapping from cartesian space $X \times Y$ to the interval $[0, 1]$, where strength of mapping is expressed by mem. f^n $\mu_R(x, y)$ of the relation

$$R = \begin{array}{c|c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} 0.5 & 0.3 \\ 0.9 & 0.7 \end{bmatrix} \end{array}$$

중간 point 존재함.

원래 1 or 0 only

A is more related to D than C

Fuzzy relation operations

two fuzzy relations R & S defined in cartesian space X x Y

union

$R \cup S \rightarrow$

$$\mu_{R \cup S}(x,y) = \max\left[\mu_R(x,y), \mu_S(x,y)\right]$$

intersection

$R \cap S \rightarrow$

$$\mu_{R \cap S}(x,y) = \min\left[\mu_R(x,y), \mu_S(x,y)\right]$$

complement

$\bar{R} \rightarrow$

$$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$$

containment

$R \subset S \rightarrow$

$$\mu_R(x,y) \leq \mu_S(x,y)$$

Ex fuzzy relations R & S on $x \times y$

cartesian space

$$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.7 & 0.5 \\ x_2 & 0.8 & 0.4 \end{array} \qquad S = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.1 & 0.9 & 0.2 \\ x_2 & 0.6 & 0.7 & 0.5 \end{array}$$

$\downarrow$

same format

$$R = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & 0.7 & 0.5 & 0 \\ x_2 & 0.8 & 0.4 & 0 \end{array}$$

now

max $(0.7, 0.1)$

$$R \cup S = \begin{bmatrix} 0.7 & 0.9 & 0.2 \\ 0.8 & 0.7 & 0.5 \end{bmatrix}$$

max of each respect dim

$$R \cap S = \begin{bmatrix} 0.1 & 0.5 & 0 \\ 0.6 & 0.4 & 0 \end{bmatrix}$$

min $(0.8, 0.6)$

$$\bar{R} = \begin{bmatrix} 0.3 & 0.5 & 1 \\ 0.2 & 0.6 & 1 \end{bmatrix}$$

$1 - 0.7$

Fuzzy cartesian product

A is a fuzzy set in universe X &

B is a fuzzy set in universe Y

$A \times B \rightarrow$

$$\mu_{A \times B}(x, y) = \min\left[\mu_A(x), \mu_B(y)\right]$$

Ex

fuzzy set A : ambient temp for a heat exchanger in univese of 3 discrete temperatures

ape set ලට

$\rightarrow X = \{x_1, x_2, x_3\}$

fuzzy set B : optimum pressure for heat exchanger in universe of 2 discrete pressures

universe දට. $\rightarrow Y = \{y_1, y_2\}$

fuzzy set A :

more ambient temp than x_1 & x_2

$$A = \frac{0.3}{x_1} + \frac{0.5}{x_2} + \frac{0.9}{x_3}$$

10°C rod

fuzzy set B:

$$B = \frac{0.2}{y_1} + \frac{1}{y_2}$$

more optimum pressure than y_1

100 kPa rod

$$A \times B = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 \\ 0.2 & 0.2 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

$\min(0.3, 1)$

efficient condition

temp = x_3

pres = y_2 //

Note

$R : A \rightarrow B \subset A \times B$

(7) fuzzy composition

R is a fuzzy relation from universe X to universe Y & S' . . .

$$R : X \rightarrow Y$$

$$S : Y \rightarrow Z$$

then

$$R \circ S' : X \rightarrow Z$$

① Max - Min composition

$$\mu_{R \cdot S}(x, z) = \max \{ \min [\mu_R(x, y), \mu_S(y, z)] \}$$

for $\forall y \in Y$

$$\mu_{R \cdot S}(x, z) = \bigvee_{y \in Y} [\mu_R(x, y) \wedge \mu_S(y, z)]$$

② Max - product composition

$$\mu_{R\cdot S}(x, z) = \max\left[\mu_R(x, y) \cdot \mu_S(y, z)\right]$$

$$\forall\, y \in Y$$

$$\mu_{R\cdot S}(x, z) = \bigvee_{y \in Y}\left[\mu_R(x, y) \cdot \mu_S(y, z)\right]$$

Ex

Consider 3 universes,

$X = \{x_1, x_2\}$ $\quad Y = \{y_1, y_2\}$ $\quad Z = \{z_1, z_2\}$

Then the relations R on $X \times Y$ and S on $Y \times Z$ are in the

$$R = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.1 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ y_1 \\ y_2 \end{array}\begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 \\ 0.1 & 0.7 \end{bmatrix} \end{array}$$

find out $R \cdot S$ max – min composition

x_1 row — z_1 column

$$R \cdot S = \begin{array}{c|cc} & z_1 & z_2 \\ \hline x_1 & 0.1 & 0.5 \\ x_2 & 0.8 & 0.6 \end{array}$$

$$\max \left[\min(0.1, 0.9), \min(0.5, 0.1) \right]$$

$$\max [0.1, 0.1]$$

$$0.1$$

find $R \cdot S$ max – product composition

$$R \cdot S = \begin{array}{c|cc} & z_1 & z_2 \\ \hline x_1 & \boxed{0.09} & 0.35 \\ x_2 & 0.72 & 0.48 \end{array}$$

(x_1, z_1)

$$\max \left[(0.1 \times 0.9), (0.5 \times 0.1) \right]$$

$$\max [0.09, 0.05]$$

$$0.09$$

∴ R ගේ x_1 row එක

S ගේ z_1 column එක ගුණ

fuzzy composition Rules

$$R \cdot S \neq S \cdot R$$

complete relation

$$E = \begin{bmatrix} 1 & & & 1 \\ 1 & & & 1 \\ & \ddots & & \\ & & \ddots & \\ 1 & & & 1 \end{bmatrix}$$

null relation

$$O = \begin{bmatrix} 0 & & & \\ 0 & & & \\ & \ddots & & 0 \\ & & & 0 \end{bmatrix}$$