

Problem ①

a) $x(t) = 3\cos(10t + \frac{\pi}{4}) - \sin(4t - \frac{\pi}{4})$

$\downarrow T$ (Period $T_1 = \frac{2\pi}{10} = \frac{\pi}{5}$) (Period $T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$)

$T = \text{LCM}(T_1, T_2)$

$$= \text{LCM}\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \pi$$

Period of $x(t) = \pi$

b) $x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$

\downarrow Period N \downarrow Period N_1 \downarrow Period N_2

$$e^{j\frac{4\pi n}{7}} = e^{j\frac{4\pi}{7}(n+N)}$$

$$\therefore 2\pi m = N_1 \frac{4\pi}{7} \quad \text{where } m \in \mathbb{Z}$$

$$\therefore N_1 = \frac{7m}{2}$$

for smallest N_1 ; $m=2$

$$\therefore N_1 = 7$$

$$e^{j\frac{2\pi n}{5}} = e^{j\frac{2\pi}{5}(n+N_2)}$$

$$\therefore 2\pi m' = N_2 \cdot \frac{2\pi}{5} \quad \text{where } m' \in \mathbb{Z}$$

for smallest N_2 ,
 $m'=1 \quad \therefore N_2 = 5$

$$\therefore N = \text{LCM}(N_1, N_2)$$

$$= \text{LCM}(7, 5)$$

$$N = 35$$

period of $x[n] = 35$ time units

Problem ②

a) $x(t) = 2 \cos\left(4t + \frac{\pi}{3}\right)$

for periodicity; if period T :

$$x(t) = x(t+T)$$

$$\begin{aligned} 2 \cos\left(4t + \frac{\pi}{3}\right) &= 2 \cos\left(4(t+T) + \frac{\pi}{3}\right) \\ &= 2 \cos\left(4t + \frac{\pi}{3} + 4T\right) \end{aligned}$$

$\underbrace{4T}_{2\pi m}$

$$4T = 2\pi m$$

for period $m=1$

periodic with $T = \frac{\pi}{2}$ time units

b) $x(t) = \sin^2\left[2t - \frac{\pi}{4}\right]$

$$= 1 - \cos\left[4t - \frac{\pi}{2}\right]$$

$$= \frac{1}{2} - \frac{1}{2} \cos\left[4t - \frac{\pi}{2}\right]$$

$$= \frac{1}{2} - \frac{1}{2} \sin(4t)$$

periodic with period of $T = \frac{2\pi}{4} = \pi/2$

$$c) x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$$

if periodic

$$x[n+N] = \sin\left(\frac{6\pi(n+N)}{7} + 1\right)$$

$$= \sin\left(\frac{6\pi n}{7} + 1 + \frac{6\pi N}{7}\right)$$

for periodicity

$$2\pi m = \frac{6\pi N}{7}; \quad m \in \mathbb{Z}$$

$$N = \frac{7m}{6}$$

for N to be smallest integer; $m \geq 0$

$$N = 7$$

∴ Periodic with period of 7

$$(d) x[n] = \cos\left(\frac{\pi}{8}n^2\right)$$

for periodic; $x[n] = x[n+N]$

$$\cos\left(\frac{\pi}{8}n^2\right) = \cos\left(\frac{\pi}{8}(n+N)^2\right)$$

$$= \cos\left(\frac{\pi}{8}n^2 + \underbrace{\frac{\pi}{4}Nn + \frac{\pi}{8}N^2}_{2\pi m}\right)$$

for periodicity;

$$2\pi m = \frac{\pi}{4}Nn + \frac{\pi}{8}N^2$$

$$m = n \cdot \left(\frac{N}{8} \right) + \frac{N^2}{16}$$

for this to be true $\forall n \in \mathbb{Z}$,

N should be divisible by 8;

\therefore smallest possible $N = 8$;

\therefore periodic with period $N = 8$

(e) $x(t) = \sin\left(\frac{\pi}{8}t^2\right)$

for periodicity with period T ,

$$x(t) = x(t+T)$$

$$= \sin\left(\frac{\pi}{8}(t+T)^2\right)$$

$$= \sin\left(\frac{\pi}{8}t + \frac{\pi}{4}T + \frac{\pi}{8}T^2\right)$$

$$\therefore \frac{\pi}{4} + T + \frac{\pi}{8} T^2 = 2\pi m \text{ for some } m \in \mathbb{Z}$$

$$m = \left(\frac{\pi}{8}\right) + \left(\frac{T^2}{8}\right)$$

with T being continuous there isn't a T which yield for m to be non zero integer for all T ;

\therefore signal is not periodic

(Q2)

$$\begin{aligned} x[n] &= \cos\left(\frac{\pi}{2}n\right) \cos\left(\frac{\pi}{4}n\right) \\ &= \frac{1}{2} \left[\cos \frac{3\pi n}{4} + \cos \frac{\pi n}{4} \right] \end{aligned}$$

(identity; $\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$)

$$x[n] = \frac{1}{2} \cos \frac{3\pi n}{4} + \frac{1}{2} \cos \frac{\pi n}{4}$$

\downarrow \downarrow \downarrow

N_1 N_1 N_2

$$\frac{3\pi}{4} N_1 = 2\pi m \quad m \in \mathbb{Z}$$

$$N_1 = \frac{8}{3} m$$

$$\text{for } m = 3$$

$$N_1 = 8$$

$$N = \text{lcm}(N_1, N_2) = \text{lcm}(8, 8) = 8$$

periodic with period of 8

$$\frac{\pi}{4} N_2 = 2\pi m' \quad m' \in \mathbb{Z}$$

$$N_2 = 8 m'$$

$$\text{for } m' = 1$$

$$N_2 = 8$$

Problem ③

a) $x[n] = \begin{cases} \cos(\pi n) & n > 0 \\ 0 & \text{otherwise.} \end{cases}$

$$E_\infty = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x[n]|$$

$$= \lim_{N \rightarrow \infty} \left[0 + \sum_{n=0}^N \cos^2 \pi n \right]$$

$$= \lim_{N \rightarrow \infty} \left[\frac{1}{2} \sum_{n=0}^N (1 + \underbrace{\cos 2\pi n}_0) \right]$$

$$= \lim_{N \rightarrow \infty} \frac{N+1}{2} \Rightarrow \infty //$$

$$P_\infty = \lim_{N \rightarrow \infty} \left[\frac{1}{2N+1} \left(\sum_{n=-N}^N |x[n]| \right) \right]$$

$$= \lim_{N \rightarrow \infty} \left(\frac{1}{2N+1} \right) \left(\frac{N+1}{2} \right)$$

$$= \lim_{N \rightarrow \infty} \frac{N+1}{4N+2}$$

$$= \frac{1}{4} //$$

$\therefore x[n]$ is a power signal

$$(b) x(t) = \begin{cases} \frac{1}{2} \cos(\omega t + 1) & -\frac{\pi}{\omega} \leq t \leq \omega \\ 0 & \text{otherwise} \end{cases}$$

$$E_\infty = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)| dt$$

$$= \lim_{T \rightarrow \infty} \left[0 + \int_{-\pi/\omega}^{\pi/\omega} \frac{\cos^2(\omega t + 1)}{2} dt \right]$$

$$= \frac{1}{8} \lim_{T \rightarrow \infty} \left[\int_{-\pi/\omega}^{\pi/\omega} (1 + \cos(2\omega t + 2)) dt \right]$$

$$= \frac{1}{8} \left[\lim_{T \rightarrow \infty} t \Big|_{-\pi/\omega}^{\pi/\omega} + \frac{\sin(2\omega t + 2)}{2\omega} \Big|_{-\pi/\omega}^{\pi/\omega} \right]$$

$$= \frac{1}{8} \lim_{T \rightarrow \infty} \left[\frac{2\pi}{\omega} + \frac{\sin(2\pi + 2) - \sin(-2\pi - 2)}{2\omega} \right]$$

$$= \frac{\pi}{4\omega}$$

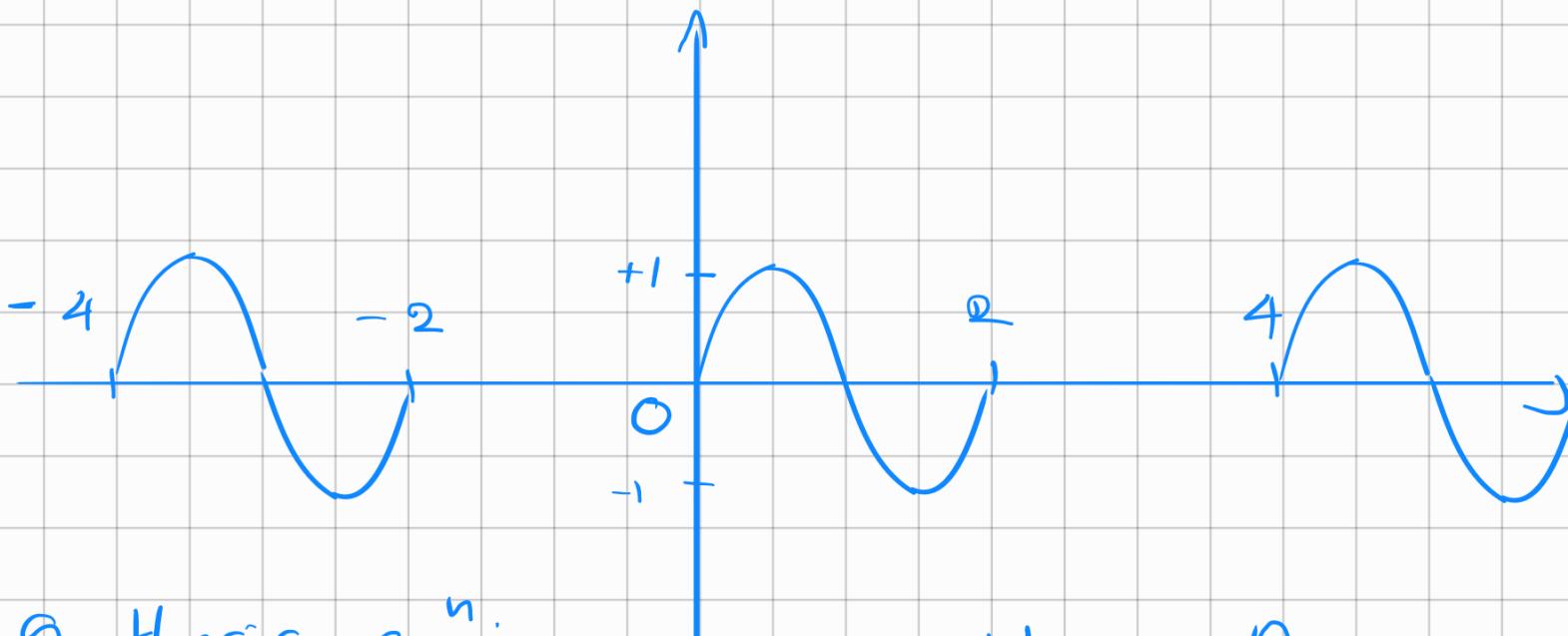
$$P_\infty = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \cdot \frac{\pi}{4\omega} = 0$$

$\therefore x(t)$ is an energy signal //

Problem 4

$$x(t) = \begin{cases} \sin \pi t & 0 < t < 2 \\ 0 & 2 \leq t \leq 4 \end{cases} \quad T = 4$$



Synthesis eqⁿ:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_c t}$$

Analytic eqⁿ

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_c t} dt$$

$$\text{dc comp;} a_0 = \frac{1}{T} \int_T x(t) dt$$

$$= \frac{1}{4} \left[\int_0^2 \sin(\pi t) dt + 0 \right]$$

$\underbrace{}_0$

$$= 0$$

$$\text{Original real;} a_k = \overline{a_{-k}}$$

for $k \neq 0$

$$\begin{aligned} a_k &= \frac{1}{4} \left[\int_0^2 \sin(\pi t) e^{-jk\frac{\pi}{2}t} dt + 0 \right] \\ &= \frac{1}{4} \left[\int_0^2 \sin(\pi t) \left(\cos \frac{jk\pi}{2}t - j \sin \frac{jk\pi}{2}t \right) dt \right] \\ &= \frac{1}{8} \left[\left(\sin \left[\pi + \frac{\pi}{2}k \right] t + \sin \left[\pi - \frac{\pi}{2}k \right] t \right) \Big|_0^2 \right. \\ &\quad \left. + j_k \int_0^2 \left(\cos \left(\pi + \frac{\pi}{2}k \right)t - \cos \left(\pi - \frac{\pi}{2}k \right)t \right) dt \right] \\ &= \frac{1}{8} \left[\left. \frac{-\cos \left(\pi + \frac{\pi}{2}k \right)t}{\pi + \frac{\pi}{2}k} \right|_0^2 - \left. \frac{\cos \left(\pi - \frac{\pi}{2}k \right)t}{\pi - \frac{\pi}{2}k} \right|_0^2 \right] \\ &\quad + \frac{j}{8} \left[\left. \frac{\sin \left(\pi + \frac{\pi}{2}k \right)t}{\pi + \frac{\pi}{2}k} \right|_0^2 - \left. \frac{\sin \left(\pi - \frac{\pi}{2}k \right)t}{\pi - \frac{\pi}{2}k} \right|_0^2 \right] \\ &= \frac{1}{8} \left[\frac{1 - \cos \pi k}{\pi + \frac{\pi}{2}k} + \frac{1 - \cos \pi k}{\pi - \frac{\pi}{2}k} \right] + \frac{j}{8} \underbrace{\left[\frac{\sin \pi k}{\pi + \frac{\pi}{2}k} - \frac{\sin \pi k}{\pi - \frac{\pi}{2}k} \right]}_0 \end{aligned}$$

undefined for $k = \pm 2$

$$\therefore a_k = \frac{1}{4} \left[\frac{1 - \cos \pi k}{(2+1)\pi} + \frac{1 - \cos \pi k}{(2-1)\pi} \right]$$

k is even $a_k = 0$

when k is odd

$$a_k = \frac{1 - (-1)}{4\pi} \left\{ \frac{1}{k+2} + \frac{1}{k-2} \right\}$$
$$= \frac{2}{4\pi} \left\{ \frac{k-2+k+2}{k^2-4} \right\}$$
$$= \frac{\cancel{4}k}{\cancel{4}\pi(k^2-4)} = \frac{k}{\pi(k^2-4)}$$

for both odd and even;

$$a_k = \frac{k(1 - (-1)^k)}{2\pi(k^2 - 1)}$$

when $k = 2$;

$$a_2 = \frac{1}{4} \int_0^2 \sin \pi t e^{-j\pi/2 \cdot 2t} dt$$
$$= \frac{1}{4} \int_0^2 (\sin \pi t \cos \pi t - j \sin^2 \pi t) dt$$
$$= -j/4 \int_0^2 \sin^2 \pi t dt$$
$$= -j/4 \int_0^2 \frac{1 - \cos 2\pi t}{2} dt$$
$$= -j/4 [\frac{t}{2} - \frac{\sin 2\pi t}{4}]_0^2 = -j/4$$

when $k = -2$

$$a_{-2} = \frac{1}{4} \int_0^2 \sin t e^{-j\pi/2(-2)t} dt +$$
$$= \frac{1}{4} \int_0^2 (\sin t \cos \pi t + j \sin^2 \pi t) dt +$$
$$= j/4$$

$$\therefore a_k = \begin{cases} \frac{-j}{2k} & k = \pm 2 \\ \frac{k(1 - (-1)^k)}{2\pi(k^2 - 4)} & k \neq \pm 2 \end{cases}$$

Problem ⑨

$$T = \frac{1}{2} \quad \omega_0 = \frac{2\pi}{T} = 4\pi$$

$$(a) x(t) = \cos 4\pi t$$

$$\begin{aligned} &= \frac{e^{j \cdot 4\pi t}}{2} + \frac{e^{-j \cdot 4\pi t}}{2} \\ &= \frac{1}{2} e^{j \cdot 4\pi t} - \frac{1}{2} e^{-j \cdot 4\pi t} \end{aligned}$$

$$a_1 = \frac{1}{2} \quad a_{-1} = +\frac{1}{2}$$

$$\therefore a_k = \begin{cases} \frac{1}{2} & |k| = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(b) y(t) = \sin(4\pi t)$$

$$\begin{aligned} &= \frac{e^{j \cdot 4\pi t} - e^{-j \cdot 4\pi t}}{2j} \\ &= \frac{1}{2j} e^{j \cdot 4\pi t} - \frac{1}{2j} e^{-j \cdot 4\pi t} \end{aligned}$$

$$b_k = \begin{cases} \frac{1}{2j} (k) & |k| = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$(c) z(t) = x(t) \cdot y(t) \xrightarrow{\text{FTS}} c_k = g_k + h_k$$

$$c_k = \left(\frac{1}{2} f[k-1] - \frac{1}{2} f[k+1] \right) + \left(\frac{1}{2} f[k+1] - \frac{1}{2} f[k-1] \right)$$

$$= \cancel{\frac{1}{4j} f[k]} + \frac{1}{2j} f[k-2] - \frac{1}{4j} f[k+2] + \cancel{\frac{1}{4j} f[k]}$$

$$= \frac{1}{4j} f[k-2] - \frac{1}{4j} f[k+2]$$

$$c_k = \begin{cases} \frac{1}{4j} & k = 2 \\ -\frac{1}{4j} & k = -2 \\ 0 & \text{otherwise} \end{cases}$$

$$(c) z(t) = x(t) \cdot y(t)$$

$$= \sin 4\pi t \cdot \cos 2\pi t$$

$$= \frac{1}{2} (\sin 8\pi t)$$

$$= \frac{1}{2} \left(\frac{e^{j8\pi t} - e^{-j8\pi t}}{2j} \right)$$

$$= \frac{1}{4j} e^{j8\pi t} - \frac{1}{4j} e^{-j8\pi t}$$

$$= \frac{1}{4j} e^{j2 \cdot 8\pi t} - \frac{1}{4j} e^{j(-2)8\pi t}$$

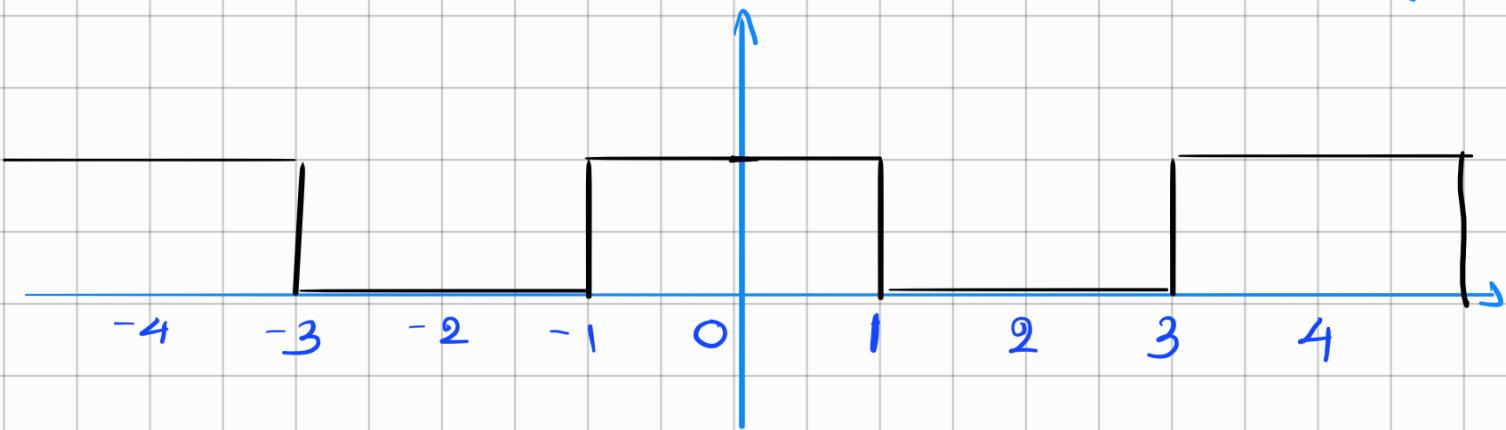
$$\therefore c_k = \begin{cases} \frac{1}{4j} & k = 2 \\ -\frac{1}{4j} & k = -2 \\ 0 & \text{otherwise} \end{cases}$$

Problem ⑥

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & |t| \geq 1 \end{cases}$$

$T = 2$

$\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$



dc component

$$a_0 = \frac{1}{T} \int x(t) dt = \frac{1}{4} (2 \times 1) = \frac{1}{2}$$

$$\begin{aligned}
 a_k &= \frac{1}{4} \int_{-2}^2 x(t) e^{-j k \pi / 2} dt \\
 &= \frac{1}{4} \left[\int_{-1}^1 e^{-j k \pi / 2} dt \right] \\
 &= \frac{1}{4} \left[\frac{e^{-j k \pi / 2}}{-j k \pi / 2} \right]_{-1}^1
 \end{aligned}$$

$$= \frac{1}{-jk\pi} \left[e^{-jk\pi/2t} - e^{jk\pi/2t} \right]$$

$$q_k = \frac{\sin k\pi/2}{k\pi}$$

$$q_k = \begin{cases} \frac{1}{2} \frac{\sin k\pi/2}{k\pi} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2} \text{sinc}\left(\frac{k}{2}\right) & k \neq 0 \\ 0 & k = 0 \end{cases}$$



(b)

```
clear all;
close all;
function a = FourierCoefficients(k)
    % computes the coefficients 'a' based on the values in vector 'k'.
    % For each element in 'k':
    %     - If the element is zero, the corresponding 'a' value is set to 1/2.
    %     - Otherwise, the value is calculated as sin(k*pi/2)/(k*pi).
    a = zeros(size(k));
    for idx = 1:length(k)
        if k(idx) == 0
            a(idx) = 1/2;
        else
            a(idx) = sin(k(idx) * pi/2) / (k(idx) * pi);
        end
    end
end

approximations = [1, 3, 7, 19, 43, 79];
maximum_approx = max(approximations);
k_values = -maximum_approx: maximum_approx;
ak = FourierCoefficients(k_values);

N = 400; %This will determine the number of points in the time domain
t = linspace(-2*5, 2*5, 4*N*5); %t stands for the time domain, 4*N*5 points in total
%Square Wave Signal
y_single = [zeros(1, N) ones(1, 2*N) zeros(1, N)];
y = [y_single y_single y_single y_single];

% plotFourier Plots the Fourier series approximation of a signal and prints the overshoot

% Inputs:
%   j           - Index for the current approximation order.
%   t           - Time vector for plotting.
%   approximations - Vector containing the number of terms for each approximation.
%   maximum_approx - Maximum order of approximation (used for indexing).
%   ak          - Vector of Fourier coefficients.
%   y           - Original signal to be approximated.
%
% The function creates a new figure, plots the original signal in white,
% overlays the Fourier approximation in green, and displays the
% approximation order in the title. It also prints the maximum and
% minimum values of the approximation to the command window.
function plotFourier (j, t, approximations, maximum_approx, ak, y)
    figure();
    plot(t, y, 'w');
    hold on;
    axis([-8 8 -0.2 1.2]);
    grid on;
    x = 0.*t; % Initialize x to zero for the current approximation
    for k = -approximations(j):approximations(j) % Loop through the range of k values for the
current approximation
        x = x + real(ak(k + maximum_approx + 1) * exp(1i * k * pi/2 .* t));
    end
```

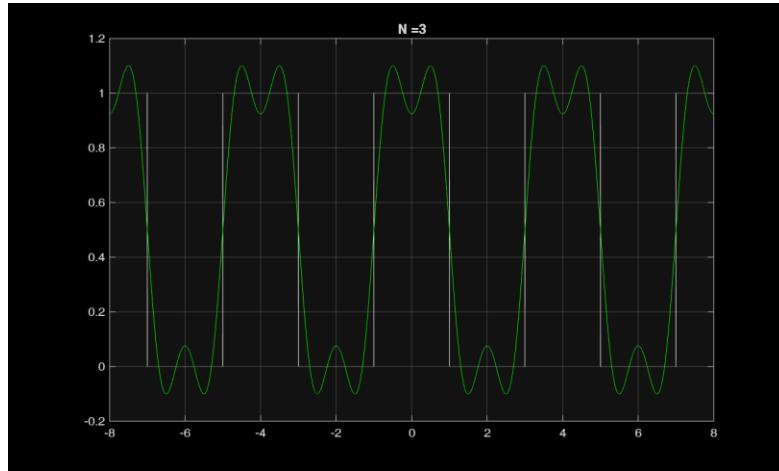
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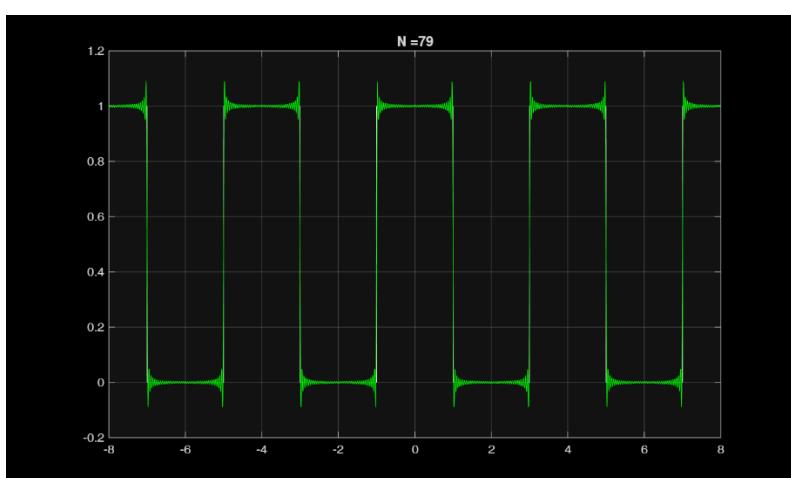
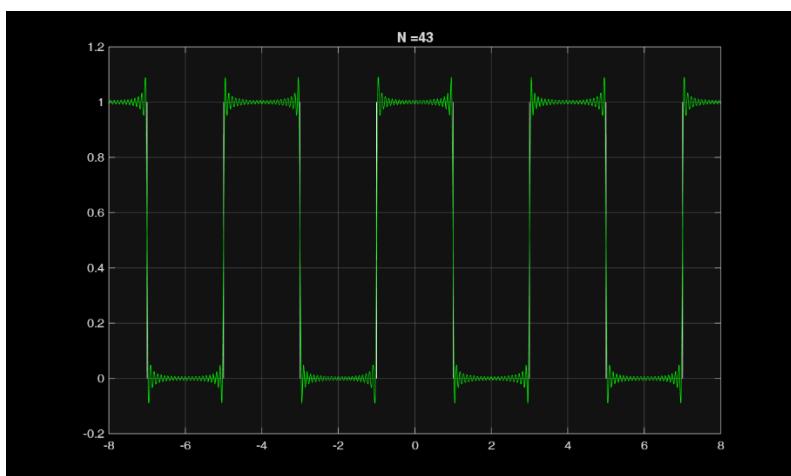
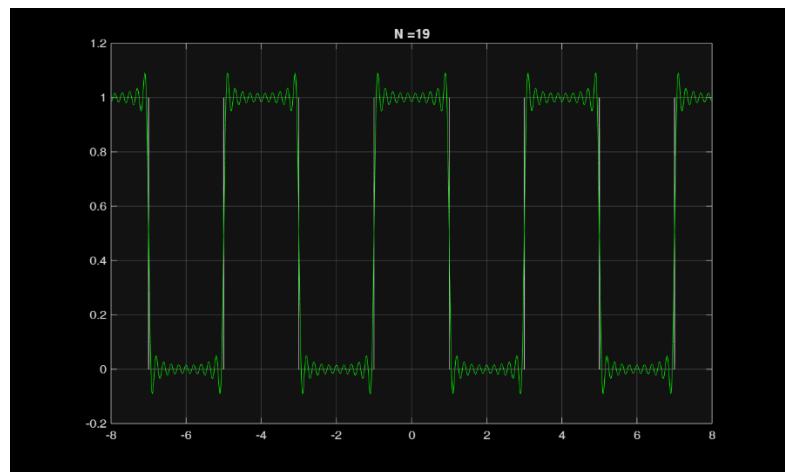
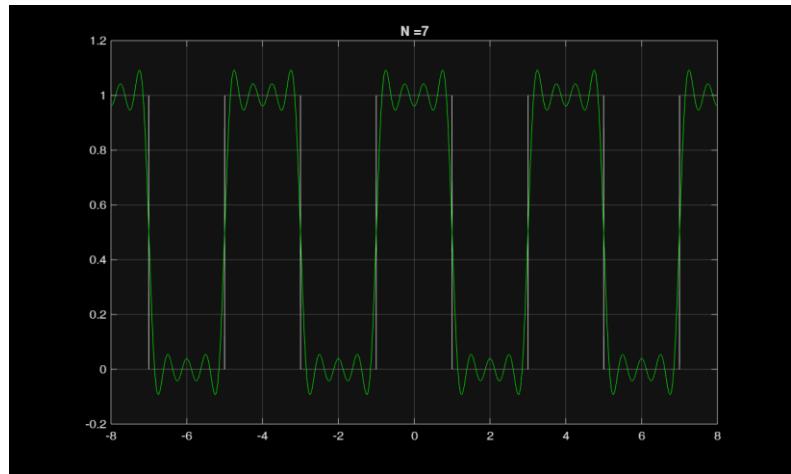
plot(t, x, 'g-');
title(['N =', num2str(approximations(j))]);
hold off;
fprintf("maximum value for %d: %f\n", approximations(j), max(x));
%Determine the percentage overshoot of the approximated square pulse for each of the above
partial sum approximations
overshoot = (max(x) - 1) / 1 * 100; % Calculate the percentage overshoot
fprintf("Percentage overshoot for N = %d: %.2f%%\n\n", approximations(j), overshoot);

end

%Creating Graphs for all approximations
for j = 1: length(approximations)
    plotFourier (j , t, approximations, maximum_approx, ak, y);
end

```





(c)

Formula used :

$$overshoot = \frac{\max(x) - 1}{1} \times 100\%$$

maximum value for 1: 1.136620

Percentage overshoot for N = 1: 13.66%

maximum value for 3: 1.100211

Percentage overshoot for N = 3: 10.02%

maximum value for 7: 1.092112

Percentage overshoot for N = 7: 9.21%

maximum value for 19: 1.089906

Percentage overshoot for N = 19: 8.99%

maximum value for 43: 1.089568

Percentage overshoot for N = 43: 8.96%

maximum value for 79: 1.089503

Percentage overshoot for N = 79: 8.95%

Problem ⑦

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) \xleftrightarrow{FT} ?$$

$$\mathcal{F}[x(-t)] = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$$

$$\text{let } -t = \tau; \quad dt = -d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau) e^{j\omega \tau} d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{(j\omega)(-\tau)} d\tau$$

$$= X(-j\omega)$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t-t_0) \xleftrightarrow{FT} ?$$

$$\mathcal{F}[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} dt$$

$$t-t_0 = \tau \quad dt = d\tau;$$

$$\therefore \mathcal{F}[x(t-t_0)] = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega(\tau+t_0)} d\tau$$
$$= X(j\omega) e^{-j\omega t_0}$$

$$\begin{aligned}
 a) x_1(t) &= x(1-t) + x(-1-t) \\
 &= x(-(t-1)) + x(-(t+1))
 \end{aligned}$$

$$x(t) \xrightleftharpoons{\text{FT}} X(j\omega)$$

$$x(1-t) = x(-(t-1))$$

$$x(t-1) \xrightleftharpoons{\text{FT}} X(j\omega) e^{-j\omega} \quad (\text{time shift})$$

$$x(-(t-1)) \xrightleftharpoons{\text{FT}} X(-j\omega) e^{+j\omega} \quad (\text{time reversal})$$

$$x(-(t+1)) \xrightleftharpoons{\text{FT}} X(-j\omega) e^{+j\omega} \quad (\text{time shift})$$

$$x_1(t) = x(t-1) - x(t+1) \xrightleftharpoons{\text{FT}} e^{-j\omega} X(-j\omega) + e^{+j\omega} X(j\omega)$$

$$\underline{x_1(t) \xrightleftharpoons{\text{FT}} e^{-j\omega} X(-j\omega) + e^{+j\omega} X(j\omega)}$$

$$\begin{aligned}
 b) x_2(t) &= x(3t-6) \\
 &= x(3(t-2))
 \end{aligned}$$

$$x(t) \xrightleftharpoons{\text{FT}} X(j\omega)$$

$$x(t-2) \xrightleftharpoons{\text{FT}} e^{-2j\omega} X(j\omega) \quad (\text{time shift})$$

$$x(3(t-2)) \xrightleftharpoons{\text{FT}} \frac{e^{-2j\omega}}{3} X\left(\frac{j\omega}{3}\right) \quad (\text{scale})$$

$$(C) x_3(t) = \frac{d^2}{dt^2} x(t-1)$$

$$x(t) \longleftrightarrow X(j\omega)$$

$$x(t-1) \longleftrightarrow e^{-j\omega} X(j\omega) \text{ (SLFT)}$$

$$x'(t-1) \longleftrightarrow j\omega e^{-j\omega} X(j\omega) \text{ (diff.)}$$

$$x''(t-1) \longleftrightarrow -\omega^2 e^{-j\omega} X(j\omega) \text{ (diff.)}$$

Problem ⑧

$$e^{-|t|} \longleftrightarrow \frac{2}{1+\omega^2}$$

(a) differentiating in frequency

$$x(t) \longleftrightarrow X(\omega)$$

$$-jt x(t) \longleftrightarrow \frac{dX(\omega)}{d\omega}$$

$$-jte^{-|t|} \longleftrightarrow \frac{2}{(1+\omega^2)^2} \left\{ 0 - 2\omega \right\}$$

$$\text{from linearity} \quad \frac{-4\omega}{(1+\omega^2)^2}$$

$$te^{-|t|} \longleftrightarrow \frac{4\omega}{j(1+\omega^2)^2}$$

$$(b) +e^{-|t|} \longleftrightarrow \frac{4\omega}{j(1+\omega^2)^2}$$

from duality

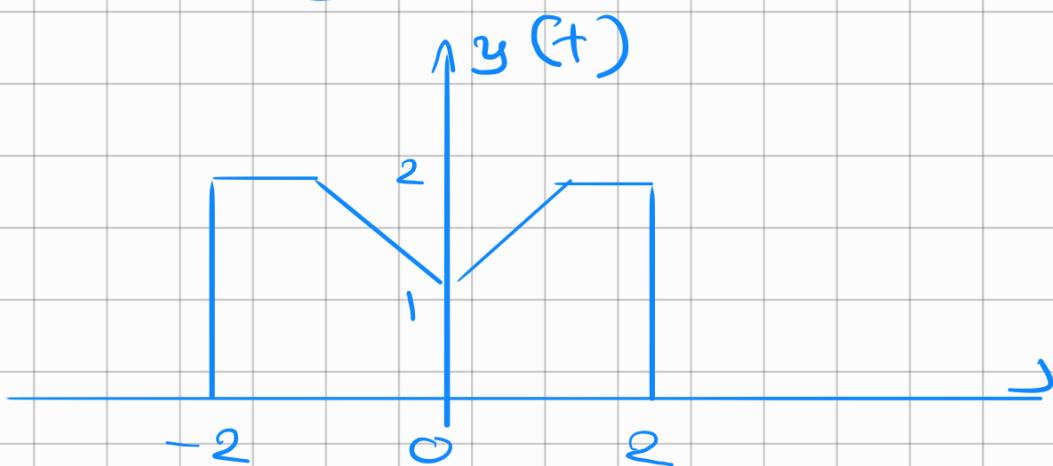
$$\frac{4t}{j(1+t^2)^2} \longleftrightarrow 2\pi(-\omega)e^{-|\omega|}$$

from linearity

$$\frac{4t}{(1+t^2)^2} \xrightarrow{\text{FT}} 2\pi j(-\omega)e^{-|\omega|}$$

Problem ⑨

$$(a) \text{ let } y(t) = x(t+1)$$



$y(t)$ is real & even;

$$\therefore y(t) \xrightarrow{\text{FT}} e^{j\omega \cdot 1} X(j\omega)$$

$\therefore Y(j\omega)$ is real & even

$$\therefore \angle Y(j\omega) = 0$$

$$\angle(e^{j\omega} X(j\omega)) = 0$$

$$\therefore \angle e^{j\omega} + \angle X(j\omega) = 0$$

$$\angle X(j\omega) = -\angle e^{j\omega}$$

$$\angle X(j\omega) = -\omega //$$

(b) $X(j_0) \equiv$ area under current

$$= (4 \times 2) - \frac{1}{2} \times 2 \times 1$$

$$= 7 //$$

(c) $\int_{-\infty}^{\infty} X(j\omega) d\omega = 0$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{-j\omega t} d\omega$$

at $t=0$ $2\pi x(0) = \int_{-\infty}^{\infty} X(j\omega) d\omega$

$$\therefore \int_{-\infty}^{\infty} X(j\omega) d\omega = 2\pi \times 1 - 2\pi //$$

$$(d) \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$x(t) = \begin{cases} 2 & -1 \leq t \leq 0 \text{ or } 2 \leq t \leq 3 \\ -t+2 & 0 \leq t \leq 1 \\ t & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \left[0 + \int_{-1}^0 2^2 dt + \int_0^3 (-t+2)^2 dt + \int_1^2 t^2 dt \right]$$

$$= \frac{76\pi}{3}$$

$$(e) \overset{\text{real}}{x(t)} = \overset{\text{Real}}{x_e(t)} + \overset{\text{Imaginary}}{x_{odd}(t)}$$

$$\mathcal{F}\{x(t)\} = \underbrace{\mathcal{F}\{x_e(t)\}}_{\text{Real}} + \underbrace{\mathcal{F}\{x_{odd}(t)\}}_{\text{Imaginary}}$$

$$\therefore x_e(t) \longleftrightarrow \operatorname{Re}\{X(j\omega)\}$$

$$x_o(t) \longleftrightarrow \operatorname{Im}\{X(j\omega)\}$$

$$\therefore x_e(t) = \frac{x(t) + x(-t)}{2}$$

