EE387 - Signal Processing E/20/420
Assignment 01 Wanasinghe J.K.

Problem ①

a) $x(t) = 3\cos(10t+1) - \sin(4t-1)$

$x(t) \rightarrow T$

$3\cos(10t+1)$: Period $T_1 = \frac{2\pi}{10}$

$\sin(4t-1)$: Period $T_2 = \frac{2\pi}{4}$

$$T = LCM(T_1, T_2)$$

$$= LCM\left(\frac{\pi}{5}, \frac{\pi}{2}\right) = \pi$$

Period of $x(t) = \pi$

b) $x[n] = 1 + e^{j\frac{4\pi n}{7}} - e^{j\frac{2\pi n}{5}}$

$x[n] \rightarrow$ Period N; $e^{j\frac{4\pi n}{7}} \rightarrow$ Period N_1; $e^{j\frac{2\pi n}{5}} \rightarrow$ Period N_2

$$e^{j\frac{4\pi n}{7}} = e^{j\frac{4\pi}{7}(n+N_1)}$$

$\therefore \cancel{2}\pi m = N_1 \frac{\overset{2}{\cancel{4}}\pi}{7}$ where $m \in \mathbb{Z}$

$\therefore N_1 = \frac{7m}{2}$

for smallest N_1; $m = 2$

$\therefore N_1 = 7$

$$e^{j\frac{2\pi n}{5}} = e^{j\frac{2\pi}{5}(n+N_2)}$$

$\therefore 2\pi m' = N_2 \cdot 2\pi/5$ where $m' \in \mathbb{Z}$

$N_2 = 5m'$

for smallest N_2;

$m' = 1 \quad \therefore N_2 = 5$

$$\therefore N = LCM(N_1, N_2)$$

$$= LCM(7, 5)$$

$$N = 35$$

period of $x[n]$ = 35 time units

Problem (2)

a) $x(t) = 2\cos\left(4t + \frac{\pi}{3}\right)$

for periodicity ; if period T ;

$$x(t) = x(t+T)$$

$$2\cos\left(4t + \frac{\pi}{3}\right) = 2\cos\left(4(t+T) + \frac{\pi}{3}\right)$$

$$= 2\cos\left(4t + \pi/3 + \underbrace{4T}_{2\pi m}\right)$$

$$4T = 2\pi m$$

for period $m = 1$

periodic with $T = \frac{\pi}{2}$ time units

---

b) $x(t) = \sin^2\left[2t - \frac{\pi}{4}\right]$

$$= \frac{1 - \cos\left[4t - \pi/2\right]}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos\left[4t - \frac{\pi}{2}\right]$$

$$= \frac{1}{2} - \frac{1}{2}\sin(4t)$$

periodic with period of $T = \frac{2\pi}{4} = \pi/2$

---

c) $x[n] = \sin\left(\frac{6\pi n}{7} + 1\right)$

if periodic

$$x[n+N] = \sin\left(\frac{6\pi}{7}(n+N) + 1\right)$$

$$= \sin\left(\frac{6\pi n}{7} + 1 + \frac{6\pi N}{7}\right)$$

for periodicity

$$\cancel{2}\pi m = \frac{\overset{3}{\cancel{6}}\pi N}{7} \quad ; \quad m \in \mathbb{Z}$$

$$N = \frac{7m}{3}$$

for N to be smallest integer; $m = 3$

$N = 7$

$\therefore$ <u>periodic with period of 7</u>

(d) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

for periodic; $x[n] = x[n+N]$

$$\cos\left(\frac{\pi}{8} n^2\right) = \cos\left(\frac{\pi}{8}(n+N)^2\right)$$

$$= \cos\left(\frac{\pi}{8} n^2 + \underbrace{\frac{\pi}{4} Nn + \frac{\pi}{8} N^2}_{2\pi m}\right)$$

for periodicity;

$$2\cancel{\pi} m = \frac{\cancel{\pi}}{4} N n + \frac{\cancel{\pi}}{8} N^2$$

$$m = n\left(\frac{N}{8}\right) + \frac{N^2}{16}$$

for this to be true $\forall\, n \in \mathbb{Z}$,

N should be divisible by 8,

$\therefore$ smallest possible $N = 8$;

$\therefore$ periodic with period $N = 8$

(e) $x(t) = \sin\left(\frac{\pi}{8} t^2\right)$

for periodicity with period T,

$$x(t) = x(t+T)$$

$$= \sin\left(\frac{\pi}{8}(t+T)^2\right)$$

$$= \sin\left(\frac{\pi}{8} t + \frac{\pi}{4} tT + \frac{\pi}{8} T^2\right)$$

$\therefore \frac{\pi}{4} tT + \frac{\pi}{8} T^2 = 2\pi m$ for some $m \in \mathbb{Z}$

$$m = \left(\frac{tT}{8}\right) + \left(\frac{T^2}{8}\right)$$

with t being continous there isn't a T which yield for m to be non zero integer for all t;

$\therefore$ signal is not periodic

(f)

$$x[n] = \cos\left(\frac{\pi}{2} n\right) \cos\left(\frac{\pi}{4} n\right)$$

$$= \frac{1}{2}\left[\cos \frac{3\pi n}{4} + \cos \frac{\pi}{4} n\right]$$

(identity; $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$\underset{\downarrow\, N}{x[n]} = \frac{1}{2}\underset{\downarrow\, N_1}{\cos \frac{3\pi n}{4}} + \frac{1}{2}\underset{\downarrow\, N_2}{\cos \frac{\pi n}{4}}$$

$\frac{3\pi}{4} N_1 = 2\pi m \quad m \in \mathbb{Z}$

$N_1 = \frac{8}{3} m$

for $m = 3$

$N_1 = 8$

$\frac{\pi}{4} N_2 = 2\pi m' \quad m \in \mathbb{Z}$

$N_2 = 8m'$

for $m' = 1$

$N_2 = 8$

$N = \text{lcm}(N_1, N_2) = \text{lcm}(8, 8) = 8$

periodic with period of 8

Problem 3

a) $$x[n] = \begin{cases} \cos(\pi n) & n \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$E_\infty = \lim_{N \to \infty} \sum_{-N}^{N} |x[n]|$$

$$= \lim_{N \to \infty} \left[0 + \sum_{n=0}^{N} \cos^2 \pi n \right]$$

$$= \lim_{N \to \infty} \left[\frac{1}{2} \sum_{n=0}^{N} \left(1 + \underbrace{\cos 2\pi n}_{0} \right) \right]$$

$$= \lim_{N \to \infty} \frac{N+1}{2} \Rightarrow \infty \; //$$

$$P_\infty = \lim_{N \to \infty} \left[\frac{1}{2N+1} \left(\sum_{-N}^{N} |x[n]| \right) \right]$$

$$= \lim_{N \to \infty} \left(\frac{1}{2N+1} \right) \left(\frac{N+1}{2} \right)$$

$$= \lim_{N \to \infty} \frac{N+1}{4N+2}$$

$$= \frac{1}{4} \; //$$

$\therefore$ <u>$x[n]$ is a power signal</u>

(b)

$$x(t) = \begin{cases} \frac{1}{2}\cos(\omega t + 1) & \frac{-\pi}{\omega} \leq t \leq \omega \\ 0 & \text{otherwise} \end{cases}$$

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)| \, dt$$

$$= \lim_{T \to \infty} \left[0 + \int_{-\pi/\omega}^{\pi/\omega} \left(\frac{\cos(\omega t + 1)}{2}\right)^2 dt \right]$$

$$= \frac{1}{8} \lim_{T \to \infty} \int \left[1 + \cos(2\omega t + 2) \right] dt$$

$$= \frac{1}{8} \left[\lim_{T \to \infty} t \Big|_{-\pi/\omega}^{\pi/\omega} + \frac{\sin(2\omega t + 2)}{2\omega} \Big|_{-\pi/\omega}^{\pi/\omega} \right]$$

$$= \frac{1}{8} \lim_{T \to \infty} \left[\frac{2\pi}{\omega} + \frac{\sin(2\pi + 2) - \sin(2 - 2\pi)}{2\omega} \right]$$

$$= \frac{\pi}{4\omega} \quad //$$

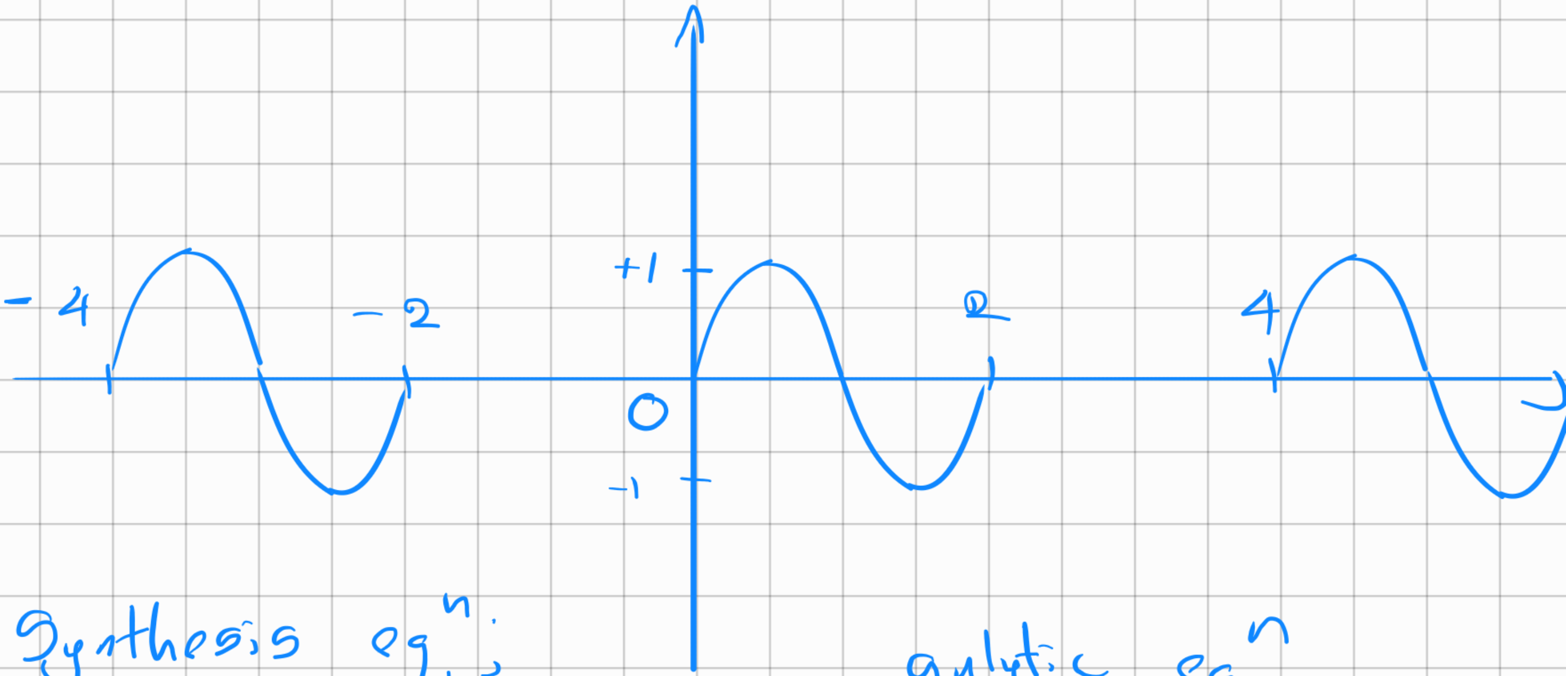
$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \cdot \frac{\pi}{4\omega} = 0$$

$\therefore x(t)$ is an energy signal //

Problem 4

$$x(t) = \begin{cases} \sin \pi t & 0 < t < 2 \\ 0 & 2 < t \leq 4 \end{cases} \qquad T = 4$$



Synthesis eq^n;

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

analytic eq^n

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

dc. comp; $a_0 = \frac{1}{T} \int_T x(t)\, dt$

$$= \frac{1}{4} \Big[\underbrace{\int_0^2 \sin(\pi t)\, dt}_{0} + 0 \Big]$$

$$= 0$$

Signal real; $a_k = a_{-k}^*$

for $k \neq 0$

$$a_k = \frac{1}{4}\left[\int_0^2 \sin(\pi t) e^{-jk\pi/2 t}\, dt + 0\right]$$

$$= \frac{1}{4}\left[\int_0^2 \sin \pi t \left(\cos \frac{k\pi t}{2} - j \sin \frac{k\pi t}{2}\right) dt\right]$$

$$= \frac{1}{8}\int_0^2 \left(\sin\left[\pi + \frac{\pi}{2}k\right]t + \sin\left[\pi - \frac{\pi}{2}k\right]t\right)dt +$$

$$j/8 \int_0^2 \left(\cos\left(\pi + \frac{\pi}{2}k\right)t - \cos\left(\pi - \frac{\pi}{2}k\right)t\right) dt$$

$$= \frac{1}{8}\left[\frac{-\cos\left(\pi + \frac{\pi}{2}k\right)t}{\pi + \frac{\pi}{2}k}\Bigg|_0^2 - \frac{\cos\left(\pi - \pi/2\, k\right)t}{\left(\pi - \frac{\pi}{2}k\right)}\Bigg|_0^2\right]$$

$$+ \frac{j}{8}\left[\frac{\sin\left(\pi + \frac{\pi}{2}k\right)t}{\pi + \pi/2\, k}\Bigg|_0^2 - \frac{\sin\left(\pi - \frac{\pi}{2}k\right)t}{\pi - \pi/2\, k}\Bigg|_0^2\right]$$

$$= \frac{1}{8}\left[\frac{1 - \cos \pi k}{\pi + \frac{\pi}{2}k} + \frac{1 - \cos \pi k}{\pi - \pi/2\, k}\right] + \frac{j}{8}\underbrace{\left[\frac{\sin \pi k}{\pi + \frac{\pi}{2}k} - \frac{\sin \pi k}{\pi - \frac{\pi}{2}k}\right]}_{0}$$

undefined for $k = \pm 2$

$$\therefore a_k = \frac{1}{4}\left[\frac{1 - \cos \pi k}{(2+k)\pi} + \frac{1 - \cos \pi k}{(2-k)\pi}\right]$$

k is even $\quad a_k = 0$

when k is odd

$$a_k = \frac{1-(-1)}{4\pi}\left\{\frac{1}{k+2} + \frac{1}{k-2}\right\}$$

$$= \frac{2}{4\pi}\left\{\frac{k\cancel{-2}+k\cancel{+2}}{k^2-4}\right\}$$

$$= \frac{\cancel{2}k}{\cancel{4}\pi(k^2-4)} = \frac{k}{\pi(k^2-4)}$$

for both odd and even;

$$a_k = \frac{k(1-(-1)^k)}{2\pi(k^2-4)}$$

when $k=2$;

$$a_2 = \frac{1}{4}\int_0^2 \sin\pi t\, e^{-j\pi/2 \cdot 2t}\, dt$$

$$= \frac{1}{4}\int_0^2 (\sin\pi t\cos\pi t - j\sin^2\pi t)\, dt$$

$$= -j/4\int_0^2 \sin^2\pi t\, dt$$

$$= -j/4\int_0^2 \frac{1-\cos 2\pi t}{2}\, dt$$

$$= -j/4\left[2/2\right] = -j/4$$

when $k = -2$

$$a_{-2} = \frac{1}{4}\int_0^2 \sin \pi t \, e^{-j\pi/2(-2)t} \, dt$$

$$= \frac{1}{4}\int_0^2 \left(\sin \pi t \cos \pi t + j \sin^2 \pi t\right) dt$$

$$= j/4$$

$$\therefore a_k = \begin{cases} \dfrac{-j}{2k} & k = \pm 2 \\[2ex] \dfrac{k\left(1-(-1)^k\right)}{2\pi\left(k^2-4\right)} & k \neq \pm 2 \end{cases}$$

Problem 9

$$T = 1/2 \qquad \omega_0 = \frac{2\pi}{T} = 4\pi$$

(a) $x(t) = \cos 4\pi t$

$$= \frac{e^{j \cdot 4\pi t} + e^{-j 4\pi t}}{2}$$

$$= \frac{1}{2} e^{j \cdot 4\pi \cdot t} - \frac{1}{2} e^{j \cdot -1 \cdot 4\pi t}$$

$$a_1 = 1/2 \qquad a_{-1} = 1/2$$

$$\therefore a_k = \begin{cases} 1/2 & |k| = 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) $y(t) = \sin(4\pi t)$

$$= \frac{e^{j4\pi t} - e^{-j 4\pi t}}{2j}$$

$$= \frac{1}{2j} e^{j4\pi t} - \frac{1}{2j} e^{-j4\pi t}$$

$$b_k = \begin{cases} \frac{1}{2j}(k) & |k| = 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) $z(t) = x(t) \cdot y(t) \xleftrightarrow{FS} c_k = a_k * b_k$

$$c_k = \left(\frac{1}{2}\delta[k-1] + \frac{1}{2}\delta[k+1]\right) * \left(\frac{1}{2j}\delta(k+1) + \frac{1}{2j}\delta(k-1)\right)$$

$$= \cancel{\frac{1}{4j}\delta[k]} + \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2] + \cancel{\frac{1}{4j}\delta[k]}$$

$$= \frac{1}{4j}\delta[k-2] - \frac{1}{4j}\delta[k+2]$$

$$c_k = \begin{cases} \frac{1}{4j} & k = 2 \\ -\frac{1}{4j} & k = -2 \\ 0 & \text{otherwise} \end{cases}$$

(c) $z(t) = x(t) \cdot y(t)$

$$= \sin 2\pi t \cdot \cos 2\pi t$$

$$= \frac{1}{2}(\sin 8\pi t)$$

$$= \frac{1}{2}\left(\frac{e^{j8\pi t} - e^{-j8\pi t}}{2j}\right)$$

$$= \frac{1}{4j}e^{j8\pi t} - \frac{1}{4j}e^{-j8\pi t}$$

$$= \frac{1}{4j}e^{j \cdot 2 \cdot 8\pi t} - \frac{1}{4j}e^{j(-2)8\pi t}$$

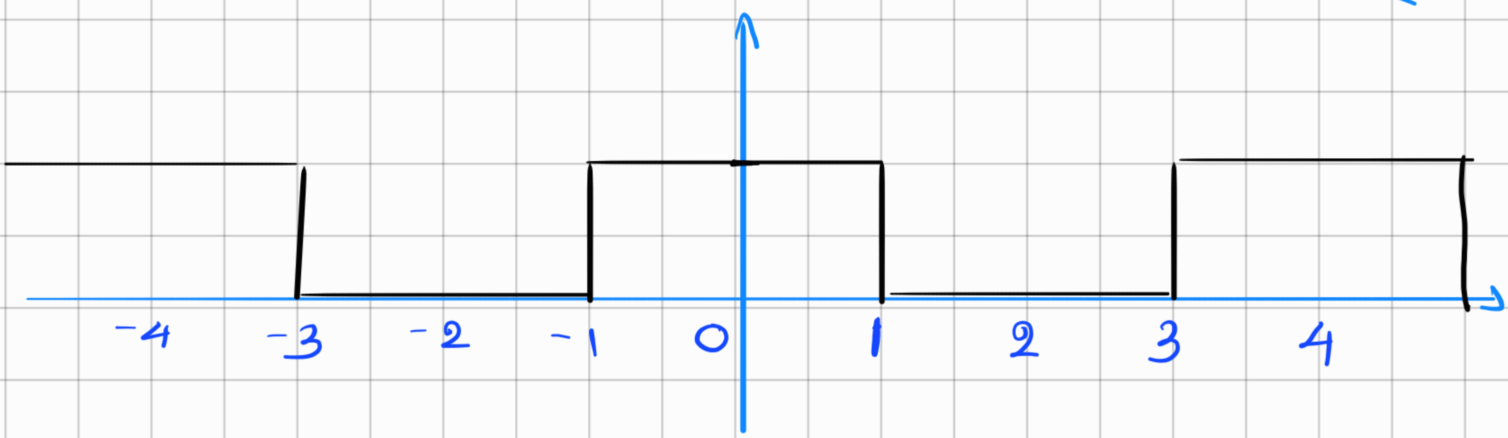
$$\therefore \; c_k = \begin{cases} \frac{1}{4j} & k = 2 \\ -\frac{1}{4j} & k = -2 \\ 0 & \text{otherwise} \end{cases}$$

Problem 6

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < 2 \end{cases}$$

$T = 4$

$\omega_0 = \frac{2\pi}{4} = \pi/2$



dc component

$$a_0 = \frac{1}{T}\int x(t)\,dt = \frac{1}{4}(2 \times 1) = \frac{1}{2}$$

$k \neq 0$

$$a_k = \frac{1}{4}\int_{-2}^{2} x(t)\, e^{-jk\pi/2\, t}\,dt$$

$$= \frac{1}{4}\left[\int_{-1}^{1} 1\, e^{-jk\pi/2\, t}\,dt\right]$$

$$= \frac{1}{4}\left[\frac{e^{-jk\pi/2\, t}}{-jk\pi/2}\right]_{-1}^{1}$$

$$= \frac{1}{-\cancel{2}jk\pi}\left[e^{-jk\pi/2\,t} - e^{jk\pi/2\,t} \right]$$

$$a_k = \frac{\sin k\pi/2}{k\pi}$$

$$a_k = \begin{cases} 2\,\dfrac{\sin k\pi/2}{\frac{k\pi}{2}} & k \neq 0 \\ 0 & k = 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2}\,\mathrm{sinc}\left(\frac{k}{2}\right) & k \neq 0 \\ 0 & k = 0 \end{cases}$$

Problem ⑦

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(-t) \xleftrightarrow{FT} ?$$

$$\mathcal{F}[x(-t)] = \int_{-\infty}^{\infty} x(-t) e^{-j\omega t} dt$$

let $-t = \tau$; $\therefore dt = -d\tau$

$$= \int_{-\infty}^{\infty} x(\tau) e^{j\omega\tau} d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) e^{(-j\omega)(-\tau)} d\tau$$

$$= X(-j\omega)$$

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t-t_0) \xleftrightarrow{FT} ?$$

$$\mathcal{F}[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} dt$$

$t - t_0 = \tau$ $dt = d\tau$;

$$\therefore \mathcal{F}(x(t-t_0)) = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega(\tau + t_0)} d\tau$$

$$= X(j\omega) e^{-j\omega t_0}$$

a) $x_1(t) = x(1-t) + x(-1-t)$

$= x(-(t-1)) + x(-(t+1))$

$x(t) \xleftrightarrow{FT} X(j\omega)$

$x(1-t) = x(-(t-1))$

$x(t-1) \xleftrightarrow{FT} X(j\omega)e^{-j\omega}$ (time shift)

$x(-(t-1)) \xleftrightarrow{FT} X(-j\omega)e^{-j\omega}$ (time reversal)

$x(-(t+1)) \xleftrightarrow{FT} X(-j\omega)e^{j\omega}$ (time shift)

$x_1(t) = x(-t+1) + x(-1-t) \xleftrightarrow{FT} e^{-j\omega}X(-j\omega) + e^{j\omega}X(-j\omega)$

$$\underline{\underline{x_1(t) \longleftrightarrow 2X(-j\omega)\cdot\cos(j\omega)}}$$

(b) $x_2(t) = x(3t-6)$

$= x(3(t-2))$

$x(t) \xleftrightarrow{F.T} X(j\omega)$

$x(t-2) \longleftrightarrow e^{-2j\omega}X(j\omega)$ (time shift)

$$\underline{\underline{x(3(t-2)) \longleftrightarrow \frac{e^{-2j\omega}}{3}X\left(\frac{j\omega}{3}\right)}}$$ (scaling)

(c) $x_3(t) = \frac{d^2}{dt^2} x(t-1)$

$$x(t) \longleftrightarrow X(j\omega)$$

$$x(t-1) \longleftrightarrow e^{-j\omega} X(j\omega) \quad (\text{shift})$$

$$x'(t-1) \longleftrightarrow j\omega e^{-j\omega} X(j\omega) \quad (\text{diff.})$$

$$x''(t-1) \longleftrightarrow -\omega^2 e^{-j\omega} X(j\omega) \quad (\text{diff.})$$

Problem (8)

$$e^{-|t|} \xleftrightarrow{F} \frac{2}{1+\omega^2}$$

(a) differentiating in frequency:

$$x(t) \longleftrightarrow X(\omega)$$

$$-jt\,x(t) \longleftrightarrow \frac{dX(\omega)}{d\omega}$$

$$-jt e^{-|t|} \longleftrightarrow \frac{2}{(1+\omega^2)^2}\{0 - 2\omega\}$$

$$-jt e^{-|t|} \longleftrightarrow \frac{-4\omega}{(1+\omega^2)^2}$$

from linearity

$$t e^{-|t|} \longleftrightarrow \frac{4\omega}{j(1+\omega^2)^2}$$

(b) $t e^{-|t|} \longleftrightarrow \dfrac{4\omega}{j(1+\omega^2)^2}$

from duality

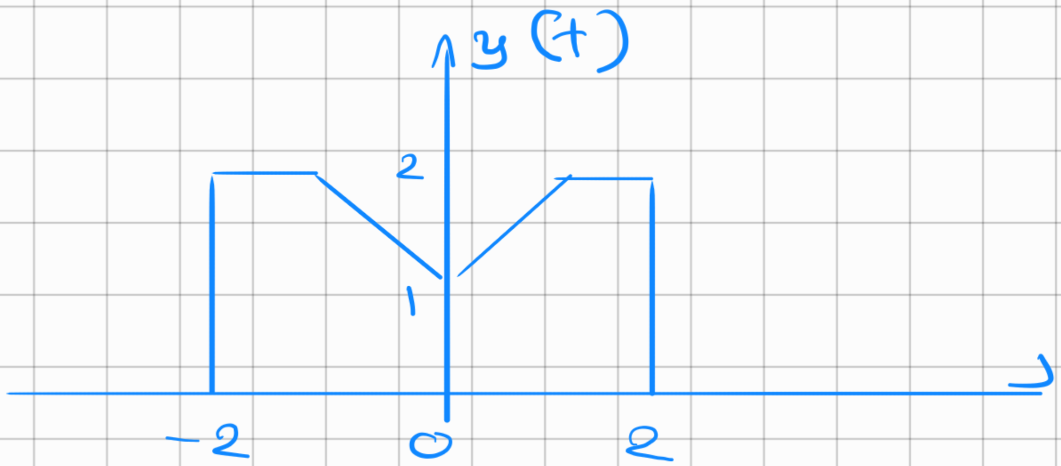
$$\frac{4t}{j(1+t^2)^2} \longleftrightarrow 2\pi(-\omega)e^{-|\omega|}$$

$\therefore$ from linearity

$$\frac{4t}{(1+t^2)^2} \xleftrightarrow{FT} 2\pi j(-\omega)e^{-|\omega|}$$

Problem 9

(a) let $y(t) = x(t+1)$



$y(t)$ is real & even;

$\therefore y(t) \xleftrightarrow{FT} e^{j\omega \cdot 1} X(j\omega)$

$\therefore Y(j\omega)$ is real & even

$\therefore \angle Y(j\omega) = 0$

$$\angle\left(e^{j\omega}X(j\omega)\right) = 0$$

$$\therefore \angle e^{j\omega} + \angle X(j\omega) = 0$$

$$\angle X(j\omega) = -\angle e^{j\omega}$$

$$\angle X(j\omega) = -\omega \;//$$

(b) $X(j0) \equiv$ area under curve

$$= (4 \times 2) - \frac{1}{2} \times 2 \times 1$$

$$= 7 \;//$$

(c) $$\int_{-\infty}^{\infty} X(j\omega)\, d\omega = ?$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{-j\omega t}\, d\omega$$

at $t = 0$ $$2\pi x(0) = \int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$$\therefore \int_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi \times 1 = 2\pi \;//$$

(d) $$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$x(t) = \begin{cases} 2 & -1 \leqslant t \leqslant 0 \text{ or } 2 \leqslant t \leqslant 3 \\ -t+2 & 0 \leqslant t \leqslant 1 \\ t & 1 \leqslant t \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \left[0 + \int_{-1}^{0} 2^2 dt + \int_{0}^{1} (-t+2)^2 dt + \int_{1}^{2} t^2 dt + \int_{2}^{3} 2^2 dt \right]$$

$$= \frac{76\pi}{3}$$

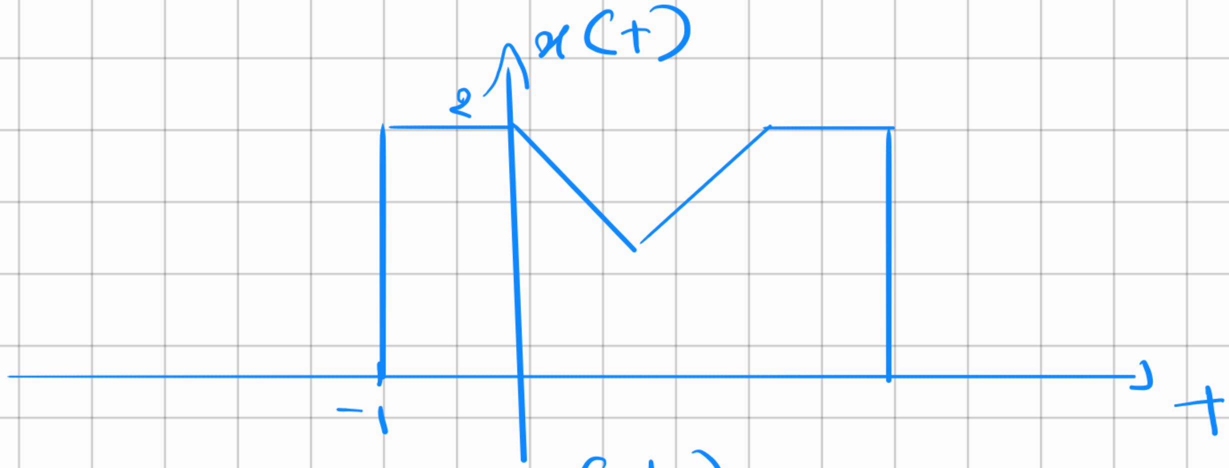
(e) $$\overset{\text{real}}{x(t)} = x_{even}(t) + x_{odd}(t)$$

$$\mathcal{F}\{x(t)\} = \underbrace{\mathcal{F}\{x_e(t)\}}_{\text{Real}} + \underbrace{\mathcal{F}\{x_{odd}(t)\}}_{\text{Imaginary}}$$

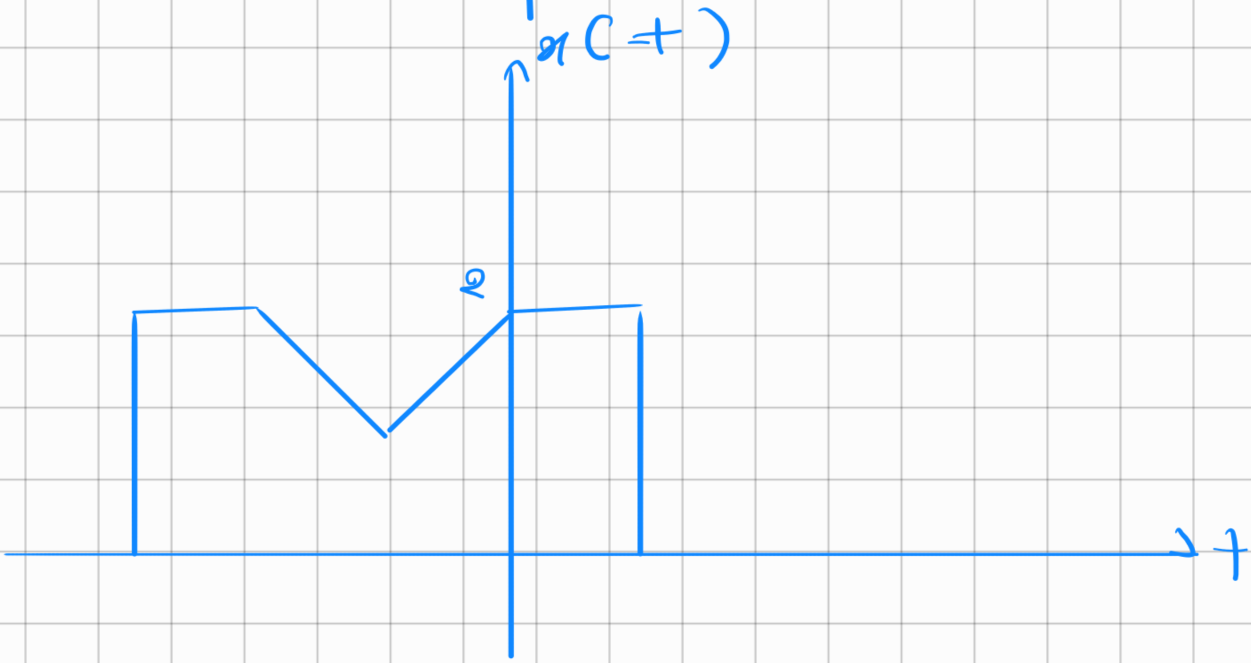
$$\therefore x_e(t) \longleftrightarrow Re\{X(j\omega)\}$$

$$x_o(t) \longleftrightarrow j\,Im\{X(j\omega)\}$$

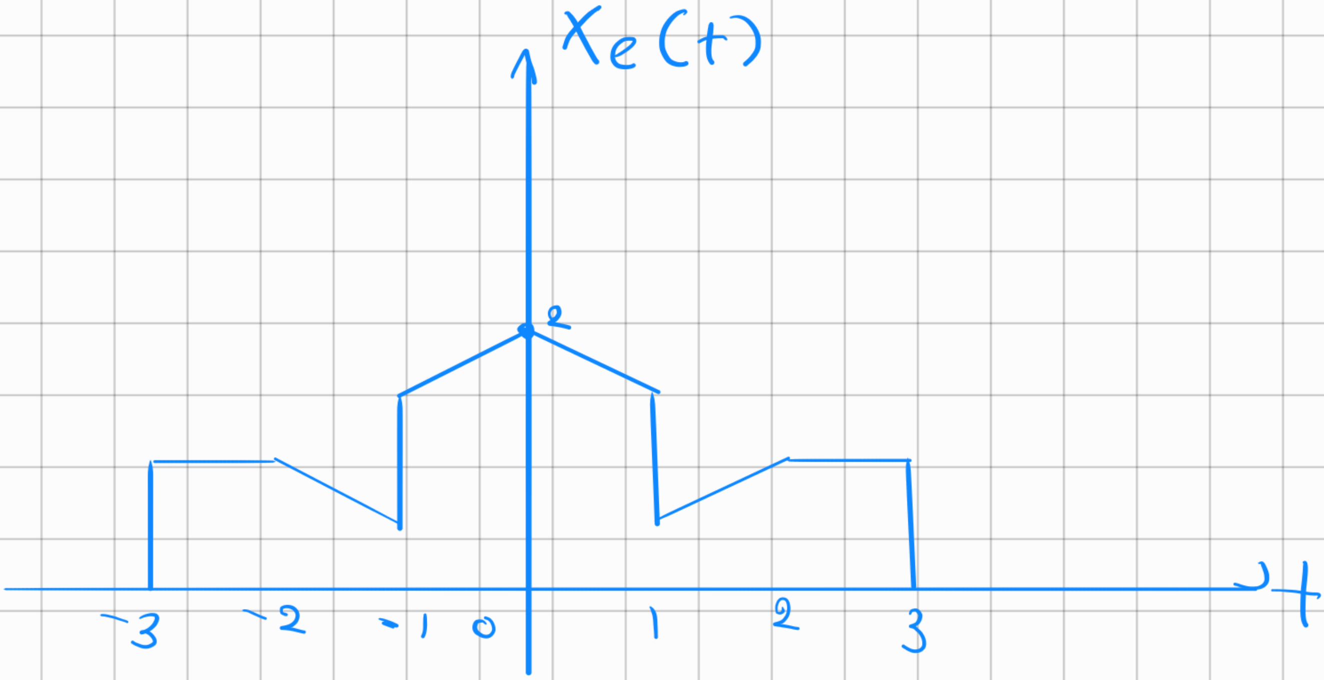
$$\therefore x_e(t) = \frac{x(t) + x(-t)}{2}$$



x(t)
2
-1
t



x(-t)
2
t



Xe(t)
2
-3
-2
-1
0
1
2
3
t