MULTIVARIATE AND ASSET-LEVEL RISK MODELS

FM **3052** - QUANTITATIVE FINANCE

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TABLE OF CONTENT

Multivariate Risk Models

- Univariate methods are useful for risk measurement.
- The univariate methods can also be used to model the return on each individual asset.
- For active **Risk Management** and **Optimal Portfolio Allocations**, multivariate models are preferable.
- The **Multivariate risk models** allow us to compute risk measures for different hypothetical portfolio allocations without having to re-estimate model parameters.
- Assumption: The risk manager knows the assets of interest.
- The asset set may include:
  - ⇒ All portfolio assets or
  - ⇒ A subset of base assets that are main drivers of risk.
- Common base assets include:
  - ⇒ Equity indices.
  - ⇒ Bond indices.
  - ⇒ Exchange rates.
  - ⇒ Economic drivers such as oil and real estate prices.
- There are different type of multivariate risk models based on different aspects. For example here below mentioned some key models and sub models.
  - ⇒ Multivariate GARCH Models
    - ♦ Constant Conditional Correlation (CCC)-GARCH Model
    - ♦ Dynamic Conditional Correlation (DCC) GARCH Model
  - ⇒ Copula Models
    - ♦ Gaussian Copula(The Normal Copula)
    - ♦ The T Copula

- **In this section, I am going to provide an in-depth analysis of the Multivariate GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, and show how its real-world applications and exploring the diverse areas and sectors where it can be effectively utilized. The Multivariate GARCH model is valuable for analyzing and forecasting volatility in multiple asset returns, making it an essential tool for risk management, portfolio optimization, and financial market analysis across various domains.**

Multivariate GARCH Models

- As earlier mention GARCH model can use in **time series data with varying volatility, commonly applied in finance, economics, and various fields requiring volatility forecasting**.
- To effectively use and udersatnd of GARCH models, it's essential to understand several key terms and concepts.
  - ⇒ **Volatility in multiple assets/markets**- The volatility of the two or more than variables are measure from the Co-Variance.(Co-Variance means relationship between variance of two variables)
  - ⇒ **Correlation** is relationship between two or more variables.
  - ⇒ **Conditional** mean which are the variables depend on some conditions that get from the historical data.
  - ⇒ **Constant** means the does not change usually we get the change over time
  - ⇒ **Dynamic** means the change usually we get the change over time

Constant Conditional Correlation (CCC) GARCH Model

- The model assumes that while individual asset volatilities change over time, the **correlations between asset returns remain constant.** That's why we call Constant Conditional Correlation (CCC) GARCH Model.
- Its stability assumption allows practitioners to estimate and predict t he impact of market volatility on portfolio risk **without recalculating correlation matrices frequently**.
- This model is especially useful in below sectors.
  - ⇒ For long-term investment decisions CCC GARCH model is critical.
    - ♦ **Asset management.**
    - ♦ **Insurance Industry.**
    - ♦ **Institutional investment funds.**
    - ♦ **Real-time currency exchange rate analysis.**

$$\boxed{\mathbf{P^*t=P}}$$

  - ⇒ P= conditional matrix / t=time

Assumptions that use to build Constant Conditional Correlation (CCC) GARCH model

- The most fundamental assumption is that the correlations between the asset returns are constant over time.
- The returns series of the assets are typically assumed to follow a multivariate normal distribution.(So **Particularly we use the over shorter time periods, log returns typically more closely resemble a normal distribution than simple returns. we discuused about mathematical terms in the presentation**)
- stationarity in returns and volatility assumes that these elements remain relatively stable over time, with their statistical characteristics, like average and variance, staying consistent.

Practical Applications of the CCC-GARCH Model in Financial Markets.

- In this section I will provide how we can apply the CCC GARCH for **currency exchange rate analysis** by using the python(machine lerning).

Eg : Currency Pair Analysis

- In the world of finance, understanding currency movements is essential for traders and analysts. This document explores a Python script designed to simulate and visualize the behavior of three major currency pairs: USD/EUR, USD/JPY, and EUR/JPY. By examining these currency pairs, we can gain valuable insights into their price dynamics over time. Using synthetic data, we mimic these concepts in a **CCC GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model framework. At the end of the docunment I attached the relevant code.**
- **The constant correlation matrix represents the correlations between different currency pairs or returns that are assumed to remain unchanged over time in the CCC GARCH framework.**

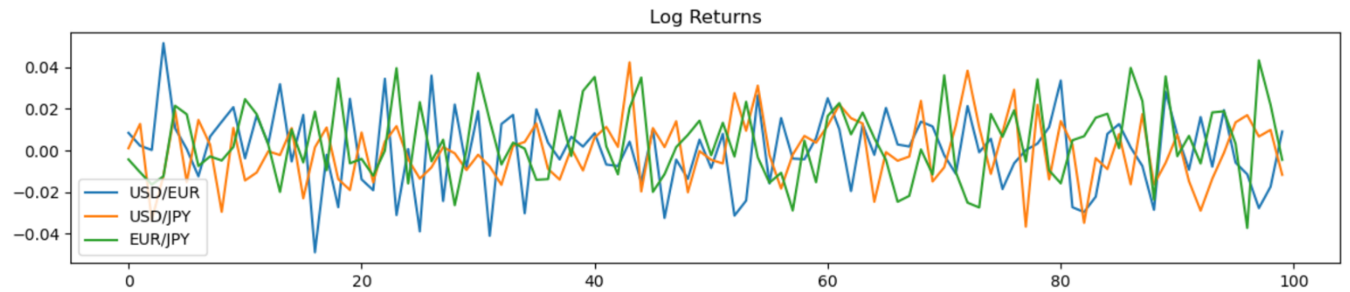
- **Data Simulation**
  - ⇒ To kick things off, we will simulate log returns using NumPy's random number generation capabilities. Specifically, we'll draw from a normal distribution to create realistic daily price changes for our chosen currency pairs. Here's how we set it up:

- **USD/EUR: Mean return of 0 with a standard deviation of 0.02.**
- **USD/JPY: Mean return of 0 with a standard deviation of 0.015.**
- **EUR/JPY: Mean return of 0 with a standard deviation of 0.018.**

⇒ These parameters are selected to reflect typical market behavior, providing a solid foundation for our analysis.

- **Visualizing Data**

  1 ***Log Returns Plot****: This graph will show how each currency pair's log returns change over 100 units of time period , allowing us to observe trends and patterns.*



Interpretation from the plot

⇒ **High Frequency of Fluctuations**

The log returns display rapid fluctuations, frequently moving above and below zero. This short-term variability is common in financial markets, where prices can change quickly due to market reactions to news and events. Such swift movements create both opportunities and challenges for traders navigating the fast-paced trading environment.

⇒ **No Clear Trend**

The log returns fluctuate around the zero line without showing a clear upward or downward trend. This is common for log returns, which typically hover near zero when prices follow a random walk. In essence, while prices may rise and fall, there isn't a consistent direction, highlighting the market's unpredictable nature.

The behavior of the log returns in this plot can be attributed to several practical factors related to financial markets and currency trading. Here I mentioned some real time causes.

⇒ **Economic Data Releases**

Economic indicators like GDP growth, employment reports, inflation data, and manufacturing indices significantly influence currency markets. For instance, positive U.S. jobs data can strengthen the U.S. dollar, affecting pairs such as USD/EUR and USD/JPY. Similarly, Japanese inflation reports can impact JPY-related pairs.

The currency markets react swiftly to these releases, leading to sudden spikes or drops in returns as traders adjust their positions. This responsiveness underscores the importance of staying informed about economic events, as they can create both opportunities and risks for traders.

⇒ **Central Bank Policies**

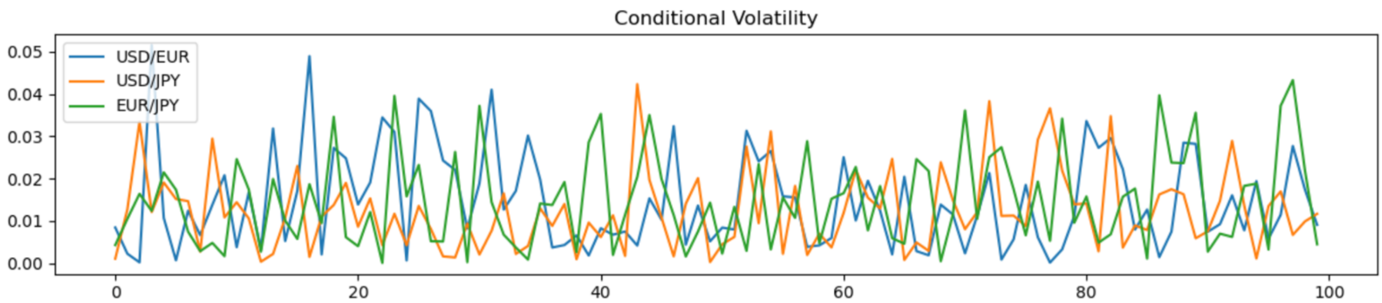
1.Role of Central Banks: Central banks like the Federal Reserve(US central bank), European Central Bank (ECB), and Bank of Japan (BoJ) are key players in the financial system, responsible for setting interest rates and conducting monetary policy.

2.Impact of Interest Rate Changes:

- When the Fed raises interest rates, it typically strengthens the U.S. dollar, affecting currency pairs such as USD/EUR and USD/JPY.
- Conversely, if the BoJ maintains low rates or implements quantitative easing, it can weaken the Japanese yen, influencing pairs like USD/JPY and EUR/JPY.

3.Market Reactions: Traders closely monitor these policy changes and often anticipate them, leading to synchronized movements across related currency pairs. This responsiveness highlights how central bank decisions can create both opportunities and risks in the forex market.

2 ***Conditional volatality Plot****: Another time series plot displays the conditional volatility, shown as absolute returns, for each currency pair. This helps identify periods of higher and lower risk, allowing traders to make more informed decisions based on market volatility*



Interpretation from the plot

⇒ **Volatility Levels**

The three currency pairs—USD/EUR, USD/JPY, and EUR/JPY—display distinct patterns of volatility, with moments of both high and low activity. While the fluctuations in volatility aren't identical for each pair, we can see some overlapping peaks at certain times. This suggests that their volatility tends to move together to some extent, reflecting shared market influences.

⇒ **Periods of High Volatility**

The plot shows distinct peaks in volatility for each currency pair, with each reaching its highest levels at different times. For example, USD/EUR may spike first, followed by EUR/JPY or USD/JPY. These sudden increases in volatility reflect the dynamic nature of the forex market, where external events and trader sentiment can quickly change risk levels. Recognizing these patterns is vital for traders navigating currency trading.

The behavior of the Volatlity in this plot can be attributed to several practical factors related to financial markets and currency trading. Here I mentioned some real time causes.

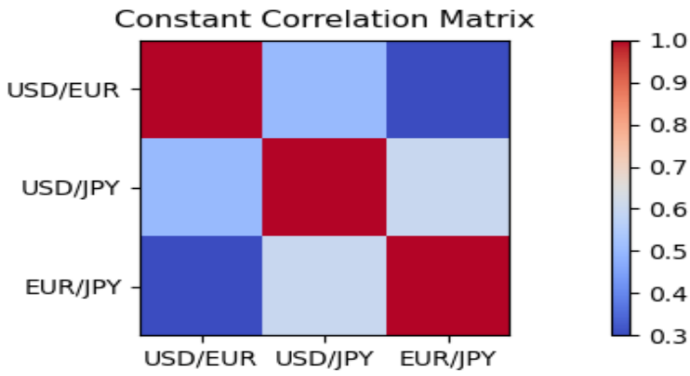
⇒ **Market Sentiment Shifts and Risk Aversion**

Investor sentiment plays a crucial role in shaping market behaviour, particularly during times of heightened risk aversion. When global economic uncertainty or crises arise, traders often flock to safe-haven currencies like the USD and JPY, which can lead to increased volatility in currency pairs involving these currencies. Significant market events—such as recessions, stock market crashes, or geopolitical tensions—can trigger synchronized spikes in volatility across multiple currency pairs as investors react to global risks.

⇒ **Political Events and Geopolitical Tensions**

Political events such as elections, trade negotiations, and conflicts can significantly increase volatility in currency markets due to uncertainty about their economic impacts.
For example, during the Brexit negotiations, currency pairs like EUR/JPY and USD/EUR likely experienced sharp spikes in volatility as traders anticipated potential effects on the Euro. These events highlight how closely intertwined geopolitics and economics are, making it essential for traders to stay informed about global developments to navigate market fluctuations effectively.

3. ***Correlation Matrix Heatmap :*** *The heatmap provides a visual representation of the constant correlation matrix, where colour intensity reflects the strength and direction of correlations between currency pairs. This makes it easy to see how closely aligned the returns of these assets are, helping traders quickly assess relationships and potential co-movements in the market.*



Interpretation from the graph

- ⇒ USD/EUR and USD/JPY (Light Blue, ~0.5 correlation): These two pairs shows a moderate positive correlation of around 0.5. This means that when USD/EUR rises or falls, USD/JPY tends to move in the same direction, although not perfectly.
- ⇒ USD/EUR and EUR/JPY (Dark Blue, ~0.3 correlation): The correlation between USD/EUR and EUR/JPY is low, at approximately 0.3. This indicates that the movements of these pairs are less synchronized and often vary independently.
- ⇒ USD/JPY and EUR/JPY (Light Red, ~0.6 correlation): These pairs show a moderately strong positive correlation of about 0.6, suggesting they frequently move together in a similar direction, but again, not perfectly aligned.

Practical Factors Contributing to Correlations

⇒ **Economic relationship Between Regions**

Currency pairs that are linked by shared currency regions or strong economic relationships typically show positive correlations. For instance, USD/JPY and EUR/JPY both involve the Japanese yen (JPY). As a result, developments in the Japanese economy—like key economic reports or geopolitical events—can impact both pairs in a similar way, creating a positive correlation. Recognizing these economic ties is essential for traders, as it allows them to predict how shifts in one area can influence multiple currencies.

⇒ **Currency Trading Strategies**

Market participants frequently trade currencies in pairs, guided by their economic outlooks. For instance, if investors are more optimistic about the U.S. dollar (USD) compared to the euro (EUR) but have a neutral stance on the Japanese yen (JPY), we might see a split in the correlations between USD/EUR and EUR/JPY. This split reflects the varying perspectives of investors and their expectations for each currency, highlighting how sentiment can influence trading behavior and market dynamics.

Dynamic Conditional Correlation (DCC) GARCH Model

- DCC GARCH model will model **dynamic covariance and correlation,** which together with the dynamic volatility models in Univariate GARCH model can be used to construct covariance matrices in DCC GARCH model.
- Once the set of assets has been determined, the next step in the multivariate model is to estimate a dynamic volatility model for each of the *n* assets. When this is complete, we can write the *n* asset returns in vector
- Here I mentioned some of the various sectors where the DCC-GARCH model is beneficial:

⇒ **Finance and Investment**
⇒ **Currency Markets and Forex Trading**
⇒ **Commodities Sector**
⇒ **Real Estate**
⇒ **Insurance and Actuarial Science**
⇒ **Banking Sector**
⇒ **Macroeconomic Analysis**
⇒ **Technology Sector**
⇒ **Public Policy and Government**
⇒ **Agricultural Economics**

Assumptions that use to build Dynamic Conditional Correlation (DCC) GARCH model

- The returns series of the assets are typically assumed to follow a multivariate normal distribution.(So **Particularly we use the over shorter time periods, log returns typically more closely resemble a normal distribution than simple returns. we discuused about mathematical terms in the presentation**)

Why we should use the log return values instead of simple returns ? *Using log returns in CCC/DCC-GARCH models is really important for getting our financial analysis right. They tend to fit a normal distribution better than simple returns, which can sometimes be skewed or misleading. This means we can make more reliable predictions about volatility and how assets correlate with each other.When we use log returns, we also get more stable estimates for our model parameters, which is crucial for making sound decisions. Plus, they help us manage risk better by capturing those extreme market events that can catch us off guard. In short, they strengthen our financial analyses and help us make smarter investment choices.*

- **Correlations and volatilities between the asset returns are change over time.**

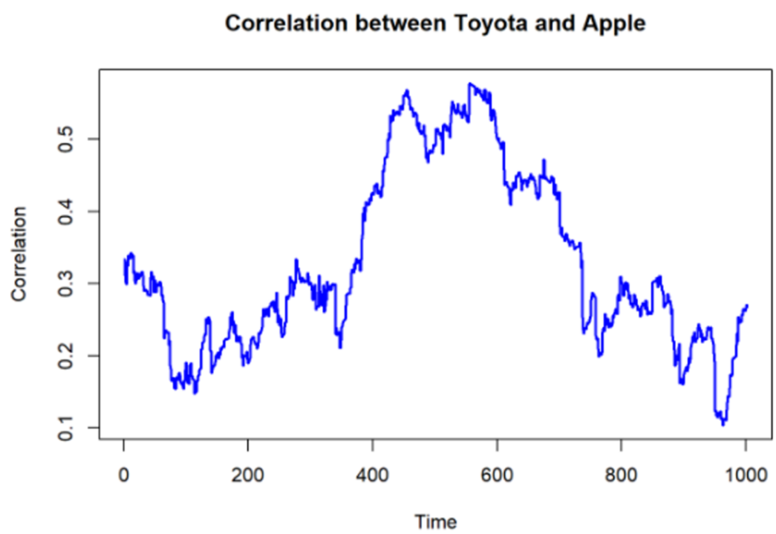
Portfolio Application using DCC GARCH Model.

- In this section I will provide how we can apply the DCC GARCH for **stock price exchange analysis** by using the R studio(R programming).

Eg : Stock prices Analysis

- I am going to analyze three types of stock prices by constructing a DCC GARCH model to assess the relationships between these variables and investigate whether one variable influences the others. The data, which includes daily share prices **from 8/28/2020 to 8/27/2024** for Apple Inc. (AAPL) representing Technology in the United States, Royal Dutch Shell plc (SHEL) for Energy in Europe, and Toyota Motor Corporation (TM) for Consumer Goods in Asia, will first be transformed into stationary data using logarithmic returns.
- In this section, I'll **focus on analyzing how the correlation between each pair of assets changes over time.** I'll also explore the practical factors and causes that influence these fluctuations. The object of the this analysis is to assess relationships between these stocks and investigate potential influences among them using the DCC GARCH model.

- **Visualizing Data**



Interpretation from the graph

⇒ **Positive Correlation**

The stock prices of Toyota and Apple generally move in the same direction, showing that positive correlation throughout the observed period.(**8/28/2020 to 8/27/2024)**

⇒ **Fluctuating Correlation**

The strength of the correlation varies over time, by showing both increasing and decreasing correlation in through out the period.

⇒ **Peak and Decline**

The correlation peaks around the midpoint(+0.5) of the time period, reaching values above 0.5, before gradually declining

Practical Factors Contributing in above Correlations

⇒ **Industry-Specific Events:**

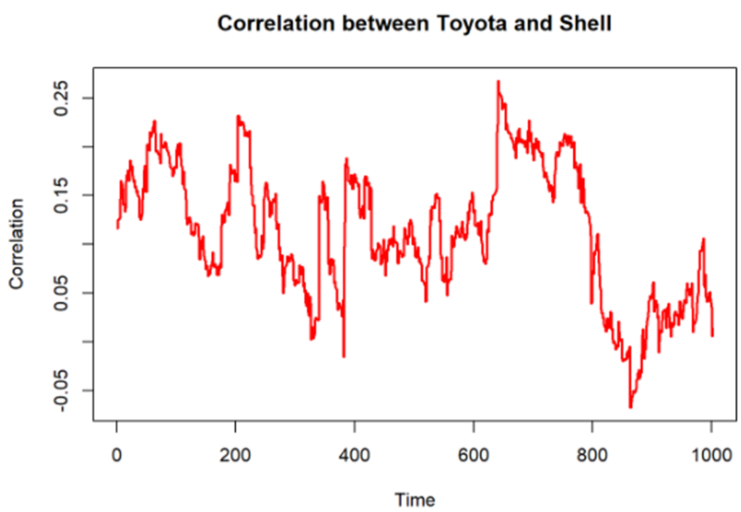
Developments in the automotive and technology sectors can impact these companies differently. For example, advancements in electric vehicles may boost Toyota's stock, while a new tech innovation could elevate Apple.(eg: innovation of electric vehicle technology might boost Toyota's stock while having little effect on Appl)

⇒ **Global Economic Conditions:**

Broader economic trends, like shifts in consumer spending or trade policies, can affect each company in unique ways. During an economic downturn, for instance, car sales might drop for Toyota, while Apple could maintain demand for its essential products.

⇒ **Company-Specific News:**

Unique news events can also drive fluctuations in correlation. If Apple launches an exciting new product, its stock might high, while negative news about Toyota—**such as production issues**—could create a disconnect between their stock movements.



Interpretation from the plot

⇒ **Lower and Volatile Correlation:**

The correlation between Toyota and Shell company stocks is show us generally low and more volatile, with both positive and negative values, reflecting the differing influences on their stock prices.

Practical Factors Contributing in above Correlations

⇒ **Industry-Specific Events**

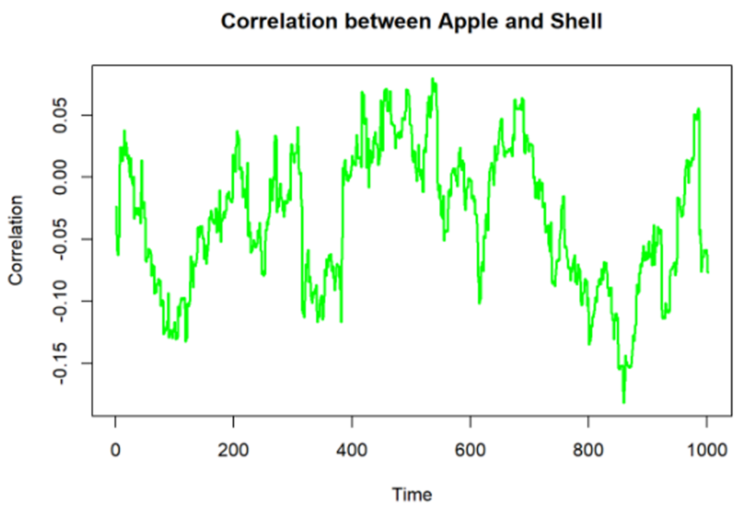
Differences in the automotive and energy sectors likely lead to the lower correlation. For example, a surge in electric vehicle sales might boost automotive stocks, whereas energy stocks could be impacted by rising oil prices or shifts towards renewable energy. This divergence in responses to market changes results in a lower correlation between companies like Toyota and those in the energy sector, as they don't always move in random with each other.

⇒ **Economic Conditions**

Broad economic events may affect each company differently, resulting in the observed fluctuations.

⇒ **Market Volatility**

The frequent ups and downs in correlation suggest that the relationship between these two stocks is less stable over time.



Interpretation from the plot

⇒ **Consistently Low Correlation:**

The correlation between Apple and Shell company stocks remains low throughout the time period(**8/28/2020 to 8/27/2024)**, mostly close to zero, indicating little to no relationship between their stock prices.

⇒ **Negative Correlation:**

There are periods where the correlation is negative, indicating that the stocks move in opposite directions during that times.(that means if the one of the return of stocks rise in company that lead to stock return is falls in other company).

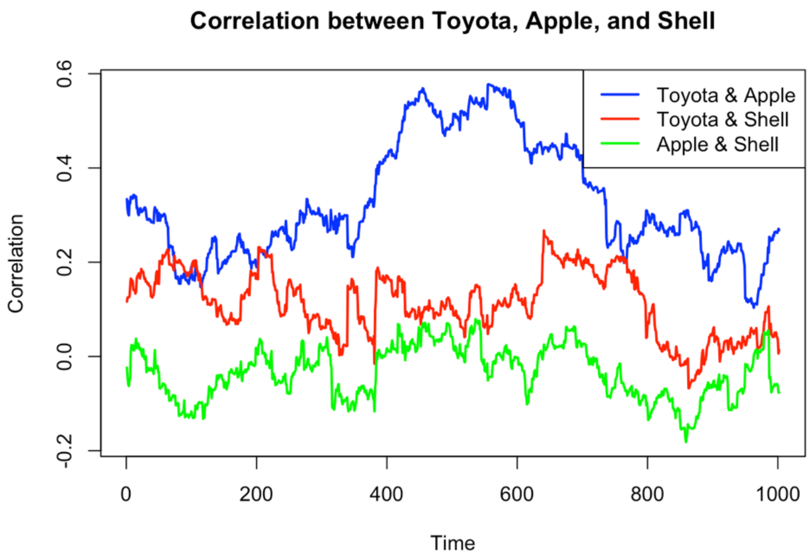
Practical Factors Contributing in above Correlations

⇒ **Industry-Specific Factors**

The difference industries of Apple and Shell (technology vs. energy) contribute to the weak correlation**.** For example, while Apple thrives on innovation and consumer demand for tech products, Shell is influenced by global oil prices and energy policies.

⇒ **Economic Conditions**

Broader economic factors may impact these companies differently, leading to the observed fluctuations in correlation, though these are generally minor .For example, a rise in fuel prices might affect Shell significantly, while Apple could see little change or no change if consumers continue to prioritize tech purchases.



- **The above plot shows us the all the pairs of correlation how changing over time.**

<u>Summary of using multivariate GARCH models</u>

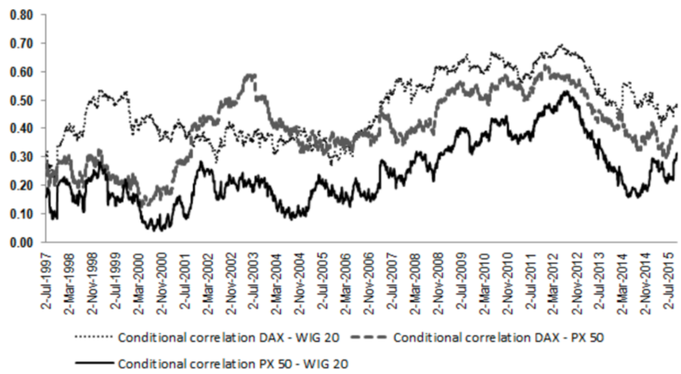
- The above models helps us to identify and understand Captures Dynamic Volatility and Correlations between the assets chaging over time or constant way.

- By modeling co-movements and volatility clustering, these models help in better assessing and managing portfolio risk, especially during periods of financial instability.
- Types of multivariate GARCH models (like CCC, and DCC), each with its strengths and weaknesses. This variety allows researchers and practitioners to choose a model that best fits for the give specific data and analytical needs.

Application(research) using DCC GARCH model

- In the research interdependencies among capital markets of Germany, Czech Republic and Poland were analysed. Thus, time series for three indices were used (PX 50, WIG20 and DAX). For this purpose a daily observations from 1 July 1997 to 30 September 2015 were taken, which gives T = 4592 observations.
- The data were obtained from the service http://www.finance.yahoo.com. In the study logarithmic returns =100* (ln(*P_t*)−ln(*P_t-1))* were used. In the estimation of parameters of DCC-GARCH model the maximum likelihood method with a fat-tailed t-distribution was applied.

Figure 1. The conditional correlation between the DAX index, the WIG 20 and the PX 50.



Source: own estimation based on data from: http://www.finance.yahoo.com.

- The article examines the growing connections between the capital markets of Germany, Poland, and the Czech Republic. As globalization deepens, these links have become increasingly important to study for effective crisis management.
- The findings reveal that while these markets are interconnected, they respond differently to external shocks. **Since 2004, correlations among them have risen, largely due to Poland and the Czech Republic joining the EU, which spurred economic growth in both countries. The global financial crisis of 20072008 further strengthened these ties through what is known as the contagion effect. However, from 2013 to 2015, there was a noticeable easing of these connections, indicating a return to stability in the markets.**
- This research underscores how understanding market dynamics is vital in our increasingly interconnected global economy.

Asset-Level Risk Models

- Asset-Level Risk Models assess the risks associated with **specific assets (individual asset**) like bonds, stocks, or real estate rather than portfolios.
- Accurate asset-level data is crucial for effective risk assessment. Without detailed information about each asset, **companies may underestimate potential losses from physical risks, leading to poor decision making regarding investments and risk management strategies.**
- By calucalting the risks associated with individual assets, investors can better understand portfolio-level risks. This **helps in calculating metrics like Value at Risk (VaR) more accurately**, ensuring that potential losses are not underestimated.
- There are seraval types model that we can use to asses the risk of individual assets,
  - ⇒ **Univariate GARCH model**
  - ⇒ **Value-at-Risk (VaR) Models**
  - ⇒ **CAPM (Capital Asset Pricing Model)**
  - ⇒ **Fama-French Three-Factor Model**
  - ⇒ **Stochastic Volatility Models**

  - ⇒ **In this section, I am going to provide an in-depth analysis of the CAPM (Capital Asset Pricing Model) model and Fama-French Three-Factor Model , show how its real-world applications and exploring the diverse areas and sectors where it can be effectively utilized.**

CAPM (Capital Asset Pricing Model)

- The capital asset pricing model (CAPM) calculates expected returns from an investment and can be used to determine prices for individual securities, such as stocks.
- As a core part of **corporate finance and investment banking**, CAPM looks at the relationship between the **investment's riskiness and the inherent risks of the market at large.**
- The CAPM helps investors determine how much they can **expect to get back from investments,** especially **risky ones**.
- Here I mentioned some of the various sectors where the CAPM model is useful:

  - ⇒ **Investment Bankers**
  - ⇒ **Investors**
  - ⇒ **Pharmaceutical and Health Research industry**
  - ⇒ **Corporate Finance industry**

- The capital asset pricing model equation looks like,(mathematical model we discussed this on our presentation slides)

$$Ra = Rrf + [Ba \times (Rm - Rrf)]$$

- Ra is *the expected rate of return on the investment* (or asset, hence the “a”)
- Rrf is *the risk-free rate of return*
- Ba is *the beta (β) of the investment* (or asset, hence the “a”)
- Rm is *the expected rate of return of the market* (hence the “m”)
- This portion of the formula — (Rm – Rrf) — is referred to as *the risk premium.*

The assumptions underlying the construction of the (CAPM)

- There is some unreasonable assumptions.
  - ⇒ For example, the formula only works if we assume that the market is dominated exclusively by rational(logical or sensible) actors who make decisions that only prioritize returns on investments.
  - ⇒ The model assumes that every actor in the market is acting on the same information.
  - ⇒ The CAPM formula assumes that the beta coefficient of an asset is constant and reflects its true systematic risk.
  - ⇒ CAPM relies only on historical data. This is an issue with many financial models and a problem that is nearly impossible to avoid. Ultimately, in the capital asset pricing model, a stock’s historical price changes are not enough to determine the overall risk of investment.
- How we can avoid or mitiagte these unreasonabale assumptions.
  - ⇒ In order to truly understand the risk of an investment, other aspects need to be considered, such as **economic conditions, the stock’s industry and competitors, and internal and external actions of the company itself.**

Real world Application using CAPM

- Eg: The CAPM formula can be applied to real-world investments to evaluate their performance and attractiveness. For example, suppose an investor is considering investing in two stocks: A and B. Stock A has a beta of 0.8 and an expected return of 9%, while stock B has a beta of 1.2 and an expected return of 11%. Using the CAPM formula, the investor can compare the **expected return and risk premium of each stock** with the risk-free rate of 2% and the expected return of the market portfolio of 10%. The results are shown in the table below:

Stock	Beta	Expected Return	Risk premium
A	0.8	9%	7%
B	1.2	11%	9%

- From the table, it can be seen that **stock B has a higher expected return and risk premium than stock A, but also a higher beta, which means it is more volatile and risky**. The investor can use the CAPM formula to decide which stock is more suitable for their risk appetite and return expectations. For example, if the investor is risk-averse, they may prefer stock A, which has a lower beta and a lower risk premium, but still offers a positive return above the risk free rate. If the investor is risk-seeking, they may prefer stock B, which has a higher beta and a higher risk premium, but also offers a higher return above the market portfolio.

⇒ **Some important key note of Beta(β) Coefficient:**

At the heart of CAPM lies the beta coefficient, a measure of an asset's volatility in relation to the market. A beta greater than 1 indicates higher volatility than the market, suggesting a riskier investment. Conversely, a beta less than 1 implies lower market correlation and potentially less risk. For instance, a company with a beta of 1.3 is expected to be 30% more volatile than the market.

Fama-French Three-Factor Model

- The Fama-French Three Factor model is a formula for calculating the rate of return on a given asset.
- It is expand from the Capital Asset Pricing Model (CAPM) by incorporating additional factors that help explain **stock returns.**
- Most of models, it offers an estimated value based on market factors at large. **In this case, investors can predict their return on investment based overall market risk , market size and market value.**
- Here I mentioned some of the various sectors where the Fama-French Three Factor model is benificial
  - ⇒ **Investors with long term investments.**
  - ⇒ **Investment Bankers.**
  - ⇒ **Corporate Finance industry.**
  - ⇒ **Share holders and equity holders.**

- The Fama-French Three Factor model equation looks like

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it}$$

where:

R_{it} = total return of a stock or portfolio i at time t

R_{ft} = risk free rate of return at time t

R_{Mt} = total market portfolio return at time t

$R_{it} - R_{ft}$ = expected excess return

$R_{Mt} - R_{ft}$ = excess return on the market portfolio (index)

SMB_t = size premium (small minus big)

HML_t = value premium (high minus low)

$\beta_{1,2,3}$ = factor coefficients

Real world Application(research) using Fama-French Three-Factor Model

- The Fama-and French Three-Factor Model is found **to outperform the CAPM on the Swedish market.**
- FF3FM(Fama-and French Three-Factor) gives higher explanatory power of the returns than the CAPM in two market conditions; for the whole **sample period of 2005-2010 and for the whole time period of 2005-2010, with the period of July 2007 to June 2008 excluded in Swedish market.**
- In the latter setting a time period characterized by economic downturn and financial turmoil is excluded and the explanatory power increases substantially for both models.

- For future studies it might be interesting to incorporate the companies on **the Swedish Small Cap list, in order to analyse how the FF3FM performs for the entire Swedish market**. Furthermore, future studies can be performed on single industries in Sweden in order to evaluate if the FF3FM performs well for all industries or if there are some industries, where it might struggle. It can also be interesting to add the momentum factor from **the Carhart Four Factor Model and analyse how it affects the level of explanation.**

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- https://finance.yahoo.com
- Relevant Data sets and code files : https://drive.google.com/file/d/1ce-_IDh8Rc9vOmQ0CiSqNoOk2B-WngUX/view?usp=sharing