Introduction of Option types and Analysis of an Option Portfolio Based on WTI Sweet Crude Oil

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Abstract

This report takes a closer look at the performance and risks tied to an option portfolio based on WTI Sweet Crude Oil. It focuses on tracking the monthly profit and loss (P&L) of the option holder, examining the option's payoff structure, and identifying potential downside risks. The analysis is centered around Asian options—contracts where the payoff depends on the average price of WTI over a set period, rather than the price on a single day at maturity. Using historical WTI price data, the report calculates the actual P&L and determines whether the option would be terminated based on its terms and market performance. Finally, from a portfolio manager's perspective, it suggests and evaluates a loss-limiting strategy aimed at minimizing exposure during unfavorable market conditions.

1 Introduction and Literature Review

Options provide a flexible means for hedging and speculative strategies. According to [3], an option is a financial derivative giving the holder the right but not the obligation to buy or sell an asset at a predetermined price(strike price). the Black-Scholes model is widely using model in to evaluate options (P&L) in European option pricing .options markets provides a significant expansion in the patterns of portfolio returns that were here to fore unavailable in combinations of equities and fixed-income instruments. When it comes to options, there are three main types based on how and when they can be exercised:**American, European, and Asian options.**

American options give the holder the most flexibility. You can exercise them at any time before they expire. This is useful if market conditions suddenly become favorable and you want to take advantage of them early.

European options, on the other hand, are more restrictive. They can only be exercised on the actual expiration date—not before. While that might seem like a limitation, these options are often simpler to handle and are commonly used in financial markets.

Asian options work a bit differently. Instead of relying on the price of the underlying asset at one point in time, they use the average price over a set period. This makes them less sensitive to sudden price swings and is especially useful in markets like crude oil, where prices can be quite volatile.Crude oil options, especially on WTI Sweet, are widely used in energy markets for hedging price volatility.

This report builds on theoretical concepts and applies them to a real-world context, focusing on how an option portfolio is constructed and analyzed for risk. When looking at related studies in this area, the study [4] provides a comprehensive explanation of options and option pricing through mathematical models. In another study [3], machine learning techniques are used to predict and price primarily stock values, aiming to optimize portfolio profits—and the approach also extends to commodities. Interestingly, [3] also highlights that there are multiple ways to derive the Black-Scholes Partial Differential Equation (PDE), including methods inspired by quantum physics [32–37] and other areas of physics. Meanwhile, the paper [5] explores the role of Indian Commodity Futures as an asset class within a traditional investment portfolio that includes equities and bonds.

2 Problem Formulation

An option portfolio is a combination of various option positions—calls and puts—that are structured to meet specific investment objectives such as hedging, speculation, or income generation. Understanding the nature of each option type is crucial before analyzing the performance of the entire portfolio.

- A **call option** gives the holder the right, but not the obligation, to *buy* an underlying asset at a specified strike price on or before the expiration date.

  When an investor **buys a call**, they anticipate the asset's price will higher above the strike price, leading to a profit. Conversely, **selling a call** (also known as writing a call) obligates the seller to deliver the asset if the buyer exercises the option, exposing the seller to potential losses if the price increases significantly.

- A **put option** gives the holder the right to *sell* the underlying asset at a specific strike price on or before the expiration date or given period.

  Buying a put benefits an investor if the asset's price drops below the strike price. In contrast, **selling a put** means the seller must buy the asset at the strike price if the buyer exercises the option—thus, the buyer lost if the price remains below the strike price.

  This report investigates a portfolio composed of ABC company of **Asian-style options**, which differ from standard options because their payoff is determined based on the **average price of the underlying asset over a defined period(in this case Monthly), rather than a single spot price at expiration**. The use of an average price helps smooth out volatility and reduce market manipulation losses.

  The underlying asset in this case is WTI Sweet Crude Oil with **volume of 100,000 barrels**, a widely traded commodity and key benchmark in the energy sector. The portfolio consists of below option types:

  – A long Asian call option with a strike price of $100,
  – A short Asian call option with a strike price of $105,
  – Two short Asian put options with a strike price of $100.

  The termination condition is the structure will continuous until ABC company receives an aggregate amount of **$15 per barrel** or till expiry(In this case One year period) This option structure is applied each month from **November 2013 to October 2014**, and **monthly average prices of WTI Sweet Crude Oil** are used to evaluate the payoff of each contract.

Objective

The objective of this analysis is to:

– Draw the pay off diagram of the option portfolio and identify the downside risk.

- Analyze the historical prices of WTI Sweet crude oil over the contract period(Compute Average WTI Crude oil price and volatlity),
- Compute the actual profit and loss (P&L) of the option holder for each month,
- Propose a strategy that could be adopted by a portfolio manager of ABC Company to limit potential losses,
- Evaluate the performance of the proposed strategy by comparing its P&L with the original position.

The findings will offer insight into managing option portfolios under varying market conditions, particularly in the energy sector.

3 Methodology

The methodology of picing the options do in two ways these are **Construct the Function and using Black Scholes Model**. In report in this analysis consists of a combination of Options and Models, historical data analysis, and visualization techniques. The steps are outlined below:

1. **Data Collection**: Historical price data for WTI Sweet Crude Oil is collected for the period from November 2013 to October 2014. [1]
2. **Return Computation**: Daily prices are used to calculate **log returns.** These returns are essential for understanding the volatility and movement trends in the underlying asset.

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

3. **Average Price and Asian Option Payoff**: Since the options are Asian-style, the average monthly price of WTI is used instead of the spot price at expiration. This average is used as the underlying settlement price in payoff calculations.The average price calculating by summing the all the values in month and divide it by days.
4. **Option Payoff Modeling using Function**: The portfolio consists of a long call at \$100, a short call at \$105, and two short puts at \$100. Payoff formulas for each component are implemented as follows:
   - *Long Call Payoff:* $\max(S_T - K, 0)$
   - *Short Call Payoff:* $-\max(S_T - K, 0)$
   - *Short Put Payoff:* $-\max(K - S_T, 0)$

   In here Python used to simulate these payoffs over the contract period and average monthly price of WTI month by month.In the function **S_T represent the Strike price** and **K is average monthly price** of oil barrel.

Month	Avg_WTI	Long_Call	Short_Call	Short_Put
2013-11	93.852857	0.000000	0.000000	-12.294286
2013-12	97.894286	0.000000	0.000000	-4.211429
2014-01	94.983478	0.000000	0.000000	-10.033043
2014-02	100.670500	0.670500	0.000000	0.000000
2014-03	100.509048	0.509048	0.000000	0.000000
2014-04	102.034762	2.034762	0.000000	0.000000
2014-05	101.907727	1.907727	0.000000	0.000000
2014-06	105.146667	5.146667	-0.146667	0.000000
2014-07	102.451304	2.451304	0.000000	0.000000
2014-08	96.076190	0.000000	0.000000	-7.847619
2014-09	93.161364	0.000000	0.000000	-13.677273
2014-10	84.511818	0.000000	0.000000	-30.976364

Figure 1: Monthly Average WTI Prices and Options Payoff (Nov 2013 – Oct 2014)

5. **Profit and Loss Calculation**: The monthly profit and loss (P&L) of the option holder is calculated by summing the individual payoffs of all option positions in the portfolio for each month. This total payoff is then multiplied by the contract volume, which is 100,000 barrels, to determine the net monthly P&L.

6. **Protective Put strategy**:A Protective Put is a risk management strategy used by an investor (or company) who already owns the underlying asset .[2].That put gives you the right to sell the asset at a fixed price ,**So if the market crashes, you're protected — like insurance.**if WTI oil prices fall too low, they'll lose money on their sales or contracts.So they buy a put option with a strike price of $95.

   – *Protective put payoff:* $\max(K - S_T, 0)$

   In the function **S_T is the Protective Strike price** and **K is average monthly price** of oil barrel.

	Month	Avg_WTI	Long_Call	Short_Call	Short_Put	Total_per_bbl	Monthly_PnL	Protective_Put	Adjusted_per_bbl	Adjusted_PnL	Cumulative_PnL
0	2013-11	93.852857	0.000000	0.000000	-12.294286	-12.294286	-1.229429e+06	1.147143	-11.147143	-1.114714e+06	-12.294286
1	2013-12	97.894286	0.000000	0.000000	-4.211429	-4.211429	-4.211429e+05	0.000000	-4.211429	-4.211429e+05	-16.505714
2	2014-01	94.983478	0.000000	0.000000	-10.033043	-10.033043	-1.003304e+06	0.016522	-10.016522	-1.001652e+06	-26.538758
3	2014-02	100.670500	0.670500	0.000000	0.000000	0.670500	6.705000e+04	0.000000	0.670500	6.705000e+04	-25.868258
4	2014-03	100.509048	0.509048	0.000000	0.000000	0.509048	5.090476e+04	0.000000	0.509048	5.090476e+04	-25.359210
5	2014-04	102.034762	2.034762	0.000000	0.000000	2.034762	2.034762e+05	0.000000	2.034762	2.034762e+05	-23.324448
6	2014-05	101.907727	1.907727	0.000000	0.000000	1.907727	1.907727e+05	0.000000	1.907727	1.907727e+05	-21.416721
7	2014-06	105.146667	5.146667	-0.146667	0.000000	5.000000	5.000000e+05	0.000000	5.000000	5.000000e+05	-16.416721
8	2014-07	102.451304	2.451304	0.000000	0.000000	2.451304	2.451304e+05	0.000000	2.451304	2.451304e+05	-13.965417
9	2014-08	96.076190	0.000000	0.000000	-7.847619	-7.847619	-7.847619e+05	0.000000	-7.847619	-7.847619e+05	-21.813036
10	2014-09	93.161364	0.000000	0.000000	-13.677273	-13.677273	-1.367727e+06	1.838636	-11.838636	-1.183864e+06	-35.490308
11	2014-10	84.511818	0.000000	0.000000	-30.976364	-30.976364	-3.097636e+06	10.488182	-20.488182	-2.048818e+06	-66.466672

Figure 2: Monthly P&L of Option Portfolio and After include the Protective strategy Monthly p&L of Option Portfolio

In the table Adjusted_per_bbl meaning is after calculate with protective put strike P&L of the one barrel in each month. after multiply it by volume of option that give by the Adjusted_PnL.

7. **Black-Scholes Model**: The Black-Scholes Model is use for comparative analytical approach and numerical technique to find the price of call option and put option and considered these two prices as buying price and selling price of stocks in the frontier markets so that we can predict the stock price (close price). It is somehow conceivable to utilize the Black-Scholes Model in this portfolio, but with some modifications, as WTI is a commodity rather than a stock.For futures and options on commodities like crude oil, the Black Model (also known as Black-76) is a modified version of Black-scholes.[3]

Black-Scholes Option Pricing Model

The Black-Scholes formula for a European **call option** is:

$$C = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

For a European **put option**, the formula is:

$$P = K \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Parameters:

- S: Current spot price of the underlying asset (e.g., average monthly WTI price)
- K: Strike price of the option
- T: Time to expiration (in years)
- r: Risk-free interest rate (annualized)
- σ: Volatility of the underlying asset (annualized)
- $N(\cdot)$: Cumulative distribution function of the standard normal distribution

```
# Log returns
wti['Log_Returns'] = np.log(wti['WTI_Price'] / wti['WTI_Price'].shift(1))

# Rolling 21-day (1-month) volatility, annualized
wti['Volatility'] = wti['Log_Returns'].rolling(window=21).std() * np.sqrt(252)
```

```
78]: from scipy.stats import norm

def bs_option_price(S, K, T, r, sigma, option_type='call'):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)

    if option_type == 'call':
        price = S * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)
    else:  # put
        price = K * np.exp(-r * T) * norm.cdf(-d2) - S * norm.cdf(-d1)

    return price

#S  Spot price  monthly_avg['Avg_WTI']
#K  Strike price    Fixed (e.g. 100, 105, 90)
#T  Time to expiry (years)  e.g. 1/12 for 1 month
#r  Risk-free interest rate Set manually, e.g., 0.01
#σ  Volatility (annualized) Use historical WTI returns
```

Figure 3: Model that use from Black-Scholes formula (for calls and puts)

Month	Avg_WTI	Volatility	Long_Call	Short_Call	Short_Put	Total_per_bbl	Monthly_PnL	Protective_Put	Adjusted_per_bbl	Adjusted_PnL	Cumu_Adj_PnL
2013-11	93.852857	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
2013-12	97.894286	0.140404	0.787219	0.073602	2.809635	-4.905653	-4.905653e+05	0.502736	-4.402917	-4.402917e+05	-4.402917
2014-01	94.983478	0.167816	0.360437	0.035640	5.293661	-10.262523	-1.026252e+06	1.803777	-8.458746	-8.458746e+05	-12.861663
2014-02	100.670500	0.127633	1.881576	0.249577	1.127778	-0.623556	-6.235558e+04	0.084463	-0.539093	-5.390928e+04	-13.400756
2014-03	100.509048	0.184234	2.435277	0.649153	1.842931	-1.899737	-1.899737e+05	0.373912	-1.525825	-1.525825e+05	-14.926581
2014-04	102.034762	0.160978	3.118333	0.812808	1.000272	0.304981	3.049807e+04	0.118021	0.423002	4.230018e+04	-14.503579
2014-05	101.907727	0.118551	2.595989	0.389724	0.604963	0.996338	9.963384e+04	0.023296	1.019635	1.019635e+05	-13.483944
2014-06	105.146667	0.111456	5.309227	1.468378	0.079262	3.682326	3.682326e+05	0.000641	3.682967	3.682967e+05	-9.800977
2014-07	102.451304	0.161551	3.414808	0.942787	0.880205	0.711611	7.116110e+04	0.097879	0.809490	8.094900e+04	-8.991487
2014-08	96.076190	0.180359	0.665732	0.097944	4.506243	-8.444698	-8.444698e+05	1.459001	-6.985697	-6.985697e+05	-15.977184
2014-09	93.161364	0.249883	0.612953	0.148222	7.368290	-14.271850	-1.427185e+06	3.675917	-10.595933	-1.059593e+06	-26.573118
2014-10	84.511818	0.237507	0.015073	0.001417	15.419956	-30.826256	-3.082626e+06	10.522921	-20.303335	-2.030334e+06	-46.876453

Figure 4: Monthly P&L of Option Portfolio and After include the Protective strategy Monthly p&L of Option Portfolio Using Black-Scholes formula

4 Results and Discussion

4.1 Monthly Payoff according to related Month

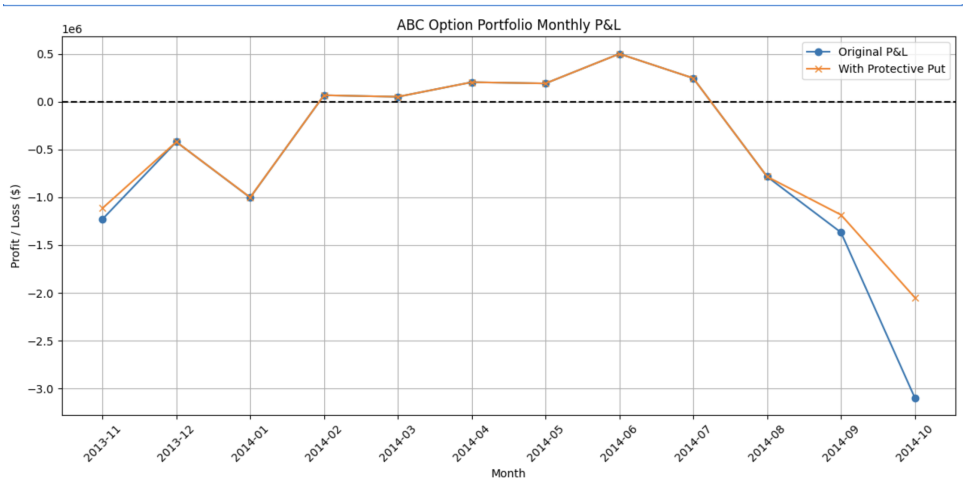


Figure 5: ABC Option Portfolio Monthly P&L

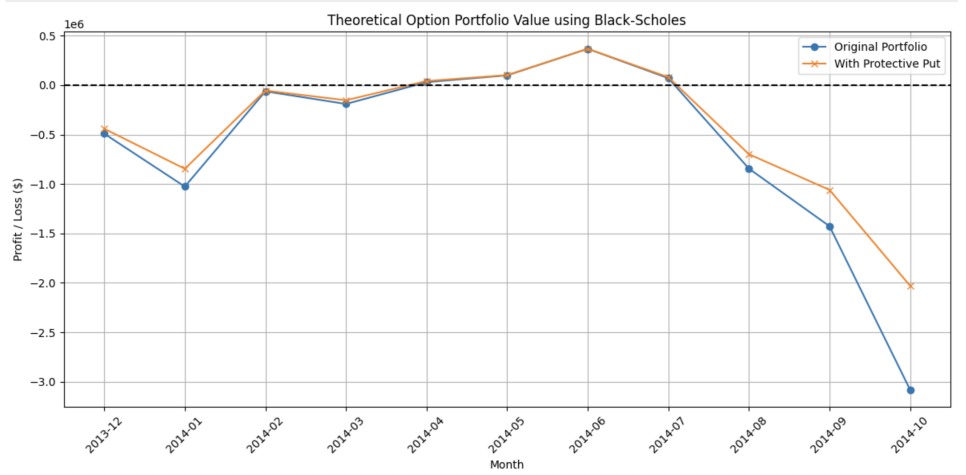


Figure 6: Theoretical Option Portfolio Value using Black-Scholes

As shown in Figure 5 Orange Line (With Protective Put) stays slightly above compared to the blue line (Original PL), especially when there's a sharp drop in price of commodity **(e.g., 2013-11, 2014-09, 2014-10)**.

That means:The protective put helps cushion the losses during high drawdown months.

As shown in Figure 6 This is a model-based estimation of the same portfolio using the Black-Scholes framework, **which doesn't fully capture the real market dynamics of commodities like crude oil**.Again, the protective put strategy shows value during large dips.The model is expected due to the model's assumptions of **constant volatility and log-normal price distribution.**

However, the gap between the original and protected portfolio appears slightly wider in some months than in the real PL.

Conclusion: According to behavior of the figures that the protective strategy is more beneficial in volatile markets, and even under theoretical valuation ,incorporating a protective put helps stabilize the portfolio performance both in real market outcomes and in theoretical models like Black-Scholes (or Black-76 when adapted for commodities like WTI crude oil).

4.2 Option Payoff Diagram with Monthly Average price of WTI

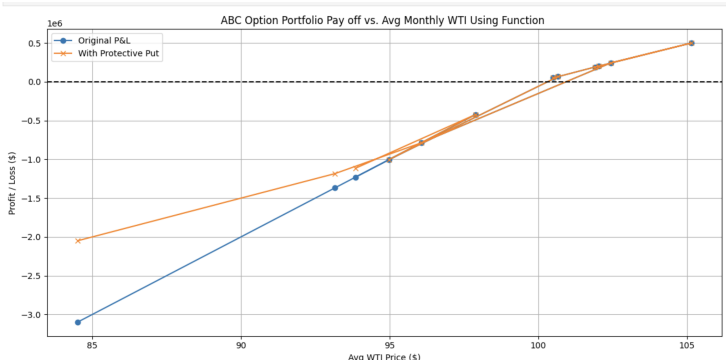


Figure 7: ABC Option Portfolio Pay off vs. Avg Monthly WTI Using Function

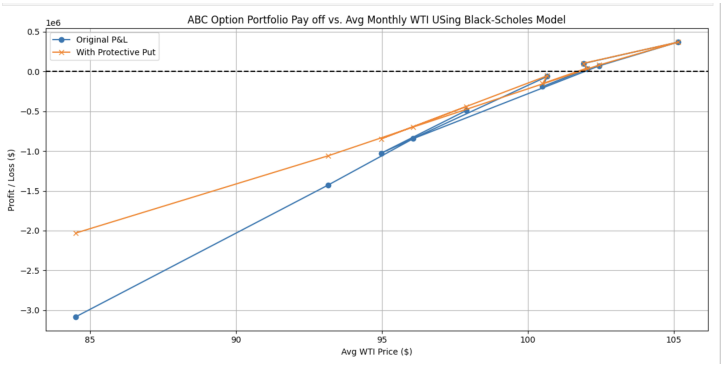


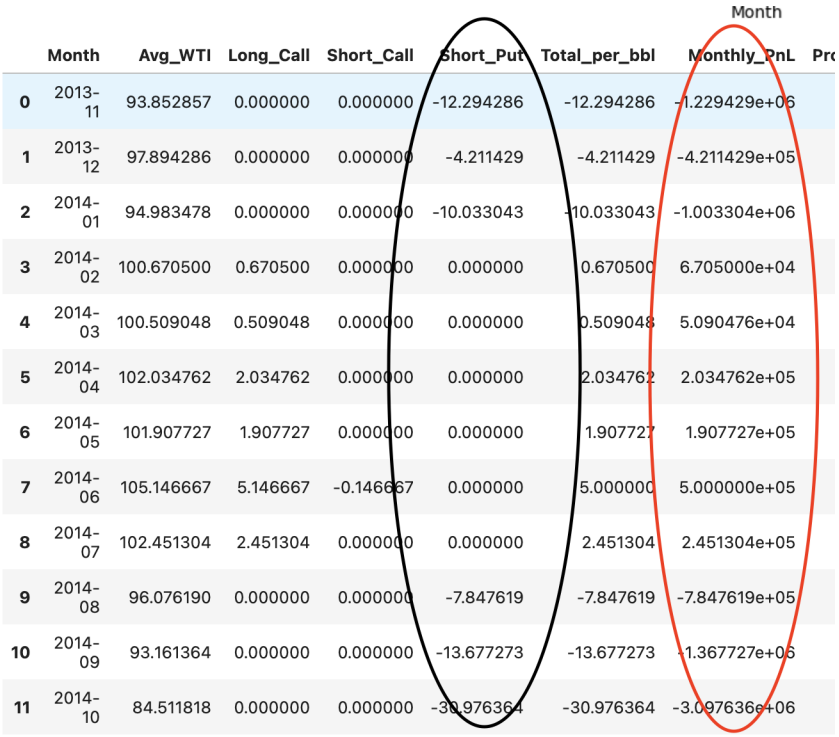
Figure 8: ABC Option Portfolio Pay off vs. Avg Monthly WTI USing Black-Scholes Model

These figures provide a price-sensitive analysis, mapping the portfolio's payoff against average monthly WTI prices. in given time period. As shown in Figure 7 payoff functions `max()`-based logic, shows that as prices drop below $95, the original portfolio incurs heavy losses, while the protected portfolio flattens down especially between $85 and $90.

Figure 8 uses the Black-Scholes Model, which, although smoother, also reflects the same risk protection below the strike price. The theoretical valuation shows a moderated slope in the loss region, confirming that downside risk.

5 Conclusion & Strategy to limit your losses

- In Figure 4 we can see all the Cumulative values in Column called **Cumulative_PnL**, according to that we can say the Option portfolio does not terminate until expiry date.
- This report has demonstrated how an option portfolio on WTI Sweet can be analyzed for risk and return. The **proposed protective strategy** offers a practical approach for portfolio managers to manage downside risk effectively so we can see that in above relevant figures.[2]
- There is another strategy we can use to minimize our loss in Theoretically is given below.Let consider below figures.



	Month	Avg_WTI	Long_Call	Short_Call	Short_Put	Total_per_bbl	Monthly_PnL	Pr
0	2013-11	93.852857	0.000000	0.000000	-12.294286	-12.294286	-1.229429e+06	
1	2013-12	97.894286	0.000000	0.000000	-4.211429	-4.211429	-4.211429e+05	
2	2014-01	94.983478	0.000000	0.000000	-10.033043	-10.033043	-1.003304e+06	
3	2014-02	100.670500	0.670500	0.000000	0.000000	0.670500	6.705000e+04	
4	2014-03	100.509048	0.509048	0.000000	0.000000	0.509048	5.090476e+04	
5	2014-04	102.034762	2.034762	0.000000	0.000000	2.034762	2.034762e+05	
6	2014-05	101.907727	1.907727	0.000000	0.000000	1.907727	1.907727e+05	
7	2014-06	105.146667	5.146667	-0.146667	0.000000	5.000000	5.000000e+05	
8	2014-07	102.451304	2.451304	0.000000	0.000000	2.451304	2.451304e+05	
9	2014-08	96.076190	0.000000	0.000000	-7.847619	-7.847619	-7.847619e+05	
10	2014-09	93.161364	0.000000	0.000000	-13.677273	-13.677273	-1.367727e+06	
11	2014-10	84.511818	0.000000	0.000000	-30.976364	-30.976364	-3.097636e+06	

Figure 9: Short put option price Analysis

According to the figure, we observe that the primary source of **loss** in the option portfolio arises from **selling put options**. In this case, we received the premium from the buyer, agreeing to buy the asset at a predetermined price of **$100**. When the **average monthly price of WTI crude oil** drops below this level, the value of the sold put increases, resulting in a loss for us as the seller.
However, it's important to remember the fundamental nature of options: the holder has the **right**, but not the **obligation**[4], to exercise the option. In our scenario, if we had chosen not to sell **two put options**, the downside exposure would have been significantly reduced. Theoretically, this adjustment would have **minimized or even eliminated the loss entirely**.
That said, this is based on **historical data**. In reality, we cannot predict future monthly averages of WTI crude oil with certainty, so these insights

should be viewed in the context of past performance and used cautiously when designing future strategies.

– To minimize our risk, one effective strategy is to adjust the strike prices of the options. However, before making such decisions, it is essential for a portfolio manager to thoroughly analyze the historical price trends of WTI crude oil. By examining the variations and calculating the average prices over time, we can gain insights into market behavior. Based on this analysis, more informed and strategic strike prices can be selected, which can significantly reduce potential losses in the portfolio.

References

[1] **Data Collection:** Historical price data was obtained from Investing.com.

[2] Merton, R.C., Scholes, M.S. and Gladstein, M.L. (1982). *The returns and risks of alternative put-option portfolio investment strategies.* Journal of Business, pp.1–55.

[3] Chowdhury, R., Mahdy, M.R.C., Alam, T.N., Al Quaderi, G.D. and Rahman, M.A., 2020. Predicting the stock price of frontier markets using machine learning and modified Black–Scholes Option pricing model. Physica A: Statistical Mechanics and its Applications, 555, p.124444.

[4] Karpman, S., 1971. Options. Transactional Analysis Journal, 1(1), pp.79-87.

[5] Bansal, Y., Kumar, S. and Verma, P., 2014. Commodity futures in portfolio diversification: Impact on investor's utility. Global Business and Management Research, 6(2), p.112.

[6] Access to relevant codes and historical data excel sheet.google_drive.