

# STA 331 2.0 Stochastic Processes

## 6. Poisson process (cont.)

---

Dr Thiyanga S. Talagala

September 15, 2020

Department of Statistics, University of Sri Jayewardenepura

# Interarrival times

---

# Inter-arrival Time Distribution for a Poisson Process

Consider a Poisson process with rate  $\lambda$ . Let,

$T_1$  be the time of the first arrival. Let  $T_2$  be the time elapsed between the first and the second arrival. Further, for  $n > 1$ , let  $T_n$  denote the elapsed time between the  $(n - 1)$ st and the  $n$ th event. The sequence  $\{T_n, n = 1, 2, 3, \dots\}$  is called the **sequence of interarrival times**.

We shall now determine the distribution of the  $T_n$ .

# Proposition

Let  $N(t)$  be a Poisson process with rate  $\lambda$ , then then the interarrival times  $T_n$ ,  $n = 1, 2, \dots$ , are independent identically distributed with  $Exponential(\lambda)$  distribution.

## Example

Let  $N(t)$  be a Poisson process with parameter  $\lambda = 2$ , and let  $T_1, T_2, \dots$ , be the corresponding interarrival times.

- i) Find the probability that the first arrival occurs after  $t = 0.5$ .
- ii) Given that we have had no arrivals before  $t = 1$ , find  $P(T_1 > 3)$ .
- iii) Given that the third arrival occurred at time  $t = 2$ , find the probability that the fourth arrival occurs after  $t = 4$ .
- iv) Suppose that we start observing the process at time  $t = 10$ . Let  $T_1$  be the time of the first arrival after we start observing the process. Find the mean and variance of the random variable  $T_1$ .

# Arrival times (Waiting time)

---

# Waiting time distribution

Let  $N(t)$  is a Poisson process with rate  $\lambda$ . We first generate i.i.d. random variables  $T_n, n = 1, 2, \dots$ , where  $T_n \sim \text{Exponential}(\lambda)$ .

Another quantity of interest is  $W_n$ , **the arrival time of the  $n$ th event**, also called **the waiting time until the  $n$ th event**. Then the arrival times are given by

$$W_1 = T_1$$

$$W_2 = T_1 + T_2$$

$$W_3 = T_1 + T_2 + T_3$$

...

## Waiting time distribution (cont.)

Since  $T_n$ 's are independent and each  $T_n$  has an exponential distribution with mean  $1/\lambda$ ,  $W_n$  has a **Gamma distribution** with shape parameter  $n$  and rate parameter  $\lambda$ . The mean waiting time is  $n/\lambda$ .



## Example

Suppose that people arrive at a territory in accordance with a Poisson process of rate  $\lambda = 10$  per day.

- i) What is the expected time until the 100th person arrives?
- ii) What is the probability that the elapsed time between the 100th and the 200th arrival exceeds 10 days?

# Acknowledgement

The contents in the slides are mainly based on  
Introduction to Probability Models by Sheldon M. Ross.  
H. Pishro-Nik, “Introduction to probability, statistics, and  
random processes”, Kappa Research LLC, 2014.