

Last Time:

- Randomized Quicksort
- Quick select

Hw 3 \rightarrow due Friday

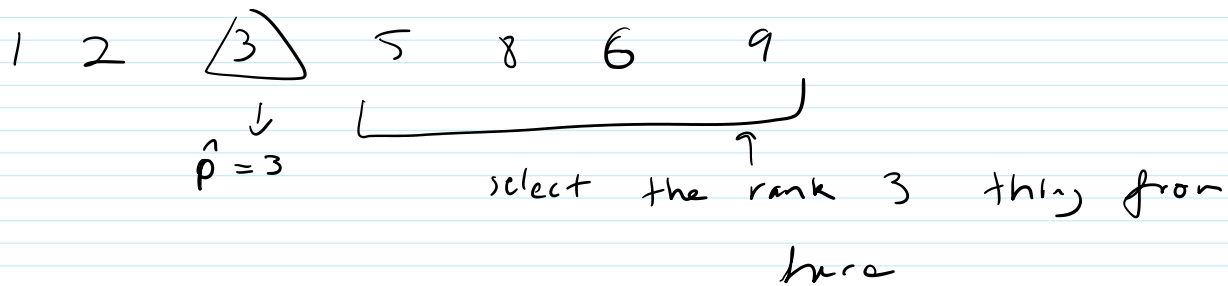
Today:

- Select

selectInput: List of items $A[1 \dots n]$ Target rank r Output: The r^{th} smallest item

1 5 8 6 2 9 3

 $r = 6$ Quickselect ($A[1 \dots n], r$)① pick a pivot a_p ② partition around the pivot $a_p \rightarrow \hat{p}$ ③ If $r = \hat{p}$, then a_p is the r^{th} smallestIf $r < \hat{p}$, then quickselect ($A[1 \dots (\hat{p}-1)], r$)If $r > \hat{p}$, then quickselect ($A[(\hat{p}+1) \dots n], r - \hat{p}$)1 5 8 6 2 9 3 $r = 6$ 1 2 3 5 8 6 9



Efficiency

$$T(n) = \underset{\substack{\uparrow \\ \text{partition}}}{\theta(n)} + \max \{ T(n - \hat{p}), T(\hat{p}) \}$$

worst-case: $T(n) = \theta(n) + T(n-1) \Rightarrow \theta(n^2)$

In practice: Randomly choose a pivot.

Randomized quickselect $\rightarrow \theta(n)$ expected runtime

If $\hat{p} = n/2 \rightarrow T(n) = \theta(n) + T(n/2)$

$\theta(n)$

$n/2$

$n/4$

$n/8$



$\theta(n)$

$\log_2 1 \rightarrow 0$

n^0 i.e. $\theta(1)$

$$T(n) = \theta(n) + 2 T(n/2)$$

mom_select($A[1 \dots n]$, r)

if $n < 1000$: do brute-force.

otherwise:

- $\theta(n) \rightarrow$ ① Divide A into $\lceil n/5 \rceil$ subarrays of 5 items each $M_1, M_2, \dots, M_{n/5}$.
- $\theta(n) \rightarrow$ ② Find the median of each of the subarrays, and make a list M_T of $n/5$.

$\Theta(n) \rightarrow$ (2) Find the median of each of the subarrays, and make a list $M[1 \dots n/5]$

$T(n/5) \rightarrow$ (3) $a_p \leftarrow \text{mom-select}(M[1 \dots n/5], r = \frac{n}{10})$

$\Theta(n) \rightarrow$ (4) $\hat{p} \leftarrow \text{partition}(A, a_p)$

\leftarrow (5) Run mom-select on $A[1 \dots (\hat{p}-1)]$, r
or
 $A[\hat{p}+1 \dots n]$, $r - \hat{p}$

We don't need $\hat{p} = n/2 \dots$

let $\hat{p} = \left(\frac{n}{\beta}\right)$ for some $\beta > 1 \Rightarrow \frac{n}{4}$

$\max\{T(n - \hat{p}), T(\hat{p})\}$

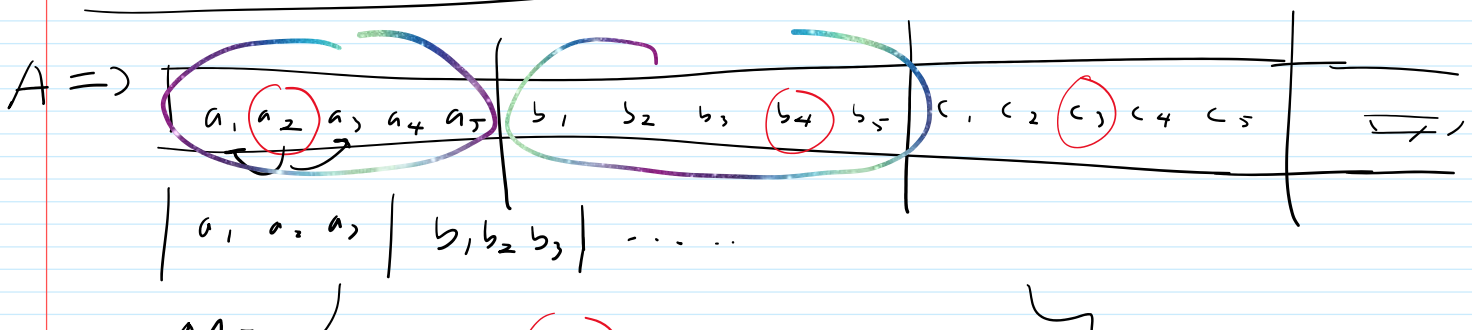
$$\downarrow$$

$$n - \frac{n}{\beta} = \frac{(\beta-1)n}{\beta} = \frac{n}{\left(\frac{\beta}{\beta-1}\right)} = \frac{n}{c}, \quad c > 1$$

$$T(n) = \Theta(n) + T\left(\frac{n}{c}\right) \quad c > 1$$

$$\log_c 1 = 0 \quad n^0 \text{ v.s. } (n^1)$$

$T(n)$ is $\Theta(n)$ by case 3.



$$M = \{ a_2, \textcircled{b_4}, c_3, \dots \}$$

$$\rightarrow \frac{5}{5/5}$$

$$a_p \rightarrow \textcircled{\underline{b_4}}$$

What is the rank of a_p ?

a_p is the median of M

$\rightarrow a_p$ is larger than $\frac{n}{10}$ items

\swarrow
each of these is bigger than 2 other things...

a_p is rank at least $\frac{3n}{10}$

a_p is rank at most $\frac{7n}{10}$

$$\underline{\underline{\frac{3n}{10} < p < \frac{7n}{10}}}$$

Efficiency

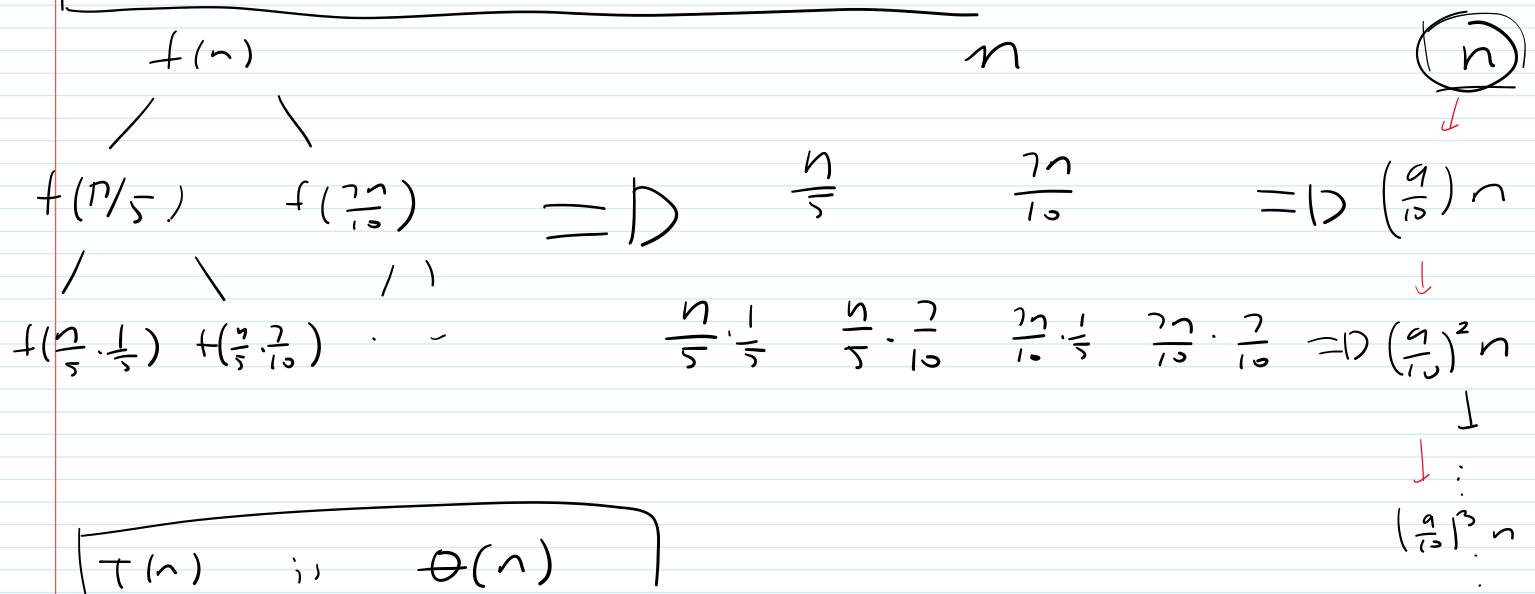
$$T(n) = \Theta(n) + T(n/5) + \max\{T(n-\hat{p}), T(\hat{p})\}$$

$$\frac{3n}{10} < \hat{p} < \frac{7n}{10}$$

$$T(n) = \Theta(n) + T(n/5) + T(7n/10)$$

$$\frac{1}{10} < p < \frac{1}{5}$$

$$T(n) = \theta(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$$



$$T(n) \in \theta(n)$$

$$5 \Rightarrow T(n) = \theta(n) + T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right)$$

$$3 \Rightarrow T(n) = \theta(n) + T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right)$$

