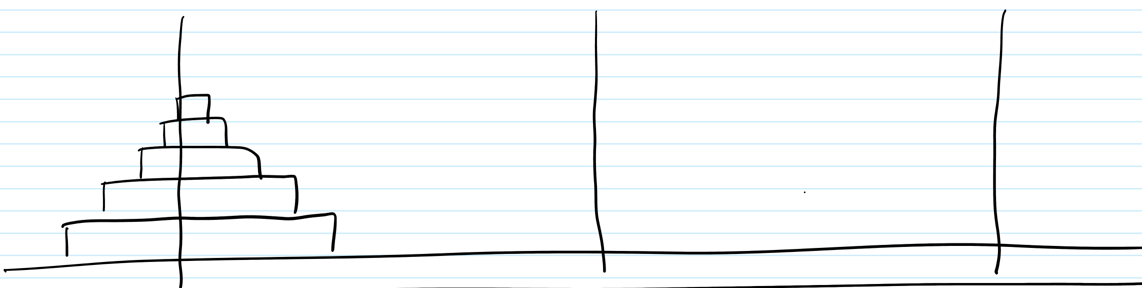
Recursion

Wednesday, January 22, 2025 11:37 AM

Last Time:
- Wrap up Big-O
- HW1 assigned!
- Reduction
  - Recursion

Today:
- Finish analysis of Hanoi
- More Recursion
- Reminder:
  - **HW1 due Friday!**

Tower of Hanoi



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Tower(n, src, dest, tmp)                              → T(n)
    if n = 0: do nothing.
    otherwise:
        Tower(n-1, src, tmp, dest)                    → T(n-1)
        move the disc n from src to dest              → 1
        Tower(n-1, tmp, dest, src)                    → T(n-1)
```

Efficiency

Count something → count single disk moves!

Let the number of moves on input n be some $T(n)$

$T(n) = ???$

$$T(n) = T(n-1) + 1 + T(n-1)$$

$$\boxed{T(n) = 2 \cdot T(n-1) + 1} \leftarrow \text{recurrence equation.}$$

I guess $T(n) = 2^n - 1$

Base case: $T(1) = 2^1 - 1 = 1$ ✓

Ind. hyp: $T(n) = 2^n - 1$ up to $n = k$.

show $\boxed{T(k+1) = 2^{k+1} - 1}$??

$$T(k+1) = 2 \cdot T(k) + 1 = 2 \cdot (2^k - 1) + 1$$

$$= 2 \cdot 2^k - 2 + 1$$

$$= 2^{k+1} - 1$$

$T(n) = 2^n - 1 \rightarrow$ # of disc moves.

$$\boxed{T(n) \text{ is } \Theta(2^n)}$$

Recursion

① If the problem instance is simple, solve it directly

② If the instance is hard, reduce it to solving easier instances of the problem.

Relative Maximum

Relative Maximum

Input: A list L of n items.

Def A relative max. or peak is an element $L[i]$ where $L[i] \geq L[i-1]$, and $L[i] \geq L[i+1]$

(At least as large as neighbors).

Output: The index of any one peak.

Example:

$L \rightarrow$ | 2 4 6 3 2 -1 4 |

6 ↑ 2 4 ↑ 6

The absolute max is always a rel. max.

Alg 1

① Find the max of the list, by iterating through all the elements.

② Output the index.

↳ $\boxed{\Theta(n)}$

Alg 2 ??

$[a \ldots \quad b \quad \boxed{c} \quad (\underbrace{d \quad e \quad \ldots \quad z})]$

peak?? T

① c is a peak

② c is not a peak.

$b > c$ or $d > c$

Pick c to be the middle to always eliminate the most elements.

$$\text{Peak}(L, \overset{\text{start}}{i}, \overset{\text{end}}{j}) \longrightarrow T(n)$$

① If $j - i < 1$: $L[i]$ is the peak.

② otherwise, we want to check whether the middle of $L[i] \ldots L[j]$ is a peak.

$m \leftarrow (i+j)/2$ $\longrightarrow O(1)$

if $L[m] < L[m-1]$:

  Peak(L, i, m) $\longrightarrow T(n/2)$

else if $L[m] < L[m+1]$:

  Peak$(L, m+1, j)$ $\longrightarrow T(n/2)$

else:

  $L[m]$ is a peak. $\longrightarrow 1$

Efficiency

$$\boxed{T(n) = T(n/2) + O(1)}$$

$$T(n) = \underline{\Theta(\log n)} \quad \checkmark$$
