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Unrestricted mixed data sampling (MIDAS): MIDAS regressions with unrestricted lag polynomials

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Summary. Mixed data sampling (MIDAS) regressions allow us to estimate dynamic equations that explain a low frequency variable by high frequency variables and their lags. When the difference in sampling frequencies between the regressand and the regressors is large, distributed lag functions are typically employed to model dynamics avoiding parameter proliferation. In macroeconomic applications, however, differences in sampling frequencies are often small. In such a case, it might not be necessary to employ distributed lag functions. We discuss the pros and cons of unrestricted lag polynomials in MIDAS regressions. We derive unrestricted-MIDAS (U-MIDAS) regressions from linear high frequency models, discuss identification issues and show that their parameters can be estimated by ordinary least squares. In Monte Carlo experiments, we compare U-MIDAS with MIDAS with functional distributed lags estimated by non-linear least squares. We show that U-MIDAS performs better than MIDAS for small differences in sampling frequencies. However, with large differing sampling frequencies, distributed lag functions outperform unrestricted polynomials. The good performance of U-MIDAS for small differences in frequency is confirmed in empirical applications on nowcasting and short-term forecasting euro area and US gross domestic product growth by using monthly indicators.

Keywords: Distributed lag polynomals; Mixed data sampling; Nowcasting; Time aggregation

1. Introduction

Economic time series differ substantially with respect to their sampling frequency. For example, financial variables are observable daily or even intradaily, whereas national accounts data such as gross domestic product (GDP) are available at quarterly frequency depending on the rules that are applied in statistical agencies. This raises the problem of how to conduct empirical analyses on the relationships between variables that are sampled at different frequencies.

The simplest solution is to work at the lowest frequency in the data, e.g. quarterly when some variables are available monthly and others quarterly. This requires time aggregation of high frequency (HF) variables with a loss of potentially relevant HF information, and a

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convolution of the dynamic relationships between the variables (see for example Marcellino (1999)).

As an alternative, mixed data sampling (MIDAS) regressions as proposed by Ghysels *et al.* (2005, 2006, 2007) and Andreou *et al.* (2010a, b), among others, directly relate variables sampled at different frequencies without losing HF information. To ensure a parsimonious specification, MIDAS regressions are typically based on distributed lag polynomials such as the exponential Almon lag (Ghysels *et al.*, 2007). Owing to the non-linearity of the lag polynomials, MIDAS regressions are typically estimated by non-linear least squares (NLS) following the literature on distributed lag models (Lütkepohl, 1981; Judge *et al.*, 1985).

MIDAS regressions have been applied in the financial literature; see for example Ghysels *et al.* (2006) in the context of volatility forecasting. In the macroeconomic literature, applications are often related to nowcasting and forecasting. For example, Clements and Galvão (2008, 2009) proposed to use MIDAS for forecasting quarterly GDP growth by using monthly business cycle indicators; see also Kuzin *et al.* (2011), Bai *et al.* (2013) and Marcellino and Schumacher (2010), among others. The recent application by Andreou *et al.* (2010b) proposed MIDAS regressions when daily financial data are used to forecast quarterly GDP.

An alternative way to handle mixed frequency data requires us to write the model in state space form with time aggregation schemes; see for example Mariano and Murasawa (2003). Kuzin *et al.* (2011) compared mixed frequency vector auto-regressive (VAR) processes estimated with the Kalman filter with MIDAS regressions, finding an unclear ranking but confirming the good performance of MIDAS. Bai *et al.* (2013) compared MIDAS regressions with state space models and discussed the approximating properties of MIDAS.

In this paper, we study the performance of a variant of MIDAS which does not resort to functional distributed lag polynomials. In particular, we discuss the pros and cons of MIDAS regressions with unrestricted linear lag polynomials, which do not require NLS but can be estimated by ordinary least squares (OLS). We shall call this approach from now on the unrestricted-MIDAS (U-MIDAS) approach and compare it with the standard MIDAS approach based on the exponential Almon lag following Ghysels *et al.* (2005, 2006), which we simply denote as MIDAS. One reason that motivates the use of U-MIDAS in macroeconomic applications is that the difference between sampling frequencies is in many applications not so high. For example, many of the references cited used monthly data, such as survey outcomes or industrial production, to predict quarterly GDP growth. In that case, the number of monthly lags that are necessary to estimate the lag polynomials might not be too large, implying that a curse of dimensionality might not be relevant. However, when financial data come into play as in Andreou *et al.* (2010b) and Monteforte and Moretti (2010), we face more severe limits in the degrees of freedom and functional lag polynomials may be preferable.

Koenig *et al.* (2003) have already proposed U-MIDAS in the context of realtime estimation. Clements and Galvão (2008, 2009) also considered U-MIDAS to forecast quarterly GDP, and Marcellino and Schumacher (2010) provided an application in a factor model framework. Rodriguez and Puggioni (2010) discussed Bayesian estimation of unrestricted MIDAS equations. However, none of these references systematically studied the role of the functional form of the lag polynomial.

We expand on the existing literature in the following respects. We discuss how U-MIDAS regressions can be derived in a general linear dynamic framework, and under which conditions the parameters of the underlying HF model can be identified.

Next, we provide Monte Carlo simulations that help to highlight the advantages and disadvantages of U-MIDAS *versus* MIDAS. The basic design of the exercise is similar to that of Ghysels and Valkanov (2006), where an HF VAR(1) process is specified. We look at the out-of-sample nowcasting performance, and we find that if the frequency mismatch is small, i.e. when mixing monthly and quarterly data, U-MIDAS is indeed better than MIDAS. With larger differences in sampling frequencies, MIDAS with exponential Almon lag polynomials is instead preferable. We also consider the case in which the restricted MIDAS model is the true data-generating process (DGP). Even in this favourable set-up for functional lag polynomials, it turns out that U-MIDAS is still preferable when the frequency mismatch is small. To robustify our results further, we consider also an HF VAR(2) process as DGP, which allows us to include more dynamics in the model. The results keep being in favour of U-MIDAS for small frequency mismatches.

Finally, we carry out an empirical exercise, where GDP growth in the USA and euro area are related to different monthly indicators. In the comparison, we find no clear-cut ranking of MIDAS and U-MIDAS on the basis of their out-of-sample nowcasting and short-term forecasting performance. However, for several indicators and horizons, U-MIDAS can outperform MIDAS. The performance of both methods substantially improves when the crisis period is included in the evaluation, highlighting the relevance of timely mixed frequency information. When there are many indicators, as in the application on the euro area, a factor U-MIDAS approach, along the lines of the factor MIDAS approach of Marcellino and Schumacher (2010), performs particularly well.

We conclude that U-MIDAS can be a strong competitor for MIDAS, because of its easier specification and estimation, and overall good empirical performance. This holds when the difference in sampling frequency in the data is small, such as when mixing quarterly and monthly data.

The paper proceeds as follows. In Section 2 we provide a theoretical motivation for U-MIDAS in a linear dynamic framework and discuss its use for nowcasting and forecasting. In Section 3 we present the results of the Monte Carlo experiments. In Section 4 we discuss the empirical forecasting exercise for the USA and in Section 5 for the euro area. In Section 6 we summarize the main results and conclude.

The data that are analysed can be obtained from

`http://wileyonlinelibrary.com/journal/rss-datasets`

2. Rationale behind unrestricted mixed data sampling and its use in forecasting

In this section we derive the U-MIDAS regression approach from a general dynamic linear model, consider its use as a forecasting device and compare it with the original MIDAS specification of Ghysels *et al.* (2005, 2006).

2.1. Unrestricted mixed data sampling regressions in dynamic linear models

We assume that y and the N variables x are generated by the VAR(p) process

$$\begin{pmatrix} \underset{1\times 1}{a(L)} & \underset{1\times N}{-b(L)} \\ \underset{N\times 1}{-d(L)} & \underset{N\times N}{C(L)} \end{pmatrix} \begin{pmatrix} \underset{1\times 1}{y_t} \\ \underset{N\times 1}{x_t} \end{pmatrix} = \begin{pmatrix} \underset{1\times 1}{e_{yt}} \\ \underset{N\times 1}{e_{xt}} \end{pmatrix}, \tag{1}$$

or

$$a(L)y_t = b_1(L)x_{1t} + \ldots + b_N(L)x_{Nt} + e_{yt}, \tag{2}$$

$$C(L)x_t = d(L)y_t + e_{xt} \tag{3}$$

where $a(L)=1-a_1L-\ldots-a_pL$, $b(L)=(b_1(L),\ldots,b_N(L))$, $b_j(L)=b_{j1}L+\ldots+b_{jp}L^p$, $j=1,\ldots,N$, $d(L)=(d_1(L),\ldots,d_N(L))'$, $d_j(L)=d_{j1}L+\ldots+d_{jp}L^p$, $C(L)=I-C_1L-\ldots-C_pL^p$ and the errors are jointly white noise. For simplicity, we suppose that the starting values $y_{-p},\ldots,y_0$ and $x_{-p},\ldots,x_0$ are all fixed and equal to 0 which coincide with the unconditional expected value of y and x. Different lag lengths of the polynomials in equations (2) and (3) can be easily handled, but at the cost of an additional complication in the notation.

We then assume that x can be observed for each period t, whereas y can be observed only every k periods. For example, $k=3$ when t refers to monthly periods and y is observed quarterly (for example, x could contain industrial production and y GDP growth), whereas $k=4$ when t refers to quarters and y is observed annually (for example, x could contain GDP growth and y fiscal variables that are typically available only annually). Let us indicate the aggregate (low) frequency by τ, whereas Z is the lag operator at τ-frequency, with $Z=L^k$ and $Zy_\tau=y_{\tau-1}$. In what follows HF indicates t and low frequency (LF) τ.

Let us then introduce the operator

$$\omega(L)=\omega_0+\omega_1L+\ldots+\omega_{k-1}L^{k-1}, \tag{4}$$

which characterizes the temporal aggregation scheme. For example, $\omega(L)=1+L+\ldots+L^{k-1}$ in the case of flow variables and $\omega(L)=1$ for stock variables.

Although general, this framework still imposes a few restrictions. In particular, y is univariate and there are no moving average components in the generating mechanism of y and x. These restrictions simplify substantially the notation and are helpful for the identification of the parameters of the HF model for y given the LF model. The framework remains sufficiently general to handle the majority of empirical applications, and the extensions are theoretically simple but notationally cumbersome.

The method that we adopt to derive the generating mechanism for y at LF is similar to that introduced by Brewer (1973), refined by Wei (1981) and Weiss (1984), and further extended by Marcellino (1999) to deal with general aggregation schemes and multivariate processes.

Let us introduce a polynomial in the lag operator, $\beta(L)$, whose degree in L is at most equal to $pk-p$ and which is such that the product $h(L)=\beta(L)\,a(L)$ contains only powers of $L^k=Z$, so that $h(L)=h(L^k)=h(Z)$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $a(L)$; see Brewer (1973), Wei (1981), Weiss (1984) and Marcellino (1999) for details.

To determine the AR component of the LF process, we then multiply both sides of equation (2) by $\omega(L)$ and $\beta(L)$ to obtain

$$h(L^k)\,\omega(L)y_t=\beta(L)\,b_1(L)\,\omega(L)x_{1t}+\ldots+\beta(L)\,b_N(L)\,\omega(L)x_{Nt}+\beta(L)\,\omega(L)e_{yt}. \tag{5}$$

Thus, the order of the LF AR component, $h(Z)$, is at most equal to p. In addition, the polynomial $h(L^k)$ can be decomposed into

$$\prod_{s=1}^{h}\prod_{i=1}^{k}\left(1-\frac{1}{h_{si}}L\right), \tag{6}$$

where $h<p$ is more precisely defined in Appendix A, and at least one h_{si} for each s must be such that $a(h_{si})=0$.

It can be shown that, in general, there is a moving average component in the LF model, $q(Z)u_{yt}$. Its order q coincides with the highest multiple of k non-zero lag in the autocovariance function of $\beta(L)\,\omega(L)e_{yt}$. The coefficients of the moving average component must be such that the implied autocovariances of $q(Z)u_{yt}$ coincide with those of $\beta(L)\,\omega(L)e_{yt}$ evaluated at all multiples of k.

Let us consider now the x-variables, which are observable at frequency t. The polynomials $\beta(L)\,b_j(L)\,\omega(L)=b_j(L)\,\beta(L)\,\omega(L)$, $j=1,\ldots,N$, are at most of order $pk+k-1$. Each term $\beta(L)\,\omega(L)x_{jt}$ is a particular combination of HF values of x_j that affects the LF values of y.

In Appendix A we show that, under certain rather strict conditions, it is possible to recover the polynomials $a(L)$ and $b_j(L)$ that appear in the HF model for y from the LF model, and therefore also $\beta(L)$ can be identified. In this case we can use the *exact MIDAS* model

$$\begin{aligned} h(L^k)\,\omega(L)y_t &= b_1(L)z_{1t}+\ldots+b_N(L)z_{Nt}+q(L^k)u_{yt}, \\ z_{jt} &= \beta(L)\,\omega(L)x_{jt}, \qquad j=1,\ldots,N, \\ t &= k, 2k, 3k, \ldots. \end{aligned} \tag{7}$$

The left-hand side of equation (7) contains the LF variable y, obtained from time aggregation $\omega(L)\,y_t=y_\tau$. The LF variable is regressed on its own LF lags and on lags of x_{jt} for $j=1,\ldots,N$. As the polynomials $a(L)$ and $b_j(L)$ are identified, there is no need for a polynomial approximation.

When $\beta(L)$ cannot be identified, we can use an *approximate U-MIDAS* model based on a linear lag polynomial such as

$$\begin{aligned} c(L^k)\,\omega(L)y_t &= \delta_1(L)x_{1t-1}+\ldots+\delta_N(L)x_{Nt-1}+\varepsilon_t, \\ t &= k, 2k, 3k, \ldots \end{aligned} \tag{8}$$

where $c(L^k)=(1-c_1L^k-\ldots-c_cL^{kc})$ and $\delta_j(L)=(\delta_{j,0}+\delta_{j,1}L+\ldots+\delta_{j,v}L^v)$, $j=1,\ldots,N$. In general, the error term ε_t has a moving average structure. However, for simplicity, we shall work with an AR approximation throughout, since this does not affect the main points that we want to make and simplifies both the notation and the estimation.

We label this approach hereafter *U-MIDAS*. The static version of U-MIDAS corresponds to the direct mixed frequency regression model of Kvedaras and Rackauskas (2010). They considered static regressions only but allowed for a larger set of aggregation schemes.

Since the polynomials $\delta_i(L)$ operate at HF whereas $c(L^k)$ operate at LF, the matrix of regressors in equation (8) is of the type

$$\begin{array}{cccccccccc} y_0 & \cdots & y_{-kc} & \delta_{1,0}x_{1,k-1} & \cdots & \delta_{1,v}x_{1,k-v-1} & \cdots & \delta_{N,0}x_{N,k-1} & \cdots & \delta_{N,v}x_{N,k-v-1}, \\ y_k & \cdots & y_{-(k-1)c} & \delta_{1,0}x_{1,2k-1} & \cdots & \delta_{1,v}x_{1,2k-v-1} & \cdots & \delta_{N,0}x_{N,2k-1} & \cdots & \delta_{N,v}x_{N,2k-v-1}, \\ \vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ y_{Tk-k} & \cdots & y_{Tk-kc} & \delta_{1,0}x_{1,Tk-1} & \cdots & \delta_{1,v}x_{1,Tk-v-1} & \cdots & \delta_{N,0}x_{N,Tk-1} & \cdots & \delta_{N,v}x_{N,Tk-v-1}. \end{array}$$

As an example, if $\omega(L)=1$, i.e. y is a stock variable, and $k=3$ (i.e. t is monthly and τ is quarterly), the matrix of regressors becomes

$$\begin{array}{cccccccccc} y_0 & \cdots & y_{-3c} & \delta_{1,0}x_{1,3-1} & \cdots & \delta_{1,v}x_{1,3-v-1} & \cdots & \delta_{N,0}x_{N,3-1} & \cdots & \delta_{N,v}x_{N,3-v-1}, \\ y_3 & \cdots & y_{-2c} & \delta_{1,0}x_{1,6-1} & \cdots & \delta_{1,v}x_{1,6-v-1} & \cdots & \delta_{N,0}x_{N,6-1} & \cdots & \delta_{N,v}x_{N,6-v-1}, \\ \vdots & & \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ y_{3T-3} & \cdots & y_{3T-3c} & \delta_{1,0}x_{1,3T-1} & \cdots & \delta_{1,v}x_{1,3T-v-1} & \cdots & \delta_{N,0}x_{N,3T-1} & \cdots & \delta_{N,v}x_{N,3T-v-1}. \end{array}$$

If we assume that the lag orders c and v are sufficiently large to make the error term ε_t uncorrelated, then all the parameters in the U-MIDAS model (8) can be estimated by simple OLS (whereas the aggregation scheme $\omega(L)$ is supposed known). From a practical point of view, the lag order v could differ across variables, and v_i and c could be selected by an information criterion such as the Bayes information criterion (BIC). We shall follow this approach in the Monte Carlo experiments and in the empirical applications, combining it with the use of an information criterion for lag length selection.

2.2. Forecasting with unrestricted mixed data sampling

To start with, let us consider the case where the forecast origin is in period $t=Tk$ and the forecast horizon measured in t time is $h=k$, namely one LF period ahead. Using standard formulae, the optimal forecast (in the mean-squared error MSE sense and assuming that ε_t is uncorrelated) can be expressed as

$$\hat{y}_{Tk+k|Tk}=(c_1L^k+\ldots+c_cL^{kc})y_{Tk+k}+\delta_1(L)\hat{x}_{1Tk+k-1|Tk}+\ldots+\delta_N(L)\hat{x}_{NTk+k-1|Tk}, \tag{9}$$

where $\hat{x}_{iTk+j|Tk}=x_{iTk+j|Tk}$ for $j\leqslant T$.

A problem with expression (9) is that forecasts of future values of the HF variables x are also required. Following for example Marcellino *et al.* (2006), a simpler approach is to use a form of direct estimation and to construct the forecast as

$$\tilde{y}_{Tk+k|Tk}=\tilde{c}(L^k)y_{Tk}+\tilde{\delta}_1(L)x_{1Tk}+\ldots+\tilde{\delta}_N(L)x_{NTk}, \tag{10}$$

where the polynomials $\tilde{c}(Z)=\tilde{c}_1L^k+\ldots+\tilde{c}_cL^{kc}$ and $\tilde{\delta}_i(L)$ are obtained by projecting y_t on information dated $t-k$ or earlier, for $t=k,2k,\ldots,Tk$. We shall use this approach in the Monte Carlo simulations and empirical applications. In general, the direct approach of equation (10) can also be extended to construct hk-step-ahead forecasts given information in Tk:

$$\bar{y}_{Tk+hk|Tk}=\bar{c}(L^k)y_{Tk}+\bar{\delta}_1(L)x_{1Tk}+\ldots+\bar{\delta}_N(L)x_{NTk}, \tag{11}$$

where the polynomials $\bar{c}(Z)$ and $\bar{\delta}_i(L)$ are obtained by projecting y_t on information dated $t-hk$ or earlier, for $t=k,2k,\ldots,Tk$.

Marcellino and Schumacher (2010) presented the details of the derivation of a direct forecasting equation for the case where the regressors are factors extracted from a large set of HF indicators. A similar approach can be used in this context to derive equation (10) from a given HF VAR DGP.

The conditioning information set for forecasting in model (10) contains HF information up to the end of the sample of the LF variable, namely period Tk. An advantage of the MIDAS approach is that it also allows for incorporating leads of the HF variable x_t for the projections. This is because observations of HF indicators are available much earlier than the observations of the LF models, e.g. surveys or industrial production. MIDAS with leads can exploit this early information and thus is in particular helpful for nowcasting, namely computing and updating projections of the LF variable for the current period given all potential HF information which is available (Giannone *et al.*, 2008; Marcellino and Schumacher, 2010; Andreou *et al.*, 2010b; Kuzin *et al.*, 2011). Nowcasting with HF indicators becomes relevant for policy makers, because the publication lags for many LF variables are quite substantial. For example, quarterly GDP in the USA is typically published after about 4 weeks in the subsequent quarter. Thus, within each quarter, the contemporaneous value of GDP growth is not available, making nowcasts necessary.

As a particular nowcasting example, suppose that the value of interest is still y_{Tk+k}, but that now HF information up to period $Tk+1$ is available, e.g. observations of monthly industrial production on the first month of a given quarter become available. Then, expression (9) can be easily modified to take the new information into account:

$$\hat{y}_{Tk+k|Tk+1}=\tilde{c}(L^k)y_{Tk}+\delta_1(L)\hat{x}_{1Tk+k-1|Tk+1}+\ldots+\delta_N(L)\hat{x}_{NTk+k-1|Tk+1}, \tag{12}$$

where $\hat{x}_{iTk+j|Tk+1}=x_{iTk+j|Tk+1}$ for $j\leqslant T+1$. Similarly, the coefficients in equation (10) would now be obtained by projecting y_t on information dated $t-k+1$ or earlier and the direct forecast becomes

$$\tilde{y}_{Tk+k|Tk+1} = \tilde{c}(L^k) y_{Tk} + \tilde{\delta}_1(L) x_{1Tk+1} + \ldots + \tilde{\delta}_N(L) x_{NTk+1}. \tag{13}$$

If time passes by and new HF information becomes available, say, in periods $Tk+1, Tk+2, \ldots$, the nowcast can be updated similarly to the one-step ahead case.

2.3. Unrestricted mixed data sampling and mixed data sampling with exponential Almon lags

It is interesting to compare the U-MIDAS approach with the original MIDAS specification of Ghysels *et al.* (2005, 2006) with functional lag polynomials; see also Clements and Galvão (2008). Assuming for simplicity that $c(L^k) = 1$, $N = 1$ and $\omega(L) = 1$, the U-MIDAS model (8) simplifies to

$$y_t = \delta_1(L) x_{t-1} + \varepsilon_t \tag{14}$$

for $t = k, 2k, \ldots, Tk$. The original MIDAS model would be

$$y_t = \beta_1 B(L, \theta) x_{t-1} + \varepsilon_t, \tag{15}$$

where the polynomial $B(L, \theta)$ is the exponential Almon lag following Lütkepohl (1981) with

$$B(L, \theta) = \sum_{j=0}^{K} b(j, \theta) L^j, \qquad b(j, \theta) = \exp(\theta_1 j + \theta_2 j^2) \Big/ \sum_{j=0}^{K} \exp(\theta_1 j + \theta_2 j^2). \tag{16}$$

Therefore, the MIDAS specification of Ghysels *et al.* (2005, 2006) is nested in U-MIDAS, since it is obtained by imposing a particular dynamic pattern. The key advantage of the original MIDAS specification is that it allows for long lags with a limited number of parameters, which can be particularly useful in financial applications with a high mismatch between the sampling frequencies of y and x, e.g. when y is monthly and x is daily. However, for macroeconomic applications with small differences in sampling frequencies, e.g. monthly and quarterly data, the specification in equation (15) can have several disadvantages. In particular, it could simply be that the functional restriction $\beta_1 B(L, \theta) = \delta_1(L)$ is not valid and that the Almon lag approximation might not be sufficiently general. Additionally, if the impulse response function is relatively short lived and only a few HF lags are needed to capture the weights, a linear unrestricted lag polynomial might suffice for estimation. Moreover, the model resulting from equation (15) is highly non-linear in the parameters, so it cannot be estimated by OLS. In summary, these considerations suggest that U-MIDAS should perform better than the original MIDAS as long as the aggregation frequency is small and U-MIDAS is not too heavily parameterized.

In general, it should be kept in mind that both MIDAS and U-MIDAS should be regarded as approximations to dynamic linear models such as that discussed in Section 2.1. Since we do not know the true model in practice, we cannot expect one of the approaches to dominate with empirical data. However, given a known DGP, it might be useful to identify conditions under which MIDAS or U-MIDAS does better. Thus, we shall consider both approaches in the simulations below.

3. Monte Carlo experiments

This section presents a set of Monte Carlo experiments that focus on the forecasting performance of alternative MIDAS regressions. We discuss, in turn, the basic simulation design, the models under comparison and the results. Next, as a robustness check, we present results for alternative simulation designs.

3.1. Simulation design

The simulation design is closely related to that in Ghysels and Valkanov (2006). We modify the exercise in a way to discuss its use in macroeconomic forecasting, in particular forecasting quarterly GDP. As predictors for this variable, the empirical literature typically uses monthly or daily indicators; see Clements and Galvão (2008, 2009) and Andreou *et al.* (2010b) respectively. Additionally, we could use weekly variables. Thus, we focus on three sampling frequencies, $k = \{3, 12, 60\}$, which represent cases of data sampled at monthly and quarterly frequency ($k = 3$), at weekly and quarterly frequency ($k = 12$) or at daily and quarterly frequency ($k = 60$).

In each case, the DGP is given by the HF VAR process

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} e_{y,t} \\ e_{x,t} \end{pmatrix}. \tag{17}$$

y_t is the LF variable and x_t is the HF variable, where t is the HF time index with $t = 1, \ldots, (T + \mathrm{ES}) \times k$. T defines the size of the estimation sample (expressed in the LF unit, e.g. quarters in our example), whereas for forecasting we generate an additional number of observations, which defines our evaluation sample ES (which is also expressed in the LF unit). k denotes the sampling frequency of the LF variable y_t, whereas x_t is sampled with $k = 1$. We further assume that $\omega(L) = 1$. Thus, the LF variable y_t is available for $t = k, 2k, \ldots, (T + \mathrm{ES}) \times k$ only.

In the VAR process (17), the shocks $e_{y,t}$ and $e_{x,t}$ are sampled independently from the normal distribution with mean 0 for all $t = 1, \ldots, (T + \mathrm{ES}) \times k$, and the variances are chosen such that the unconditional variance of y is equal to 1, given the specifications of the other parameters: for the persistence parameter, we choose $\rho = \{0.1, 0.5, 0.9\}$ and, following Ghysels and Valkanov (2006), we fix $\delta_l = \{0.1, 0.5, 1.0\}$ and $\delta_h = 0$. Thus, the DGP is recursive, as the HF variable affects the LF variable, but not vice versa. Later on in Section 3.4, we shall also consider non-recursive DGPs. Overall, we cover a broad range of DGPs representing different degrees of persistence and correlation between the HF and the LF variable.

Initially, y_t and x_t are simulated for all $t = 1, \ldots, T \times k$; see Ghysels and Valkanov (2006). To estimate the MIDAS regressions, the available data are defined as y_t with $t = k, 2k, \ldots, T \times k$ and x_t with $t = 1, 2, \ldots, T \times k$, representing mixed frequency data which are typical in empirical applications. The number of observations for the LF variable is $T = 100$. To conduct a forecast comparison, we also simulate both variables $\mathrm{ES} \times k$ HF periods ahead for $t = T \times k + 1, \ldots, T \times k + \mathrm{ES} \times k$. ES is set equal to $T/2 = 50$. The final values of the LF variable, from $y_{T \times k + k}$ to $y_{T \times k + \mathrm{ES} \times k}$, will be used as the actual values to be compared with the alternative forecasts. Regarding the information set that is available for forecasting, we assume that we know values up to period $(T + \mathrm{es} - 1) \times k$, with $\mathrm{es} = 1, \ldots, \mathrm{ES}$, for the LF variable and $(T + \mathrm{es} - 1) \times k + k - 1$ for the HF variable x_t. This yields forecasts of the LF variable k HF periods ahead for each date in the evaluation sample, $y_{T \times k + \mathrm{es} \times k | T \times k + \mathrm{es} \times k - 1}$, conditional on HF information within the LF forecast period. The corresponding forecast error is $y_{T \times k + \mathrm{es} \times k | T \times k + \mathrm{es} \times k - 1} - y_{T \times k + \mathrm{es} \times k}$, which is used to compute the MSE over the evaluation sample for each replication r, as

$$\mathrm{MSE}_r = \frac{1}{\mathrm{ES}} \sum_{\mathrm{es}=1}^{\mathrm{ES}} (y_{T \times k + \mathrm{es} \times k | T \times k + \mathrm{es} \times k - 1, r} - y_{T \times k + \mathrm{es} \times k, r})^2,$$

where $r = 1, \ldots, R$, and in our experiment $R = 500$. The number of replications is 500 for computational issues. In fact, for each replication we need to estimate the models and then to compute the forecast 50 times: one for each quarter of the evaluation sample. Therefore, even with 500 replications, we obtain 25000 forecasts.

The alternative mixed frequency models will be evaluated according to their relative MSE performance.

3.2. *Models under comparison*

We consider empirical MIDAS regressions that are based on estimated coefficients and possibly misspecified functional forms. In particular, we evaluate the following two types of models.

(a) MIDAS with an AR term as used in Clements and Galvão (2008, 2009): this follows from the fact that the HF VAR process also implies an AR term. To rule out periodic movements in the impulse response function from the HF variable x_t on the LF variable y_t, a common factor specification is imposed, yielding the model

$$y_{t\times k}=\beta_0+\lambda y_{t\times k-k}+\beta_1(1-\lambda L^k)B(L,\theta)x_{t\times k-1}+\varepsilon_{t\times k}, \tag{18}$$

where the polynomial $B(L,\theta)$ is the exponential Almon lag defined as in equation (16). The model is estimated by using NLS, with the additional coefficient λ and the common factor structure imposed. We apply the coefficient restrictions $-100<\theta_1<5$ and $-100<\theta_2<0$. As starting values might matter, we compute the residual sum of squares in each replication for alternative parameter pairs in the sets $\theta_1=\{-0.5,0.0,0.5\}$ and $\theta_2=\{-0.01,\ -0.1,\ -0.5,\ -1\}$. Note that, when x_t is available until $T\times k+\mathrm{es}\times k-1$, this is the final date that is used to estimate the coefficients of the model. Given the NLS estimates of the parameters, the forecast can be computed as

$$y_{T\times k+\mathrm{es}\times k|T\times k+\mathrm{es}\times k-1}=\hat{\beta}_0+\hat{\lambda}y_{T\times k+\mathrm{es}\times k-k}+\hat{\beta}_1(1-\hat{\lambda}L^k)B(L,\hat{\theta})x_{T\times k+\mathrm{es}\times k-1}. \tag{19}$$

(b) U-MIDAS as introduced in Section 2: in particular, U-MIDAS is estimated without considering the Almon lag polynomial. Rather, we leave the lag polynomial of the indicator HF variable x_t unrestricted. Furthermore, we do not impose the common factor restriction as in Clements and Galvão (2008) for the AR term. Thus, the model becomes

$$y_{t\times k}=\mu_0+\mu_1 y_{t\times k-k}+\psi(L)x_{t\times k-1}+\varepsilon_{t\times k}, \tag{20}$$

with lag polynomial $\psi(L)=\Sigma_{j=0}^{K}\psi_j L^j=\psi_0+\psi_1 L+\ldots+\psi_K L^K$. The coefficients μ_0, μ_1 and $\psi(L)$ are estimated by OLS. To specify the lag order, we use the BIC. Given the selected BIC order $\hat{K}$ and OLS estimated parameters $\hat{\mu}_0$, $\hat{\mu}_1$ and $\hat{\psi}(L)$, the U-MIDAS forecast can be computed as

$$y_{T\times k+\mathrm{es}\times k|T\times k+\mathrm{es}\times k-1}=\hat{\mu}_0+\hat{\mu}_1 y_{T\times k+\mathrm{es}\times k-k}+\hat{\psi}(L)x_{T\times k+\mathrm{es}\times k-1}. \tag{21}$$

As in Clements and Galvão (2009), we consider the inclusion of a different amount of past information. In particular, in our analysis, we choose to include up to either $K_{\max}=k$ or $K_{\max}=4k$ lags of x_t. In the case of $k=60$, we provide results for up to $K=60$ lags only since the sample size (in LF units) includes only 100 observations.

3.3. *Monte Carlo forecast comparison results*

The results of the Monte Carlo forecast experiments are summarized in Table 1. We compare the forecasting performance of U-MIDAS and MIDAS on the basis of their (out-of-sample) MSEs, computed over the 50 periods of the evaluation sample. Table 1 reports summary statistics for the distribution of the MSE of U-MIDAS relative to that of MIDAS as described in the previous subsection for alternative parameter values and sampling frequencies. In more detail, we first compute the ratio MSE(U-MIDAS)/MSE(MIDAS) for each replication; then we report the

Table 1. Out-of-sample MSE of U-MIDAS relative to the MSE of MIDAS (DGP: recursive HF VAR(1) process)†

ρ	δ_l	δ_h	k	Relative performance ($K_{max}=k$)						Relative performance ($K_{max}=4k$)					
				Mean	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*	*Mean*	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*
0.1	0.1	0	3	1.01	0.96	0.99	1.01	1.03	1.07	1.01	0.96	0.99	1.01	1.04	1.07
0.1	0.1	0	12	1.01	0.97	0.99	1.01	1.03	1.06	1.01	0.96	0.98	1.01	1.03	1.06
0.1	0.1	0	60	1.01	0.97	0.99	1.01	1.03	1.06	—	—	—	—	—	—
0.1	0.5	0	3	1.01	0.96	0.99	1.01	1.03	1.07	1.01	0.97	0.99	1.01	1.03	1.07
0.1	0.5	0	12	1.01	0.97	0.99	1.01	1.03	1.06	1.02	0.97	0.99	1.01	1.04	1.07
0.1	0.5	0	60	1.01	0.97	0.99	1.01	1.04	1.07	—	—	—	—	—	—
0.1	1.0	0	3	1.01	0.97	0.99	1.01	1.03	1.07	1.01	0.96	0.98	1.01	1.03	1.07
0.1	1.0	0	12	1.01	0.96	0.99	1.01	1.04	1.07	1.01	0.96	0.99	1.01	1.03	1.08
0.1	1.0	0	60	1.01	0.97	0.99	1.01	1.04	1.07	—	—	—	—	—	—
0.5	0.1	0	3	0.94	0.83	0.89	0.94	0.98	1.04	1.01	0.96	0.99	1.01	1.04	1.07
0.5	0.1	0	12	1.05	0.97	1.00	1.05	1.09	1.13	1.05	0.97	1.00	1.04	1.09	1.13
0.5	0.1	0	60	1.05	0.97	1.00	1.04	1.10	1.14	—	—	—	—	—	—
0.5	0.5	0	3	0.93	0.82	0.88	0.93	0.98	1.02	1.01	0.96	0.98	1.01	1.04	1.07
0.5	0.5	0	12	1.05	0.97	1.01	1.05	1.09	1.13	1.05	0.96	1.01	1.05	1.08	1.13
0.5	0.5	0	60	1.05	0.96	1.00	1.05	1.09	1.15	—	—	—	—	—	—
0.5	1.0	0	3	0.94	0.83	0.89	0.94	0.99	1.04	1.02	0.97	0.99	1.01	1.04	1.08
0.5	1.0	0	12	1.05	0.97	1.00	1.04	1.08	1.14	1.05	0.97	1.01	1.05	1.09	1.14
0.5	1.0	0	60	1.05	0.97	1.01	1.05	1.09	1.14	—	—	—	—	—	—
0.9	0.1	0	3	0.91	0.80	0.86	0.91	0.96	1.02	0.99	0.90	0.94	0.99	1.03	1.07
0.9	0.1	0	12	1.09	0.93	1.01	1.08	1.18	1.25	1.12	0.98	1.04	1.10	1.18	1.28
0.9	0.1	0	60	1.23	1.03	1.11	1.21	1.32	1.45	—	—	—	—	—	—
0.9	0.5	0	3	0.91	0.81	0.85	0.91	0.97	1.01	0.99	0.90	0.94	0.99	1.03	1.07
0.9	0.5	0	12	1.07	0.91	0.98	1.07	1.16	1.24	1.13	0.98	1.05	1.13	1.21	1.29
0.9	0.5	0	60	1.24	1.05	1.13	1.23	1.34	1.46	—	—	—	—	—	—
0.9	1.0	0	3	0.92	0.81	0.86	0.92	0.98	1.03	0.99	0.91	0.95	0.99	1.03	1.07
0.9	1.0	0	12	1.08	0.92	0.99	1.07	1.15	1.23	1.12	0.98	1.04	1.12	1.19	1.28
0.9	1.0	0	60	1.22	1.04	1.11	1.21	1.31	1.44	—	—	—	—	—	—

†The first four columns show the parameter specification for the DGP in equation (17). The entries of the middle six columns report the performance of the out-of-sample MSE(U-MIDAS) relative to out-of-sample MSE(MIDAS), when $K_{max}=k$ and the actual lag order is selected by the BIC for U-MIDAS. MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication; then in the fifth column the mean of the distribution of these ratios is reported, and in the next five columns the main percentiles (10th, 25th, 50th,75th, 90th) of the distribution are reported. The final six columns report the same entries of the sixth to 10th columns, when $K_{max}=4k$.

mean, the median and selected percentiles of the distribution across replications of these ratios. Hence, median, mean and percentile ratio values that are smaller than 1 indicate a superior performance of U-MIDAS.

The results can be summarized as follows: in the majortiy of cases, U-MIDAS forecasts perform better than MIDAS for small differences in sampling frequency, $k=3$. Exceptions can be found in those simulations with low persistence and a large maximum number of high frequency lags ($K_{\max}=4\times 3=12$). If the difference in sampling frequencies is large ($k=12, 60$), MIDAS outperforms U-MIDAS for almost each value of persistence and interrelatedness. A likely reason for this pattern of results is that, when k is large, U-MIDAS becomes heavily parameterized, notwithstanding BIC lag length selection, and imprecise estimation affects the forecasting accuracy. In contrast, when $k=3$, the number of U-MIDAS parameters is limited and their estimates precise, and the additional flexibility that is allowed by U-MIDAS yields a better forecasting performance than MIDAS.

An alternative explanation for the poor performance of U-MIDAS with large k could be that the BIC selects models that are too parsimonious, and therefore relevant regressors are omitted. To assess whether this is so, we have repeated the entire exercise by using the Akaike information criterion, which puts a lower loss on the number of parameters than does the BIC and therefore in general yields a larger estimate for k. It turns out (in results that are available on request) that the performance of U-MIDAS further deteriorates, confirming that the problem is overparameterization.

3.4. Alternative data-generating process with non-recursive vector auto-regressive structure

To check the robustness of the results that we have obtained so far, we now consider an alternative HF DGP that allows for reverse causality from y to x by setting $\delta_h \neq 0$. The values of δ_l and δ_h cannot be chosen freely but must be selected to ensure a non-explosive solution, which ensures stationarity of both y and x. To ensure a stable solution, the condition

$$\det\left\{ I_2 - \begin{pmatrix} \rho & \delta_l \\ \delta_h & \rho \end{pmatrix} z \right\} \neq 0 \qquad \text{for } |z| \leqslant 1 \tag{22}$$

must hold. In general, the solution is

$$z_{1,2} = \frac{1}{\rho^2 - \delta_h\delta_l}\{\rho \pm \sqrt{(\delta_h\delta_l)}\},$$

depending heavily on the relative size of δ_l and δ_h. For simplicity, we assume that both processes are equally important as each other, so that $\delta_l=\delta_h=\delta$. This implies that the solutions for the characteristic roots are

$$z_{1,2} = \frac{\rho \pm \delta}{(\rho+\delta)(\rho-\delta)}. \tag{23}$$

These roots lie outside the unit circle, if $1+\delta>\rho$ and $1-\delta>\rho$. Thus, if we further assume that the series y_t and x_t have a positive effect on each other, $\delta>0$, only the restriction $1-\delta>\rho$ is binding. Hence, depending on the selection of $\rho=\{0.1, 0.5, 0.9\}$, we select the (ρ, δ)-couples

$$\left.\begin{array}{c} \{0.1, 0.1\}, \{0.1, 0.4\}, \{0.1, 0.8\}, \\ \{0.5, 0.1\}, \{0.5, 0.2\}, \{0.5, 0.4\}, \\ \{0.9, 0.01\}, \{0.9, 0.04\}, \{0.9, 0.08\}, \end{array}\right\} \tag{24}$$

with varying degrees of persistence and interrelatedness. The results on the relative forecasting performance of U-MIDAS and MIDAS can be found in Table 2.

Table 2. Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: non-recursive HF VAR(1) process)†

ρ	δ_l	δ_h	k	Relative performance ($K_{max}=k$)						Relative performance ($K_{max}=4k$)					
				Mean	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*	*Mean*	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*
0.1	0.1	0.1	3	1.01	0.96	0.99	1.01	1.03	1.07	1.01	0.96	0.99	1.01	1.04	1.07
0.1	0.1	0.1	12	1.01	0.97	0.99	1.01	1.03	1.06	1.01	0.96	0.98	1.01	1.03	1.06
0.1	0.1	0.1	60	1.01	0.97	0.99	1.01	1.03	1.06	—	—	—	—	—	—
0.1	0.4	0.4	3	1.01	0.97	0.99	1.01	1.03	1.06	1.01	0.97	0.99	1.01	1.03	1.07
0.1	0.4	0.4	12	1.01	0.96	0.99	1.01	1.03	1.06	1.02	0.97	0.99	1.01	1.04	1.07
0.1	0.4	0.4	60	1.01	0.97	0.99	1.01	1.03	1.06	—	—	—	—	—	—
0.1	0.8	0.8	3	1.02	0.96	0.99	1.01	1.04	1.08	1.01	0.96	0.99	1.01	1.05	1.08
0.1	0.8	0.8	12	1.01	0.97	0.99	1.01	1.04	1.07	1.02	0.96	0.99	1.01	1.04	1.08
0.1	0.8	0.8	60	1.01	0.97	0.99	1.01	1.04	1.07	—	—	—	—	—	—
0.5	0.1	0.1	3	0.94	0.83	0.89	0.94	0.99	1.04	1.01	0.96	0.99	1.01	1.04	1.07
0.5	0.1	0.1	12	1.05	0.97	1.00	1.05	1.09	1.13	1.05	0.97	1.00	1.04	1.08	1.13
0.5	0.1	0.1	60	1.05	0.97	1.00	1.04	1.09	1.14	—	—	—	—	—	—
0.5	0.2	0.2	3	0.94	0.83	0.89	0.94	0.98	1.03	1.01	0.96	0.99	1.01	1.04	1.07
0.5	0.2	0.2	12	1.05	0.97	1.01	1.05	1.09	1.12	1.05	0.96	1.01	1.05	1.08	1.13
0.5	0.2	0.2	60	1.05	0.96	1.00	1.05	1.09	1.14	—	—	—	—	—	—
0.5	0.4	0.4	3	0.98	0.90	0.93	0.98	1.03	1.07	1.02	0.96	0.99	1.02	1.05	1.09
0.5	0.4	0.4	12	1.04	0.96	1.00	1.04	1.07	1.12	1.04	0.97	1.00	1.04	1.08	1.13
0.5	0.4	0.4	60	1.04	0.96	1.00	1.03	1.07	1.11	—	—	—	—	—	—
0.9	0.0	0.01	3	0.91	0.81	0.86	0.91	0.96	1.02	0.99	0.90	0.94	0.99	1.03	1.07
0.9	0.0	0.01	12	1.09	0.93	1.01	1.08	1.18	1.25	1.12	0.98	1.03	1.11	1.18	1.28
0.9	0.0	0.01	60	1.23	1.03	1.11	1.21	1.32	1.45	—	—	—	—	—	—
0.9	0.0	0.04	3	0.93	0.83	0.88	0.92	0.98	1.03	0.99	0.90	0.94	0.98	1.02	1.07
0.9	0.0	0.04	12	1.06	0.91	0.98	1.06	1.14	1.22	1.12	0.97	1.03	1.11	1.19	1.27
0.9	0.0	0.04	60	1.22	1.04	1.11	1.20	1.32	1.44	—	—	—	—	—	—
0.9	0.1	0.08	3	0.99	0.93	0.96	0.99	1.02	1.05	1.00	0.94	0.97	0.99	1.02	1.05
0.9	0.1	0.08	12	0.99	0.92	0.96	0.99	1.03	1.06	0.99	0.92	0.96	0.99	1.02	1.06
0.9	0.1	0.08	60	1.00	0.93	0.97	1.00	1.03	1.06	—	—	—	—	—	—

†For notes to the table, see Table 1.

The results are very similar to those for the benchmark case. With a high degree of persistence, the forecasting performance of U-MIDAS is better than that of MIDAS when $k=3$, whereas in general MIDAS dominates for $k=\{12, 60\}$.

3.5. Using mixed data sampling as data-generating process

As another robustness check, we now carry out Monte Carlo simulations using a MIDAS regression equation with exponential Almon lag as the DGP. Thus, we are in a case that favours *a priori* the restricted MIDAS regression over the U-MIDAS. The DGP is

$$y_{t\times k+k}=\beta_0+\beta_1 B(L,\theta)x_{t\times k+k-1}+\varepsilon_{t\times k+k}, \tag{25}$$

with the lag polynomial $B(L,\theta)$ defined as in equation (16). We set $k=\{3, 12, 60\}$ to mimic the design in the previous sections. When simulating $y_{t\times k+\mathrm{es}\times k}$, we use $T=100$ and the sets $\theta_1=0.7$ and $\theta_2=\{-0.025, -0.05, -0.3\}$. The monthly indicator x_t is generated as an AR(1) process, with persistence equal to 0.9. Given these nine different DGPs, we again use U-MIDAS and MIDAS as before to forecast $y_{t\times k+\mathrm{es}\times k}$ and we evaluate their performance by MSE.

To fix the starting values of θ_1 and θ_2, we compute the residual sum of squares in each replication for alternative parameter pairs in the sets $\theta_1=\{-0.5, 0.0, 0.5\}$ and $\theta_2=\{-0.01, -0.1, -0.5, -1\}$. As initial values, we then choose those θ_1 and θ_2 that minimize the residual sum of squares. Results on the forecasting performance can be found in Table 3.

Interestingly, Table 3 highlights that, even in a set-up that is favourable to restricted MIDAS, as long as the frequency mismatch is small ($k=3$) and therefore the number of parameters to be estimated is low, U-MIDAS still yields a better forecasting performance than MIDAS. Restricted MIDAS is of course strongly outperforming in the case of very large discrepancies in the sampling frequency.

A likely cause for the pattern detected is as follows. U-MIDAS provides more flexibility than MIDAS, since as we have seen the latter is typically nested in the former. Hence, as long as the number of parameters is quite limited with respect to the sample size, as in the case $k=3$, U-MIDAS is a nesting model and as such it is not surprising that it forecasts slightly better than MIDAS. However, in some cases U-MIDAS is much better than MIDAS. In general, in these cases MIDAS estimation is problematic. When we do not know the DGP, even starting from reasonable starting values can lead to misleading results, especially when misspecification occurs or when a substantial amount of past information is included. Therefore, computational problems for the NLS estimator of the MIDAS parameters can further add to the advantages of OLS-based U-MIDAS.

Both positive U-MIDAS features (nesting and simplicity of estimation) are, however, more than counterbalanced by the curse of dimensionality when k is large, thus making MIDAS the clear winner in that case.

From Table 3, we can also note that results change substantially when considering $K_{\max}=4k$ instead than $K_{\max}=k$. An explanation for this evidence can be that when $K_{\max}$ is much larger than the actual number of lags there can be problems in the estimation of the MIDAS polynomial, which can end up assigning too much weight to lags that should instead receive zero weight.

3.6. Adding more dynamics: a VAR(2) process as data-generating process

As a final robustness check, we consider as DGP an HF VAR(2) process:

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix}=\begin{pmatrix} \rho_1 & \delta_{1l} \\ \delta_{1h} & \rho_1 \end{pmatrix}\begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix}+\begin{pmatrix} \rho_2 & \delta_{2l} \\ \delta_{2h} & \rho_2 \end{pmatrix}\begin{pmatrix} y_{t-2} \\ x_{t-2} \end{pmatrix}+\begin{pmatrix} e_{y,t} \\ e_{x,t} \end{pmatrix}. \tag{26}$$

Table 3. Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: MIDAS)†

θ_1	θ_2	k	Relative performance ($K_{max}=k$)						Relative performance ($K_{max}=4k$)					
			Mean	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*	*Mean*	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*
0.7	−0.025	3	0.95	0.86	0.91	0.96	1.01	1.04	0.49	0.37	0.42	0.48	0.56	0.64
0.7	−0.025	12	1.20	1.00	1.07	1.19	1.31	1.43	0.89	0.72	0.78	0.88	0.98	1.09
0.7	−0.025	60	1.27	1.06	1.15	1.25	1.37	1.48	—	—	—	—	—	—
0.7	−0.05	3	0.90	0.80	0.85	0.90	0.96	1.00	0.51	0.38	0.44	0.50	0.58	0.65
0.7	−0.05	12	1.09	0.98	1.03	1.09	1.15	1.21	1.14	1.01	1.08	1.14	1.20	1.28
0.7	−0.05	60	1.14	1.01	1.07	1.14	1.21	1.28	—	—	—	—	—	—
0.7	−0.3	3	0.98	0.91	0.95	0.98	1.01	1.04	1.02	0.96	0.99	1.01	1.05	1.08
0.7	−0.3	12	1.03	0.96	1.00	1.03	1.07	1.10	1.03	0.96	0.99	1.03	1.06	1.11
0.7	−0.3	60	1.03	0.96	0.99	1.03	1.07	1.10	—	—	—	—	—	—

†The first three columns show the parameter specification for the DGP in equation (25). The entries of the middle six columns report the performance of the out-of-sample MSE(U-MIDAS) relative to out-of-sample MSE(MIDAS), when $K_{max}=k$, with the actual lag length determined by the BIC for U-MIDAS. MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication; then in the fourth column the mean of the distribution of these ratios is reported, and in the next five columns the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported. The final six columns report the same entries of the fourth to ninth columns, when $K_{max}=4k$.

Table 4. Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: a recursive HF VAR(2) process)†

ρ_1	δ_{1l}	δ_{1h}	ρ_2	δ_{2l}	δ_{2h}	m	*Relative performance ($K_{\max}=4k$)*					
							Mean	*10th percentile*	*25th percentile*	*Median*	*75th percentile*	*90th percentile*
0.10	0.50	0	0.10	0.25	0	3	0.97	0.87	0.93	0.98	1.03	1.07
0.10	0.50	0	0.10	0.25	0	12	0.96	0.86	0.92	0.98	1.03	1.06
0.10	1.00	0	0.10	0.50	0	3	0.96	0.85	0.92	0.97	1.02	1.07
0.10	1.00	0	0.10	0.50	0	12	0.96	0.86	0.93	0.98	1.02	1.08
0.25	0.25	0	0.50	0.50	0	3	0.96	0.83	0.90	0.97	1.03	1.09
0.25	0.25	0	0.50	0.50	0	12	1.06	0.91	0.99	1.06	1.14	1.24
0.25	0.50	0	0.50	1.00	0	3	0.97	0.83	0.91	0.98	1.04	1.08
0.25	0.50	0	0.50	1.00	0	12	1.07	0.89	0.99	1.07	1.16	1.24
0.50	0.50	0	0.25	0.25	0	3	0.83	0.43	0.62	0.93	1.01	1.06
0.50	0.50	0	0.25	0.25	0	12	0.93	0.38	0.84	1.04	1.12	1.21
0.50	1.00	0	0.25	0.50	0	3	0.83	0.43	0.62	0.94	1.01	1.06
0.50	1.00	0	0.25	0.50	0	12	0.90	0.34	0.61	1.04	1.14	1.22

†The first three columns show the parameter specification for the DGP in equation (26). The entries of the final six columns report the performance of the out-of-sample MSE(U-MIDAS) relative to the out-of-sample MSE(MIDAS), when $K_{\max}=4k$, with actual lag length determined by the BIC for U-MIDAS. MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication; then in the eighth column the mean of the distribution of these ratios is reported, and in the ninth to 13th columns the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.

We choose different combinations of persistence parameters ρ_1 and ρ_2, and for the interaction parameters δ_{l1} and δ_{l2}, whereas we fix δ_{h1} and δ_{h2} equal to 0, such that the HF variable is not affected by the LF variable. All the parameters combinations (which are reported in Table 4) guarantee the stability of the system. The experimental design then follows closely that described in Section 3.1. Since the dynamic of the processes is designed to involve more lags, we consider for our results only the case in which up to $4k$ months of past information are included, with the actual lag length determined by the BIC for U-MIDAS. Therefore, for computational issues, we limit our analysis to the sampling frequencies $k=\{3,12\}$.

To allow for the additional dynamics, the models that we shall compare are as follows.

(a) MIDAS with an AR component, with a common factor specification imposed:

$$y_{t\times k}=\beta_0+\lambda_1 y_{t\times k-k}+\lambda_2 y_{t\times k-2k}+\beta_1(1-\lambda_1 L^k-\lambda_2 L^{2k})B(L,\theta)x_{t\times k-1}+\varepsilon_{t\times k}, \tag{27}$$

where the polynomial $B(L,\theta)$ is the exponential Almon lag defined as in equation (16). The model is estimated by using NLS. We apply the same coefficient restrictions and starting values as in Section 3.1.

(b) U-MIDAS with an AR term:

$$y_{t\times k}=\mu_0+\mu_1 y_{t\times k-k}+\mu_2 y_{t\times k-2k}+\psi(L)x_{t\times k-1}+\varepsilon_{t\times k}, \tag{28}$$

with lag polynomial $\psi(L)=\Sigma_{j=0}^{K}\psi_j L^j=\psi_0+\psi_1 L+\ldots+\psi_K L^K$. The coefficients μ_0, μ_1, μ_2 and $\psi(L)$ are estimated by OLS. To specify the lag order of $\psi(L)$, K, we use the BIC.

The results in Table 4 show that U-MIDAS performs quite well, not only for $k=3$ as in the previous cases, but also for $k=12$, with some parameter combinations. The worst performances of U-MIDAS coincide with the case in which the BIC selects a lag length that is too high and therefore causes overparameterization. The underperformance of MIDAS in some cases is again mostly due to computational issues related to the non-linear specification and estimation method.

3.7. Summary of the simulation results

As a general summary of the simulation results, we can say that, as long as the dependent variable is sufficiently persistent and the frequency mismatch with the explanatory variables limited, there is strong evidence that the U-MIDAS specification is better than MIDAS. The main issues with MIDAS models are related to the non-linearity in the restrictions, which make estimation problematic. U-MIDAS models, in contrast, are easily estimated by OLS and do not require any kind of common factor restriction, making estimation very fast and non-problematic. However, sometimes the models can be overparameterized, in particular with large mismatches in sampling frequency, which causes their forecasting performance to deteriorate.

4. Nowcasting and short-term forecasting US gross domestic product growth

In this section we assess the MIDAS and U-MIDAS methods in terms of their empirical nowcasting and forecasting performance. Specifically, we consider nowcasting and forecasting quarterly US GDP growth up to four quarters ahead, using a set of selected monthly indicators as in Clements and Galvão (2009) and Stock and Watson (2003). The indicators are the 10 components of the composite leading indicator provided by the Conference Board, starting in January 1959 and ending in July 2011. Table 5 provides a complete description of the data. Since these are leading indicators, they can be expected to perform better at forecasting than at nowcasting GDP growth. However, since we also consider their lagged values, up to $K_{\max}$ periods, they can also be useful for nowcasting.

We adopt a recursive approach, with the first evaluation quarter fixed at 1985, quarter 1, and the last at 2011, quarter 1, for a total of 105 evaluation samples. For each quarter, we compute three nowcasts for monthly horizons $h_m = 0, 1, 2$, at the release date of the Conference Board leading indicator, when the Conference Board releases the updated values for the 10 components (the Conference Board releases its indicator around the 20th of each month). Moreover, we compute forecasts for GDP growth up to four quarters ahead (in terms of monthly horizons $h_m = 3, 6, 9, 12$). The information that is included in our exercise therefore includes values for

Table 5. Monthly US data

Name	*Monthly indicator*
M2	Real money supply M2
stock	Stock price index (500 common stocks)
hours	Average weekly hours in manufacturing
ordersn	Orders: non-defence capital goods
ordersc	Orders: consumer goods and materials
building	Building permits
claims	New claims for unemployment insurance
vendor	Vendor performance diffusion index
spread	Term spread (10 year—Federal funds)
expect	Consumer confidence index (University of Michigan)

all the components up to the previous month, except for the University of Michigan consumer confidence indicator, which is already available for the same month. The data set is a final data set, but we replicate the ragged edge structure due to different publication lags of the monthly series, as just described. Thus, we take into account the various information sets that are available at each point in time in which the nowcasts are computed. Each month, the models are re-estimated by using all the data that are available at that point in time. To specify the MIDAS and U-MIDAS regressions, we consider a maximum number of monthly lags of the indicators equal to $K_{\max} = 6, 12, 24$ months.

To compare the two mixed frequency approaches, we consider their out-of-sample performance for nowcasting and forecasting GDP growth by using the various monthly indicators, relative to a benchmark. In our exercise, the benchmark is a quarterly AR model with lag length recursively selected according to the BIC and maximum lag set respectively at two, four and eight quarters.

We compute MSE for each mixed frequency model and for the benchmark over two evaluation samples. Both start in 1985, quarter 1, but one ends in 2006, quarter 4, and the other in 2011, quarter 1, including also the recent crisis period to assess whether this influences the results substantially. A relative MSE that is smaller than 1 indicates a superior performance of the mixed frequency approach with respect to the benchmark AR model. The results are reported in Tables 6 and 7.

The figures that are reported in Table 6 and Table 7 indicate that the mixed frequency models can outperform the quarterly AR benchmark only for few monthly indicators. In particular, according to the results for the first evaluation period, which are depicted in Table 6, it is difficult to beat the benchmark in stable times. However, when the period of crisis is included, the orders of consumption goods, ordersc, building permits, build, new claims for unemployment insurance, claims, the vendor performance diffusion index, vendor, and consumer expectations, expect, yield a relative MSE that is lower than 1 for most of the horizons, except $h_m = 12$, hinting at the usefulness of mixed frequency approaches especially in crisis times.

It is not clear cut from Tables 6 and 7 which one of the mixed frequency approaches is better. Neither U-MIDAS nor restricted MIDAS has a clearly superior performance, with their ranking depending on the horizon and indicator. This could be considered an advantage for U-MIDAS, given its easier specification and estimation.

Finally, about the amount of past information that is included, the best results across specifications are often obtained with $K_{\max} = 12$ or $K_{\max} = 24$. The discrepancies between the results that are obtained with $K_{\max} = 12$ and $K_{\max} = 24$ are minimal, suggesting that there is no need for a very large number of lags to improve the forecasting performance.

Confirming the evidence in the Monte Carlo simulations, MIDAS improves its performance when the maximum number of lags for the HF variable is higher ($K_{\max} = 12$ or $K_{\max} = 24$), whereas the opposite happens to U-MIDAS. For example, in Table 6 U-MIDAS overperforms MIDAS 34 times when $K_{\max} = 6$, but only 26 times when $K_{\max} = 12$ and 13 times when $K_{\max} = 24$. Similarly, in Table 7, U-MIDAS's performance deteriorates when increasing $K_{\max}$, beating MIDAS 40 times when $K_{\max} = 6$, 27 times when $K_{\max} = 12$ and 17 times when $K_{\max} = 24$.

5. Nowcasting and short-term forecasting euro area gross domestic product growth

To provide additional evidence on the relative performance of MIDAS and U-MIDAS we also conduct an evaluation for the euro area GDP growth rate. The data set in this case is an updated version of that in Foroni and Marcellino (2013), extracted from the Eurostat database of principal European economic indicators and downloaded at the end of May 2011. Quarterly

Table 6. Results for individual indicators relative to an AR benchmark: forecasting performance for US GDP growth†

Indicator	*Method*	*Results for the following values of h_m ($K_{max}=6$):*							*Results for the following values of h_m ($K_{max}=12$):*							*Results for the following values of h_m ($K_{max}=24$):*						
		0	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*
M2	MIDAS	1.44	1.41	1.43	1.44	1.28	1.15	1.25	1.20	1.34	1.38	1.40	1.29	1.13	1.25	1.32	1.39	1.38	1.39	1.27	1.13	1.26
	U-MIDAS	1.17	1.04	1.47	1.42	1.28	1.16	1.21	1.17	1.16	1.58	1.52	1.42	1.16	1.23	1.19	1.22	1.54	1.51	1.31	1.13	1.21
stock	MIDAS	1.14	1.18	1.09	1.10	1.12	1.02	1.16	1.14	1.16	1.04	1.11	1.13	**0.98**	1.14	1.09	1.23	1.02	1.09	1.10	0.98	1.17
	U-MIDAS	1.26	1.24	1.20	1.05	1.17	1.04	1.04	1.24	1.23	1.21	1.07	1.17	1.05	1.07	1.31	1.21	1.25	1.12	1.16	0.99	***1.03***
hours	MIDAS	1.29	1.18	1.05	0.98	1.07	1.09	1.12	1.25	1.17	1.04	0.98	1.06	1.07	1.10	1.25	1.19	1.05	0.97	1.04	1.06	1.10
	U-MIDAS	1.54	1.43	1.07	0.99	1.06	1.11	1.08	1.49	1.36	1.04	1.01	1.06	1.11	1.09	1.47	1.40	1.11	1.00	1.08	1.08	1.06
ordersn	MIDAS	1.37	1.14	1.10	0.97	1.10	1.02	1.10	1.28	1.15	1.11	0.96	1.10	1.00	1.09	1.29	1.18	1.10	0.95	1.07	1.00	1.06
	U-MIDAS	1.58	1.36	1.30	0.93	1.01	1.02	***1.03***	1.53	1.32	1.26	0.92	1.02	1.20	1.06	1.52	1.34	1.28	0.93	1.03	1.15	***1.03***
ordersc	MIDAS	0.93	1.14	1.02	1.11	1.07	**0.99**	1.05	***0.90***	1.05	1.01	1.11	1.07	**0.98**	1.04	**0.92**	1.05	1.01	1.13	1.05	***0.97***	1.05
	U-MIDAS	0.95	1.14	1.00	1.11	1.09	1.00	1.05	1.03	1.18	0.98	1.13	1.08	1.02	1.07	1.04	1.21	1.04	1.13	1.08	1.00	1.04
building	MIDAS	1.01	0.99	1.03	1.00	**0.96**	1.08	1.21	1.00	***0.89***	0.91	**0.89**	**0.97**	1.05	1.21	1.02	***0.89***	**0.90**	***0.87***	***0.95***	1.04	1.21
	U-MIDAS	1.02	0.99	**0.91**	0.97	1.16	1.13	1.22	1.00	0.98	***0.89***	0.93	1.16	1.12	1.26	1.01	0.99	0.92	0.98	1.10	1.08	1.22
claims	MIDAS	0.95	1.04	1.15	1.03	1.01	1.00	1.04	1.00	1.00	1.13	1.01	1.01	**0.98**	***1.03***	1.00	1.00	1.12	1.00	1.01	0.98	1.03
	U-MIDAS	***0.90***	1.00	1.11	**0.88**	1.05	0.99	1.03	1.00	0.94	1.06	0.96	1.05	1.01	1.05	0.96	0.94	1.06	0.96	1.05	0.98	1.03
vendor	MIDAS	0.98	0.99	0.95	0.94	1.15	1.15	1.16	0.97	0.98	0.95	0.95	1.17	1.13	1.15	0.98	0.99	0.94	0.94	1.14	1.11	1.15
	U-MIDAS	0.98	**0.94**	1.05	1.07	1.06	1.09	1.16	0.92	0.93	1.05	1.04	1.07	1.10	1.19	0.94	0.94	1.10	1.08	1.12	1.08	1.15
spread	MIDAS	1.59	1.59	1.59	1.58	1.77	1.50	1.42	1.57	1.58	1.65	1.64	1.75	1.44	1.41	1.65	1.65	1.59	1.59	1.68	1.45	1.42
	U-MIDAS	1.47	1.56	1.53	1.54	1.76	1.48	1.41	1.48	1.53	1.50	1.50	1.76	1.55	1.47	1.63	1.67	1.68	1.63	1.76	1.44	1.40
expect	MIDAS	0.99	1.00	1.00	0.98	1.23	1.16	1.19	0.99	1.01	1.06	1.03	1.22	1.12	1.19	1.05	1.06	1.05	1.01	1.19	1.12	1.19
	U-MIDAS	1.11	0.99	1.02	0.99	1.14	1.09	1.15	1.09	0.98	1.02	0.99	1.20	1.17	1.24	1.18	1.08	1.11	1.07	1.24	1.12	1.20

†The table reports the performance of MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single indicator model and is reported in the corresponding row. The table reports the results for the evaluation sample ending in December 2006 (before the financial crisis). The table reports the results for various specifications of past information included. The benchmark is an AR model with lag length selected according to the BIC. The numbers in bold represent the best performance for each horizon and each specification. The numbers in bold and italics are the best for each horizon across different specifications. (Sample: first quarter 1985 to fourth quarter 2006.)

Table 7. Results for individual indicators relative to an AR benchmark: forecasting performance for US GDP growth†

Indicator	*Method*	*Results for the following values of h_m ($K_{\max}=6$):*							*Results for the following values of h_m ($K_{\max}=12$):*							*Results for the following values of h_m ($K_{\max}=24$):*						
		0	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*
M2	MIDAS	1.58	1.55	1.44	1.42	1.26	1.16	1.19	1.39	1.47	1.42	1.40	1.35	1.15	1.18	1.44	1.47	1.34	1.33	1.28	1.14	1.19
	U-MIDAS	1.27	1.17	1.38	1.33	1.24	1.17	1.17	1.29	1.25	1.49	1.45	1.38	1.18	1.19	1.28	1.27	1.41	1.39	1.28	1.14	1.17
stock	MIDAS	1.11	1.02	0.97	0.92	1.00	1.00	1.10	1.10	0.99	0.89	0.97	1.02	0.98	1.09	1.04	1.03	0.90	0.92	0.98	0.96	1.10
	U-MIDAS	1.08	1.06	0.95	0.88	1.06	0.99	1.02	1.08	1.07	0.99	0.92	1.08	1.02	1.05	1.12	1.05	0.98	0.92	1.06	0.96	***1.02***
hours	MIDAS	1.23	1.15	1.01	0.96	1.01	0.99	1.04	1.22	1.16	1.04	0.99	1.04	0.98	**1.03**	1.19	1.15	1.00	0.95	0.99	0.97	1.03
	U-MIDAS	1.38	1.28	0.86	0.88	1.00	0.99	1.03	1.35	1.24	0.86	0.91	1.02	1.00	1.05	1.30	1.24	0.88	0.88	1.00	0.98	1.03
ordersn	MIDAS	1.16	1.05	0.92	0.89	1.03	1.00	1.10	1.09	1.06	0.97	0.92	1.04	0.99	1.07	1.08	1.05	0.92	0.87	0.99	0.98	1.05
	U-MIDAS	1.30	1.24	1.15	0.93	0.99	1.00	1.04	1.29	1.23	1.14	0.95	1.02	1.12	1.06	1.26	1.22	1.11	0.92	1.00	1.07	1.03
ordersc	MIDAS	**0.71**	0.90	0.78	0.86	0.94	0.97	1.05	***0.69***	0.85	0.81	0.89	0.96	0.96	1.04	***0.69***	0.84	***0.77***	0.86	0.93	0.95	1.04
	U-MIDAS	0.73	0.91	**0.78**	0.89	0.97	0.97	1.05	0.80	0.95	**0.80**	0.91	0.99	1.00	1.07	0.79	0.95	0.80	0.87	0.97	0.97	1.04
building	MIDAS	1.01	0.90	0.88	0.83	**0.83**	0.96	1.09	1.03	**0.82**	0.82	**0.75**	**0.84**	**0.95**	1.08	1.01	***0.80***	***0.77***	***0.70***	***0.81***	***0.94***	1.08
	U-MIDAS	0.92	0.89	0.78	0.81	0.95	0.98	1.11	0.90	0.87	**0.80**	0.82	0.97	0.98	1.14	0.88	0.86	0.78	0.81	0.94	0.95	1.14
claims	MIDAS	0.85	0.92	0.87	0.86	0.95	0.99	1.05	0.88	0.90	0.90	0.89	0.96	0.98	1.04	0.86	0.87	0.86	0.84	0.94	0.97	1.04
	U-MIDAS	0.80	0.88	0.83	0.84	1.00	0.98	1.04	0.89	0.86	0.84	0.84	1.02	1.01	1.07	0.84	0.84	0.81	0.80	1.00	0.96	1.04
vendor	MIDAS	1.00	1.00	0.93	0.92	1.07	1.08	1.11	0.99	1.01	0.97	0.97	1.10	1.06	1.10	0.98	1.00	0.92	0.92	1.06	1.04	1.10
	U-MIDAS	1.01	0.97	1.02	1.02	1.02	1.01	1.09	0.95	0.97	1.04	1.03	1.05	1.06	1.14	0.94	0.96	1.03	1.01	1.06	1.04	1.11
spread	MIDAS	1.54	1.54	1.42	1.41	1.54	1.30	1.23	1.55	1.56	1.52	1.52	1.55	1.27	1.22	1.58	1.58	1.41	1.41	1.47	1.26	1.22
	U-MIDAS	1.42	1.51	1.39	1.39	1.53	1.33	1.24	1.44	1.50	1.41	1.40	1.57	1.38	1.29	1.52	1.54	1.47	1.43	1.52	1.29	1.23
expect	MIDAS	0.88	0.88	0.82	**0.80**	0.97	0.98	1.04	0.90	0.90	0.89	0.87	0.98	0.97	1.04	0.92	0.93	0.84	0.83	0.95	0.97	1.04
	U-MIDAS	0.96	**0.87**	0.84	0.82	0.92	**0.95**	**1.02**	0.96	0.88	0.87	0.85	0.96	0.99	1.07	1.00	0.94	0.89	0.87	0.97	0.97	1.04

†The table reports the performance of MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single indicator model and is reported in the corresponding row. The table reports the results for the evaluation sample ending in March 2011 (with the financial crisis included in the sample). The table reports the results for various specifications of past information included. The benchmark is an AR model with lag length selected according to the BIC. The numbers in bold represent the best performance for each horizon and each specification. The numbers in bold and italics are the best for each horizon across different specifications. (Sample: first quarter of 1985 to first quarter of 2011.)

GDP is available from 1996, quarter 1, until 2010, quarter 4, whereas the roughly 140 monthly indicators are from January 1996 to at most May 2011 (depending on the publication delay, there is a different number of missing observations for each series at the end of the sample). Generally, the monthly series include consumer and producer price index by sector, industrial production and (deflated) turnover indices by sector, car registrations, new orders received index, business and consumers surveys with their components, sentiment indicators, unemployment indices, monetary aggregates, interest and exchange rates. The main source of the indicators is the Eurostat data set 'Euro indicators/PEEIs'. The complete list of variables is available on request.

When analysing the nowcasting performance, we adopt a recursive approach as for the USA, with the first evaluation quarter fixed at 2003, quarter 1, and the last at 2010, quarter 4, for a total of 32 evaluation samples. We also consider a shorter sample, starting in 2003, quarter 1, and ending in 2006, quarter 4, to exclude the crisis period. For each quarter, we compute three nowcasts ($h_m = 0, 1, 2$) and four forecasts, up to 1 year ahead ($h_m = 3, 6, 9, 12$).

To specify the MIDAS and U-MIDAS regressions, we consider a maximum number of monthly lags of the indicators equal to $K_{\max} = 6$. We also evaluated $K_{\max} = 12$, but because of the short sample size the results in terms of MSE were worse.

To start with, we compute the ratio of the MSE of each U-MIDAS single-indicator model relative to the MSE of the corresponding MIDAS model. Then, we report in panel (a) of Table 8 different statistics of the distribution of these ratios (the mean, median and key percentiles). A relative MSE that is smaller than 1 indicates a superior performance of the U-MIDAS approach. In panel (b), instead, we compare the nowcasting and forecasting performance for GDP growth of the two mixed frequency approaches relative to a quarterly AR benchmark, whose lag length is recursively selected according to the BIC with a maximum lag set at two quarters (corresponding to $K_{\max} = 6$). Again, we first compute the ratio of the MSE of each model relative to the benchmark, and then report in Table 8 summary statistics of the distribution. A relative MSE that is smaller than 1 indicates a superior performance of the mixed frequency approach.

The median results in panel (a) of Table 8 indicate that at very short horizons more than half of the U-MIDAS models perform better than the corresponding restricted models, in the precrisis sample. Including the crisis, the restricted MIDAS performs better at short horizons (up to one quarter ahead). This happens because the estimates of the U-MIDAS parameters are influenced substantially by the dramatic drop and subsequent recovery of the GDP in 2008, quarter 4, and 2009, quarter 1.

Overall, the results are in line with those with simulated data and similar to those obtained for the USA. It is also worth noting that for at least 25% of the indicators the U-MIDAS approach produces more precise nowcasts than MIDAS, with even larger values in the precrisis period.

The evidence of a slightly better performance of U-MIDAS than MIDAS for nowcasting in the precrisis sample emerges also from panel (b) of Table 8, reporting the average and median MSE of MIDAS and U-MIDAS relative to the AR benchmark. However, before the crisis, on average both methods were dominated by the AR process, stressing the importance of an appropriate indicator selection.

Table 8 also highlights striking differences before and after the crisis. As already noted in Foroni and Marcellino (2013), it is impossible to outperform a naive benchmark in the period up to 2006 on average across all indicators. However, during the crisis, the use of monthly information becomes very important, and both mixed frequency approaches clearly outperform the benchmark, for both nowcasting and forecasting.

Given the large set of monthly indicators under analysis, the poor average performance before the crisis, and the difficulty of selection methods in this large data set context, it is of interest to summarize the information that is contained in the data set via the use of factor models.

Table 8. Results for individual indicators: forecasting performance for euro area GDP growth†

Statistic	*Results for the following values of h_m ($K_{max}=6$)‡:*							*Results for the following values of h_m ($K_{max}=6$)§:*						
	0	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*
(a)														
Average	1.02	1.02	1.08	1.06	1.09	0.96	0.85	1.11	1.02	1.01	0.98	0.96	0.95	0.94
10th percentile	0.77	0.73	0.86	0.86	0.80	0.56	0.37	0.91	0.71	0.79	0.76	0.81	0.79	0.77
25th percentile	0.89	0.85	0.96	0.92	0.90	0.78	0.55	1.02	0.92	0.92	0.90	0.89	0.89	0.87
Median	0.98	0.98	1.02	0.99	1.01	0.95	0.78	1.09	1.04	1.03	1.00	0.95	0.95	0.95
75th percentile	1.08	1.14	1.18	1.10	1.15	1.07	1.00	1.22	1.14	1.11	1.08	1.03	1.00	1.02
90th percentile	1.32	1.36	1.37	1.42	1.52	1.36	1.35	1.33	1.25	1.17	1.16	1.11	1.08	1.09
(b)														
Average U-MIDAS	1.14	1.12	1.19	1.25	1.37	1.53	1.78	0.87	0.80	0.86	0.88	0.71	0.64	0.57
Average MIDAS	1.15	1.13	1.13	1.19	1.32	1.91	3.11	0.81	0.79	0.86	0.90	0.75	0.69	0.64
Median U-MIDAS	1.07	1.04	1.09	1.10	1.17	1.38	1.62	0.94	0.85	0.92	0.93	0.72	0.64	0.57
Median MIDAS	1.08	1.07	1.06	1.10	1.15	1.48	2.09	0.88	0.84	0.88	0.92	0.74	0.67	0.60

†In panel (a), the table reports the performance of MSE(U-MIDAS) relative to MSE(MIDAS). The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each single-indicator model; then the mean, median and the main percentiles of the distribution of the ratio are reported. Panel (b) reports the performance of MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single-indicator model, and then the mean and median are computed for each class of models. The table reports the results for the evaluation sample ending in December 2006 (before the financial crisis) and for the evaluation sample ending in December 2010 (including the crisis). The table reports the results for a maximum lag order of 6. The benchmark is an AR model with lag length selected according to the BIC.
‡Sample: first quarter of 2003 to fourth quarter of 2006.
§Sample: first quarter of 2003 to first quarter of 2010.

Table 9. Results for factor models: forecasting performance for euro area GDP growth†

Model	*Results for the following values of h_m ($K_{\max}=6$)‡:*							*Results for the following values of h_m ($K_{\max}=6$)§:*						
	0	*1*	*2*	*3*	*6*	*9*	*12*	*0*	*1*	*2*	*3*	*6*	*9*	*12*
Factor MIDAS	0.96	0.99	1.52	1.19	1.17	2.03	3.85	0.74	0.56	0.67	0.80	0.84	0.66	0.72
Factor U-MIDAS	1.12	2.61	1.72	1.04	1.07	1.38	1.51	0.42	0.50	0.30	0.49	0.64	0.64	0.62
Quarterly factor	1.31	1.30	1.30	1.30	1.23	1.52	1.66	1.04	0.72	0.72	0.72	0.61	0.69	0.58

†The table reports the performance of MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each type of factor models considered. The table reports the results for the evaluation sample ending in December 2006 (before the financial crisis) and for the evaluation sample ending in December 2010 (including the crisis). The table reports the results for 6 months of past information included. The benchmark is an AR model with lag length selected according to the BIC.
‡Sample: first quarter of 2003 to fourth quarter of 2006.
§Sample: first quarter of 2003 to first quarter of 2010.

The underlying idea is that the HF indicators, being all related to the GDP growth, share a substantial common component, driven by one or few common factors, which can therefore be used as a summary of the relevant HF information. Therefore, we now consider the same mixed frequency approaches but with factors as explanatory variables. The merging of factors models with MIDAS approaches has been proposed by Marcellino and Schumacher (2010), to which we refer for additional details.

Specifically, we extract one factor from the large set of monthly euro area indicators, and estimate equations (18) and (20) with the estimated monthly factor as explanatory variable. Since the data set presents a ragged edge structure, to estimate the factor we adopt a procedure which combines principal component analysis and the EM algorithm, as outlined by Stock and Watson (2002); see also Marcellino and Schumacher (2010) for more details. The number of lags in the unrestricted approach is chosen according to the BIC, again with $K_{\max}=6$.

Table 9 reports the results of the factor MIDAS and factor U-MIDAS, relative to the same AR benchmark as in the case of single-indicator models.

For completeness, we also report the results of a standard quarterly factor model (relative to the benchmark). We employ the standard factor model that was proposed by Stock and Watson (2002) estimated on time-aggregated quarterly data. Below, the variables will be indexed by quarterly time periods t_q . The direct equation for forecasting h_q quarters ahead is

$$y_{t_q+h_q}=\beta_0+\beta(L_q)\hat{f}_{t_q}+\lambda(L_q)y_{t_q}+\varepsilon_{t_q+h_q}, \tag{29}$$

where $\beta(L_q)$ is an unrestricted lag polynomial of lag order P in the quarterly lag operator L_q, and $\lambda(L_q)$ is of order R. The estimation is conducted with a two-step procedure. First, the quarterly data set, which is obtained by aggregating the monthly indicators over time, is used to estimate the quarterly factors $\hat{f}_{t_q}$ by principal component analysis. Second, the estimators $\hat{\beta}_0$, $\hat{\beta}(L_q)$ and $\hat{\lambda}(L_q)$ are obtained by regressing $y_{t_q+h_q}$ onto a constant, $\hat{f}_{t_q}$ and y_{t_q} and lags. Given information in quarter T_q, the forecast then is formed as

$$\hat{y}_{T_q+h_q|T_q}=\hat{\beta}_0+\hat{\beta}(L_q)\hat{f}_{T_q}+\hat{\lambda}(L_q)y_{T_q}. \tag{30}$$

In our application, we choose a quarterly model with one lag of GDP growth ($R=1$) and the number of lags of the factor chosen by the BIC.

The results in Table 9 show that in the sample which ends in 2006 it is very difficult to beat the benchmark, confirming the evidence that was obtained from single-indicator models. Only the restricted version of the MIDAS approach displays some improvements over the benchmark at very short horizons. The picture is completely different when we look at the sample including the crisis. In this case, summarizing the information by means of factors hugely improves the forecasting performance relative to the AR benchmark. Even the standard quarterly factor model has a better performance than the AR model (except at the shortest horizon). Exploiting the monthly information helps even more; the gains with mixed frequency factor models are remarkable at all horizons. Differently from the single-indicator case, the U-MIDAS approach now outperforms the MIDAS model even at short horizons.

In summary, and as for the USA, there is mixed empirical evidence on the relative performance of U-MIDAS and MIDAS for the euro area. However, generally, U-MIDAS is a strong competitor to restricted MIDAS, as it performs better for several HF indicators and forecast horizons. In our euro area application, since small samples are more problematic for heavily parameterized models, the short sample length might penalize U-MIDAS more than MIDAS, even when the specification of the former is based on the BIC. Finally, a factor U-MIDAS approach seems very promising: possibly even better than the factor-MIDAS method that already performed well according to Marcellino and Schumacher (2010).

6. Conclusions

In the recent literature, MIDAS regressions have turned out to be useful reduced form tools for nowcasting LF variables with HF indicators. To avoid parameter proliferation in the case of long HF lags, functional lag polynomials have been proposed. In this paper, we have discussed a variant of the MIDAS approach which does not resort to functional lag polynomials, but rather to simple linear lag polynomials. Compared with the standard MIDAS approach in the literature, these polynomials are not restricted by a certain functional form, and we therefore call the approach U-MIDAS.

We derive U-MIDAS regressions from a linear dynamic model, obtaining a simple and flexible specification to handle mixed frequency data. It can be expected to perform better for forecasting than MIDAS as long as it is not too heavily parameterized, in particular, as long as the differences in sampling frequencies and the maximum number of HF lags are not too large. We have shown that this is indeed so by means of Monte Carlo simulations. U-MIDAS is particularly suited to provide macroeconomic nowcasts and forecasts of quarterly variables, such as GDP growth, given timely observations of monthly indicators like industrial production. In contrast, when daily data are available, our simulation results indicate that MIDAS with functional lag polynomials is preferable to predict quarterly variables.

In the empirical applications on nowcasting and short-term forecasting US and euro area GDP growth, we find no clear-cut ranking of MIDAS and U-MIDAS. However, the latter outperforms the former for several indicators and horizons.

Overall, we do not expect one polynomial specification to be dominant in every case. As U-MIDAS might be a strong competitor, we rather suggest considering it as an alternative to the existing MIDAS approaches.

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Appendix A: Identification of the disaggregate process

Let us consider the LF exact MIDAS model for y (equation (7)):

$$h(L^k)\,\omega(L)y_t = b_1(L)\,\beta(L)\,\omega(L)x_{1t} + \ldots + b_N(L)\,\beta(L)\,\omega(L)x_{Nt} + q(L^k)u_{yt}, \qquad t = k, 2k, 3k, \ldots. \tag{31}$$

We want to determine what and how many HF models are compatible with this LF model, namely, whether the parameters of the generating mechanism of y at HF can be uniquely identified from those at LF. The following discussion is based on Marcellino (1998), to whom we refer for additional details.

To start with, assuming that y follows model (31), we try to identify the $a(L)$ polynomials that can have generated $h(L^k)$. This requires us to analyse all the possible decompositions of $h(L^k)$ into $\beta(L)\,a(L)$.

We have said that at least one h_{si} for each s in expression (6) must be such that $a(h_{si}) = 0$. The other $k-1$ h_{si}s can instead solve either $\beta(h_{si}) = 0$ or also $a(h_{si}) = 0$. Thus, for each s, there are $2^k - 1$ possible 'distributions' of the h_{si}s as roots of $\beta(L)$ and $a(L)$. Hence, we obtain a total of $(2^k - 1)^h$ potential disaggregated AR components, which can be written as

$$\prod_m \left(1 - \frac{1}{h_m} L\right), \tag{32}$$

where the h_ms are the h_{si}s which are considered as roots of $a(l) = 0$. The possible degree of $a(L)$ ranges from h to hk, with $h < p$.

A simple but quite stringent sufficient condition for exact identification of the disaggregate process in our context is the following proposition.

Proposition 1. All the roots of $a(l)=0$ are distinct and positive, or distinct and possibly negative if k is even.

Proof. If $a(l)=0$ has distinct and positive roots, or distinct and possibly negative roots if k is even, then they coincide with those of $h(z)=0$ raised to power of $1/k$, and this exactly identifies the AR component. Once $a(L)$ has been exactly identified, $\beta(L)$ is also unique. Finally, given $\beta(L)$ and since the aggregation operator $\omega(L)$ is known, the polynomials $b_j(L)$ can be also recovered, $j=1,\ldots,N$. □

To conclude, it is worth making a few comments on this result. First, Wei and Stram (1990) discussed more general sufficient *a priori* conditions for one disaggregate model to be identifiable from an aggregate model. Second, the hypothesis of no MA component at the disaggregate level can be relaxed. Marcellino (1998) showed that such an MA component can be uniquely identified if its order is smaller than $p-1$ and the condition in proposition 1 holds. Third, the condition in proposition 1 could be relaxed by imposing constraints on the $b_i(L)$ polynomials. Fourth, when y is multivariate the link between the disaggregate and aggregate models is much more complicated (see Marcellino (1999)), even though conceptually the procedure to recover the disaggregate model is similar to the procedure that we have proposed for the univariate case. Finally, note that the identification problem does not emerge clearly within a Kalman filter approach to interpolation and forecasting, where the underlying assumption is that the disaggregate model is known.

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