Intelligent Systems Programming

Lecture 3: **Adversarial Search**

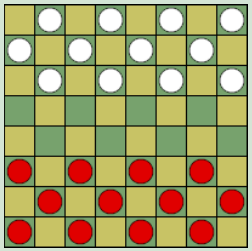
IT University of Copenhagen

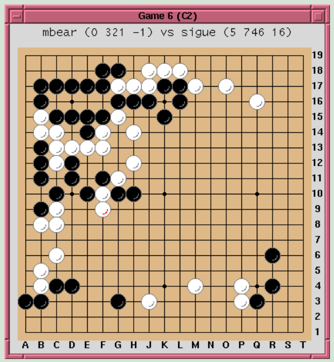
Why Learn About Game Algorithms?

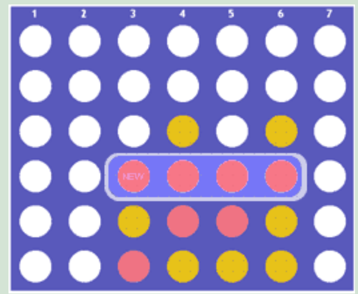
- Game playing is considered intellectual behavior
- **Gamification**
- The algorithms are relevant **control of non-deterministic systems** like nuclear power plants
- The stock market is adversarial

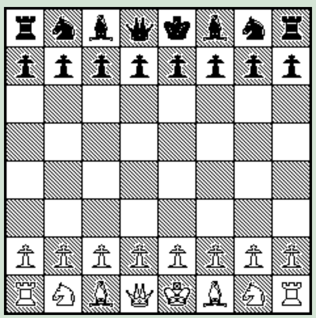
Types of Games

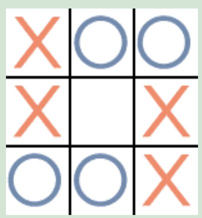
Board Games

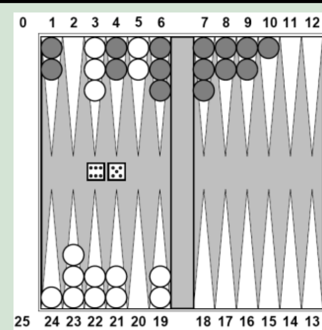


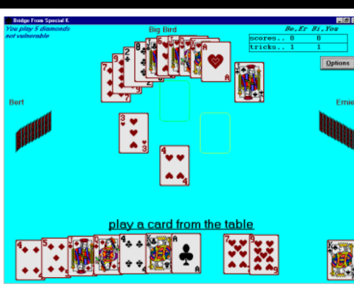












"Physical" Games

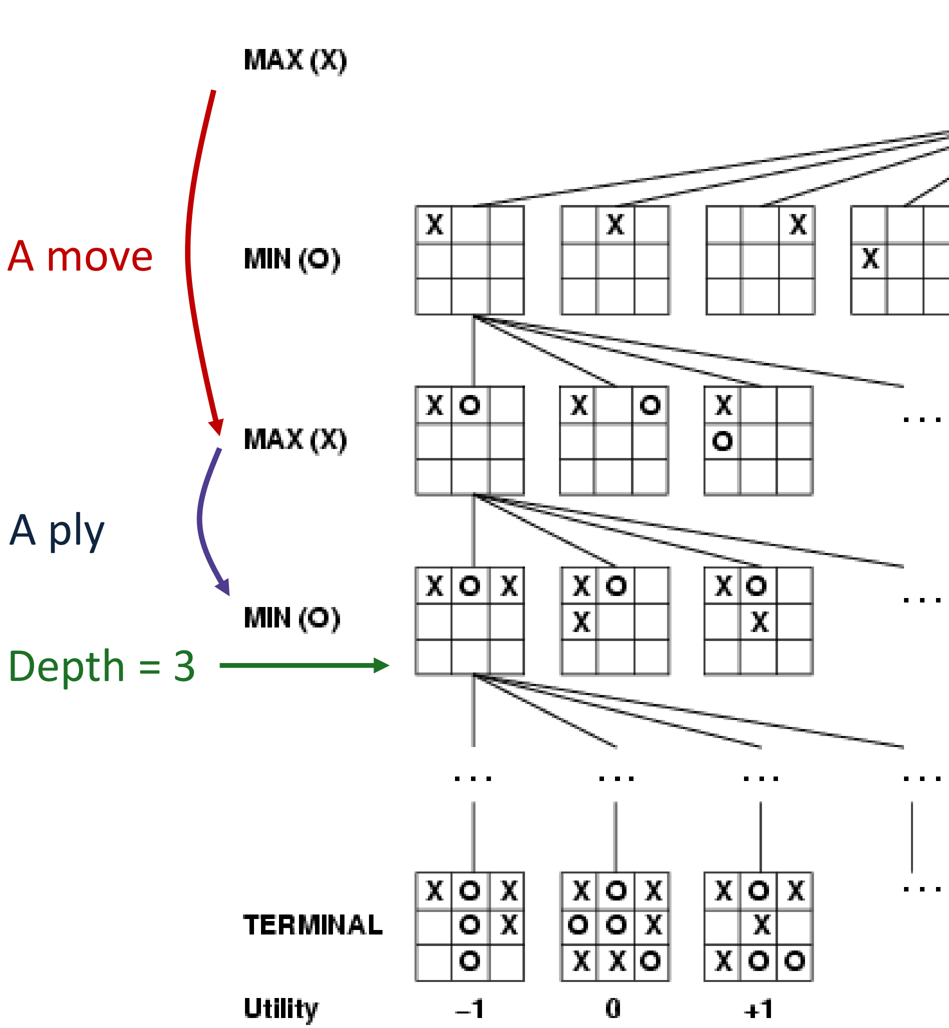
Our Assumptions

- Discrete states
- Turn-taking
- Observable state
- Zero-Sum utility
- Mainly 2-Player
- Mainly deterministic

Today’s Program

- **[12:10-13:00]**
  - Game trees
  - Minimax optimal strategy
    - MINIMAX
    - MINIMAX with alpha-beta pruning
- **[13:10-14:00]**
  - Evaluation functions
  - Cut-off strategies
  - EXPECTIMINIMAX
- **[14:00-16:00]**
  - Exercises

Tic-Tac-Toe Game Tree



- We assume the initial node to be a MAX node
- **Search tree: part of game tree we search to decide an action**

Is a solution a path to a terminal state?

Game Tree

- Initial state S_0
- PLAYER(s)
  - Max node ▲ (us)
  - Min node ▼ (opponent)
- ACTIONS(s)
- RESULT (s, a)
- TERMINAL-TEST(s)
- UTILITY(s, p) (e.g., win +1, draw 0, loss -1)

Optimal Decisions in Games

Maximize worst-case utility for Max!

$$\textsc{Minimax}(s) = \begin{cases} \textsc{Utility}(s, \textsc{Player}(s)) & \text{if } \textsc{Terminal-Test}(s) \\ max_{a \in \textsc{Actions}(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ min_{a \in \textsc{Actions}(s)} \textsc{Minimax}(\textsc{Result}(s,a)) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$

Minimax decision at root =

Action to *a* with highest MINIMAX VALUE

Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state,a))
```

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state, PLAYER(state))
  v ← -∞
  for each a in ACTIONS(state) do
    v ← MAX(v ,MIN-VALUE(RESULT(s,a))
  return v
```

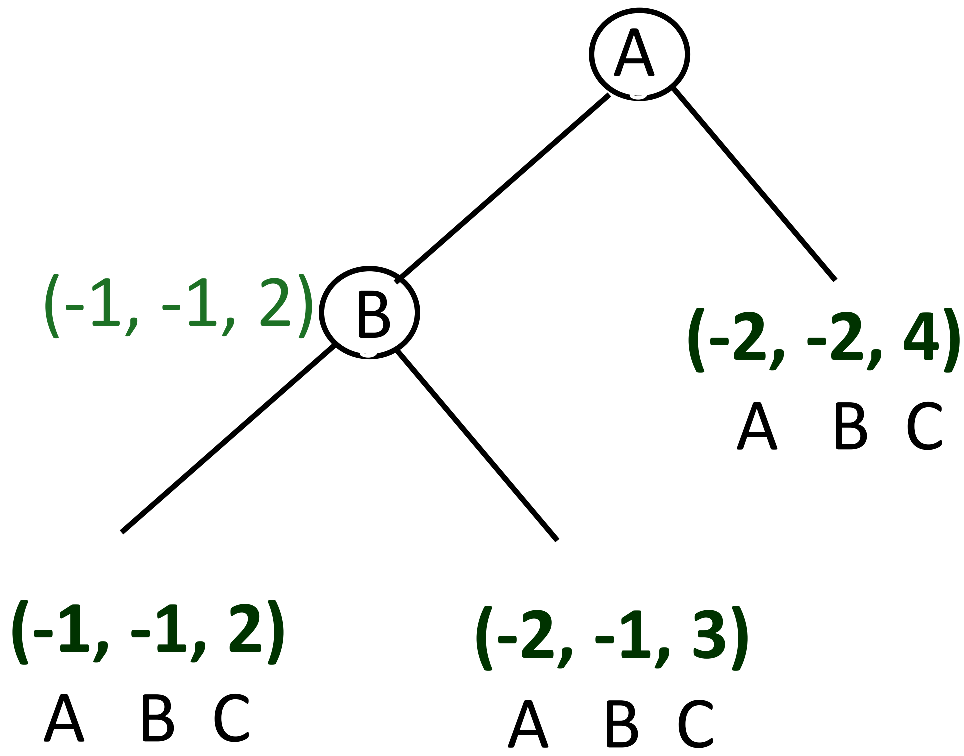
```
function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state, PLAYER(state))
  v ← ∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a))
  return v
```

Complexity

- *b* : max number of successors of a node
- *m* : max depth of tree

- Time: ?
- Space: ?

Multi-Player Alliances



α-β Pruning

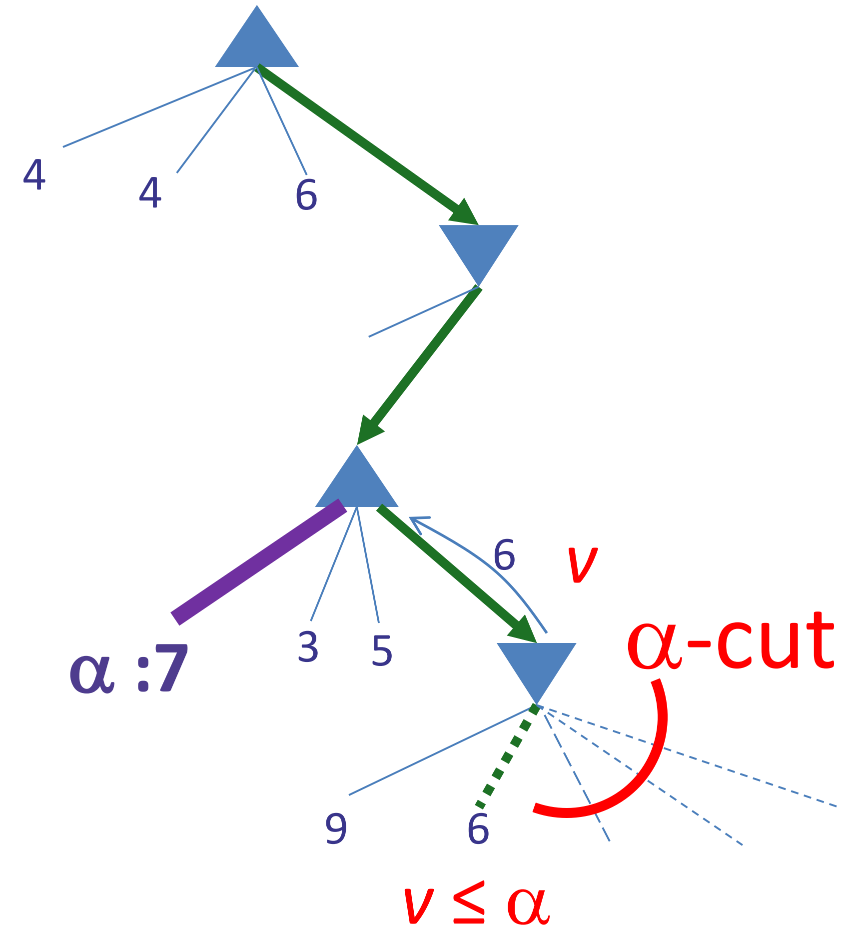
Idea: keep track of MAX and MIN's best choice found so far on the explored path

- α = the value (i.e. highest value) of the best choice for MAX
- β = the value (i.e. lowest value) of the best choice for MIN

Use this to prune the tree

α-cut

$\alpha = \max\{4,4,6,7,3,5\}$
$= 7$



Minimax with α-β Pruning

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, -∞ , +∞ )
  return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state, PLAYER(state))
  v ← -∞
  for each a in ACTIONS(state) do
    v ← MAX(v , MIN-VALUE(RESULT(s , a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state, PLAYER(state))
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v , MAX-VALUE(RESULT(s , a), α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Properties of α-β Pruning

- Does not affect the final result
- Exploring "best" nodes first improve pruning (killer heuristic)
- $O(b^{m/2})$ with perfect ordering
  - doubles reachable search depth!

Break

Imperfect Real-Time Decisions

Standard Approach for MINIMAX

- Use CUTOFF-TEST instead of TERMINAL-TEST
  - e.g., depth-limit
- Use EVAL instead of UTILITY
  - I.e., evaluation function that estimates desirability of position
    1) Order terminal state in the same way as UTILITY
    2) Value strongly correlated with chance of winning
    3) Efficiently computable

Minimax with EVAL and CUTOFF-TEST

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state,a))
```

```
function MAX-VALUE(state) returns a utility value
  if CUTOFF-TEST(state) then return EVAL(state, PLAYER(state))
  v ← -∞
  for each a in ACTIONS(state) do
    v ← MAX(v ,MIN-VALUE(RESULT(s , a))
  return v
```

```
function MIN-VALUE(state) returns a utility value
  if CUTOFF-TEST(state) then return EVAL(state, PLAYER(state))
  v ← ∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s , a))
  return v
```

Evaluation Functions

Idea to reduce complexity

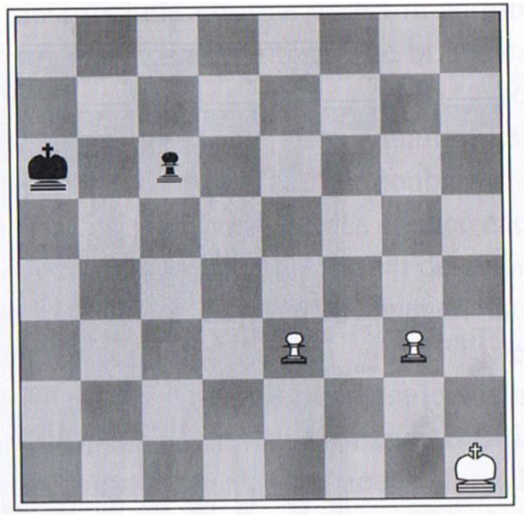
1. Simplify states by extracting features
2. Use features as input to evaluation functions

Examples

- Chess: # of pawns, rooks, queens,… of each side
- Tic-tac-toe: # adjacent tokens of each side
- Etc…

Expected Utility

- Eval = expected utility of a category (=all states with same state features)



- Category example:

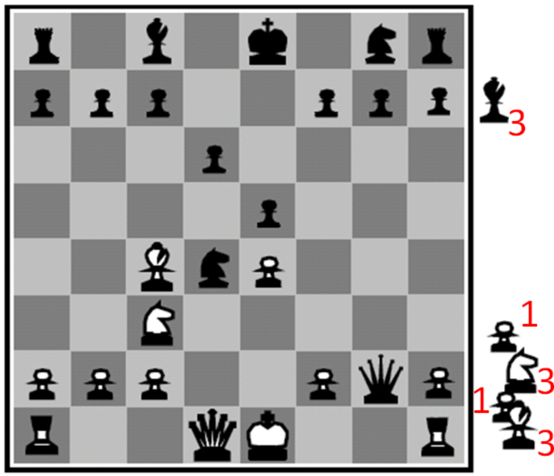
  two pawns vs. one

  From experience : 72% win, 20% loss 8% draw

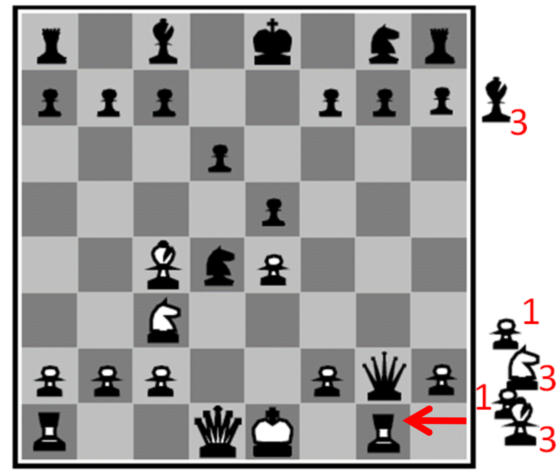
  Expected Value: (0.72 * 1)+(0.20 * 0)+(0.08 * ½) = 0.76

But: too many states of each category in practice!

Material Value



White to move

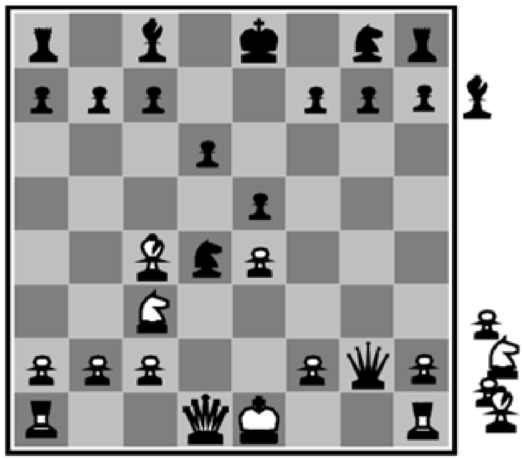


White to move

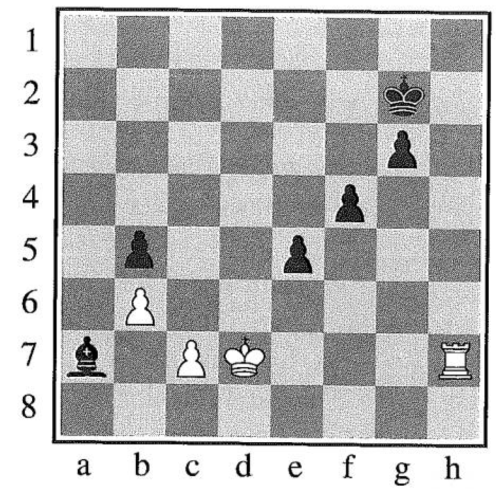
- Eval = sum of feature values
- For chess typically linear weighted sum of features
  - $\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
- E.g. $w_1 = 9$ with
  - $f_1(s)$ =(# of white queens) - (# of black queens), etc.

CUT-OFF Function

White to move



Quiescence State

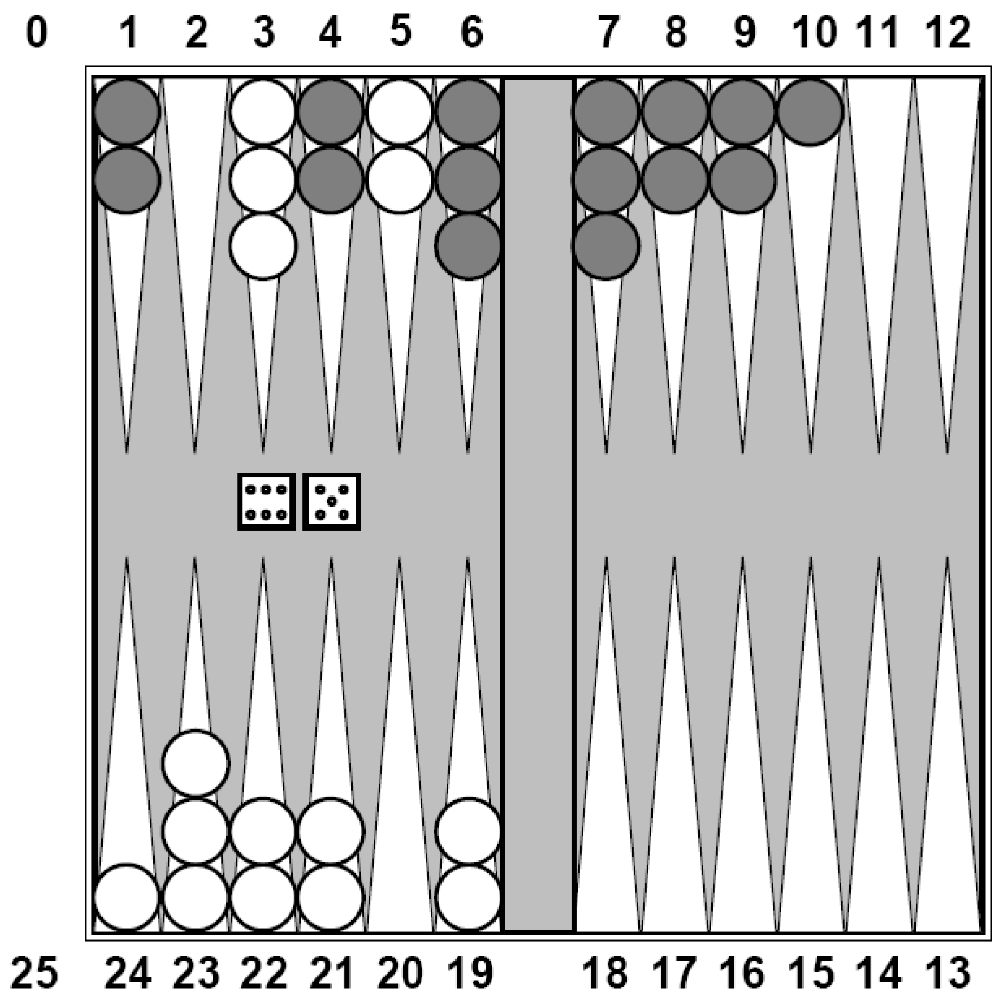


Horizon Effect

Iterative Deepening, possibly extended with

- Quiescence search: only make cut-off in quiescence states
- Singular extensions: apply singular extension to avoid horizon effect
- Transposition table: memorize previously evaluated states

Nondeterministic Games



Backgammon

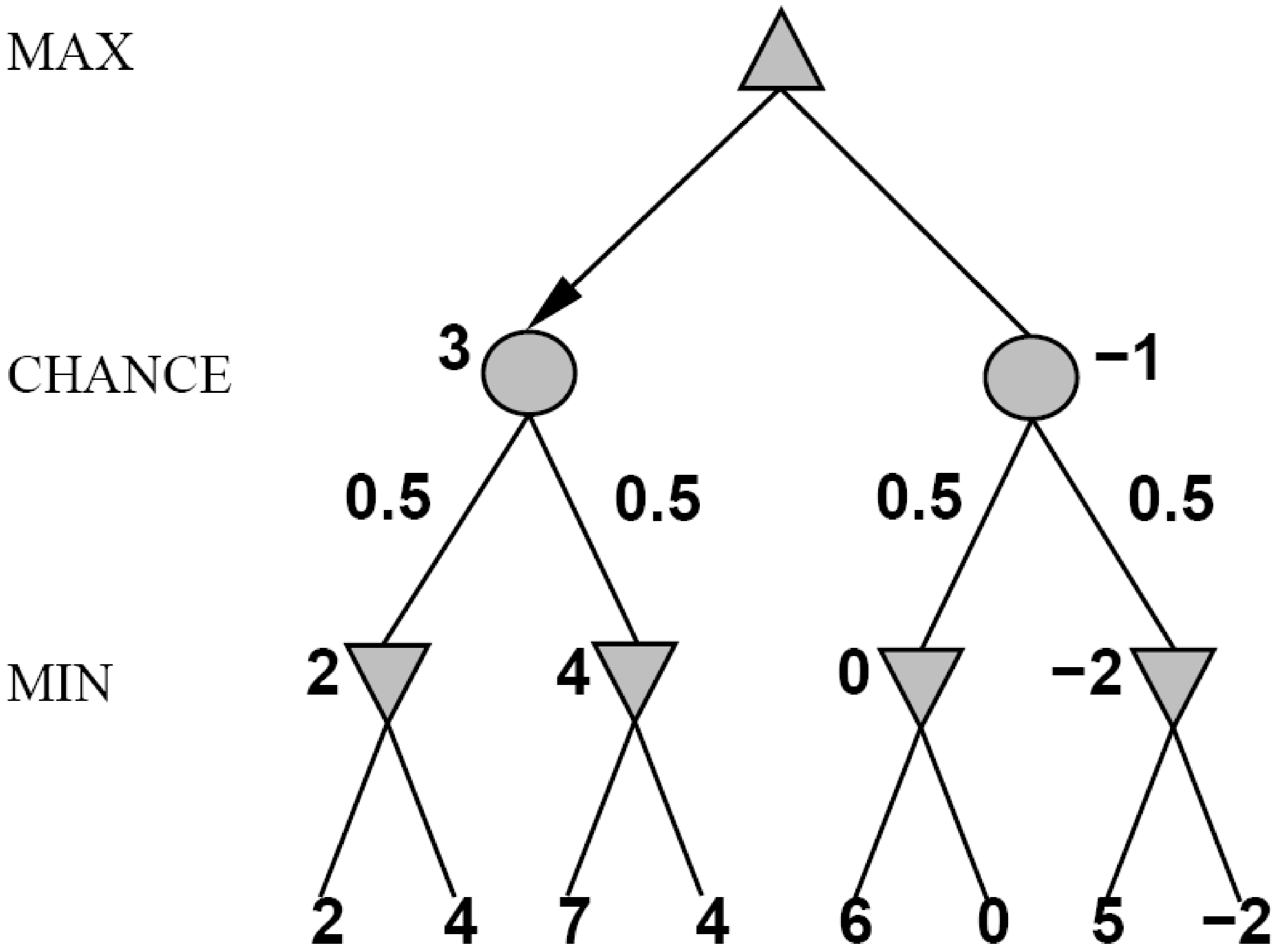
Branching factor ≈ 20

Distinct dice rolls = 21

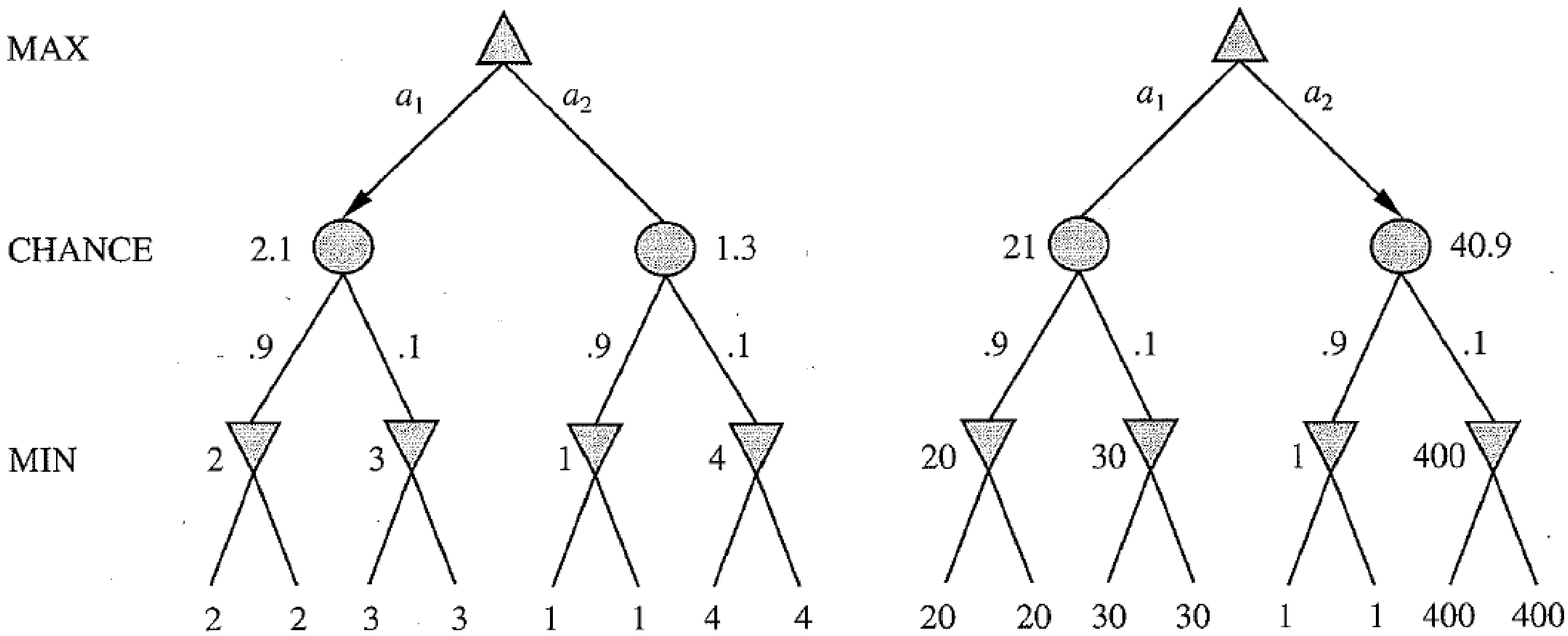
Total outcomes 420

Too high branching factor!

Expectiminimax



Evaluation Function

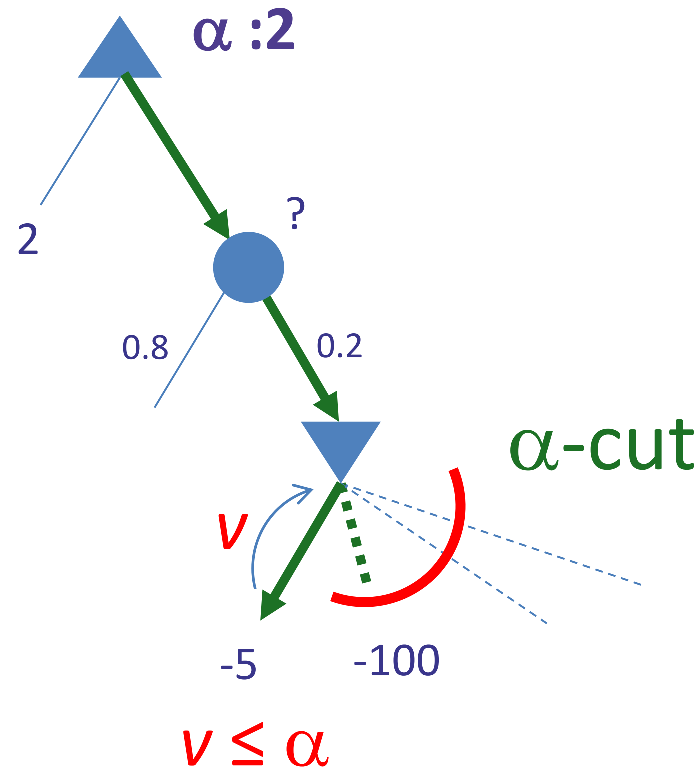


- Only linear transformations of utility function allowed!

Expectiminimax

$$\text{EXPECTIMINIMAX}(s) = \begin{cases} \text{UTILITY}(s, \text{PLAYER}(s)) & \text{if TERMINAL-TEST}(s) \\ max_{a \in \text{ACTIONS}(s)} \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) & \text{if PLAYER}(s) = \text{MAX} \\ min_{a \in \text{ACTIONS}(s)} \text{EXPECTIMINIMAX}(\text{RESULT}(s, a)) & \text{if PLAYER}(s) = \text{MIN} \\ \Sigma_r P(r) \cdot \text{EXPECTIMINIMAX}(\text{RESULT}(s, r)) & \text{if PLAYER}(s) = \text{CHANCE} \end{cases}$$

Does simple α-β pruning still work?



Problem: due uncertainty the value of the chance node may be better than -5.

So MIN cannot conclude that the cut can be made!