Assignment 0515

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Given the formula of least square as $y = Ax + B$, we want to find the smallest of deviation between estimation data and the observed data.
We think the minimum of the distance should be minimum when the distance is squared.
$\Sigma(y_i - Ax_i - B)^2$
$= A^2\Sigma x_i^2 + nB^2 + \Sigma y_i^2 - 2A\Sigma x_i y_i - 2B\Sigma y_i + 2AB\Sigma x_i$

Now, partial derivative with respect to A,
$2A\Sigma x_i^2 - 2\Sigma x_i y_i + 2B\Sigma x_i$
then partial derivative with respect to B,
$2nB - 2\Sigma y_i + 2A\Sigma x_i$
When these two are equal 0, the maximum will be given.
Now solve this simultaneous equation,

$$\begin{cases} 2A\Sigma x_i^2 - 2\Sigma x_i y_i + 2B\Sigma x_i = 0 & (1) \\ 2nB - 2\Sigma y_i + 2A\Sigma x_i = 0 & (2) \end{cases}$$

$$\begin{cases} A\Sigma x_i^2 - \Sigma x_i y_i + B\Sigma x_i = 0 & (3) \\ nB - \Sigma y_i + A\Sigma x_i = 0 & (4) \end{cases}$$

Then,

$$B = \frac{\Sigma y_i - A\Sigma x_i}{n} \tag{5}$$

Inserting this to (3),

$A\Sigma x_i^2 - \Sigma x_i y_i + \frac{\Sigma y_i - A\Sigma x_i}{n}\Sigma x_i = 0$

$nA\Sigma x_i^2 - n\Sigma x_i y_i + \Sigma y_i - A(\Sigma x_i)^2 = 0$

$$A = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2},$$

Inserting this to (5)

$$B = \frac{1}{n}(\Sigma y_i - A\Sigma x_i)$$
$$= \frac{1}{n}\left(\Sigma y_i - \Sigma x_i (\tfrac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2})\right)$$
$$= \frac{1}{n}\frac{n\Sigma x_i^2 \Sigma y_i - (\Sigma x_i)^2 \Sigma y_i - n\Sigma x_i \Sigma x_i y_i + (\Sigma x_i)^2 \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2})$$
$$= \frac{\Sigma x_i^2 \Sigma y_i - n\Sigma x_i \Sigma x_i y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

Therefore given (x_i, y_i), we can calculate the gradients and intercepts from formula.