Assignment (May 15)

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> Today we've learned linear algebra formulas to solve least square regression. Partial differentiation of least square formula will give you the solution formula. Check the transformation yourself and comment on it. Read "Systems of Linear Equations with Matrix 行列で連立方程式" and "PCA with Matrix 行列でPCA" before the class on May 22.

Solution of least square formula

Least square regression formula is to obtain β with the best fitting of estimation into the real sample data, given by;

$$\sum_{i=1}^{n} |y_i - \hat{y}|^2 = \sum_{i=1}^{n} |y_i - (\alpha x_i + \beta)|^2 \tag{1}$$

In general, you can use differentiation to solve the problem to minimize something. So in this case you can differentiate the formula (1) by β.

If $\alpha \neq 0$ and $\beta \neq 0$, you can perform partial differentiation by α and β, and the left side should be zero when the distance between the estimation and the sample values;

$$\frac{d}{d\alpha} \sum_{i=1}^{n} |y_i - \hat{y}|^2 = \frac{d}{d\alpha} \sum_{i=1}^{n} |y_i - (\alpha x_i + \beta)|^2$$

$$0 = \sum_{i=1}^{n} 2(y_i - \alpha x_i - \beta)(-x_i))$$

$$0 = \alpha \sum_{i=1}^{n} x_i^2 + \beta \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i y_i \tag{2}$$

$$\frac{d}{d\beta} \sum_{i=1}^{n} |y_i - \hat{y}|^2 = \frac{d}{d\beta} \sum_{i=1}^{n} |y_i - (\alpha x_i + \beta)|^2$$

$$0 = \sum_{i=1}^{n} 2(y_i - \alpha x_i - \beta)$$

$$0 = \alpha \sum_{i=1}^{n} x_i^2 + \beta n - \sum_{i=1}^{n} y_i \tag{3}$$

2. and (3) are solved as simultaneous equations, and described using matrices as;

$$\begin{pmatrix} \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n y_i \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i^2 & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i^2 & n \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n y_i \end{pmatrix} \tag{4}$$

You can solve (4) by calculating the matrices on the right side determined by each x, y.

If $\beta = 0$, (1) is simplified as;

$$\sum_{i=1}^{n} |y_i - \hat{y}|^2 = \sum_{i=1}^{n} |y_i - \alpha x_i|^2 \tag{5}$$

Like above, you can solve (5) by differentiation;

$$\frac{d}{d\alpha} \sum_{i=1}^{n} |y_i - \hat{y}|^2 = \frac{d}{d\alpha} \sum_{i=1}^{n} |y_i - \alpha x_i|^2$$

$$0 = \sum_{i=1}^{n} |-2x_i(y_i - \alpha x_i)|$$

$$\alpha = \sum_{i=1}^{n} y_i / \sum_{i=1}^{n} x_i \tag{6}$$