CS 570: Data Structures

Stacks

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CHAPTER 3

Stacks

Chapter Objectives

- To learn about the stack data type and how to use its four methods:
  - push
  - pop
  - peek
  - empty
- To learn how to implement a stack using an underlying array or linked list
- To see how to use a stack to perform various applications, including finding palindromes, testing for balanced (properly nested) parentheses, and evaluating arithmetic expressions

Week 6

- Reading Assignment: Koffman and Wolfgang, Sections 4.1-4.4

Stack Abstract Data Type

Section 4.1

Stack Abstract Data Type

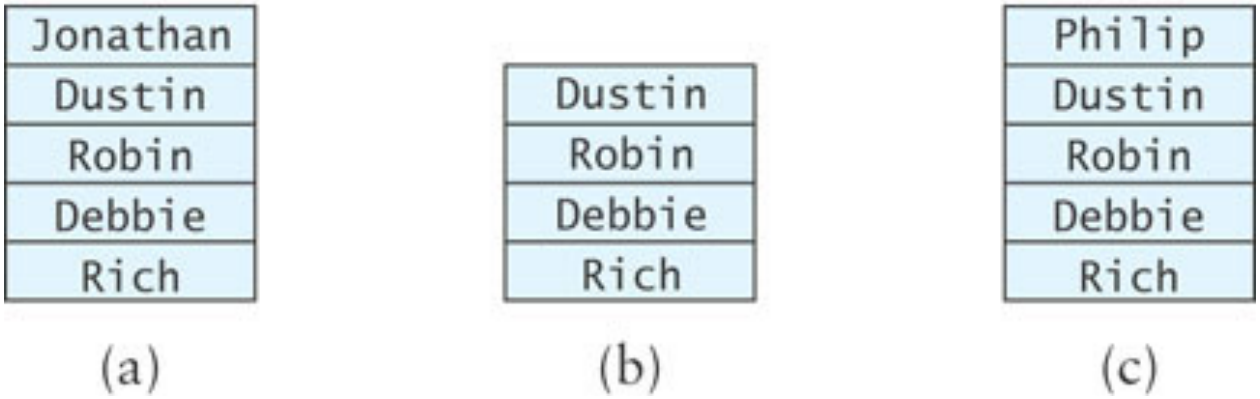
- A stack is one of the most commonly used data structures in computer science
- A stack can be compared to a Pez dispenser
  - Only the top item can be accessed
  - You can extract only one item at a time
- The top element in the stack is the one added to the stack most recently
- The stack's storage policy is *Last-In, First-Out*, or *LIFO*

Specification of the Stack Abstract Data Type

- Only the top element of a stack is visible; therefore the number of operations performed by a stack are few
- We need the ability to
  - test for an empty stack (empty)
  - inspect the top element (peek)
  - retrieve the top element (pop)
  - put a new element on the stack (push)

Methods	Behavior
`boolean empty()`	Returns **`true`** if the stack is empty; otherwise, returns **`false`**.
`E peek()`	Returns the object at the top of the stack without removing it.
`E pop()`	Returns the object at the top of the stack and removes it.
`E push(E obj)`	Pushes an item onto the top of the stack and returns the item pushed.

A Stack of Strings



- "Rich" is the oldest element on the stack and "Jonathan" is the youngest (Figure a)
- `String last = names.peek();` stores a reference to "Jonathan" in `last`
- `String temp = names.pop();` removes "Jonathan" and stores a reference to it in `temp` (Figure b)
- `names.push("Philip");` pushes "Philip" onto the stack (Figure c)

Stack Applications

Section 4.2

Finding Palindromes

- Palindrome: a string that reads identically in either direction, letter by letter (ignoring case)
  - kayak
  - "I saw I was I"
  - “Able was I ere I saw Elba”
  - "Level, madam, level"

- Problem: Write a program that reads a string and determines whether it is a palindrome

Finding Palindromes (cont.)

Data Fields	Attributes
`private String inputString`	The input string.
`private Stack<Character> charStack`	The stack where characters are stored.
Methods	**Behavior**
`public PalindromeFinder(String str)`	Initializes a new `PalindromeFinder` object, storing a reference to the parameter `str` in `inputString` and pushing each character onto the stack.
`private void fillStack()`	Fills the stack with the characters in `inputString`.
`private String buildReverse()`	Returns the string formed by popping each character from the stack and joining the characters. Empties the stack.
`public boolean isPalindrome()`	Returns **`true`** if `inputString` and the string built by `buildReverse` have the same contents, except for case. Otherwise, returns **`false`**.

Finding Palindromes (cont.)

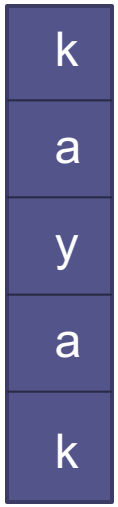

```
import java.util.*;

public class PalindromeFinder {
  private String inputString;
  private Stack<Character> charStack = new
                                       Stack<Character>();

 public PalindromeFinder(String str) {
   inputString = str;
   fillStack();
 }
 ...
```

Finding Palindromes (cont.)

- Solving using a stack:
  - Push each string character, from left to right, onto a stack



k a y a k

```
private void fillStack() {
  for(int i = 0; i < inputString.length(); i++) {
    charStack.push(inputString.charAt(i));
  }
}
```

Finding Palindromes (cont.)

- Solving using a stack:
  - Pop each character off the stack, appending each to the `StringBuilder result`

k a y a k

```
private String buildReverse(){
 StringBuilder result = new StringBuilder();
 while(!charStack.empty()) {
  result.append(charStack.pop());
 }
 return result.toString();
}
```

Finding Palindromes (cont.)


```
   ...

    public boolean isPalindrome() {
      return inputString.equalsIgnoreCase(buildReverse());
    }
}
```

Testing

- We can test this class using the following inputs:
  - a single character (always a palindrome)
  - multiple characters in a word
  - multiple words
  - different cases
  - even-length strings
  - odd-length strings
  - the empty string (considered a palindrome)

Balanced Parentheses

- When analyzing arithmetic expressions, it is important to determine whether an expression is balanced with respect to parentheses

```
( a + b * ( c / ( d - e ) ) ) + ( d / e )
```

- The problem is further complicated if braces or brackets are used in conjunction with parentheses
- The solution is to use stacks!

Balanced Parentheses (cont.)

Method	Behavior
`public static boolean isBalanced(String expression)`	Returns **`true`** if `expression` is balanced with respect to parentheses and **`false`** if it is not.
`private static boolean isOpen(char ch)`	Returns **`true`** if `ch` is an opening parenthesis.
`private static boolean isClose(char ch)`	Returns **`true`** if `ch` is a closing parenthesis.

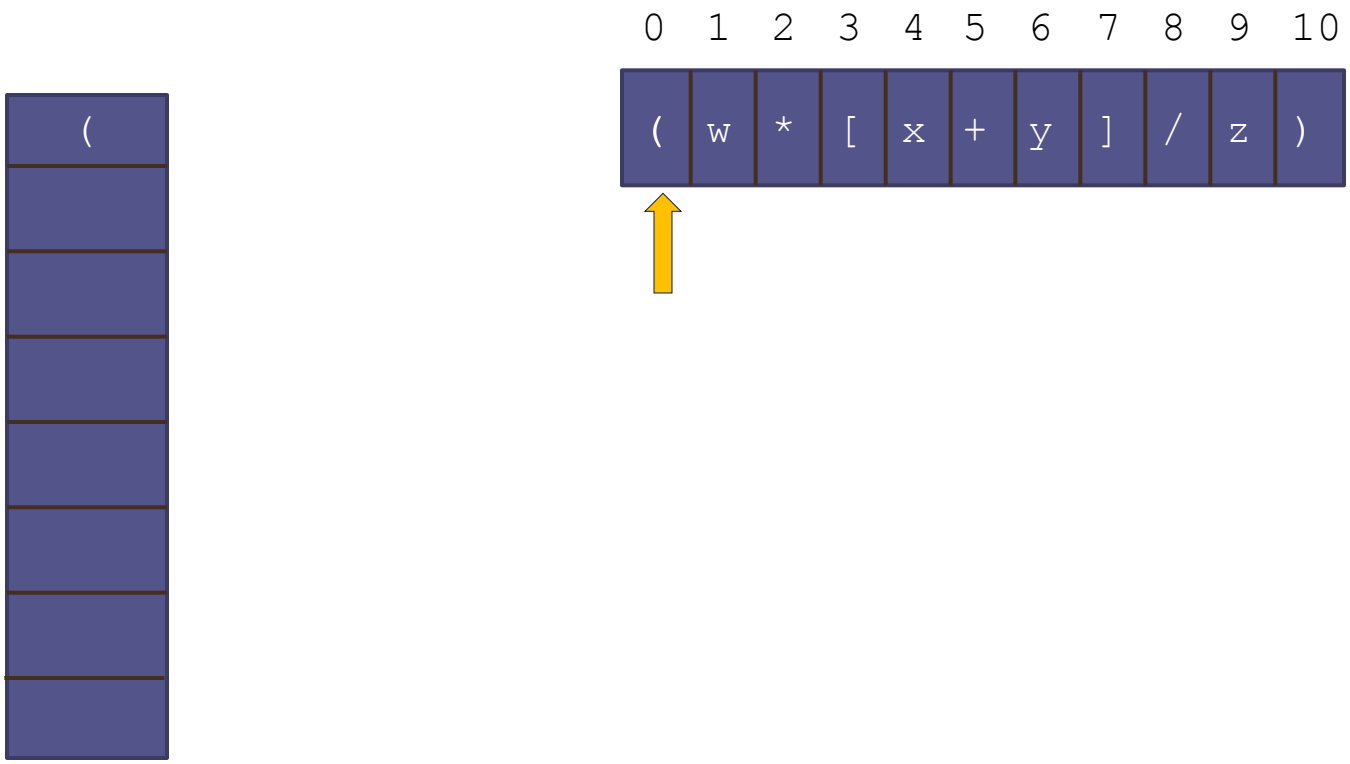
Balanced Parentheses (cont.)

Algorithm for method `isBalanced`

```
1.  Create an empty stack of characters.
2.  Assume that the expression is balanced (balanced is true).
3.  Set index to 0.
4.  while balanced is true and index < the expression's length
5.      Get the next character in the data string.
6.      if the next character is an opening parenthesis
7.          Push it onto the stack.
8.      else if the next character is a closing parenthesis
9.          Pop the top of the stack.
10.         if stack was empty or its top does not match the closing
            parenthesis
11.             Set balanced to false.
12.     Increment index.
13. Return true if balanced is true and the stack is empty.
```

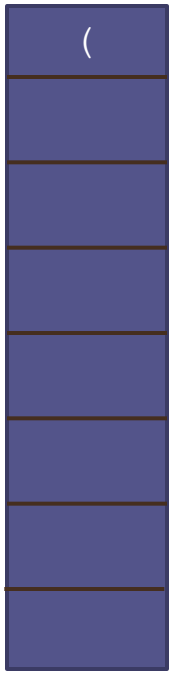
Balanced Parentheses (cont.)

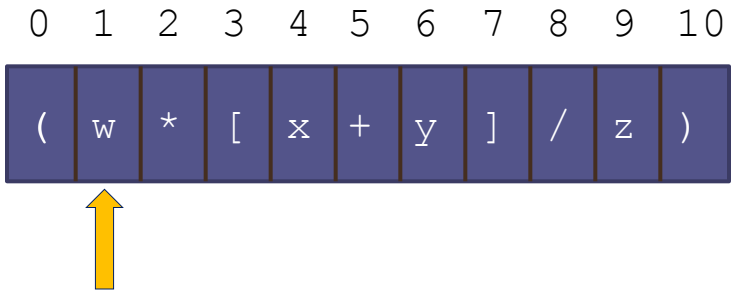
Expression: `(w * [x + y] / z)`



Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

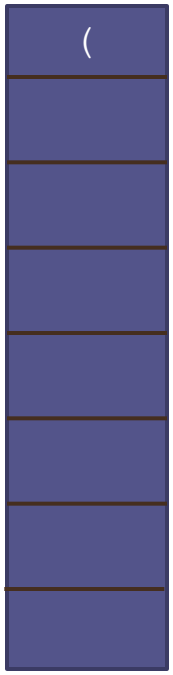


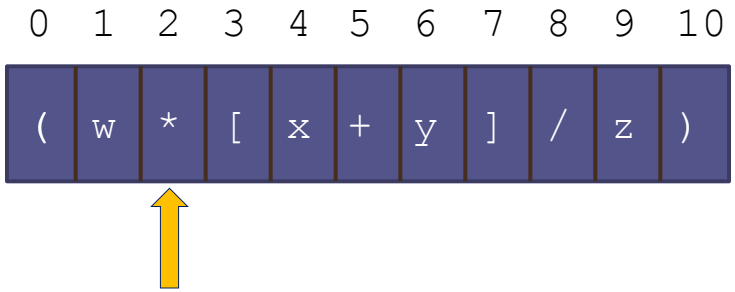



```
balanced : true
index    : 1
```

Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

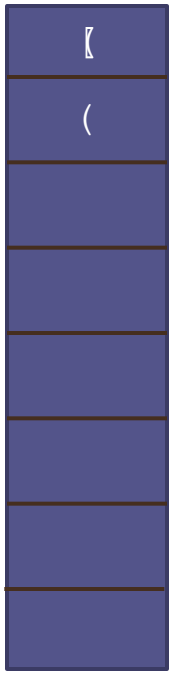


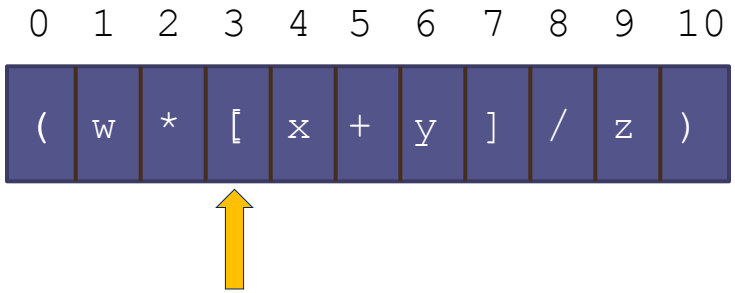



```
balanced : true
index    : 2
```

Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

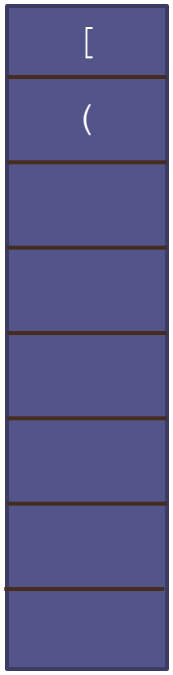


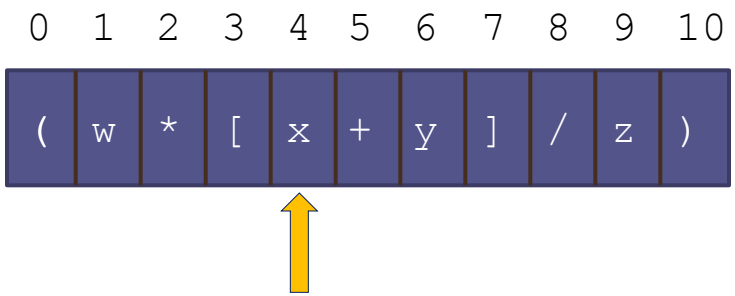



```
balanced : true
index    : 3
```

Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

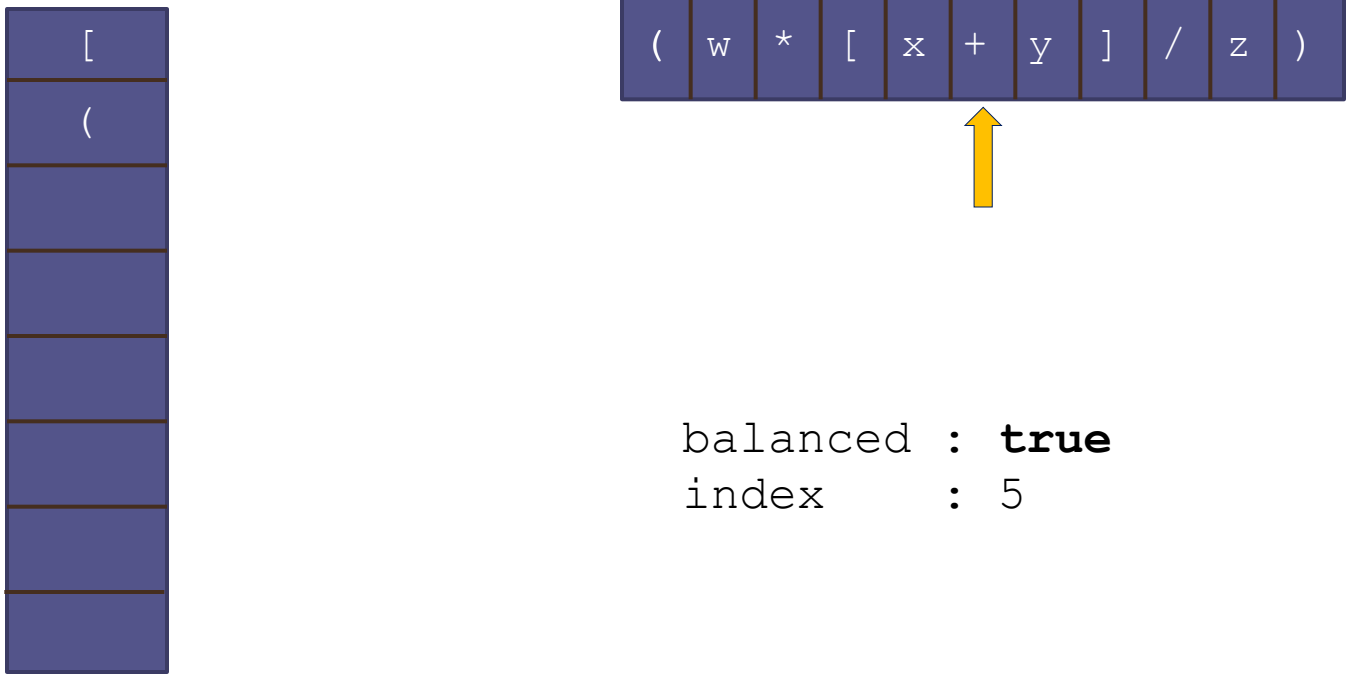





```
balanced : true
index    : 4
```

Balanced Parentheses (cont.)

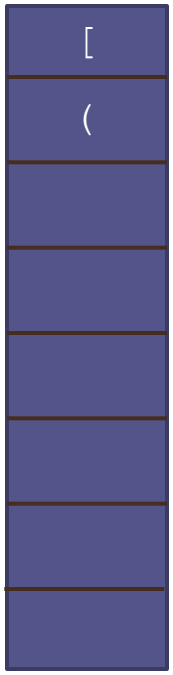
Expression: `(w * [x + y] / z)`

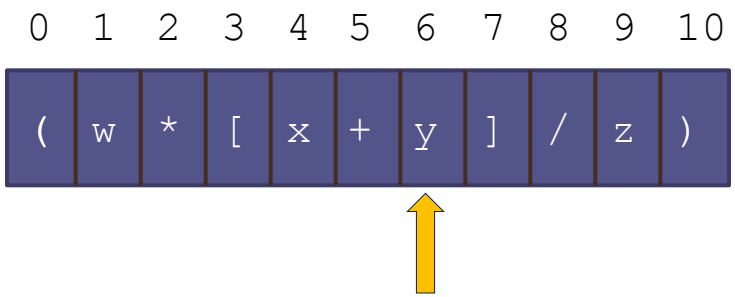



```
balanced : true
index    : 5
```

Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

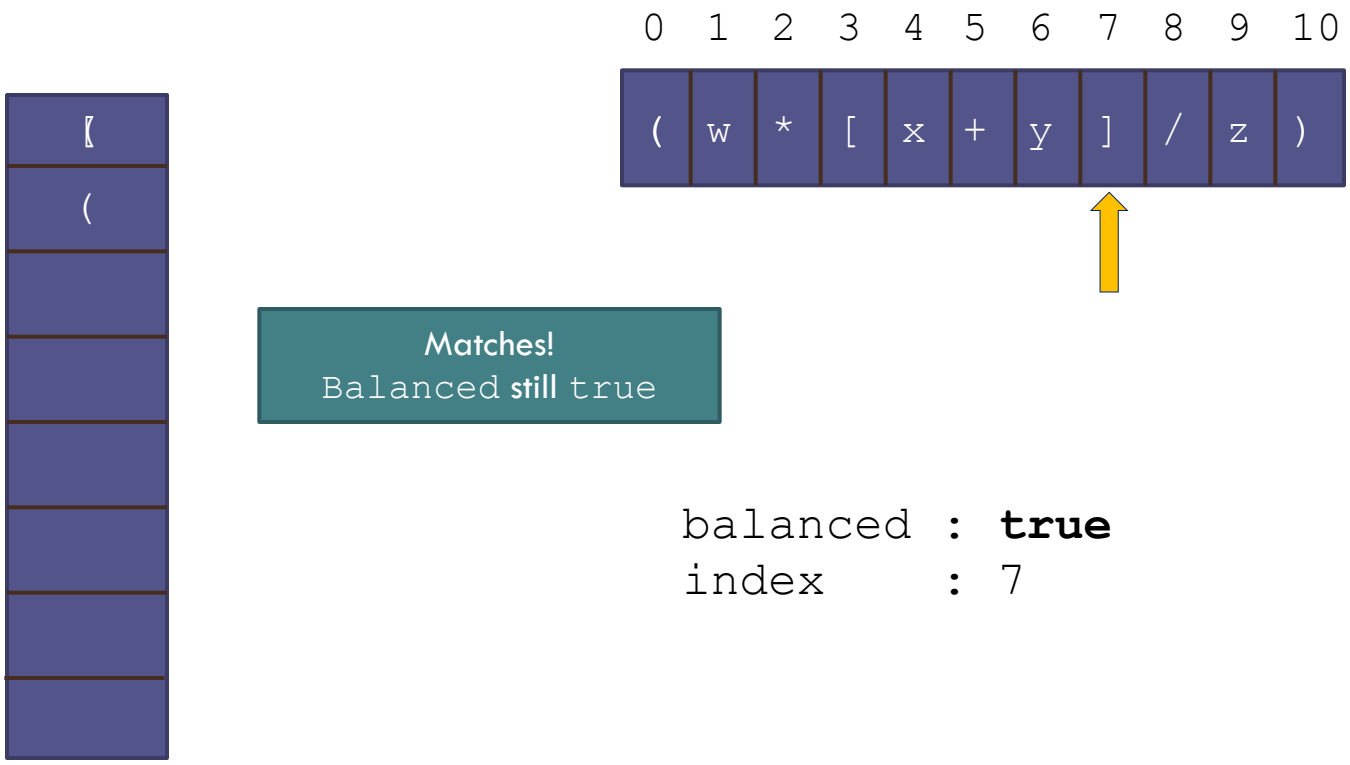





```
balanced : true
index    : 6
```

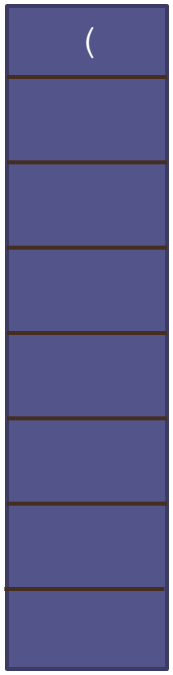
Balanced Parentheses (cont.)

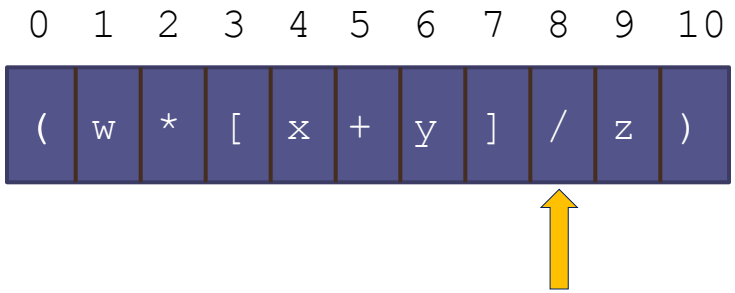
Expression: `(w * [x + y] / z)`



Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`

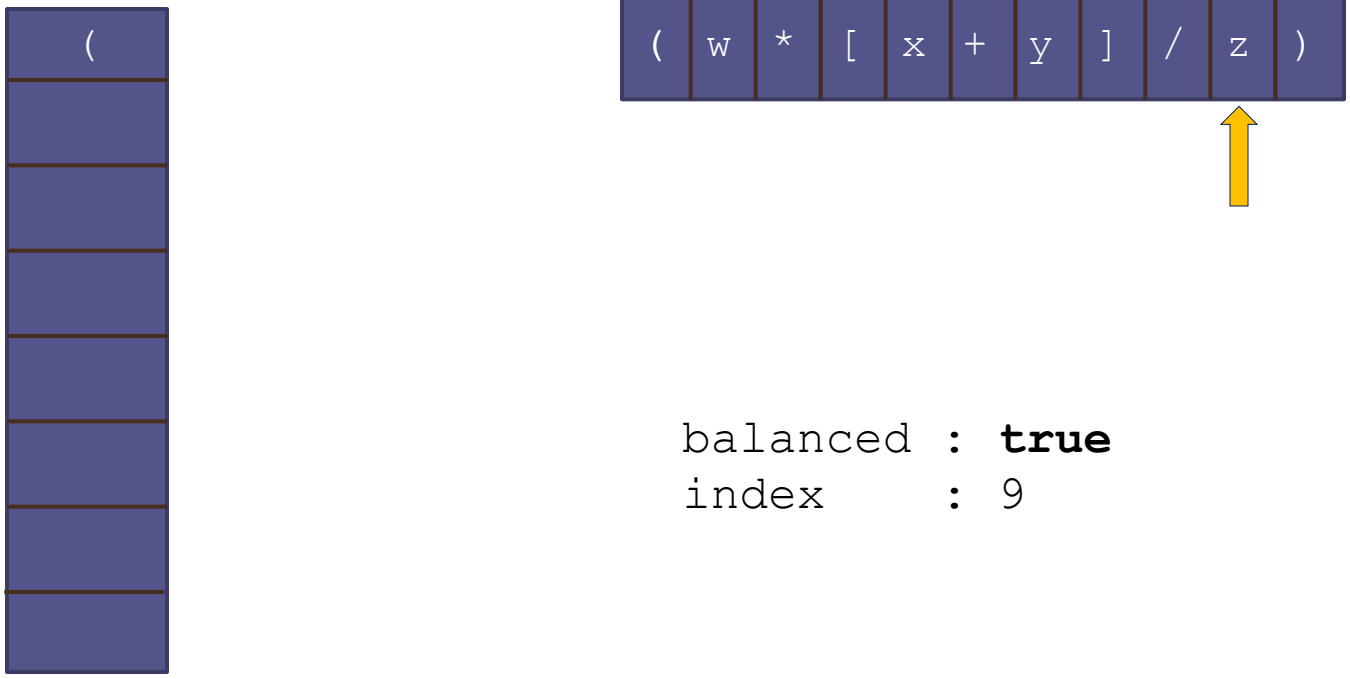





```
balanced : true
index    : 8
```

Balanced Parentheses (cont.)

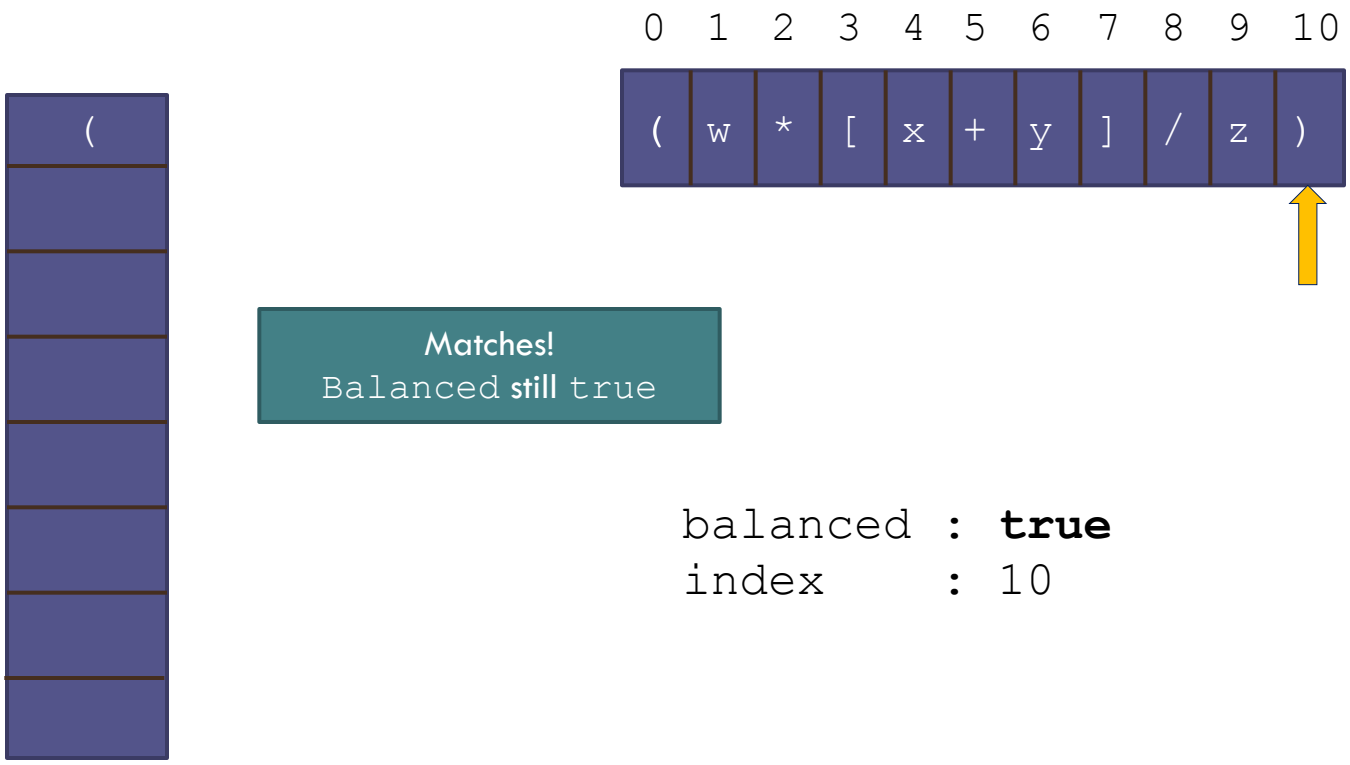
Expression: `(w * [x + y] / z)`




```
balanced : true
index    : 9
```

Balanced Parentheses (cont.)

Expression: `(w * [x + y] / z)`



Testing

- Provide a variety of input expressions displaying the result `true` or `false`
- Try several levels of nested parentheses
- Try nested parentheses where corresponding parentheses are not of the same type
- Try unbalanced parentheses

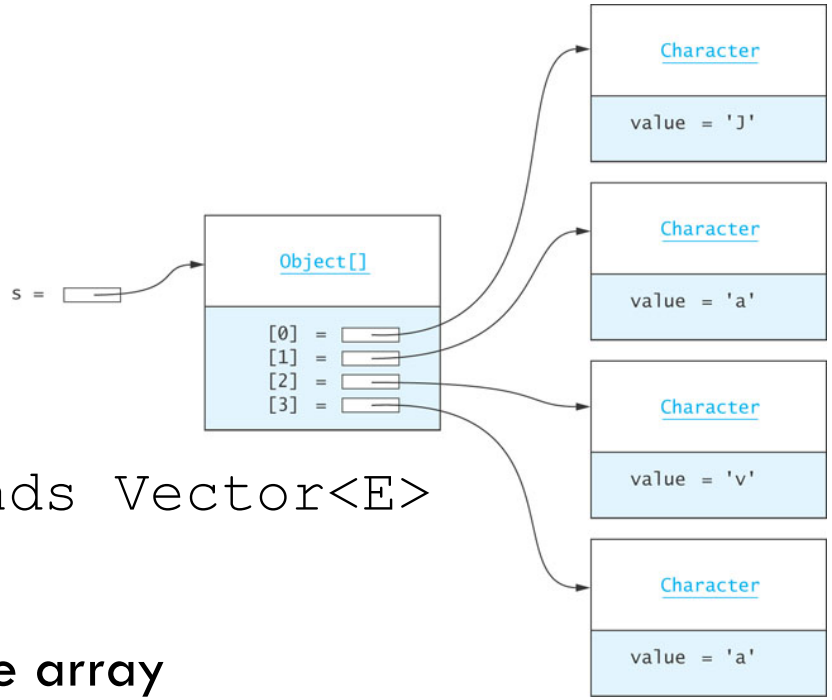
- PITFALL: attempting to pop an empty stack will throw an `EmptyStackException`. You can guard against this by either testing for an empty stack or catching the exception

Implementing a Stack

Section 4.3

Implementing a Stack as an Extension of Vector

- The Java API includes a `Stack` class as part of the package `java.util`:




```
public class Stack<E> extends Vector<E>
```

- The `Vector` class implements a growable array of objects
- Elements of a `Vector` can be accessed using an integer index and the size can grow or shrink as needed to accommodate the insertion and removal of elements

Implementing a Stack as an Extension of Vector (cont.)

- We can use `Vector`'s `add` method to implement `push`:

```
public E push(obj E) {
  add(obj);
  return obj;
}
```

- pop can be coded as

```
public E pop() throws EmptyStackException {
  try {
   return remove (size() – 1);
  } catch (ArrayIndexOutOfBoundsException ex) {
   throw new EmptyStackException();
  }
}
```

Implementing a Stack as an Extension of Vector (cont.)

- Because a `Stack` *is a* `Vector`, all of `Vector` operations can be applied to a `Stack` (such as searches and access by index)
- But, since only the top element of a stack should be accessible, this violates the principle of information hiding

Implementing a Stack with a List Component

- As an alternative to a stack as an extension of `Vector`, we can write a class, `ListStack`, that has a `List` component (in the example below, `theData`)
- We can use either the `ArrayList`, `Vector`, or the `LinkedList` classes, as all implement the `List` interface. The `push` method, for example, can be coded as

```
public E push(E obj) {
   theData.add(obj);
   return obj;
}
```

- A class which adapts methods of another class by giving different names to essentially the same methods (`push` instead of `add`) is called an *adapter class*
- Writing methods in this way is called *method delegation*

Implementing a Stack Using an Array

- If we implement a stack as an array, we would need . . .

```
public class ArrayStack<E> implements StackInt<E> {
  private E[] theData;
  int topOfStack = -1;
  private static final int INITIAL_CAPACITY = 10;

  @SupressWarnings("unchecked")
  public ArrayStack() {
    theData = (E[])new Object[INITIAL_CAPACITY];
  }
```

Implementing a Stack Using an Array

- If we implement a stack as an arr
  we would need . . .

```
public class ArrayStack<E> implements S
  private E[] theData;
  int topOfStack = -1;
  private static final int INITIAL_CA

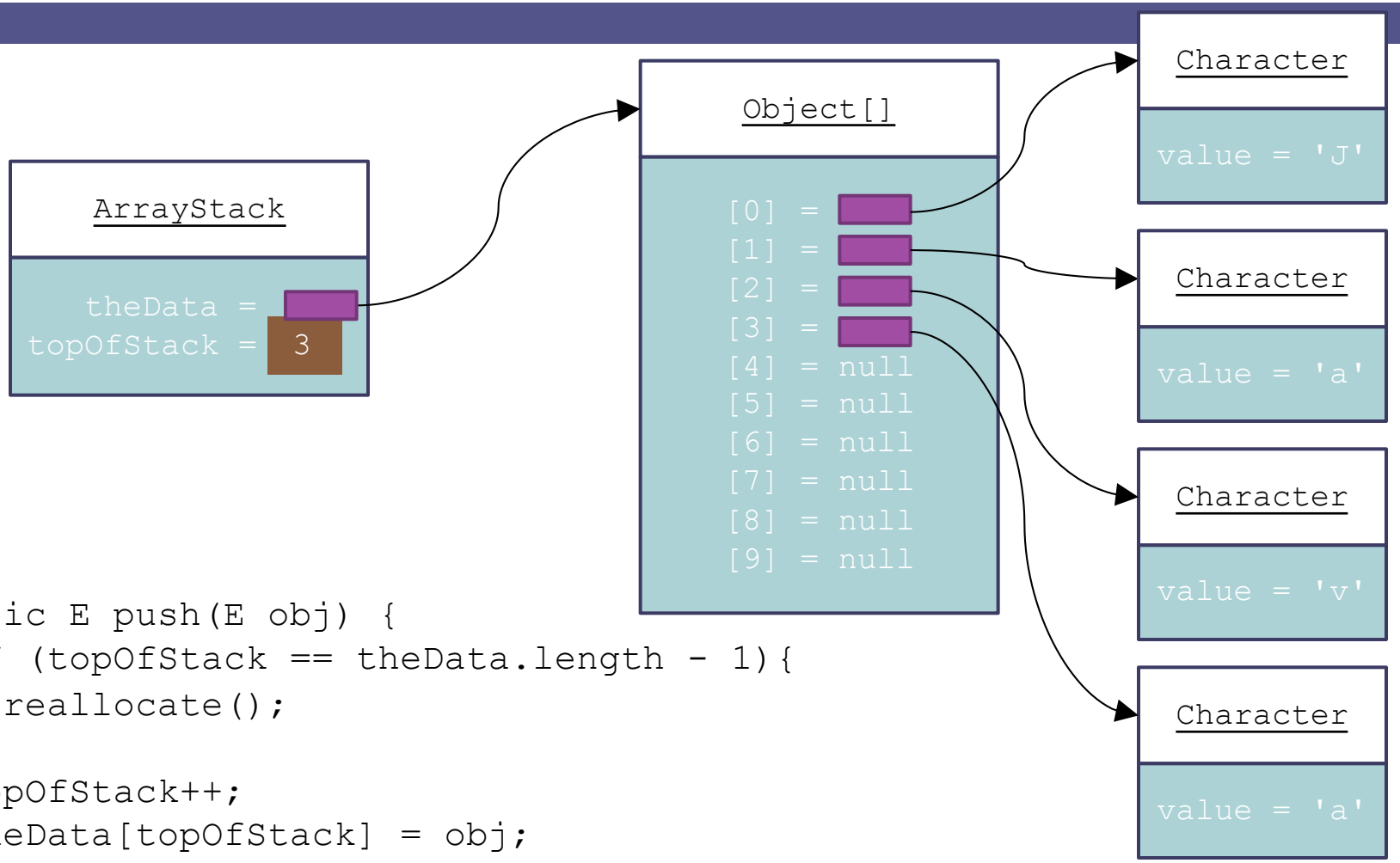
  @SupressWarnings("unchecked")
  publ
    th
  }
```

Allocate storage for an array with a default capacity

Keep track of the top of the stack (subscript of the element at the top of the stack; for empty stack = -1)

We do not need a `size` variable or method

Implementing a Stack Using an Array (cont.)




```
public E push(E obj) {
  if (topOfStack == theData.length - 1){
    reallocate();
  }
  topOfStack++;
  theData[topOfStack] = obj;
  return obj;
}
```

Implementing a Stack Using an Array (cont.)

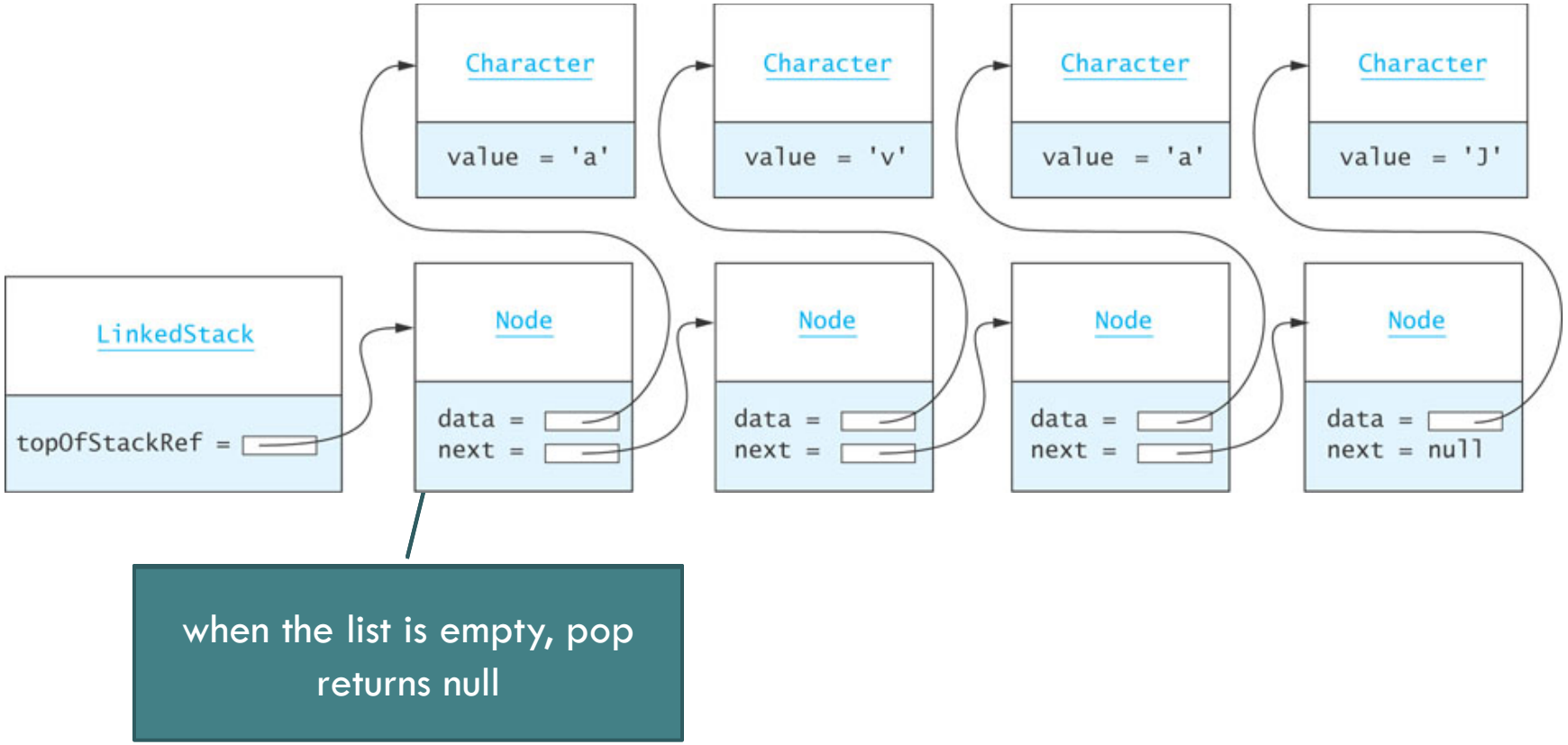

```
@Override
public E pop() {
  if (empty()) {
    throw new EmptyStackException();
  }
  return theData[topOfStack--];
}
```

Implementing a Stack Using an Array (cont.)

- This implementation is O(1), in contrast to the Pez analogy and the "kayak" example, which are both O(n)

Implementing a Stack as a Linked Data Structure

- We can also implement a stack using a linked list of nodes



Implementing a Stack as a Linked Data Structure (cont.)


```
import java.util.EmptyStackException;

public class LinkedStack < E > implements StackInt < E > {

  /** A Node is the building block for a single-linked list. */
  private static class Node < E > {
    // Data Fields
    /** The reference to the data. */
    private E data;
    /** The reference to the next node. */
    private Node next;
```

Implementing a Stack as a Linked Data Structure (cont.)


```
  // Constructors
  /** Creates a new node with a null next field.
      @param dataItem The data stored
   */
  private Node(E dataItem) {
    data = dataItem;
    next = null;
  }
  /** Creates a new node that references another node.
      @param dataItem The data stored
      @param nodeRef The node referenced by new node
   */
  private Node(E dataItem, Node < E > nodeRef) {
    data = dataItem;
    next = nodeRef;
  }
} //end class Node
```

Implementing a Stack as a Linked Data Structure (cont.)


```
// Data Fields
/** The reference to the first stack node. */
private Node < E > topOfStackRef = null;

/** Insert a new item on top of the stack.
    post: The new item is the top item on the stack.
          All other items are one position lower.
    @param obj The item to be inserted
    @return The item that was inserted
 */
public E push(E obj) {
  topOfStackRef = new Node < E > (obj, topOfStackRef);
  return obj;
}
```

Implementing a Stack as a Linked Data Structure (cont.)


```
/** Remove and return the top item on the stack.
    pre: The stack is not empty.
    post: The top item on the stack has been
          removed and the stack is one item smaller.
    @return The top item on the stack
    @throws EmptyStackException if the stack is empty
 */
public E pop() {
  if (empty()) {
    throw new EmptyStackException();
  }
  else {
    E result = topOfStackRef.data;
    topOfStackRef = topOfStackRef.next;
    return result;
  }
}
```

Implementing a Stack as a Linked Data Structure (cont.)


```
/** Return the top item on the stack.
    pre: The stack is not empty.
    post: The stack remains unchanged.
    @return The top item on the stack
    @throws EmptyStackException if the stack is empty
 */
public E peek() {
  if (empty()) {
    throw new EmptyStackException();
  }
  else {
    return topOfStackRef.data;
  }
}
```

Implementing a Stack as a Linked Data Structure (cont.)


```
  /** See whether the stack is empty.
      @return true if the stack is empty
   */
  public boolean empty() {
    return topOfStackRef == null;
  }
}
```

Comparison of Stack Implementations

- Extending a `Vector` (as is done by Java) is a poor choice for stack implementation, since all `Vector` methods are accessible
- The easiest implementation uses a `List` component (`ArrayList` is the simplest) for storing data
  - An underlying array requires reallocation of space when the array becomes full, and
  - an underlying linked data structure requires allocating storage for links
  - As all insertions and deletions occur at one end, they are constant time, O(1), regardless of the type of implementation used

Additional Stack Applications

Section 4.4

Additional Stack Applications

- Postfix and infix notation
  - Expressions normally are written in infix form, but
  - it easier to evaluate an expression in postfix form since there is no need to group sub-expressions in parentheses or worry about operator precedence

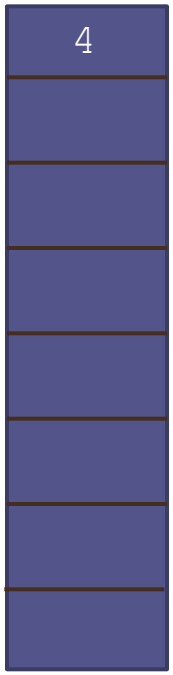
Postfix Expression	Infix Expression	Value
4 7 *	4 * 7	28
4 7 2 + *	4 * (7 + 2)	36
4 7 * 20 -	(4 * 7) - 20	8
3 4 7 * 2 / +	3 + ((4 * 7) / 2)	17

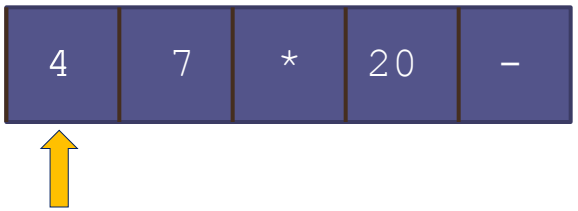
Evaluating Postfix Expressions

- Write a class that evaluates a postfix expression
- Use the space character as a delimiter between tokens

Data Field	Attribute
`Stack<Integer> operandStack`	The stack of operands (`Integer` objects).
Method	**Behavior**
`public int eval(String expression)`	Returns the value of `expression`.
`private int evalOp(char op)`	Pops two operands and applies operator `op` to its operands, returning the result.
`private boolean isOperator(char ch)`	Returns **`true`** if `ch` is an operator symbol.

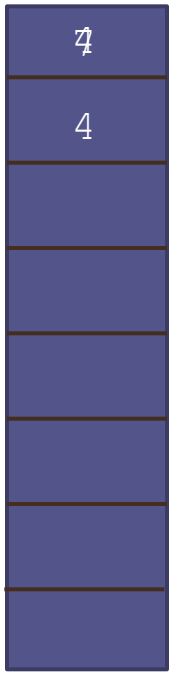
Evaluating Postfix Expressions (cont.)

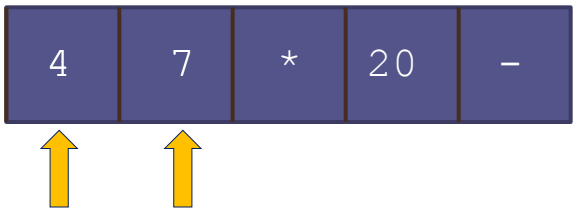





```
1. create an empty stack of integers
2. while there are more tokens
3.    get the next token
4.    if the first character of the token is a digit
5.       push the character on the stack
6.    else if the token is an operator
7.       pop the right operand off the stack
8.       pop the left operand off the stack
9.       evaluate the operation
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)






```
 1. create an empty stack of integers
 2. while there are more tokens
 3.    get the next token
 4.    if the first character of the token is a digit
 5.       push the character on the stack
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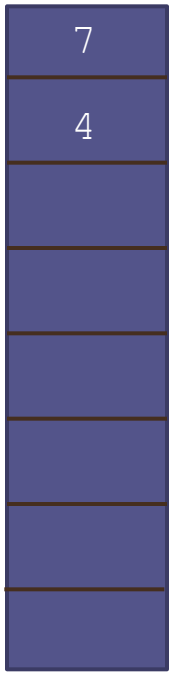
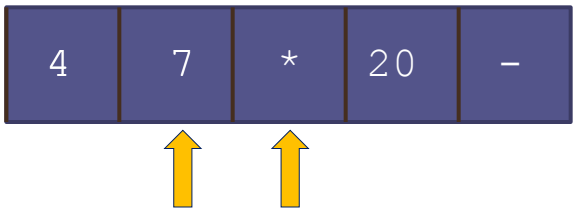
```

Evaluating Postfix Expressions (cont.)



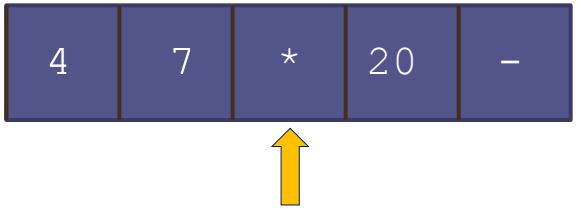
4 * 7

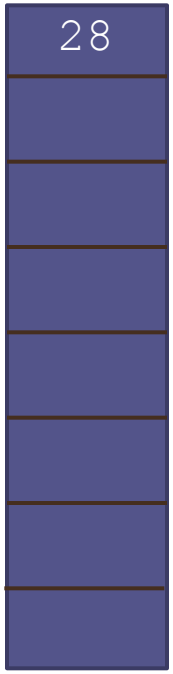



```
1. create an empty stack of integers
2. while there are more tokens
3.    get the next token
4.    if the first character of the token is a digit
5.       push the character on the stack
6.    else if the token is an operator
7.       pop the right operand off the stack
8.       pop the left operand off the stack
9.       evaluate the operation
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)

28



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5.       push the character on the stack
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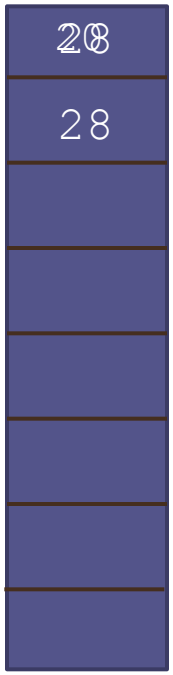
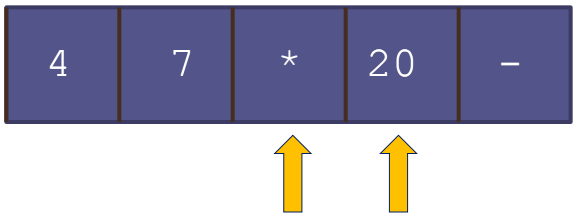
7.       pop the right operand off the stack
8.       pop the left operand off the stack
9.       evaluate the operation
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)






```
 1. create an empty stack of integers
 2. while there are more tokens
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 4.   if the first character of the token is a digit
 5.      push the character on the stack
 6.   else if the token is an operator
 7.      pop the right operand off the stack
 8.      pop the left operand off the stack
 9.      evaluate the operation
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11. pop the stack and return the result
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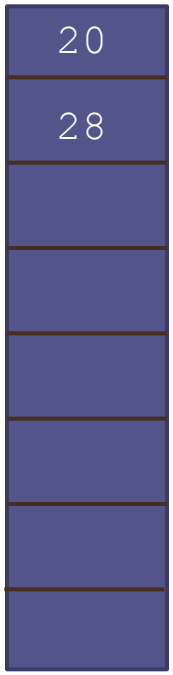
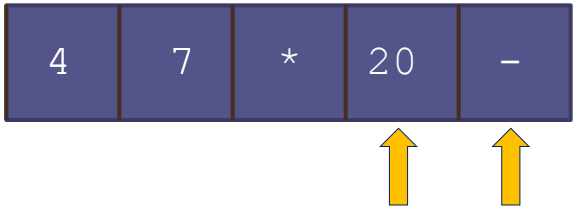
Evaluating Postfix Expressions (cont.)



28 - 20

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8.       pop the left operand off the stack
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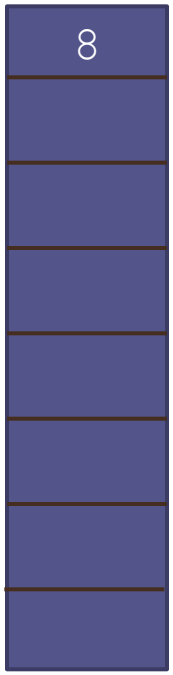
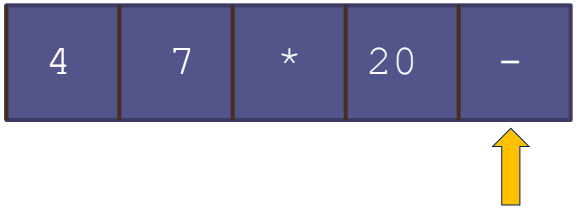
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)

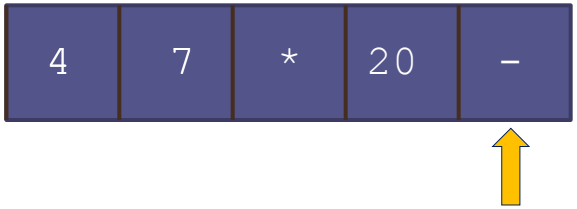


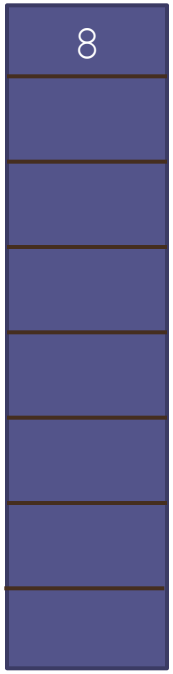
8




```
1.  create an empty stack of integers
2.  while there are more tokens
3.    get the next token
4.    if the first character of the token is a digit
5.       push the character on the stack
6.    else if the token is an operator
7.       pop the right operand off the stack
8.       pop the left operand off the stack
9.       evaluate the operation
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)






```
1.  create an empty stack of integers
2.  while there are more tokens
3.    get the next token
4.    if the first character of the token is a digit
5.       push the number on the stack
6.    else if the token is an operator
7.       pop the right operand off the stack
8.       pop the left operand off the stack
9.       evaluate the operation
10.      push the result onto the stack
11. pop the stack and return the result
```

Evaluating Postfix Expressions (cont.)

- Testing: write a driver which
  - creates a `PostfixEvaluator` object
  - reads one or more expressions and reports the result
  - catches `PostfixEvaluator.SyntaxErrorException`
  - exercises each path by using each operator
  - exercises each path through the method by trying different orderings and multiple occurrences of operators
  - tests for syntax errors:
    - an operator without any operands
    - a single operand
    - an extra operand
    - an extra operator
    - a variable name
    - the empty string

Converting from Infix to Postfix

- Convert infix expressions to postfix expressions
- Assume:
  - expressions consists of only spaces, operands, and operators
  - space is a delimiter character
  - all operands that are identifiers begin with a letter or underscore
  - all operands that are numbers begin with a digit

Data Field	Attribute
`private Stack<Character> operatorStack`	Stack of operators.
`private StringBuilder postfix`	The postfix string being formed.
Method	**Behavior**
`public String convert(String infix)`	Extracts and processes each token in `infix` and returns the equivalent postfix string.
`private void processOperator(char op)`	Processes operator `op` by updating `operatorStack`.
`private int precedence(char op)`	Returns the precedence of operator `op`.
`private boolean isOperator(char ch)`	Returns **`true`** if `ch` is an operator symbol.

Converting from Infix to Postfix (cont.)

- Example: convert

  w – 5.1 / sum * 2

  to its postfix form

  w 5.1 sum / 2 * -

Converting from Infix to Postfix (cont.)

Next Token	Action	Effect on operatorStack	Effect on postfix
w	Append w to postfix.	[ ]	w
-	The stack is empty Push - onto the stack	[-]	w
5.1	Append 5.1 to postfix	[-]	w 5.1
/	precedence(/) > precedence(-), Push / onto the stack	[/] [-]	w 5.1
sum	Append sum to postfix	[/] [-]	w 5.1 sum
*	precedence(*) equals precedence(/) Pop / off of stack and append to postfix	[-]	w 5.1 sum /

Converting from Infix to Postfix (cont.)

Next Token	Action	Effect on operatorStack	Effect on postfix
*	`precedence(*) > precedence(-)`, Push * onto the stack	* -	`w 5.1 sum /`
2	Append 2 to `postfix`	* -	`w 5.1 sum / 2`
End of input	Stack is not empty, Pop * off the stack and append to `postfix`	-	`w 5.1 sum / 2 *`
End of input	Stack is not empty, Pop - off the stack and append to `postfix`		`w 5.1 sum / 2 * -`

Converting from Infix to Postfix (cont.)

Algorithm for Method convert

1. Initialize `postfix` to an empty `StringBuilder`.
2. Initialize the operator stack to an empty stack.
3. **`while`** there are more tokens in the infix string
4. Get the next token.
5. **`if`** the next token is an operand
6. Append it to `postfix`.
7. **`else if`** the next token is an operator
8. Call `processOperator` to process the operator.
9. **`else`**
10. Indicate a syntax error.
11. Pop remaining operators off the operator stack and append them to `postfix`.

Converting from Infix to Postfix (cont.)

Algorithm for Method processOperator

```
1.  if the operator stack is empty
2.      Push the current operator onto the stack.
    else
3.      Peek the operator stack and let topOp be the top operator.
4.      if the precedence of the current operator is greater than the
        precedence of topOp
5.          Push the current operator onto the stack.
        else
6.          while the stack is not empty and the precedence of the current
            operator is less than or equal to the precedence of topOp
7.              Pop topOp off the stack and append it to postfix.
8.              if the operator stack is not empty
9.                  Peek the operator stack and let topOp be the top
                    operator.
10.         Push the current operator onto the stack.
```