


```

In[245]:= M = ConstantArray[0, {8, 8}];

M[[1 ;; 2, 1 ;; 2]] =  $\frac{13}{35}$  IdentityMatrix[2];

M[[3 ;; 4, 3 ;; 4]] =  $\frac{1}{105} L^2$  IdentityMatrix[2];

M[[5 ;; 6, 5 ;; 6]] =  $\frac{13}{35}$  IdentityMatrix[2];

M[[7 ;; 8, 7 ;; 8]] =  $\frac{1}{105} L^2$  IdentityMatrix[2];

M[[1 ;; 2, 3 ;; 4]] = M[[3 ;; 4, 1 ;; 2]] =  $\frac{11}{210} L$  IdentityMatrix[2];

M[[1 ;; 2, 5 ;; 6]] = M[[5 ;; 6, 1 ;; 2]] =  $\frac{9}{70}$  IdentityMatrix[2];

M[[1 ;; 2, 7 ;; 8]] = M[[7 ;; 8, 1 ;; 2]] =  $-\frac{13}{420} L$  IdentityMatrix[2];

M[[3 ;; 4, 7 ;; 8]] = M[[7 ;; 8, 3 ;; 4]] =  $-\frac{1}{140} L^2$  IdentityMatrix[2];

M[[5 ;; 6, 7 ;; 8]] = M[[7 ;; 8, 5 ;; 6]] =  $-\frac{11}{210} L$  IdentityMatrix[2];

M[[3 ;; 4, 5 ;; 6]] = M[[5 ;; 6, 3 ;; 4]] =  $\frac{13}{420} L$  IdentityMatrix[2];

Mfun[m_] := m * M;

Mi = Transpose[TBi].Mfun[m].TBi /. vals;

Mj = Transpose[TBj].Mfun[m].TBj /. vals;

M = Mi + Mj;

MatrixForm[M]

MatrixForm[N[M]]

```

Out[260]//MatrixForm=

$$\begin{pmatrix}
 \frac{26}{35} & 0 & \frac{11}{105} & 0 & \frac{9}{35} & 0 & -\frac{13}{140\sqrt{3}} & 0 & -\frac{13}{420} \\
 0 & \frac{26}{35} & 0 & \frac{11}{105} & 0 & \frac{9}{35} & 0 & -\frac{13}{140\sqrt{3}} & 0 \\
 \frac{11}{105} & 0 & \frac{2}{105} & 0 & \frac{13}{210} & 0 & -\frac{\sqrt{3}}{140} & 0 & -\frac{1}{140} \\
 0 & \frac{11}{105} & 0 & \frac{2}{105} & 0 & \frac{13}{210} & 0 & -\frac{\sqrt{3}}{140} & 0 \\
 \frac{9}{35} & 0 & \frac{13}{210} & 0 & \frac{52}{35} & 0 & \frac{11}{210} - \frac{11}{70\sqrt{3}} & 0 & -\frac{11}{210} + \frac{1}{70} \\
 0 & \frac{9}{35} & 0 & \frac{13}{210} & 0 & \frac{52}{35} & 0 & \frac{11}{210} - \frac{11}{70\sqrt{3}} & 0 \\
 -\frac{13}{140\sqrt{3}} & 0 & -\frac{\sqrt{3}}{140} & 0 & \frac{11}{210} - \frac{11}{70\sqrt{3}} & 0 & \frac{2}{105} & 0 & \frac{1}{35\sqrt{3}} \\
 0 & -\frac{13}{140\sqrt{3}} & 0 & -\frac{\sqrt{3}}{140} & 0 & \frac{11}{210} - \frac{11}{70\sqrt{3}} & 0 & \frac{2}{105} & 0 \\
 -\frac{13}{420} & 0 & -\frac{1}{140} & 0 & -\frac{11}{210} + \frac{11}{70\sqrt{3}} & 0 & \frac{1}{35\sqrt{3}} & 0 & \frac{2}{105} \\
 0 & -\frac{13}{420} & 0 & -\frac{1}{140} & 0 & -\frac{11}{210} + \frac{11}{70\sqrt{3}} & 0 & \frac{1}{35\sqrt{3}} & 0 \\
 0 & 0 & 0 & 0 & \frac{9}{35} & 0 & \frac{13}{420} & 0 & \frac{13}{140\sqrt{3}} \\
 0 & 0 & 0 & 0 & 0 & \frac{9}{35} & 0 & \frac{13}{420} & 0 \\
 0 & 0 & 0 & 0 & -\frac{13}{210} & 0 & -\frac{1}{140} & 0 & -\frac{\sqrt{3}}{140} \\
 0 & 0 & 0 & 0 & 0 & -\frac{13}{210} & 0 & -\frac{1}{140} & 0
 \end{pmatrix}$$

Out[261]//MatrixForm=

$$\begin{pmatrix}
 0.742857 & 0. & 0.104762 & 0. & 0.257143 & 0. & -0.0536 \\
 0. & 0.742857 & 0. & 0.104762 & 0. & 0.257143 & 0. \\
 0.104762 & 0. & 0.0190476 & 0. & 0.0619048 & 0. & -0.0123 \\
 0. & 0.104762 & 0. & 0.0190476 & 0. & 0.0619048 & 0. \\
 0.257143 & 0. & 0.0619048 & 0. & 1.48571 & 0. & -0.0383 \\
 0. & 0.257143 & 0. & 0.0619048 & 0. & 1.48571 & 0. \\
 -0.0536111 & 0. & -0.0123718 & 0. & -0.0383455 & 0. & 0.01904 \\
 0. & -0.0536111 & 0. & -0.0123718 & 0. & -0.0383455 & 0. \\
 -0.0309524 & 0. & -0.00714286 & 0. & 0.0383455 & 0. & 0.01649 \\
 0. & -0.0309524 & 0. & -0.00714286 & 0. & 0.0383455 & 0. \\
 0. & 0. & 0. & 0. & 0.257143 & 0. & 0.03095 \\
 0. & 0. & 0. & 0. & 0. & 0.257143 & 0. \\
 0. & 0. & 0. & 0. & -0.0619048 & 0. & -0.00714 \\
 0. & 0. & 0. & 0. & 0. & -0.0619048 & 0.
 \end{pmatrix}$$

ksi = x1 / 1;

```

s1 = 1 - 3 ksi^2 + 2 ksi^3;
s2 = 1 (ksi - 2 ksi^2 + ksi^3);
s3 = 3 ksi^2 - 2 ksi^3;
s4 = 1 (-ksi^2 + ksi^3);
S[ksi_] =  $\begin{pmatrix} s1 & 0 & s2 & 0 & s3 & 0 & s4 & 0 \\ 0 & s1 & 0 & s2 & 0 & s3 & 0 & s4 \end{pmatrix}$ 
eVector = {e1, e2, e3, e4, e5, e6, e7, e8};
{ {1 - 3 ksi^2 + 2 ksi^3, 0, (ksi - 2 ksi^2 + ksi^3) 1, 0, 3 ksi^2 - 2 ksi^3, 0, (-ksi^2 + ksi^3) 1, 0},
  {0, 1 - 3 ksi^2 + 2 ksi^3, 0, (ksi - 2 ksi^2 + ksi^3) 1, 0, 3 ksi^2 - 2 ksi^3, 0, (-ksi^2 + ksi^3) 1} }

dS[ksi_] = D[S[ksi], ksi];
MatrixForm[%]

$$\begin{pmatrix} -6 \text{ ksi} + 6 \text{ ksi}^2 & 0 & (1 - 4 \text{ ksi} + 3 \text{ ksi}^2) 1 & 0 & 6 \text{ ksi} - 6 \text{ ksi}^2 \\ 0 & -6 \text{ ksi} + 6 \text{ ksi}^2 & 0 & (1 - 4 \text{ ksi} + 3 \text{ ksi}^2) 1 & 0 \end{pmatrix}$$


ddS[ksi_] = D[S[ksi], {ksi, 2}];
MatrixForm[%]

$$\begin{pmatrix} -6 + 12 \text{ ksi} & 0 & (-4 + 6 \text{ ksi}) 1 & 0 & 6 - 12 \text{ ksi} & 0 & (-2 + 6 \text{ ksi}) 1 \\ 0 & -6 + 12 \text{ ksi} & 0 & (-4 + 6 \text{ ksi}) 1 & 0 & 6 - 12 \text{ ksi} & 0 \end{pmatrix}$$


ddST[ksi_] = Transpose[ddS[ksi]];
MatrixForm[%]

$$\begin{pmatrix} -6 + 12 \text{ ksi} & 0 \\ 0 & -6 + 12 \text{ ksi} \\ (-4 + 6 \text{ ksi}) 1 & 0 \\ 0 & (-4 + 6 \text{ ksi}) 1 \\ 6 - 12 \text{ ksi} & 0 \\ 0 & 6 - 12 \text{ ksi} \\ (-2 + 6 \text{ ksi}) 1 & 0 \\ 0 & (-2 + 6 \text{ ksi}) 1 \end{pmatrix}$$


Dot[ddST[ksi], ddS[ksi]];
MatrixForm[%]

$$\begin{pmatrix} (-6 + 12 \text{ ksi})^2 & 0 & (-4 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 \\ 0 & (-6 + 12 \text{ ksi})^2 & 0 & (-4 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 \\ (-4 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 & 0 & (-4 + 6 \text{ ksi})^2 1^2 & 0 \\ 0 & (-4 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 & 0 & (-4 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 \\ (6 - 12 \text{ ksi}) (-6 + 12 \text{ ksi}) & 0 & (6 - 12 \text{ ksi}) (-4 + 6 \text{ ksi}) 1 & 0 \\ 0 & (6 - 12 \text{ ksi}) (-6 + 12 \text{ ksi}) & 0 & (6 - 12 \text{ ksi}) (-4 + 6 \text{ ksi}) 1 \\ (-2 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 & 0 & (-4 + 6 \text{ ksi}) (-2 + 6 \text{ ksi}) 1^2 & 0 \\ 0 & (-2 + 6 \text{ ksi}) (-6 + 12 \text{ ksi}) 1 & 0 & (-2 + 6 \text{ ksi}) (-4 + 6 \text{ ksi}) 1 \end{pmatrix}$$


```

```
Integrate[Dot[ddST[ksi], ddS[ksi]], ksi];
MatrixForm[%]
```

$$\begin{pmatrix} 36 \text{ ksi} - 72 \text{ ksi}^2 + 48 \text{ ksi}^3 & 0 & (24 \text{ ksi} - 42 \text{ ksi}^2 + 24 \text{ ksi}^3) \text{ l} \\ 0 & 36 \text{ ksi} - 72 \text{ ksi}^2 + 48 \text{ ksi}^3 & 0 \\ (24 \text{ ksi} - 42 \text{ ksi}^2 + 24 \text{ ksi}^3) \text{ l} & 0 & -\frac{1}{18} (4 - 6 \text{ ksi})^3 \text{ l}^2 \\ 0 & (24 \text{ ksi} - 42 \text{ ksi}^2 + 24 \text{ ksi}^3) \text{ l} & 0 \\ -36 \text{ ksi} + 72 \text{ ksi}^2 - 48 \text{ ksi}^3 & 0 & (-24 \text{ ksi} + 42 \text{ ksi}^2 - 24 \text{ ksi}^3) \text{ l} \\ 0 & -36 \text{ ksi} + 72 \text{ ksi}^2 - 48 \text{ ksi}^3 & 0 \\ (12 \text{ ksi} - 30 \text{ ksi}^2 + 24 \text{ ksi}^3) \text{ l} & 0 & (8 \text{ ksi} - 18 \text{ ksi}^2 + 12 \text{ ksi}^3) \text{ l}^2 \\ 0 & (12 \text{ ksi} - 30 \text{ ksi}^2 + 24 \text{ ksi}^3) \text{ l} & 0 \end{pmatrix}$$

```
Integrate[Dot[ddST[ksi], ddS[ksi]], {ksi, 0, 1}];
MatrixForm[%]
```

$$\begin{pmatrix} 12 & 0 & 6 \text{ l} & 0 & -12 & 0 & 6 \text{ l} & 0 \\ 0 & 12 & 0 & 6 \text{ l} & 0 & -12 & 0 & 6 \text{ l} \\ 6 \text{ l} & 0 & 4 \text{ l}^2 & 0 & -6 \text{ l} & 0 & 2 \text{ l}^2 & 0 \\ 0 & 6 \text{ l} & 0 & 4 \text{ l}^2 & 0 & -6 \text{ l} & 0 & 2 \text{ l}^2 \\ -12 & 0 & -6 \text{ l} & 0 & 12 & 0 & -6 \text{ l} & 0 \\ 0 & -12 & 0 & -6 \text{ l} & 0 & 12 & 0 & -6 \text{ l} \\ 6 \text{ l} & 0 & 2 \text{ l}^2 & 0 & -6 \text{ l} & 0 & 4 \text{ l}^2 & 0 \\ 0 & 6 \text{ l} & 0 & 2 \text{ l}^2 & 0 & -6 \text{ l} & 0 & 4 \text{ l}^2 \end{pmatrix}$$

```
D[s1, {ksi, 2}]
```

```
D[s2, {ksi, 2}]
```

```
D[s3, {ksi, 2}]
```

```
D[s4, {ksi, 2}]
```

```
-6 + 12 ksi
```

```
(-4 + 6 ksi) l
```

```
6 - 12 ksi
```

```
(-2 + 6 ksi) l
```

```
D[s1, ksi]
```

```
D[s2, ksi]
```

```
D[s3, ksi]
```

```
D[s4, ksi]
```

```
-6 ksi + 6 ksi^2
```

```
(1 - 4 ksi + 3 ksi^2) l
```

```
6 ksi - 6 ksi^2
```

```
(-2 ksi + 3 ksi^2) l
```

```
D[S[ksi], ksi];
```

```
MatrixForm[%]
```

$$\begin{pmatrix} -6 \text{ ksi} + 6 \text{ ksi}^2 & 0 & (1 - 4 \text{ ksi} + 3 \text{ ksi}^2) \text{ l} & 0 & 6 \text{ ksi} - 6 \text{ ksi}^2 \\ 0 & -6 \text{ ksi} + 6 \text{ ksi}^2 & 0 & (1 - 4 \text{ ksi} + 3 \text{ ksi}^2) \text{ l} & 0 & 6 \end{pmatrix}$$

```
STS = Dot[Transpose[S[x1]], S[x1]]
```

[illegible]

```
STS = Transpose[STS]
```

True

MatrixForm[%]

$$\begin{pmatrix} \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right)^2 & 0 & 1 \left(\frac{x1}{1} - \frac{2x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) \\ 0 & \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right)^2 & 0 \\ 1 \left(\frac{x1}{1} - \frac{2x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 & 1^2 \left(\frac{x1}{1} - \frac{2x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) \\ 0 & 1 \left(\frac{x1}{1} - \frac{2x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 \\ \left(\frac{3x1^2}{1^2} - \frac{2x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 & 1 \left(\frac{3x1^2}{1^2} - \frac{2x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) \\ 0 & \left(\frac{3x1^2}{1^2} - \frac{2x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 \\ 1 \left(-\frac{x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 & 1^2 \left(-\frac{x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) \\ 0 & 1 \left(-\frac{x1^2}{1^2} + \frac{x1^3}{1^3}\right) \left(1 - \frac{3x1^2}{1^2} + \frac{2x1^3}{1^3}\right) & 0 \end{pmatrix}$$

FullSimplify[%]

$$\begin{aligned} & \left\{ \left\{ \frac{(1-x1)^4 (1+2x1)^2}{1^6}, 0, \frac{(1-x1)^4 x1 (1+2x1)}{1^5}, 0, \right. \right. \\ & \quad \left. \frac{(31-2x1)(1-x1)^2 x1^2 (1+2x1)}{1^6}, 0, -\frac{(1-x1)^3 x1^2 (1+2x1)}{1^5}, 0 \right\}, \\ & \left\{ 0, \frac{(1-x1)^4 (1+2x1)^2}{1^6}, 0, \frac{(1-x1)^4 x1 (1+2x1)}{1^5}, 0, \right. \\ & \quad \left. \frac{(31-2x1)(1-x1)^2 x1^2 (1+2x1)}{1^6}, 0, -\frac{(1-x1)^3 x1^2 (1+2x1)}{1^5} \right\}, \\ & \left\{ \frac{(1-x1)^4 x1 (1+2x1)}{1^5}, 0, \frac{(1-x1)^4 x1^2}{1^4}, 0, \frac{(31-2x1)(1-x1)^2 x1^3}{1^5}, \right. \\ & \quad \left. 0, -\frac{(1-x1)^3 x1^3}{1^4}, 0 \right\}, \left\{ 0, \frac{(1-x1)^4 x1 (1+2x1)}{1^5}, 0, \right. \\ & \quad \left. \frac{(1-x1)^4 x1^2}{1^4}, 0, \frac{(31-2x1)(1-x1)^2 x1^3}{1^5}, 0, -\frac{(1-x1)^3 x1^3}{1^4} \right\}, \\ & \left\{ \frac{(31-2x1)(1-x1)^2 x1^2 (1+2x1)}{1^6}, 0, \frac{(31-2x1)(1-x1)^2 x1^3}{1^5}, \right. \\ & \quad \left. 0, \frac{(31-2x1)^2 x1^4}{1^6}, 0, -\frac{(31-2x1)(1-x1)x1^4}{1^5}, 0 \right\}, \\ & \left\{ 0, \frac{(31-2x1)(1-x1)^2 x1^2 (1+2x1)}{1^6}, 0, \frac{(31-2x1)(1-x1)^2 x1^3}{1^5}, \right. \\ & \quad \left. 0, \frac{(31-2x1)^2 x1^4}{1^6}, 0, -\frac{(31-2x1)(1-x1)x1^4}{1^5} \right\}, \\ & \left\{ -\frac{(1-x1)^3 x1^2 (1+2x1)}{1^5}, 0, -\frac{(1-x1)^3 x1^3}{1^4}, 0, -\frac{(31-2x1)(1-x1)x1^4}{1^5}, \right. \\ & \quad \left. 0, \frac{(1-x1)^2 x1^4}{1^4}, 0 \right\}, \left\{ 0, -\frac{(1-x1)^3 x1^2 (1+2x1)}{1^5}, 0, \right. \\ & \quad \left. -\frac{(1-x1)^3 x1^3}{1^4}, 0, -\frac{(31-2x1)(1-x1)x1^4}{1^5}, 0, \frac{(1-x1)^2 x1^4}{1^4} \right\} \end{aligned}$$

MatrixForm[%]

$$\begin{pmatrix} \frac{(1-x_1)^4 (1+2 x_1)^2}{1^6} & 0 & \frac{(1-x_1)^4 x_1 (1+2 x_1)}{1^5} & 0 & \frac{(3 1-2 x_1)}{1^6} \\ 0 & \frac{(1-x_1)^4 (1+2 x_1)^2}{1^6} & 0 & \frac{(1-x_1)^4 x_1 (1+2 x_1)}{1^5} & 0 \\ \frac{(1-x_1)^4 x_1 (1+2 x_1)}{1^5} & 0 & \frac{(1-x_1)^4 x_1^2}{1^4} & 0 & \frac{(3 1-x_1)}{1^6} \\ 0 & \frac{(1-x_1)^4 x_1 (1+2 x_1)}{1^5} & 0 & \frac{(1-x_1)^4 x_1^2}{1^4} & 0 \\ \frac{(3 1-2 x_1) (1-x_1)^2 x_1^2 (1+2 x_1)}{1^6} & 0 & \frac{(3 1-2 x_1) (1-x_1)^2 x_1^3}{1^5} & 0 & \frac{(3 1-2 x_1) (1-x_1)^2 x_1^3}{1^5} \\ 0 & \frac{(3 1-2 x_1) (1-x_1)^2 x_1^2 (1+2 x_1)}{1^6} & 0 & \frac{(3 1-2 x_1) (1-x_1)^2 x_1^3}{1^5} & 0 \\ -\frac{(1-x_1)^3 x_1^2 (1+2 x_1)}{1^5} & 0 & -\frac{(1-x_1)^3 x_1^3}{1^4} & 0 & -\frac{(1-x_1)^3 x_1^3}{1^4} \\ 0 & -\frac{(1-x_1)^3 x_1^2 (1+2 x_1)}{1^5} & 0 & -\frac{(1-x_1)^3 x_1^3}{1^4} & 0 \end{pmatrix}$$