Označíme

$$C_\alpha(\theta) = \left(\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x \right)^{\frac{\alpha}{1+\alpha}} \tag{1}$$

pak Rényiho odhady po dosazení empirického rozdělení pravděpodobnosti P_n vypadají

$$\theta_{\alpha,n} = \begin{cases} \arg\max\limits_{\theta\in\Theta} C_\alpha(\theta)^{-1} \dfrac{1}{n} \sum\limits_{i=1}^{n} p_\theta^\alpha(X_i) & \text{pro } 0 < \alpha \leq \beta, \\ \arg\max\limits_{\theta\in\Theta} \dfrac{1}{n} \sum\limits_{i=1}^{n} p_\theta^\alpha(X_i) & \text{pro } \alpha = 0 \end{cases} \tag{2}$$

1 Exponenciální rozdělení

$$\begin{aligned} \int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \int_\mu^\infty \left(\frac{1}{\theta} \exp\left[-\frac{(x-\mu)}{\theta} \right] \right)^{1+\alpha} \mathrm{d}x \\ &= \int_\mu^\infty \frac{1}{\theta^{1+\alpha}} \exp\left[-\frac{(1+\alpha)(x-\mu)}{\theta} \right] \mathrm{d}x \end{aligned}$$

substituujeme $y = \frac{(1+\alpha)(x-\mu)}{\theta}$, tedy $\mathrm{d}y = \frac{1+\alpha}{\theta}\mathrm{d}x$, pak

$$\begin{aligned} \int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \frac{1}{\theta^{1+\alpha}} \int_0^\infty \frac{\theta}{1+\alpha} \exp[-y]\,\mathrm{d}x \\ &= \frac{\theta^{-\alpha}}{1+\alpha} \end{aligned} \tag{3}$$

tedy

$$\begin{aligned} \theta_{\alpha,n} &= \arg\max_{\theta\in\Theta} \left(\frac{\theta^{-\alpha}}{1+\alpha} \right)^{-\frac{\alpha}{1+\alpha}} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\theta} \exp\left[-\frac{x-\mu}{\theta} \right] \\ &= \arg\max_{\theta\in\Theta} \theta^{\frac{\alpha^2-\alpha-1}{1+\alpha}} \frac{1}{n} \sum_{i=1}^{n} \exp\left[\frac{x-\mu}{\theta} \right] \end{aligned} \tag{4}$$

2 Laplaceovo rozdělení

$$\begin{aligned}\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \int_{-\infty}^{\infty}\left(\frac{1}{2\theta}\exp\left[-\frac{|x-\mu|}{\theta}\right]\right)^{1+\alpha}\mathrm{d}x \\ &= \int_{-\infty}^{\mu}\frac{1}{(2\theta)^{1+\alpha}}\exp\left[\frac{(1+\alpha)(x-\mu)}{\theta}\right]\mathrm{d}x \\ &+ \int_{\mu}^{\infty}\frac{1}{(2\theta)^{1+\alpha}}\exp\left[-\frac{(1+\alpha)(x-\mu)}{\theta}\right]\mathrm{d}x\end{aligned}$$

substituujeme $y = \frac{(1+\alpha)(x-\mu)}{\theta}$, tedy $\mathrm{d}y = \frac{1+\alpha}{\theta}\mathrm{d}x$, pak

$$\begin{aligned}\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \frac{1}{(2\theta)^{1+\alpha}}\frac{\theta}{1+\alpha}\left(\int_{-\infty}^{0}\exp[y]\,\mathrm{d}x + \int_{0}^{\infty}\exp[-y]\,\mathrm{d}x\right) \\ &= \frac{1}{(2\theta)^{1+\alpha}}\frac{\theta}{1+\alpha}\cdot 2 \\ &= \frac{(2\theta)^{-\alpha}}{(1+\alpha)} \qquad (5)\end{aligned}$$

tedy

$$\begin{aligned}\theta_{\alpha,n} &= \arg\max_{\theta\in\Theta}\left(\frac{(2\theta)^{-\alpha}}{(1+\alpha)}\right)^{-\frac{\alpha}{1+\alpha}}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{2\theta}\exp\left[-\frac{|x-\mu|}{\theta}\right] \\ &= \arg\max_{\theta\in\Theta}(2\theta)^{\frac{\alpha^2-\alpha-1}{1+\alpha}}\frac{1}{n}\sum_{i=1}^{n}\exp\left[-\frac{|x-\mu|}{\theta}\right] \qquad (6)\end{aligned}$$

3 Rovnoměrné rozdělení

$$\begin{aligned}\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \int_a^b\left(\frac{1}{b-a}\right)^{1+\alpha}\mathrm{d}x \\ &= \frac{b-a}{(b-a)^{\alpha+1}}\end{aligned}$$

tedy

$$\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x = (b-a)^{-\alpha} \qquad (7)$$

tedy

$$
\begin{aligned}
\theta_{\alpha,n} &= \arg\max_{\theta\in\Theta} \left((b-a)^{-\alpha}\right)^{-\frac{\alpha}{1+\alpha}} \frac{1}{n}\sum_{i=1}^{n} \frac{1}{b-a} \\
&= \arg\max_{\theta\in\Theta} (b-a)^{\frac{\alpha^2-\alpha-1}{1+\alpha}} \\
&= \arg\max_{\theta\in\Theta} (b-a) \qquad (8)
\end{aligned}
$$

4 Cauchyovo rozdělení

$$
\begin{aligned}
\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \int_{-\infty}^{\infty} \left(\frac{1}{\pi\sigma} \left[1 + \left(\frac{x-\mu}{\sigma}\right)^2 \right]^{-1} \right)^{1+\alpha} \mathrm{d}x \\
&= \ldots
\end{aligned}
$$

pak

$$
\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x = \frac{1}{\pi^{\frac{1}{2}+\alpha}\sigma^\alpha} \frac{\Gamma(\frac{1}{2}+\alpha)}{\alpha\Gamma(\alpha)} \qquad (9)
$$

5 Weibullovo rozdělení

$$
\begin{aligned}
\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x &= \int_{-\mu}^{\infty} \left(\frac{k}{\lambda} \left(\frac{x-\mu}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{x-\mu}{\lambda}\right)^k \right] \right)^{1+\alpha} \mathrm{d}x \\
&= \ldots
\end{aligned}
$$

pak

$$
\int p_\theta^{1+\alpha}(x)\,\mathrm{d}x = \frac{k^\alpha}{\lambda^\alpha} (1+\alpha)^{-\frac{1+\alpha+k}{k}} \Gamma\left(\frac{1+\alpha+k}{k}\right) \qquad (10)
$$