Weibull :

In[266]:= ClearAll[α, σ, μ, x]

In[267]:= $p = \frac{k}{\lambda} e^{-\left(\frac{x-\mu}{\lambda}\right)^k} \left(\frac{x-\mu}{\lambda}\right)^{k-1}$

Out[267]= $\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}$

In[268]:= $\theta = k;$

In[296]:=

ss = FullSimplify[D[Log[p], θ]]

Out[296]= $\frac{1 - k \left(-1 + \left(\frac{x-\mu}{\lambda}\right)^k\right) \mathrm{Log}\left[\frac{x-\mu}{\lambda}\right]}{k}$

In[297]:= ss' = FullSimplify[D[ss, θ]]

Out[297]= $-\frac{1}{k^2} - \left(\frac{x-\mu}{\lambda}\right)^k \mathrm{Log}\left[\frac{x-\mu}{\lambda}\right]^2$

In[298]:= csIntCitatel1 = FullSimplify[p ^ (1 + α) * ss]

Out[298]= $\frac{\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \left(1 - k \left(-1 + \left(\frac{x-\mu}{\lambda}\right)^k\right) \mathrm{Log}\left[\frac{x-\mu}{\lambda}\right]\right)}{k}$

In[299]:= csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x - μ) → y * λ]

Out[299]= $\frac{\left(\frac{e^{-y^k} k\, y^{-1+k}}{\lambda}\right)^{1+\alpha} \left(1 - k \left(-1 + y^k\right) \mathrm{Log}[y]\right)}{k}$

In[300]:= csIntCitatel3 = FullSimplify[csIntCitatel2 * λ /. y → t ^ (1 / k)]

Out[300]= $\frac{\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^k} k \left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \lambda \left(1 - k \left(-1 + \left(t^{\frac{1}{k}}\right)^k\right) \mathrm{Log}\left[t^{\frac{1}{k}}\right]\right)}{k}$

In[301]:= csIntCitatel4 = FullSimplify[csIntCitatel3 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]

Out[301]= $\frac{e^{-t} \left(\frac{e^{-t} k\, t^{\frac{-1+k}{k}}}{\lambda}\right)^{\alpha} \left(1 + \mathrm{Log}[t] - t\, \mathrm{Log}[t]\right)}{k}$

In[302]:= `csIntCitatel5 = FullSimplify[Integrate[csIntCitatel4, {t, 0, ∞}]]`

Out[302]= ConditionalExpression[

$$\frac{\alpha\,(1+\alpha)^{-2+\left(-1+\frac{1}{k}\right)\alpha}\left(\frac{k}{\lambda}\right)^{\alpha}\,\mathrm{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right]\left(k-\mathrm{Log}[1+\alpha]+\mathrm{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)}{k^{2}},$$

$$\mathrm{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \mathrm{Re}\left[\frac{(-1+k)\,\alpha}{k}\right]\geq -1\ \&\&\ \mathrm{Re}[\alpha]>-1\Big]$$

In[269]:= `csIntJmenovatel1 = FullSimplify[p^(1 + α)]`

Out[269]=
$$\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,k\,\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[271]:= `csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x - μ) → y * λ]`

Out[271]=
$$\left(\frac{e^{-y^{k}}\,k\,y^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[293]:= `csIntJmenovatel3 = FullSimplify[csIntJmenovatel2 * λ /. y → t^(1/k)]`

Out[293]=
$$\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}}\,k\,\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}\lambda$$

In[294]:= `csIntJmenovatel4 = FullSimplify[csIntJmenovatel3 / k * t^((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]`

Out[294]=
$$e^{-t}\left(\frac{e^{-t}\,k\,t^{\frac{-1+k}{k}}}{\lambda}\right)^{\alpha}$$

In[295]:= `csIntJmenovatel5 = FullSimplify[Integrate[csIntJmenovatel4, {t, 0, ∞}]]`

Out[295]=
$$\mathrm{ConditionalExpression}\left[(1+\alpha)^{-1+\left(-1+\frac{1}{k}\right)\alpha}\left(\frac{k}{\lambda}\right)^{\alpha}\mathrm{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right],\ \mathrm{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \mathrm{Re}[\alpha]>-1\right]$$

In[303]:= `cs = FullSimplify[csIntCitatel5 / csIntJmenovatel5]`

Out[303]= ConditionalExpression[

$$\frac{\alpha\left(k-\mathrm{Log}[1+\alpha]+\mathrm{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)}{k^{2}\,(1+\alpha)},$$

$$\mathrm{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \mathrm{Re}\left[\frac{(-1+k)\,\alpha}{k}\right]\geq -1\ \&\&\ \mathrm{Re}[\alpha]>-1\Big]$$

In[304]:= `cs' = FullSimplify[D[cs, θ]]`

Out[304]= ConditionalExpression[

$$\frac{\alpha\left(-k\left(k-2\,\mathrm{Log}[1+\alpha]+2\,\mathrm{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)+\alpha\,\mathrm{PolyGamma}\left[1,1+\alpha-\frac{\alpha}{k}\right]\right)}{k^{4}\,(1+\alpha)},$$

$$\mathrm{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \mathrm{Re}\left[\frac{(-1+k)\,\alpha}{k}\right]\geq -1\ \&\&\ \mathrm{Re}[\alpha]>-1\Big]$$

In[305]:= `Ia = FullSimplify[(ss' - cs' - α (ss - cs) (cs - ss)) * p^(1 + α)]`

Out[305]=
$$\text{ConditionalExpression}\Bigg[\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \Bigg(-\frac{1}{k^2} - \left(\frac{x-\mu}{\lambda}\right)^k \text{Log}\left[\frac{x-\mu}{\lambda}\right]^2 + \frac{\alpha \left(-\alpha\, \text{Log}[1+\alpha] + k \left(-1 + k (1+\alpha) \left(-1 + \left(\frac{x-\mu}{\lambda}\right)^k\right) \text{Log}\left[\frac{x-\mu}{\lambda}\right]\right) + \alpha\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right)^2}{k^4 (1+\alpha)^2} + \frac{\alpha \left(k \left(k - 2\, \text{Log}[1+\alpha] + 2\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right) - \alpha\, \text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\right)}{k^4 (1+\alpha)}\Bigg),$$
$$\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right] < 1\ \&\&\ \text{Re}\left[\frac{(-1+k)\,\alpha}{k}\right] \geq -1\ \&\&\ \text{Re}[\alpha] > -1\Bigg]$$

In[307]:= `Ia1 = FullSimplify[Ia /. (x - μ) → y * λ]`

Out[307]=
$$\text{ConditionalExpression}\Bigg[\left(\frac{e^{-y^k} k\, y^{-1+k}}{\lambda}\right)^{1+\alpha} \Bigg(-\frac{1}{k^2} - y^k\, \text{Log}[y]^2 + \frac{\alpha \left(k \left(-1 + k \left(-1 + y^k\right) (1+\alpha)\, \text{Log}[y]\right) - \alpha\, \text{Log}[1+\alpha] + \alpha\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right)^2}{k^4 (1+\alpha)^2} + \frac{\alpha \left(k \left(k - 2\, \text{Log}[1+\alpha] + 2\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right) - \alpha\, \text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\right)}{k^4 (1+\alpha)}\Bigg),$$
$$\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right] < 1\ \&\&\ \text{Re}\left[\frac{(-1+k)\,\alpha}{k}\right] \geq -1\ \&\&\ \text{Re}[\alpha] > -1\Bigg]$$

In[308]:= `Ia2 = FullSimplify[Ia1 * λ /. y → t^(1 / k)]`

Out[308]=
$$\text{ConditionalExpression}\Bigg[\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^k} k \left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \lambda \Bigg(-\frac{1}{k^2} - \left(t^{\frac{1}{k}}\right)^k \text{Log}\left[t^{\frac{1}{k}}\right]^2 + \frac{\alpha \left(k \left(-1 + k \left(-1 + \left(t^{\frac{1}{k}}\right)^k\right) (1+\alpha)\, \text{Log}\left[t^{\frac{1}{k}}\right]\right) - \alpha\, \text{Log}[1+\alpha] + \alpha\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right)^2}{k^4 (1+\alpha)^2} + \frac{\alpha \left(k \left(k - 2\, \text{Log}[1+\alpha] + 2\, \text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right) - \alpha\, \text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\right)}{k^4 (1+\alpha)}\Bigg),$$
$$\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right] < 1\ \&\&\ \text{Re}\left[\frac{(-1+k)\,\alpha}{k}\right] \geq -1\ \&\&\ \text{Re}[\alpha] > -1\Bigg]$$

In[309]:= **Ia3 = FullSimplify[Ia2 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]**

Out[309]= $$\text{ConditionalExpression}\Big[\frac{1}{k^5}\, t^{-1+\frac{1}{k}} \left(\frac{e^{-t}\, k\, t^{\frac{-1+k}{k}}}{\lambda}\right)^{1+\alpha} \lambda \left(-k^2 - k^2\, t\, \text{Log}[t]^2 + \frac{\alpha \left(k - k\,(-1+t)\,(1+\alpha)\,\text{Log}[t] + \alpha\,\text{Log}[1+\alpha] - \alpha\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right)^2}{(1+\alpha)^2} + \frac{\alpha \left(k \left(k - 2\,\text{Log}[1+\alpha] + 2\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right) - \alpha\,\text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\right)}{1+\alpha}\right), k + k\,\alpha > \alpha\Big]$$

In[310]:= **Ia4 = FullSimplify[Integrate[Ia3, {t, 0, ∞}]]**

Out[310]= $$\text{ConditionalExpression}\Big[-\frac{1}{k^4}\,(1+\alpha)^{-3+\left(-1+\frac{1}{k}\right)\alpha} \left(\frac{k}{\lambda}\right)^{\alpha} \text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right] \Big(k\,(k + \text{Log}[1+\alpha]\,(-2\,k + 2\,\alpha + (k + (-1+k)\,\alpha)\,\text{Log}[1+\alpha])) - 2\,k\,(-k + \alpha + (k + (-1+k)\,\alpha)\,\text{Log}[1+\alpha])\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right] + k\,(k + (-1+k)\,\alpha)\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]^2 + \left(k^2 + (-1+k)\,k\,\alpha + \alpha^2\right)\text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\Big), k + k\,\alpha > \alpha\ \&\&\ \alpha > -1\ \&\&\ k > 0\Big]$$

In[311]:= **IF = FullSimplify[-Ia4 ^ (-1) * (p ^ α) * (ss - cs)]**

Out[311]= $$\text{ConditionalExpression}\Bigg[\left(k^2\,(1+\alpha)^{2+\alpha-\frac{\alpha}{k}} \left(\frac{1}{\lambda}\right)^{-\alpha} \left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{\alpha} \left(\alpha\,\text{Log}[1+\alpha] + k\left(1 - k\,(1+\alpha)\left(-1 + \left(\frac{x-\mu}{\lambda}\right)^k\right)\text{Log}\left[\frac{x-\mu}{\lambda}\right]\right) - \alpha\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]\right)\right) \Big/ \left(\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right] \Big(k\,(k + \text{Log}[1+\alpha]\,(-2\,k + 2\,\alpha + (k + (-1+k)\,\alpha)\,\text{Log}[1+\alpha])) - 2\,k\,(-k + \alpha + (k + (-1+k)\,\alpha)\,\text{Log}[1+\alpha])\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right] + k\,(k + (-1+k)\,\alpha)\,\text{PolyGamma}\left[0, 1+\alpha-\frac{\alpha}{k}\right]^2 + \left(k^2 + (-1+k)\,k\,\alpha + \alpha^2\right)\text{PolyGamma}\left[1, 1+\alpha-\frac{\alpha}{k}\right]\Big)\right), k > 0\ \&\&\ 1 + \alpha > 0\ \&\&\ \alpha < k + k\,\alpha\Bigg]$$

In[312]:= **IF1 = FullSimplify[IF /. μ -> 0]**

Out[312]= $$\text{ConditionalExpression}\Bigg[\Bigg(k^2\,(1+\alpha)^{2+\alpha-\frac{\alpha}{k}}\left(\frac{e^{-\left(\frac{x}{\lambda}\right)^k}\left(\frac{x}{\lambda}\right)^k}{x}\right)^{\alpha}\left(\frac{1}{\lambda}\right)^{-\alpha}\left(\alpha\,\text{Log}[1+\alpha]+k\left(1-k\,(1+\alpha)\left(-1+\left(\frac{x}{\lambda}\right)^k\right)\text{Log}\left[\frac{x}{\lambda}\right]\right)-\alpha\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)\Bigg)\Big/\left(\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right]\left(k\,(k+\text{Log}[1+\alpha]\,(-2\,k+2\,\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha]))-2\,k\,(-k+\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha])\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]+k\,(k+(-1+k)\,\alpha)\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]^2+\left(k^2+(-1+k)\,k\,\alpha+\alpha^2\right)\text{PolyGamma}\left[1,1+\alpha-\frac{\alpha}{k}\right]\right)\right),\ k>0\,\&\&\,1+\alpha>0\,\&\&\,\alpha<k+k\,\alpha\Bigg]$$

In[313]:= **IF2 = FullSimplify[IF1 /. λ → 1]**

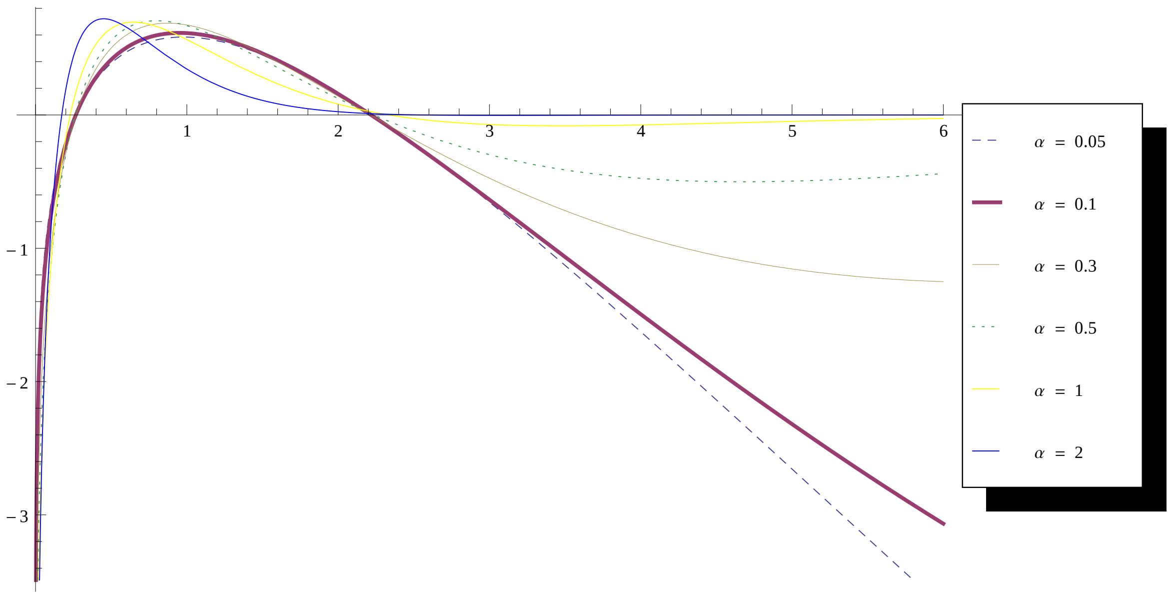
Out[313]= $$\text{ConditionalExpression}\Bigg[\Bigg(k^2\left(e^{-x^k}\,x^{-1+k}\right)^{\alpha}(1+\alpha)^{2+\alpha-\frac{\alpha}{k}}\left(k-k^2\left(-1+x^k\right)(1+\alpha)\,\text{Log}[x]+\alpha\,\text{Log}[1+\alpha]-\alpha\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)\Bigg)\Big/\left(\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right]\left(k\,(k+\text{Log}[1+\alpha]\,(-2\,k+2\,\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha]))-2\,k\,(-k+\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha])\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]+k\,(k+(-1+k)\,\alpha)\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]^2+\left(k^2+(-1+k)\,k\,\alpha+\alpha^2\right)\text{PolyGamma}\left[1,1+\alpha-\frac{\alpha}{k}\right]\right)\right),\ k>0\,\&\&\,1+\alpha>0\,\&\&\,\alpha<k+k\,\alpha\Bigg]$$

In[314]:= **IFun = Function[{k, α}, $\left(k^2\left(e^{-x^k}\,x^{-1+k}\right)^{\alpha}(1+\alpha)^{2+\alpha-\frac{\alpha}{k}}\left(k-k^2\left(-1+x^k\right)(1+\alpha)\,\text{Log}[x]+\alpha\,\text{Log}[1+\alpha]-\alpha\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]\right)\right)\Big/\left(\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right]\left(k\,(k+\text{Log}[1+\alpha]\,(-2\,k+2\,\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha]))-2\,k\,(-k+\alpha+(k+(-1+k)\,\alpha)\,\text{Log}[1+\alpha])\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]+k\,(k+(-1+k)\,\alpha)\,\text{PolyGamma}\left[0,1+\alpha-\frac{\alpha}{k}\right]^2+\left(k^2+(-1+k)\,k\,\alpha+\alpha^2\right)\text{PolyGamma}\left[1,1+\alpha-\frac{\alpha}{k}\right]\right)\right)$];**

Needs["PlotLegends`"]

In[317]:=
```
Plot[{
  IFun[1, 0.05],
  IFun[1, 0.1],
  IFun[1, 0.3],
  IFun[1, 0.5],
  IFun[1, 1],
  IFun[1, 2]},
 {x, 0, 6},
 PlotLegend → {"α = 0.05", "α = 0.1", "α = 0.3", "α = 0.5", "α = 1", "α = 2"},
 LegendPosition → {1, -0.4},
 PlotStyle → {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]
```

Out[317]=



Weibull :

In[320]:= `ClearAll[α, σ, μ, x]`

In[321]:= $p = \frac{k}{\lambda} e^{-\left(\frac{x-\mu}{\lambda}\right)^k} \left(\frac{x-\mu}{\lambda}\right)^{k-1}$

Out[321]= $$\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}$$

In[322]:= $\theta = \lambda;$

In[323]:=

```
ss = FullSimplify[D[Log[p], θ]]
```

Out[323]= $$\frac{k\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)}{\lambda}$$

In[324]:=

```
ss' = FullSimplify[D[ss, θ]]
```

Out[324]= $$\frac{k-k\,(1+k)\left(\frac{x-\mu}{\lambda}\right)^{k}}{\lambda^{2}}$$

In[325]:=

```
csIntCitatel1 = FullSimplify[p^(1+α)*ss]
```

Out[325]= $$\frac{k\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,k\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda}$$

In[326]:=

```
csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x - μ) → y*λ]
```

Out[326]= $$\frac{k\left(-1+y^{k}\right)\left(\frac{e^{-y^{k}}\,k\,y^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda}$$

In[327]:=

```
csIntCitatel3 = FullSimplify[csIntCitatel2*λ /. y → t^(1/k)]
```

Out[327]= $$k\left(-1+\left(t^{\frac{1}{k}}\right)^{k}\right)\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}}\,k\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[328]:=

```
csIntCitatel4 = FullSimplify[csIntCitatel3/k*t^((1-k)/k), {k > 0, t > 0, λ > 0, α > 0}]
```

Out[328]= $$(-1+t)\,t^{\frac{(-1+k)\,\alpha}{k}}\left(\frac{e^{-t}\,k}{\lambda}\right)^{1+\alpha}$$

In[329]:=

```
csIntCitatel5 = FullSimplify[Integrate[csIntCitatel4, {t, 0, ∞}]]
```

Out[329]= $$\text{ConditionalExpression}\left[-\frac{\alpha\,(1+\alpha)^{-2+\left(-1+\frac{1}{k}\right)\alpha}\left(\frac{k}{\lambda}\right)^{\alpha}\,\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right]}{\lambda},\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[331]:=

```
csIntJmenovatel1 = FullSimplify[p^(1+α)]
```

Out[331]= $$\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,k\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[332]:=

```
csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x - μ) → y*λ]
```

Out[332]= $$\left(\frac{e^{-y^{k}}\,k\,y^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[333]:= **csIntJmenovatel3 = FullSimplify[csIntJmenovatel2 * λ /. y → t ^ (1 / k)]**

Out[333]= $$\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}}\, k\, \left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \lambda$$

In[334]:= **csIntJmenovatel4 = FullSimplify[csIntJmenovatel3 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]**

Out[334]= $$e^{-t}\left(\frac{e^{-t}\, k\, t^{\frac{-1+k}{k}}}{\lambda}\right)^{\alpha}$$

In[335]:= **csIntJmenovatel5 = FullSimplify[Integrate[csIntJmenovatel4, {t, 0, ∞}]]**

Out[335]= $$\text{ConditionalExpression}\left[(1+\alpha)^{-1+\left(-1+\frac{1}{k}\right)\alpha}\left(\frac{k}{\lambda}\right)^{\alpha}\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right],\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[336]:= **cs = FullSimplify[csIntCitatel5 / csIntJmenovatel5]**

Out[336]= $$\text{ConditionalExpression}\left[-\frac{\alpha}{\lambda+\alpha\,\lambda},\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[337]:= **cs' = FullSimplify[D[cs, θ]]**

Out[337]= $$\text{ConditionalExpression}\left[\frac{\alpha}{(1+\alpha)\,\lambda^{2}},\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[338]:= **Ia = FullSimplify[(ss' - cs' - α (ss - cs) (cs - ss)) * p ^ (1 + α)]**

Out[338]= $$\text{ConditionalExpression}\Bigg[\frac{\left(-1+k+\frac{1}{1+\alpha}+\frac{\alpha\left(\alpha+k\,(1+\alpha)\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)^{2}}{(1+\alpha)^{2}}-k\,(1+k)\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\, k\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda^{2}},$$
$$\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\Bigg]$$

In[339]:= **Ia1 = FullSimplify[Ia /. (x - μ) → y * λ]**

Out[339]= $$\text{ConditionalExpression}\Bigg[\frac{\left(-1+k-k\,(1+k)\,y^{k}+\frac{1}{1+\alpha}+\frac{\alpha\left(\alpha+k\left(-1+y^{k}\right)(1+\alpha)\right)^{2}}{(1+\alpha)^{2}}\right)\left(\frac{e^{-y^{k}}\,k\,y^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda^{2}},$$
$$\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\Bigg]$$

In[340]:= `Ia2 = FullSimplify[Ia1 * λ /. y → t ^ (1 / k)]`

Out[340]= ConditionalExpression[

$$\frac{\left(-1+k-k\,(1+k)\,\left(t^{\frac{1}{k}}\right)^{k}+\frac{1}{1+\alpha}+\frac{\alpha\,\left(\alpha+k\,\left(-1+\left(t^{\frac{1}{k}}\right)^{k}\right)\,(1+\alpha)\right)^{2}}{(1+\alpha)^{2}}\right)\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}}\,k\,\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda},$$

$\mathrm{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \mathrm{Re}[\alpha]>-1$]

In[341]:= `Ia3 = FullSimplify[Ia2 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]`

Out[341]= ConditionalExpression[

$$e^{-t\,(1+\alpha)}\,k^{\alpha}\,t^{\frac{(-1+k)\,\alpha}{k}}\left(-1+k-k\,(1+k)\,t+\frac{1}{1+\alpha}+\frac{\alpha\,(\alpha+k\,(-1+t)\,(1+\alpha))^{2}}{(1+\alpha)^{2}}\right)\lambda^{-2-\alpha},\ \alpha<k+k\,\alpha$$]

In[342]:= `Ia4 = FullSimplify[Integrate[Ia3, {t, 0, ∞}]]`

Out[342]= ConditionalExpression$\left[-k^{2+\alpha}\,(1+\alpha)^{-3+\left(-1+\frac{1}{k}\right)\alpha}\,\lambda^{-2-\alpha}\,\mathrm{Gamma}\left[2+\alpha-\frac{\alpha}{k}\right],\ \alpha<k+k\,\alpha\ \&\&\ \alpha>-1\ \&\&\ k>0\right]$

In[343]:= `IF = FullSimplify[-Ia4 ^ (-1) * (p ^ α) * (ss - cs)]`

Out[343]= ConditionalExpression[

$$\frac{(1+\alpha)^{2+\alpha-\frac{\alpha}{k}}\,\lambda^{1+\alpha}\,\left(\alpha+k\,(1+\alpha)\,\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{\alpha}}{k^{2}\,\mathrm{Gamma}\left[2+\alpha-\frac{\alpha}{k}\right]},\ \alpha<k+k\,\alpha\ \&\&\ \alpha>-1\ \&\&\ k>0$$]

In[344]:= `IF1 = FullSimplify[IF /. μ -> 0]`

Out[344]= ConditionalExpression[

$$\frac{(1+\alpha)^{2+\alpha-\frac{\alpha}{k}}\,\left(\alpha+k\,(1+\alpha)\,\left(-1+\left(\frac{x}{\lambda}\right)^{k}\right)\right)\left(\frac{e^{-\left(\frac{x}{\lambda}\right)^{k}}\,\left(\frac{x}{\lambda}\right)^{k}}{x}\right)^{\alpha}\lambda^{1+\alpha}}{k^{2}\,\mathrm{Gamma}\left[2+\alpha-\frac{\alpha}{k}\right]},\ \alpha<k+k\,\alpha\ \&\&\ \alpha>-1\ \&\&\ k>0$$]

In[345]:= `IF2 = FullSimplify[IF1 /. k → 1]`

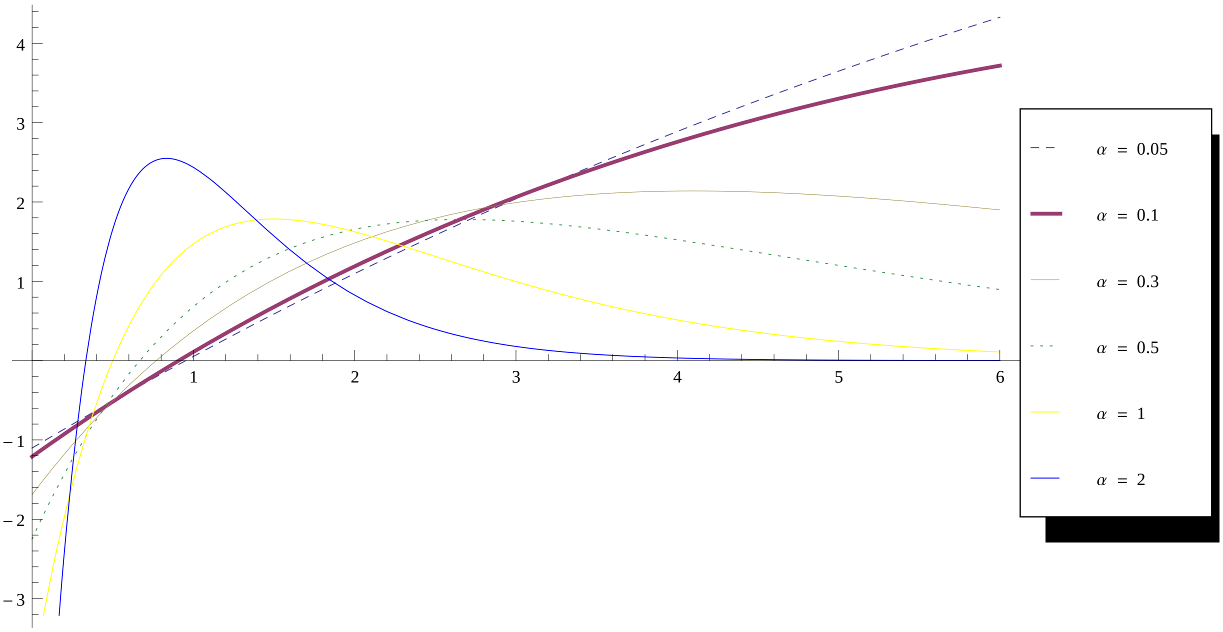
Out[345]= ConditionalExpression$\left[(1+\alpha)^{2}\,(x+x\,\alpha-\lambda)\left(\frac{e^{-\frac{x}{\lambda}}}{\lambda}\right)^{\alpha}\lambda^{\alpha},\ \alpha>-1\right]$

In[346]:= **IFun = Function**$\left[\{\lambda,\alpha\},\ (1+\alpha)^{2}\,(x+x\,\alpha-\lambda)\left(\frac{e^{-\frac{x}{\lambda}}}{\lambda}\right)^{\alpha}\lambda^{\alpha}\right]$;

`Needs["PlotLegends`"]`

In[347]:=
```
Plot[{
  IFun[1, 0.05],
  IFun[1, 0.1],
  IFun[1, 0.3],
  IFun[1, 0.5],
  IFun[1, 1],
  IFun[1, 2]},
 {x, 0, 6},
 PlotLegend → {"α = 0.05", "α = 0.1", "α = 0.3", "α = 0.5", "α = 1", "α = 2"},
 LegendPosition → {1, -0.4},
 PlotStyle → {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]
```

Out[347]=



Weibull :

In[348]:= `ClearAll[α, σ, μ, x]`

In[349]:= $$p = \frac{k}{\lambda} e^{-\left(\frac{x-\mu}{\lambda}\right)^k} \left(\frac{x-\mu}{\lambda}\right)^{k-1}$$

Out[349]= $$\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^k} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}$$

In[350]:= $\theta = \mu;$

In[351]:=

`ss = FullSimplify[D[Log[p], θ]]`

Out[351]= $$\frac{1 + k \left(-1 + \left(\frac{x-\mu}{\lambda}\right)^k\right)}{x - \mu}$$

In[352]:=
```
ss' = FullSimplify[D[ss, θ]]
```

Out[352]=
$$-\frac{(-1+k)\left(1+k\left(\frac{x-\mu}{\lambda}\right)^{k}\right)}{(x-\mu)^{2}}$$

In[353]:=
```
csIntCitatel1 = FullSimplify[p^(1 + α) * ss]
```

Out[353]=
$$\frac{\left(1+k\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}} k \left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{x-\mu}$$

In[354]:=
```
csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x - μ) → y * λ]
```

Out[354]=
$$\frac{\left(1+k\left(-1+y^{k}\right)\right)\left(\frac{e^{-y^{k}} k\, y^{-1+k}}{\lambda}\right)^{1+\alpha}}{y\,\lambda}$$

In[355]:=
```
csIntCitatel3 = FullSimplify[csIntCitatel2 * λ /. y → t^(1/k)]
```

Out[355]=
$$t^{-1/k}\left(1+k\left(-1+\left(t^{\frac{1}{k}}\right)^{k}\right)\right)\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}} k\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[356]:=
```
csIntCitatel4 = FullSimplify[csIntCitatel3 / k * t^((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]
```

Out[356]=
$$e^{-t(1+\alpha)} k^{\alpha} (1+k(-1+t))\, t^{\frac{-1+(-1+k)\alpha}{k}} \lambda^{-1-\alpha}$$

In[357]:=
```
csIntCitatel5 = FullSimplify[Integrate[csIntCitatel4, {t, 0, ∞}]]
```

Out[357]=
$$\text{ConditionalExpression}\left[0,\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[358]:=
```
csIntJmenovatel1 = FullSimplify[p^(1 + α)]
```

Out[358]=
$$\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}} k\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[332]:=
```
csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x - μ) → y * λ]
```

Out[332]=
$$\left(\frac{e^{-y^{k}} k\, y^{-1+k}}{\lambda}\right)^{1+\alpha}$$

In[333]:=
```
csIntJmenovatel3 = FullSimplify[csIntJmenovatel2 * λ /. y → t^(1/k)]
```

Out[333]=
$$\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}} k\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha} \lambda$$

In[334]:= `csIntJmenovatel4 = FullSimplify[csIntJmenovatel3 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]`

Out[334]= $$e^{-t}\left(\frac{e^{-t}\,k\,t^{\frac{-1+k}{k}}}{\lambda}\right)^{\alpha}$$

In[335]:= `csIntJmenovatel5 = FullSimplify[Integrate[csIntJmenovatel4, {t, 0, ∞}]]`

Out[335]= $$\text{ConditionalExpression}\left[(1+\alpha)^{-1+\left(-1+\frac{1}{k}\right)\alpha}\left(\frac{k}{\lambda}\right)^{\alpha}\text{Gamma}\left[1+\alpha-\frac{\alpha}{k}\right],\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[359]:= `cs = FullSimplify[csIntCitatel5 / csIntJmenovatel5]`

Out[359]= $$\text{ConditionalExpression}\left[0,\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[360]:= `cs' = FullSimplify[D[cs, θ]]`

Out[360]= $$\text{ConditionalExpression}\left[0,\ \text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[361]:= `Ia = FullSimplify[(ss' - cs' - α (ss - cs) (cs - ss)) * p ^ (1 + α)]`

Out[361]= $$\text{ConditionalExpression}\left[\frac{\left(\alpha\left(1+k\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)^{2}-(-1+k)\left(1+k\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,k\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{(x-\mu)^{2}},\right.$$
$$\left.\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[362]:= `Ia1 = FullSimplify[Ia /. (x - μ) → y * λ]`

Out[362]= $$\text{ConditionalExpression}\left[\frac{\left(-(-1+k)\left(1+k\,y^{k}\right)+\left(1+k\left(-1+y^{k}\right)\right)^{2}\alpha\right)\left(\frac{e^{-y^{k}}\,k\,y^{-1+k}}{\lambda}\right)^{1+\alpha}}{y^{2}\,\lambda^{2}},\right.$$
$$\left.\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[363]:= `Ia2 = FullSimplify[Ia1 * λ /. y → t ^ (1 / k)]`

Out[363]= ConditionalExpression[
$$\frac{t^{-2/k}\left(-(-1+k)\left(1+k\left(t^{\frac{1}{k}}\right)^{k}\right)+\left(1+k\left(-1+\left(t^{\frac{1}{k}}\right)^{k}\right)\right)^{2}\alpha\right)\left(\frac{e^{-\left(t^{\frac{1}{k}}\right)^{k}}\,k\left(t^{\frac{1}{k}}\right)^{-1+k}}{\lambda}\right)^{1+\alpha}}{\lambda},$$
$$\left.\text{Re}\left[\left(-1+\frac{1}{k}\right)\alpha\right]<1\ \&\&\ \text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\ \&\&\ \text{Re}[\alpha]>-1\right]$$

In[364]:= `Ia3 = FullSimplify[Ia2 / k * t ^ ((1 - k) / k), {k > 0, t > 0, λ > 0, α > 0}]`

Out[364]= $$\text{ConditionalExpression}\left[\frac{e^{-t}\,t^{-2/k}\left(-(-1+k)(1+k\,t)+(1+k\,(-1+t))^{2}\alpha\right)\left(\frac{e^{-t}\,k\,t^{\frac{-1+k}{k}}}{\lambda}\right)^{\alpha}}{\lambda^{2}},\ k>1\right]$$

In[365]:= `Ia4 = FullSimplify[Integrate[Ia3, {t, 0, ∞}]]`

Out[365]= $\text{ConditionalExpression}\left[-(-1+k)\,k^{\alpha}\,(1+\alpha)^{\frac{2+\alpha-k\,(3+\alpha)}{k}}\,(-1+k+k\,\alpha)\,\left(\frac{1}{\lambda}\right)^{2+\alpha}\,\text{Gamma}\left[\frac{-2+k+(-1+k)\,\alpha}{k}\right],\ k>1\,\&\&\,2+\text{Re}[\alpha]<k+k\,\text{Re}[\alpha]\right]$

In[366]:= `IF = FullSimplify[-Ia4^(-1) * (p^α) * (ss - cs)]`

Out[366]= $\text{ConditionalExpression}\left[\frac{(1+\alpha)^{-\frac{2+\alpha-k\,(3+\alpha)}{k}}\,\left(\frac{1}{\lambda}\right)^{-2-\alpha}\,\left(1+k\,\left(-1+\left(\frac{x-\mu}{\lambda}\right)^{k}\right)\right)\,\left(\frac{e^{-\left(\frac{x-\mu}{\lambda}\right)^{k}}\,\left(\frac{x-\mu}{\lambda}\right)^{-1+k}}{\lambda}\right)^{\alpha}}{(-1+k)\,(-1+k+k\,\alpha)\,(x-\mu)\,\text{Gamma}\left[\frac{-2+k+(-1+k)\,\alpha}{k}\right]},\ 2+\text{Re}[\alpha]<k+k\,\text{Re}[\alpha]\,\&\&\,\text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\,\&\&\,\text{Re}[\alpha]>-1\right]$

In[368]:= `IF1 = FullSimplify[IF /. λ → 1]`

Out[368]= $\text{ConditionalExpression}\left[\frac{(1+\alpha)^{-\frac{2+\alpha-k\,(3+\alpha)}{k}}\,\left(1+k\,\left(-1+(x-\mu)^{k}\right)\right)\,\left(e^{-(x-\mu)^{k}}\,(x-\mu)^{-1+k}\right)^{\alpha}}{(-1+k)\,(-1+k+k\,\alpha)\,(x-\mu)\,\text{Gamma}\left[\frac{-2+k+(-1+k)\,\alpha}{k}\right]},\ 2+\text{Re}[\alpha]<k+k\,\text{Re}[\alpha]\,\&\&\,\text{Re}\left[\frac{1+\alpha-k\,\alpha}{k}\right]<1\,\&\&\,\text{Re}[\alpha]>-1\right]$

In[373]:= `IF2 = FullSimplify[IF1 /. k → 2]`

Out[373]= $\text{ConditionalExpression}\left[\frac{(1+\alpha)^{2+\frac{\alpha}{2}}\,\left(-1+2\,(x-\mu)^{2}\right)\,\left(e^{-(x-\mu)^{2}}\,(x-\mu)\right)^{\alpha}}{(1+2\,\alpha)\,(x-\mu)\,\text{Gamma}\left[\frac{\alpha}{2}\right]},\ \text{Re}[\alpha]>0\right]$

In[375]:= $\text{IFun} = \text{Function}\left[\{\mu,\alpha\},\ \frac{(1+\alpha)^{2+\frac{\alpha}{2}}\,\left(-1+2\,(x-\mu)^{2}\right)\,\left(e^{-(x-\mu)^{2}}\,(x-\mu)\right)^{\alpha}}{(1+2\,\alpha)\,(x-\mu)\,\text{Gamma}\left[\frac{\alpha}{2}\right]}\right];$

```
Needs["PlotLegends`"]
```

In[376]:=
```
Plot[{
  IFun[0, 0.05],
  IFun[0, 0.1],
  IFun[0, 0.3],
  IFun[0, 0.5],
  IFun[0, 1],
  IFun[0, 2]},
 {x, 0, 6},
 PlotLegend → {"α = 0.05", "α = 0.1", "α = 0.3", "α = 0.5", "α = 1", "α = 2"},
 LegendPosition → {1, -0.4},
 PlotStyle → {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]
```

Out[376]=