

Laplace :



In[262]:= **ClearAll**[α , σ , μ , x]

p = $1 / (2 \sigma) \text{Exp}[-\text{Abs}[x - \mu] / \sigma]$

$$\frac{e^{-\frac{\text{Abs}[x - \mu]}{\sigma}}}{2 \sigma}$$

θ = σ ;

ss = **FullSimplify**[**D**[**Log**[**p**], **θ**]]

$$\frac{-\sigma + \text{Abs}[x - \mu]}{\sigma^2}$$

ss' = **FullSimplify**[**D**[**ss**, **θ**]]

$$\frac{\sigma - 2 \text{Abs}[x - \mu]}{\sigma^3}$$

csIntCitatel1 = **FullSimplify**[**p**^(1 + α) * **ss**]

$$\frac{2^{-1-\alpha} \left(\frac{e^{-\frac{\text{Abs}[x - \mu]}{\sigma}}}{\sigma} \right)^{1+\alpha} (-\sigma + \text{Abs}[x - \mu])}{\sigma^2}$$

csIntCitatel2 = **FullSimplify**[**csIntCitatel1** /. ($x - \mu$) $\rightarrow y * \sigma$, $\sigma \geq 0$]

$$\frac{2^{-1-\alpha} (e^{\text{Abs}[y]} \sigma)^{-1-\alpha} (-1 + \text{Abs}[y])}{\sigma}$$

csIntCitatel3 = **FullSimplify**[**Integrate**[2 * **csIntCitatel2** * σ , {**y**, 0, ∞ }]]

$$\text{ConditionalExpression}\left[-\frac{2^{-\alpha} \alpha \sigma^{-1-\alpha}}{(1 + \alpha)^2}, \text{Re}[\alpha] > -1\right]$$

csIntJmenovatel1 = **FullSimplify**[**p**^(1 + α)]

$$2^{-1-\alpha} \left(\frac{e^{-\frac{\text{Abs}[x - \mu]}{\sigma}}}{\sigma} \right)^{1+\alpha}$$

csIntJmenovatel2 = **FullSimplify**[**csIntJmenovatel1** /. ($x - \mu$) $\rightarrow y * \sigma$, $\sigma \geq 0$]

$$2^{-1-\alpha} (e^{\text{Abs}[y]} \sigma)^{-1-\alpha}$$

csIntJmenovatel3 = **FullSimplify**[**Integrate**[2 * **csIntJmenovatel2** * σ , {**y**, 0, ∞ }]]

$$\text{ConditionalExpression}\left[\frac{2^{-\alpha} \sigma^{-\alpha}}{1 + \alpha}, \text{Re}[\alpha] > -1\right]$$

cs = **FullSimplify**[**csIntCitatel3** / **csIntJmenovatel3**]

$$\text{ConditionalExpression}\left[-\frac{\alpha}{\sigma + \alpha \sigma}, \text{Re}[\alpha] > -1\right]$$

cs' = **FullSimplify**[**D**[**cs**, **θ**]]

$$\text{ConditionalExpression}\left[\frac{\alpha}{(1 + \alpha) \sigma^2}, \text{Re}[\alpha] > -1\right]$$

```
Ia = FullSimplify[ (ss' - cs' - α (ss - cs) (cs - ss)) * p ^ (1 + α)]
```

$$\text{ConditionalExpression}\left[\frac{1}{(1+\alpha)^2 \sigma^4} 2^{-1-\alpha} \left(\frac{e^{-\frac{\text{Abs}[x-\mu]}{\sigma}}}{\sigma}\right)^{1+\alpha} \left((1+2\alpha)\sigma^2 + (1+\alpha)\text{Abs}[x-\mu](-2(\sigma+2\alpha\sigma) + \alpha(1+\alpha)\text{Abs}[x-\mu])\right), \text{Re}[\alpha] > -1\right]$$

```
Ia1 = FullSimplify[Ia /. (x - μ) → y * σ, σ ≥ 0]
```

$$\text{ConditionalExpression}\left[\frac{1}{(1+\alpha)^2} 2^{-1-\alpha} e^{-(1+\alpha)\text{Abs}[y]} \sigma^{-3-\alpha} \left(1+2\alpha-2(1+\alpha)(1+2\alpha)\text{Abs}[y] + y\alpha(1+\alpha)^2 \text{Conjugate}[y]\right), \text{Re}[\alpha] > -1\right]$$

```
Ia2 = FullSimplify[Integrate[2 * Ia1 * σ, {y, 0, ∞}]]
```

$$\text{ConditionalExpression}\left[-\frac{2^{-\alpha} \sigma^{-2-\alpha}}{(1+\alpha)^3}, \text{Re}[\alpha] > -1\right]$$

```
IF = FullSimplify[-Ia2 ^ (-1) * (p ^ α) * (ss - cs)]
```

$$\text{ConditionalExpression}\left[(1+\alpha)^2 \left(\frac{e^{-\frac{\text{Abs}[x-\mu]}{\sigma}}}{\sigma}\right)^\alpha \sigma^\alpha (-\sigma + (1+\alpha)\text{Abs}[x-\mu]), \text{Re}[\alpha] > -1\right]$$

```
IF1 = FullSimplify[IF /. μ -> 0]
```

$$\text{ConditionalExpression}\left[(1+\alpha)^2 \left(\frac{e^{-\frac{\text{Abs}[x]}{\sigma}}}{\sigma}\right)^\alpha \sigma^\alpha (-\sigma + (1+\alpha)\text{Abs}[x]), \text{Re}[\alpha] > -1\right]$$

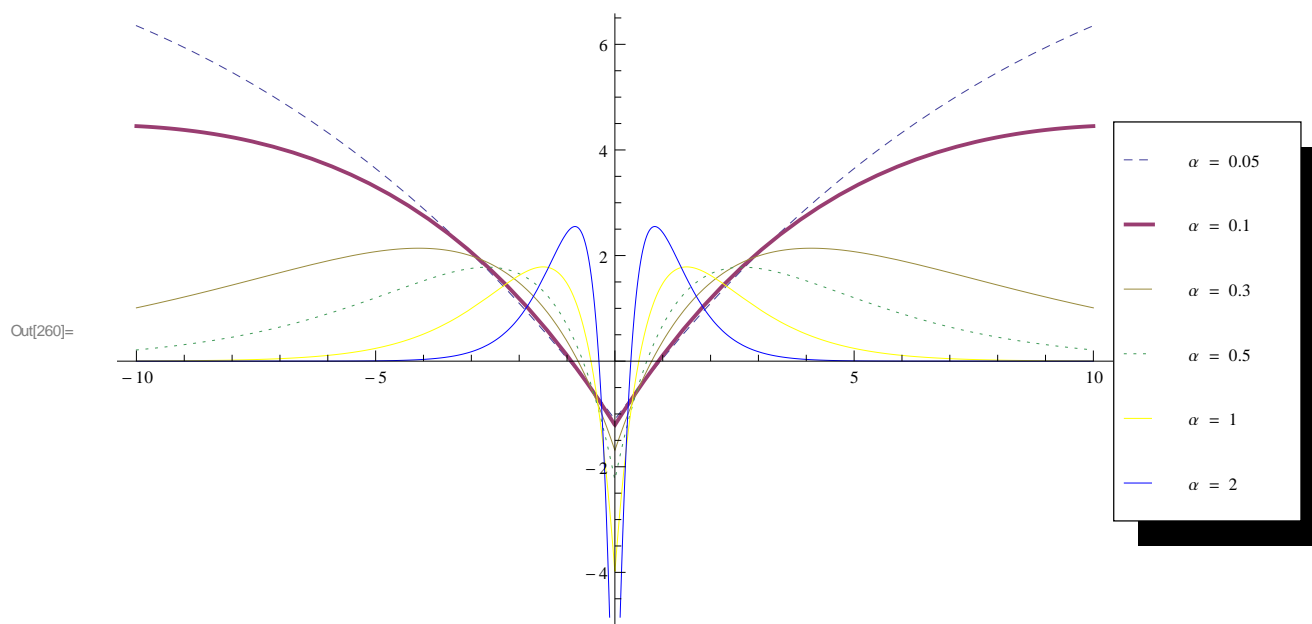
$$\text{In[259]:= IFun} = \text{Function}\left[\{\sigma, \alpha\}, (1+\alpha)^2 \left(\frac{e^{-\frac{\text{Abs}[x]}{\sigma}}}{\sigma}\right)^\alpha \sigma^\alpha (-\sigma + (1+\alpha)\text{Abs}[x])\right];$$

```
Needs["PlotLegends`"]
```

```

In[260]:= Plot[{
  IFun[1, 0.05],
  IFun[1, 0.1],
  IFun[1, 0.3],
  IFun[1, 0.5],
  IFun[1, 1],
  IFun[1, 2]},
{x, -10, 10},
PlotLegend -> {"α = 0.05", "α = 0.1", "α = 0.3", "α = 0.5", "α = 1", "α = 2"},
LegendPosition -> {1, -0.4},
PlotStyle -> {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]

```



Normalni :

ClearAll[α, σ, μ, x]

$\theta = \mu$

μ

ss = FullSimplify[D[Log[p], θ]]

$\frac{\text{Abs}'[x - \mu]}{\sigma}$

ss' = FullSimplify[D[ss, θ]]

$-\frac{\text{Abs}''[x - \mu]}{\sigma}$

```
csIntCitatel1 = FullSimplify[p^(1 + α) * ss]
```

$$\frac{2^{-1-\alpha} \left(\frac{e^{-\frac{\text{Abs}[x-\mu]}{\sigma}}}{\sigma} \right)^{1+\alpha} \text{Abs}'[x-\mu]}{\sigma}$$

```
csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x - μ) → y * σ, {σ ≥ 0, y ≥ 0}]
```

$$2^{-1-\alpha} e^{-y(1+\alpha)} \sigma^{-2-\alpha} \text{Sign}[y] \text{Sign}[\sigma]$$

```
csIntCitatel3 = FullSimplify[Integrate[2 * csIntCitatel2 * σ, {y, 0, ∞}]]
```

$$\text{ConditionalExpression}\left[\frac{2^{-\alpha} \sigma^{-1-\alpha} \text{Sign}[\sigma]}{1+\alpha}, \text{Re}[\alpha] > -1\right]$$

```
csIntJmenovatel1 = FullSimplify[p^(1 + α)]
```

$$2^{-1-\alpha} \left(\frac{e^{-\frac{\text{Abs}[x-\mu]}{\sigma}}}{\sigma} \right)^{1+\alpha}$$

```
csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x - μ) → y * σ, {σ ≥ 0, y ≥ 0}]
```

$$2^{-1-\alpha} (e^y \sigma)^{-1-\alpha}$$

```
csIntJmenovatel3 = FullSimplify[Integrate[csIntJmenovatel2 * σ * 2, {y, 0, ∞}]]
```

$$\text{ConditionalExpression}\left[\frac{2^{-\alpha} \sigma^{-\alpha}}{1+\alpha}, \text{Re}[\alpha] > -1\right]$$

```
cs = FullSimplify[csIntCitatel3 / csIntJmenovatel3]
```

$$\text{ConditionalExpression}\left[\frac{\text{Sign}[\sigma]}{\sigma}, \text{Re}[\alpha] > -1\right]$$

```
cs' = FullSimplify[D[cs, θ]]
```

```
0
```

```
Ia = FullSimplify[(ss' - cs' - α (ss - cs) (cs - ss)) * p^(1 + α)]
```

$$\text{ConditionalExpression}\left[\frac{2^{-1-\alpha} \left(\frac{e^{-\frac{-x+\mu}{\sigma \text{Sign}[x-\mu]}}}{\sigma} \right)^{1+\alpha} (\alpha (\text{Sign}[\sigma] - \text{Abs}'[x-\mu])^2 - \sigma \text{Abs}''[x-\mu])}{\sigma^2}, \text{Re}[\alpha] > -1\right]$$

```
Ia1 = FullSimplify[Ia /. (x - μ) → y * σ, {σ ≥ 0}]
```

$$\text{ConditionalExpression}\left[\frac{2^{-1-\alpha} \left(\frac{e^{-\frac{-x+\mu}{\sigma \text{Sign}[y]}}}{\sigma} \right)^{1+\alpha} (\alpha (\text{Sign}[\sigma] - \text{Abs}'[y \sigma])^2 - \sigma \text{Abs}''[y \sigma])}{\sigma^2}, \text{Re}[\alpha] > -1\right]$$

```
Ia2 = FullSimplify[Ia1 /. (-x + μ) → -y * σ, {σ > 0, y > 0}]
```

$$\text{ConditionalExpression}[-2^{-1-\alpha} e^y (e^y \sigma)^{-2-\alpha} \text{Abs}''[y \sigma], \text{Re}[\alpha] > -1]$$

```
Ia2 = FullSimplify[Integrate[Ia1 * σ * 2, {y, 0, ∞}]]
```

Integrate::idiv: Integral of $\alpha (\text{Sign}[\sigma] - \text{Abs}'[y \sigma])^2 - \sigma \text{Abs}''[y \sigma]$ does not converge on $\{0, \infty\}$. >>

$$\int_0^{\infty} \text{ConditionalExpression}\left[\frac{2^{-\alpha} \left(\frac{e^{-\frac{-x+\mu}{\sigma \text{Sign}[y]}}}{\sigma} \right)^{1+\alpha} (\alpha (\text{Sign}[\sigma] - \text{Abs}'[y \sigma])^2 - \sigma \text{Abs}''[y \sigma])}{\sigma}, \text{Re}[\alpha] > -1\right] d y$$

```
IF = FullSimplify[-Ia2^(-1) * (p^alpha) * (ss - cs)]
```

```
ConditionalExpression[ $(1 + \alpha)^{3/2} (x - \mu) \sigma (\sigma^2)^{\frac{1}{2}(-1 + \alpha)} \left( \frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^\alpha, \text{Re}[\alpha] > -1]$ 
```

```
IF1 = FullSimplify[IF /. sigma -> 1]
```

```
ConditionalExpression[ $\left( e^{-\frac{1}{2}(x - \mu)^2} \right)^\alpha (1 + \alpha)^{3/2} (x - \mu), \text{Re}[\alpha] > -1]$ 
```

```
In[263]:= IFun = Function[{mu, alpha},  $\left( e^{-\frac{1}{2}(x - \mu)^2} \right)^\alpha (1 + \alpha)^{3/2} (x - \mu)$ ];
```

```
Needs["PlotLegends`"]
```

```
In[264]:= Plot[{
  IFun[0, 0.05],
  IFun[0, 0.1],
  IFun[0, 0.3],
  IFun[0, 0.5],
  IFun[0, 1],
  IFun[0, 2]},
{x, -10, 10},
PlotLegend -> {"alpha = 0.05", "alpha = 0.1", "alpha = 0.3", "alpha = 0.5", "alpha = 1", "alpha = 2"},
LegendPosition -> {1, -0.4},
PlotStyle -> {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]
```

