

Normalni :



ClearAll[α, σ, μ, x]

p = 1/Sqrt[2*π*σ^2]*Exp[-(x-μ)^2/(2*σ^2)]

$$\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sqrt{\sigma^2}}$$

θ = σ;

ss = FullSimplify[D[Log[p], θ]]

$$\frac{(x-\mu)^2 - \sigma^2}{\sigma^3}$$

ss' = FullSimplify[D[ss, θ]]

$$\frac{-3(x-\mu)^2 + \sigma^2}{\sigma^4}$$

csIntCitatel1 = FullSimplify[p^(1+α)*ss]

$$\frac{(2\pi)^{\frac{1}{2}}(-1-\alpha)\left(\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{\sigma^2}}\right)^{1+\alpha}\left((x-\mu)^2 - \sigma^2\right)}{\sigma^3}$$

csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x-μ) → y*σ]

$$\frac{(2\pi)^{\frac{1}{2}}(-1-\alpha)(-1+y^2)\left(\frac{e^{-\frac{y^2}{2}}}{\sqrt{\sigma^2}}\right)^{1+\alpha}}{\sigma}$$

csIntCitatel3 = FullSimplify[Integrate[csIntCitatel2*σ, {y, -∞, ∞}]]

$$\text{ConditionalExpression}\left[-\frac{(2\pi)^{-\alpha/2}\alpha(\sigma^2)^{-\frac{1}{2}-\frac{\alpha}{2}}}{(1+\alpha)^{3/2}}, \text{Re}[\alpha] > -1\right]$$

csIntJmenovatel1 = FullSimplify[p^(1+α)]

$$(2\pi)^{\frac{1}{2}}(-1-\alpha)\left(\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{\sigma^2}}\right)^{1+\alpha}$$

csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x-μ) → y*σ]

$$(2\pi)^{\frac{1}{2}}(-1-\alpha)\left(\frac{e^{-\frac{y^2}{2}}}{\sqrt{\sigma^2}}\right)^{1+\alpha}$$

csIntJmenovatel3 = FullSimplify[Integrate[csIntJmenovatel2*σ, {y, -∞, ∞}]]

$$\text{ConditionalExpression}\left[\frac{(2\pi)^{-\alpha/2}\sigma(\sigma^2)^{-\frac{1}{2}-\frac{\alpha}{2}}}{\sqrt{1+\alpha}}, \text{Re}[\alpha] > -1\right]$$

cs = FullSimplify[csIntCitatel3 / csIntJmenovatel3]

ConditionalExpression $\left[-\frac{\alpha}{\sigma + \alpha \sigma}, \text{Re}[\alpha] > -1 \right]$

cs' = FullSimplify[D[cs, \theta]]

ConditionalExpression $\left[\frac{\alpha}{(1 + \alpha) \sigma^2}, \text{Re}[\alpha] > -1 \right]$

Ia = FullSimplify[(ss' - cs' - \alpha (ss - cs) (cs - ss)) * p^(1 + \alpha)]

ConditionalExpression $\left[\frac{(2 \pi)^{\frac{1}{2}} (-1 - \alpha) \left(\frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha} \left(\alpha (x - \mu)^4 - \frac{(3 + 5 \alpha) (x - \mu)^2 \sigma^2}{1 + \alpha} + \frac{(1 + 2 \alpha) \sigma^4}{(1 + \alpha)^2} \right)}{\sigma^6}, \text{Re}[\alpha] > -1 \right]$

Ia1 = FullSimplify[Ia /. (x - \mu) \rightarrow y * \sigma]

ConditionalExpression $\left[\frac{(2 \pi)^{\frac{1}{2}} (-1 - \alpha) (1 + 2 \alpha + y^2 (1 + \alpha) (-3 + \alpha (-5 + y^2 (1 + \alpha)))) \left(\frac{e^{-\frac{y^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}}{(1 + \alpha)^2 \sigma^2}, \text{Re}[\alpha] > -1 \right]$

Ia2 = FullSimplify[Integrate[Ia1 * \sigma, {y, -\infty, \infty}]]

ConditionalExpression $\left[-\frac{2^{1 - \frac{\alpha}{2}} \pi^{-\alpha/2} (\sigma^2)^{\frac{1}{2} - \frac{\alpha}{2}}}{(1 + \alpha)^{5/2} \sigma^3}, \text{Re}[\alpha] > -1 \right]$

IF = FullSimplify[-Ia2^(-1) * (p^\alpha) * (ss - cs)]

ConditionalExpression $\left[\frac{1}{2} (1 + \alpha)^{3/2} (\sigma^2)^{\frac{1}{2} (-1 + \alpha)} \left(\frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^\alpha ((1 + \alpha) (x - \mu)^2 - \sigma^2), \text{Re}[\alpha] > -1 \right]$

IF1 = FullSimplify[IF /. \mu \rightarrow 0]

ConditionalExpression $\left[\frac{1}{2} (1 + \alpha)^{3/2} (\sigma^2)^{\frac{1}{2} (-1 + \alpha)} \left(\frac{e^{-\frac{x^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^\alpha (x^2 (1 + \alpha) - \sigma^2), \text{Re}[\alpha] > -1 \right]$

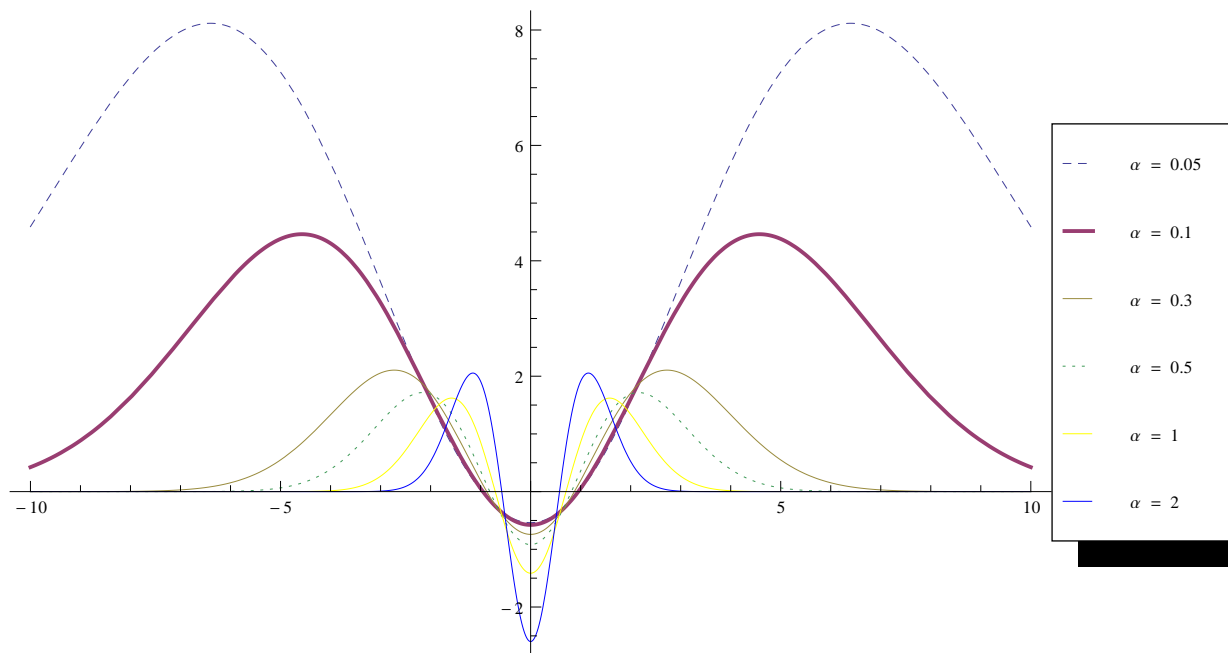
IFun = Function[{ \sigma, \alpha }, \frac{1}{2} (1 + \alpha)^{3/2} (\sigma^2)^{\frac{1}{2} (-1 + \alpha)} \left(\frac{e^{-\frac{x^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^\alpha (x^2 (1 + \alpha) - \sigma^2)];

Needs["PlotLegends`"]

```

Plot[{
  IFun[1, 0.05],
  IFun[1, 0.1],
  IFun[1, 0.3],
  IFun[1, 0.5],
  IFun[1, 1],
  IFun[1, 2]},
{x, -10, 10},
PlotLegend -> {"α = 0.05", "α = 0.1", "α = 0.3", "α = 0.5", "α = 1", "α = 2"},
LegendPosition -> {1, -0.4},
PlotStyle -> {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]

```



Normalni :

ClearAll[α, σ, μ, x]

p = 1 / Sqrt[2 * π * σ ^ 2] * Exp[-(x - μ) ^ 2 / (2 * σ ^ 2)]

$$e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\sqrt{2\pi} \sqrt{\sigma^2}$$

θ = μ

μ

ss = FullSimplify[D[Log[p], θ]]

$$\frac{x - \mu}{\sigma^2}$$

ss' = FullSimplify[D[ss, θ]]

$$-\frac{1}{\sigma^2}$$

csIntCitatel1 = FullSimplify[p^(1 + α) * ss]

$$\frac{(2 \pi)^{\frac{1}{2}} (-1 - \alpha) (x - \mu) \left(\frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}}{\sigma^2}$$

csIntCitatel2 = FullSimplify[csIntCitatel1 /. (x - μ) → y * σ]

$$\frac{(2 \pi)^{\frac{1}{2}} (-1 - \alpha) y \left(\frac{e^{-\frac{y^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}}{\sigma}$$

csIntCitatel3 = FullSimplify[Integrate[csIntCitatel2 * σ, {y, -∞, ∞}]]

ConditionalExpression[0, Re[α] > -1]

csIntJmenovatel1 = FullSimplify[p^(1 + α)]

$$(2 \pi)^{\frac{1}{2}} (-1 - \alpha) \left(\frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}$$

csIntJmenovatel2 = FullSimplify[csIntJmenovatel1 /. (x - μ) → y * σ]

$$(2 \pi)^{\frac{1}{2}} (-1 - \alpha) \left(\frac{e^{-\frac{y^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}$$

csIntJmenovatel3 = FullSimplify[Integrate[csIntJmenovatel2 * σ, {y, -∞, ∞}]]

$$\text{ConditionalExpression} \left[\frac{(2 \pi)^{-\alpha/2} \sigma (\sigma^2)^{-\frac{1}{2} - \frac{\alpha}{2}}}{\sqrt{1 + \alpha}}, \text{Re}[\alpha] > -1 \right]$$

cs = FullSimplify[csIntCitatel3 / csIntJmenovatel3]

ConditionalExpression[0, Re[α] > -1]

cs' = FullSimplify[D[cs, θ]]

ConditionalExpression[0, Re[α] > -1]

Ia = FullSimplify[(ss' - cs' - α (ss - cs) (cs - ss)) * p^(1 + α)]

$$\text{ConditionalExpression} \left[(2 \pi)^{\frac{1}{2}} (-1 - \alpha) \left(\frac{\alpha (x - \mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right) \left(\frac{e^{-\frac{(x - \mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}} \right)^{1 + \alpha}, \text{Re}[\alpha] > -1 \right]$$

```
Ia1 = FullSimplify[Ia /. (x - μ) → y * σ]
```

$$\text{ConditionalExpression}\left[\frac{(2 \pi)^{\frac{1}{2}(-1-\alpha)}(-1+y^2 \alpha)\left(\frac{e^{-\frac{y^2}{2}}}{\sqrt{\sigma^2}}\right)^{1+\alpha}}{\sigma^2}, \operatorname{Re}[\alpha] > -1\right]$$

```
Ia2 = FullSimplify[Integrate[Ia1 * σ, {y, -∞, ∞}]]
```

$$\text{ConditionalExpression}\left[-\frac{(2 \pi)^{-\alpha/2}(\sigma^2)^{\frac{1}{2}-\frac{\alpha}{2}}}{(1+\alpha)^{3/2} \sigma^3}, \operatorname{Re}[\alpha] > -1\right]$$

```
IF = FullSimplify[-Ia2 ^ (-1) * (p ^ α) * (ss - cs)]
```

$$\text{ConditionalExpression}\left[(1+\alpha)^{3/2}(\mathbf{x}-\mu) \sigma(\sigma^2)^{\frac{1}{2}(-1+\alpha)}\left(\frac{e^{-\frac{(\mathbf{x}-\mu)^2}{2 \sigma^2}}}{\sqrt{\sigma^2}}\right)^{\alpha}, \operatorname{Re}[\alpha] > -1\right]$$

```
IF1 = FullSimplify[IF /. σ → 1]
```

$$\text{ConditionalExpression}\left[\left(e^{-\frac{1}{2}(\mathbf{x}-\mu)^2}\right)^{\alpha}(1+\alpha)^{3/2}(\mathbf{x}-\mu), \operatorname{Re}[\alpha] > -1\right]$$

```
IFun = Function[{μ, α}, (e^{-1/2 (x-μ)^2})^α (1+α)^{3/2} (x-μ)];
```

```
Needs["PlotLegends`"]
```

```

Plot[{
  IFun[0, 0.05],
  IFun[0, 0.1],
  IFun[0, 0.3],
  IFun[0, 0.5],
  IFun[0, 1],
  IFun[0, 2]},
{x, -10, 10},
PlotLegend -> {" $\alpha = 0.05$ ", " $\alpha = 0.1$ ", " $\alpha = 0.3$ ", " $\alpha = 0.5$ ", " $\alpha = 1$ ", " $\alpha = 2$ "},
LegendPosition -> {1, -0.4},
PlotStyle -> {Dashed, Thick, Thin, Dotted, Yellow, Blue}
]

```

