generally:

$$\mu = \frac{\tau}{\rho} \qquad \tau = \frac{\mu}{\sigma^2} \tag{1}$$

$$\sigma^2 = \frac{1}{\rho} \qquad \rho = \frac{1}{\sigma^2} \tag{2}$$

scalingfactor

$$\mu' = \frac{\mu}{a}$$
$$\sigma'^2 = \frac{\sigma^2}{a^2}$$

$$\tau' = \frac{\mu'}{\sigma'^2} = \frac{\frac{\mu}{a}}{\frac{\sigma^2}{a^2}} = \frac{\mu}{a} \cdot \frac{a^2}{\sigma^2} = \frac{\mu \cdot a}{\sigma^2} = \frac{\frac{\tau}{\rho} \cdot a}{\frac{1}{\rho}} = \frac{\tau}{\rho} \cdot a \cdot \rho = \tau \cdot a$$

$$\rho' = \frac{1}{\sigma'^2} = \frac{1}{\frac{\sigma^2}{a^2}} = \frac{a^2}{\sigma^2} = \frac{a^2}{\frac{1}{\rho}} = a^2 \cdot \rho$$

scalingfactor with bias b

$$y = ax + b$$

$$m_{f \to y} = \mathcal{N}(z; a\mu_x + b, a^2\sigma_x^2)$$

$$x = \frac{y - b}{a}$$

$$a = 1, b = -b$$

(ignore the $\frac{1}{a}$, while reassigning a and b, and just add it to the message Distribution, because $E[c \cdot X] = c \cdot E[X]$ and $Var[c \cdot X] = c^2 \cdot Var[X]$)

$$m_{f \to x} = \mathcal{N}\left(z_i; \left(\frac{\mu_y - b}{a}\right), \frac{\sigma_y^2}{a^2}\right)$$

$$\mu' = \frac{\mu - b}{a}$$

$$\sigma'^2 = \frac{\sigma^2}{a^2}$$

$$\tau' = \frac{\mu'}{\sigma'^2} = \frac{\frac{\mu-b}{a}}{\frac{\sigma^2}{a^2}} = \frac{\mu - b}{a} \cdot \frac{a^2}{\sigma^2}$$
$$= \frac{\frac{\tau}{\rho} - b}{a} \cdot \frac{a^2}{\frac{1}{\rho}} = (\frac{\tau}{\rho} - b) \cdot a \cdot \rho$$
$$= a\tau - ab\rho$$

$$\rho' = \frac{1}{\sigma'^2} = \frac{1}{\frac{\sigma^2}{a^2}} = \frac{a^2}{\sigma^2} = \frac{a^2}{\frac{1}{\rho}} = a^2\rho$$

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weightedsumfactor with bias c

$$z = a \cdot x + b \cdot y + c$$
$$\rightarrow a = a, b = b, c = c$$
$$m_{f\rightarrow z} = \mathcal{N}(z; a\mu_x + b\mu_y + c, a^2\sigma_x^2 + b^2\sigma_y^2)$$

$$x = \frac{z - b \cdot y - c}{a}$$
$$\rightarrow a = 1, b = -1, c = -c,$$
handle constant factor same as before, in scalingfactor
$$m_{f\rightarrow x} = \mathcal{N}(z; \frac{\mu_z - b\mu_y - c}{a}, \frac{\sigma_z^2 + b^2\sigma_y^2}{a^2})$$

$$y = \frac{z - a \cdot x - c}{b}$$
$$\rightarrow a = 1, b = -1, c = -c,$$
handle constant factor same as before, in scalingfactor
$$m_{f\rightarrow y} = \mathcal{N}(z; \frac{\mu_z - a\mu_x - c}{b}, \frac{\sigma_z^2 + a^2\sigma_y^2}{b^2})$$