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Applying FMCDM to evaluate financial performance of domestic airlines in Taiwan

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Abstract

Many previous researches concerning the performance of airlines usually focus on operation. Financial performance, which would influence the survival of an airline, is often ignored. To evaluate financial performance, financial ratios obtained form balance sheet, income statement and cash flow must be partitioned into several clusters and found the representative indicators from these clusters to be criteria. In this paper, we utilize grey relation analysis to cluster financial ratios and find representative indicators. Then we apply a fuzzy multi-criteria decision-making (FMCDM) method to evaluate financial performance of airlines. Finally, an empirical study of financial performance of three domestic airlines in Taiwan is illustrated.

Keywords: Airlines; Clustering; Grey relation; Financial performance; FMCDM

1. Introduction

For the deregulation of air transportation in 1987 and Asia financial crisis in 1997, the operation of an airline on air transportation market had been a challenge. Recently, the passenger or cargo load factor was decreased in Taiwan. The situation will be more worse gradually as high-speed rail is working. Therefore, the domestic airlines should enhance their performance. Based on the concept, the performance evaluation is important especially for the financial field. It is due to large capital being critical to these airlines. However, some previous researches concerning airline performance focus on the operation performance. Financial performance, which directly influences the survival of an airline, is often ignored. In fact, an airline should evaluate the financial performance to realize the financial location between these competitive companies.

In Taiwan, the main domestic airlines are Eva Air, China Airlines and Far Eastern Air Transport etc. To evaluate financial performance of these domestic airlines, we must have evaluation criteria which are usually financial ratios (Walter & Robert, 1988). Financial ratios coming from balance sheet, income statement and cash flow in stock market are so many, whereas some of financial ratios are identified on similar patterns. To avoid evaluating on similar financial ratios repeatedly, financial ratios should be partitioned into several financial clusters, and a representative indicator is selected from one cluster to be as an evaluation criterion (Deogun, Kratsch, & Steiner, 1997; Dubes & Jains, 1988; Duda & Hart, 1973; Eom, 1999; Hirano, Sun, & Tsumoto, 2004; Kaufman & Rousseeuw, 1990; Krishnapuram & Keller, 1993; Lee, 1999; Miyamoto, 2003; Pedrycz & Vukovich, 2002). Then a fuzzy multi-criteria decision-making (FMCDM) method is applied to evaluate the financial performance of domestic airlines.

Based on these ideas, the representative indicators have to be found from financial ratios (Feng & Wang, 2000). We apply a clustering method to partition financial ratios into several clusters and select the representative ones from these clusters. From Taiwan stock market, we know that the number of major domestic airlines is merely 3, that is, Eva Air, China Airlines and Far Eastern Air Transport.

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The number is small and the distribution is unknown, thus these classical clustering methods such as cluster analysis, discriminant analysis, factor analysis and principal component analysis (Johnson & Wichern, 1992) are not suitable for the situation. To reflect the scarce data and unknown distribution, we utilize grey relation analysis (Deng, 1989) to partition financial ratios into several clusters and then find representative indicators from these clusters to be evaluation criteria. Because several evaluation criteria are found, the financial performance evaluation of airlines belongs to multi-criteria decision-making (MCDM) problems (Hwang & Yoon, 1981; Keeney & Raiffa, 1976). To illustrate MCDM, we will evaluate the financial performance of airlines from 2001 to 2005. Since these financial ratios of the 5 years are expressed with fuzzy numbers on calculation, the evaluation problem should be a FMCDM problem (Boender, de Graan, & Lootsma, 1989; Chang & Yeh, 2002; Chen, 2000; Chen & Hwang, 1992; Hsu & Chen, 1997; Jain, 1978; Liang, 1999; Ostrowski, O'Brien, & Gordon, 1993; Parasurman, Zeithaml, & Berry, 1985; Truitt & Haynes, 1994; Tsaur, Chang, & Yen, 2002; Wang, Lee, & Lin, 2003). In this paper, we apply the FMCDM method called fuzzy TOPSIS (Wang et al., 2003) to evaluate the financial performance of Taiwan domestic airlines. With the FMCDM method, the evaluation problem of financial performance can be easily solved.

For the sake of clarity, finding representative indicators with grey relation analysis are presented in Section 2. The notions of fuzzy sets and fuzzy numbers are introduced in Section 3. The FMCDM method is presented in Section 4. Finally, an empirical study of three domestic airlines in Taiwan is presented in Section 5.

2. Finding the representative indicators from financial ratios

In stock market, financial ratios are usually partitioned into some categories, because accounting experts suppose that the financial ratios in one category are partially similar. Thus the financial ratios of airlines are initially divided into four categories according to their related patterns. We review financial ratios of domestic airlines in Taiwan and present them as follows.

In Table 1, fixed assets to stockholder's equity ratio, debt to total assets ratio and accounts payable turnover belong to cost items, and the rest ratios belong to benefit items.

From Table 1, we evaluate the financial performance of domestic airlines in Taiwan based on these financial ratios. Since some of ratios have similar patterns, we apply grey relation analysis to partition them into several clusters and then find representative indicators to be evaluation criteria from these clusters. Grey theory was first introduced by Deng (1989). The fundamental definition of greyness is information being incomplete or unknown, thus an element from the incomplete message is taken as grey elements. Grey relation analysis is the method to measure the relations among the elements, and its definitions in mathematics and application to the clustering of financial ratios are stated as follows.

Assume that m airlines are evaluated on s financial ratios. Let $X_i = \{x_i(k)\} \in X$ denote the sequence of the financial ratio i, where $k = 1, 2, \ldots, m$; $i = 1, 2, \ldots, s$, and X is the set consisting of all financial ratio sequences. First, these financial ratios are divided into two situations to be normalized. If $x_i(k)$ belongs to benefit items, then

Table 1
The financial ratios on four categories

Category	Code	Formula	Ratio
Financial structure	F1	Fixed assets/total stockholder's equity	Fixed assets to stockholder's equity ratio
	F2	Fixed assets/long-term liabilities	Fixed assets to long-term liabilities ratio
	F3	Fixed assets/long-term capital	Fixed assets to long-term capital ratio
	F4	Total liabilities/total assets	Debt to total assets ratio
	F5	Total stockholder's equity/total liabilities	Stockholder's equity to total liabilities ratio
	F6	Working capital/total assets	Working capital to total assets ratio
Solvency	S1	Current assets/current liabilities	Current ratio
	S2	Quick assets/current liabilities	Quick ratio
	S3	Cash and cash equivalent/current assets	Cash ratio
	S4	Net cash provided by operating activities/current liabilities	Cash flow ratio
	S5	Working capital/current assets	Working capital to current assets ratio
Turnover	T1	Operation cost/accounts payable	Accounts payable turnover
	T2	Operation cost/accounts receivable	Accounts receivable turnover
	T3	Operation revenue/fixed assets	Fixed assets turnover
	T4	Operation revenue/total assets	Total assets turnover
	T5	Net income(loss)/operation revenue	Net income(loss) turnover
Profitability	P1	(Operation revenue – operation cost)/operation revenue	Gross profit ratio
	P2	Operation income(loss)/operation revenue	Operation profit ratio
	P3	Income(loss) before tax/operation revenue	Income before tax ratio
	P4	Net income(loss)/operation revenue	Net income ratio
	P5	Net income(loss)/total assets	Return on total assets

$$y_i(k) = \frac{x_i(k)}{\sqrt{\sum_{t=1}^{m}[x_i(t)]^2}}.$$

Otherwise, $x_i(k)$ belongs to cost items, then

$$y_i(k) = \frac{1/x_i(k)}{\sqrt{\sum_{t=1}^{m}[1/x_i(t)]^2}}.$$

In above equations, $y_i(k)$ is the normalized value of the financial ratio i on the period k, where $k = 1, 2, \ldots, m$; $i = 1, 2, \ldots, s$.

Assume Y to be the set composed of all normalized financial ratio sequences. $Y_i = \{y_i(k)\} \in Y$ denotes the sequence of normalized financial ratio i. Let Y be a factor set of grey relation, $y_0 \in Y$ represent the referential sequence and $y_i \in Y$ represent the comparative sequence. $y_0(k)$ and $y_i(k)$ represent the financial ratio values of k on y_0 and y_i, respectively. If the average value $\gamma(y_0, y_i)$ of a set $\{\gamma(y_0(k), y_i(k)) | k = 1, 2, \ldots, m\}$ is a real number, then the grey relation $\gamma(y_0, y_i)$ is defined as:

$$\gamma(y_0, y_i) = \frac{1}{m}\sum_{k=1}^{m}\gamma(y_0(k), y_i(k)) = r_{0i},$$

where

$$\gamma(y_0(k), y_i(k)) = \frac{\min_{y_i(\neq y_0)\in Y}\min_k|y_0(k) - y_i(k)| + \zeta\max_{y_i(\neq y_0)\in Y}\max_k|y_0(k) - y_i(k)|}{|y_0(k) - y_i(k)| + \zeta\max_{y_i(\neq y_0)\in Y}\max_k|y_0(k) - y_i(k)|},$$

and ζ is the distinguished coefficient ($\zeta \in [0, 1]$).

With grey relation analysis, we can obtain grey relation matrix:

$$R = (r_{ij}),$$

where

$$i = 1, 2, \ldots, s; \quad j = 1, 2, \ldots, s.$$

The clustering definitions of financial ratios according to the entries of grey relation matrix are presented as follows.

Definition 2.1. If $r_{ij} \geqslant r$ and $r_{ji} \geqslant r$ then Y_i and Y_j belong to the same cluster, where r is the threshold value of clustering.

Definition 2.2. When $r_{ij} \geqslant r$ and $r_{ji} \geqslant r$, $r_{ik} \geqslant r$ and $r_{ki} \geqslant r$, but $r_{jk} < r$ or $r_{kj} < r$. If $\min\{r_{ij}, r_{ji}\} \geqslant \min\{r_{ik}, r_{ki}\}$ then Y_i and Y_j belong to the same cluster.

After partitioning financial ratios into several clusters, the finding of representative indicators is stated as follows.

Definition 2.3. As Y_i and Y_j belong to the same cluster, the representative indicator of the cluster is found according to the maximum value of r_{ij} and r_{ji}. If $r_{ij} \geqslant r_{ji}$ then the representative indicator of the cluster is financial ratio i.

Definition 2.4. As Y_i, Y_j and Y_k are in the same cluster, the representative indicator of the cluster is decided according to the maximum value of $r_{ij} + r_{ik}$, $r_{ji} + r_{jk}$ and $r_{ki} + r_{kj}$. If $r_{ij} + r_{ik}$ is the maximum value, then the representative indicator of the cluster is financial ratio i.

Sometimes, one cluster has more than three financial ratios. Definition 2.4 can extends on four or more financial ratios belonging to the same cluster to find the representative indicator.

Definition 2.5. As Y_i belongs to the cluster T and the element number of T is more than 3. The representative indicator of T is financial ratio i if

$$\sum_{j(\neq i)\in T} r_{ij} \geqslant \sum_{j(\neq k)\in T} r_{kj}, \quad \forall k \in T, \quad \text{but } k \neq i.$$

Therefore, we can find the representative indicators of financial ratios from these above definitions.

3. Fuzzy sets and fuzzy numbers

In this section, we review some basic notions of fuzzy sets and fuzzy numbers (Zadeh, 1965; Zimmermann, 1987; Zimmermann, 1991). These notions are presented as follows:

Definition 3.1. Let U be a universe set. A fuzzy set A of U is defined with a membership function $\mu_A(x) \to [0, 1]$, where $\mu_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 3.2. The support A is the crisp set $\mathrm{Supp}(A) = \{x \mid \mu_A(x) > 0\}$. A is normal iff $\sup_{x\in U}\mu_A(x) = 1$, where U is the universe set.

Definition 3.3. A fuzzy set A of the universe set U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geqslant (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U$, $\forall \lambda \in [0, 1]$, where $\wedge$ denotes the minimum operator.

Definition 3.4. A is a fuzzy number iff A is the normal and convex fuzzy set of U.

Definition 3.5. A triangular fuzzy number A is a fuzzy number with piecewise linear membership function μ_A defined by

$$\mu_X(x) = \begin{cases} (x - a^l)/(a^m - a^l), & a^l \leqslant x < a^m, \\ 1, & x = a^m, \\ (a^r - x)/(a^r - a^m), & a^m < x \leqslant a^r, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a triplet (a^l, a^m, a^r).

Definition 3.6. Let $A = (a^l, a^m, a^r)$ and $B = (b^l, b^m, b^r)$ be two triangular fuzzy numbers. A distance measure function $d(A, B)$ can be defined (Chang & Yeh, 2002):

$$d(A, B) = \sqrt{\frac{1}{3}[(a^l - b^l)^2 + (a^m - b^m)^2 + (a^r - b^r)^2}.$$

Besides these above notions, we utilize a operator $\succeq$ satisfying the partial order relation on triangular fuzzy numbers (Wang et al., 2003).

Definition 3.7. Let $\succeq$ be a binary relation on triangular fuzzy numbers. Assume $A=(a^l,a^m,a^r)$ and $B=(b^l,b^m,b^r)$ to be two triangular fuzzy numbers. Then $A\succeq B$ iff $a^l \geqslant b^l$, $a^m \geqslant b^m$, $a^r \geqslant b^r$.

Lemma 3.1. *$\succeq$ is a partial order relation on triangular fuzzy numbers.*

Definition 3.8. Let MIN and MAX be two operators on a set of triangular fuzzy numbers $\{X_1,X_2,\ldots,X_n\}$. Define

$$MIN\{X_1,X_2,\ldots,X_n\}=MIN_{1\leqslant j\leqslant n}\{X_j\}=X^- \\ =(x^{l-},x^{m-},x^{r-})$$

and

$$MAX\{X_1,X_2,\ldots,X_n\}=MAX_{1\leqslant j\leqslant n}\{X_j\}=X^+ \\ =(x^{l+},x^{m+},x^{r+}),$$

where

$$X_j=(x_j^l,x_j^m,x_j^r),$$
$$x_1^-=\min_{1\leqslant j\leqslant n}\{x_j^l\},\quad x_2^-=\min_{1\leqslant j\leqslant n}\{x_j^m\},\quad x_3^-=\min_{1\leqslant j\leqslant n}\{x_j^r\},$$
$$x_1^+=\max_{1\leqslant j\leqslant n}\{x_j^l\},\quad x_2^+=\max_{1\leqslant j\leqslant n}\{x_j^m\},\quad x_3^+=\max_{1\leqslant j\leqslant n}\{x_j^r\}.$$

Lemma 3.2. *$X_t\succeq MIN_{1\leqslant j\leqslant n}\{X_j\}$ and $MAX_{1\leqslant j\leqslant n}\{X_j\}\succeq X_t$ where $t=1,2,\ldots,n$.*

Based on the two operators MAX and MIN, we can extend TOPSIS into fuzzy TOPSIS and then solve the FMCDM problem (see Fig. 1).

4. Fuzzy TOPSIS applied in financial performance evaluation

Fuzzy TOPSIS (Liang, 1999; Miyamoto, 2003; Wang et al., 2003) is the generalization of TOPSIS on fuzzy environment. TOPSIS was first proposed from Hwang and Yoon (1981). The underlying logic of TOPSIS is to define the ideal and anti-ideal solutions. The ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the anti-ideal solution is the solution that maximizes the cost criteria and minimizes the benefit criteria. The optimal alternative is the one being closest to the ideal solution and farthest to the anti-ideal solution. In short, the ideal solution is composed of all best values attainable of criteria, whereas the anti-ideal solution consists of all worst values attainable of criteria.

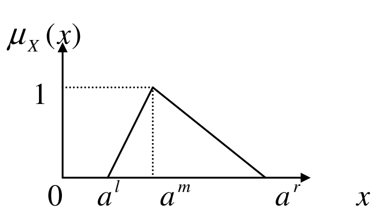

Fig. 1. The membership function of the triangular fuzzy number A.

Generally, the performance ratings and weights of criteria are known precisely in TOPSIS. As the ratings and weights are on uncertain environment or expressed in several periods, TOPSIS still solve the FMCDM problems, whereas the ratings and weights will be transferred into crisp values and some messages are lost. To avoid losing these messages, TOPSIS should be generalized on uncertain environment. Thus we propose the method called fuzzy TOPSIS to solve FMCDM problems in the condition of reserving the messages. In fuzzy TOPSTS, the performance ratings and criteria weights are measured under vagueness or fuzzy environment. Most of the steps of TOPSIS can be easily generalized to fuzzy environment except the max and min operations in finding ideal and anti-ideal solutions. In Section 3, we express two operators, MAX and MIN, being the generalization of max and min on fuzzy environment. The fuzzy TOPSIS is constructed on the two operators. To evaluate financial performance of airlines, we assume that the ratings are presented with fuzzy numbers on a given period and the weights are expressed with linguistic values, thus fuzzy TOPSIS can be applied to evaluate the problem. To describe the evaluation method clearly, let us present the procedure of fuzzy TOPSIS.

We first formulate a FMCDM problem about the comparative evaluation of financial performance. The FMCDM problem involves a set of m alternatives, which are evaluated on n financial indicators and related weights. The problem can be modeled:

$$G=\left[G_{ij}\right]_{m\times n}=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{array},$$

and

$$W=[W_1,W_2,\ldots,W_n],$$

where $A_1,A_2,\ldots,A_m$ are the possible alternatives among which experts have to evaluate, $C_1,C_2,\ldots,C_n$ are the criteria against performance of alternatives are measured, G_{ij} is the financial performance rating of alternative A_i against criteria C_j, and W_j is the related weight of C_j.

In the evaluating process, these weights represent the important degrees of criteria presented with the linguistic terms (Delgado, Verdegay, & Vila, 1992; Herrera & Herrera-Viedma, 1992) from finance experts via surveys on subjective assessments. These linguistic weights are divided into very low (VL), low (L), medium (M), high (H) and very high (VH). Assume that all linguistic terms can be presented with triangular fuzzy numbers, and fuzzy terms can be limited in the interval of [0, 1]. Thus these fuzzy numbers would be not normalized as the operation of TOPSIS. On the other hand, the financial performance ratings are measured easily as representative indicators are determined, because the values of these representative indicators are taken as financial performance ratings. Let $b_{ij}(e)$ indicate the value of representative indicator j of alternative i on the period e, where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $e = 1, 2, \ldots, t$.

Define

$$G_{ij} = (g_{ij}^l, g_{ij}^m, g_{ij}^r),$$

where

$$g_{ij}^l = \min\{b_{ij}(e) | e = 1, 2, \ldots, t\},$$

$$g_{ij}^m = \frac{1}{t}\sum_{e=1}^{t} b_{ij}(e)$$

and

$$g_{ij}^r = \max\{b_{ij}(e) | e = 1, 2, \ldots, t\}.$$

Then, $[G_{i1}, G_{i2}, \ldots, G_{in}]$ denotes the performance ratings of alternative A_i on n criteria.

With MAX and MIN operations, anti-ideal and ideal solutions of the set of alternatives are found.

Let

$$A^- = [G_1^-, G_2^-, \ldots, G_n^-]$$

and

$$A^+ = [G_1^+, G_2^+, \ldots, G_n^+]$$

denote the anti-ideal solution and ideal solution respectively, where

$$G_j^- = MIN_{1 \leqslant i \leqslant m}\{G_{ij}\} = (g_j^{l-}, g_j^{m-}, g_j^{r-}) = (\min_{1 \leqslant i \leqslant m}\{g_{ij}^l\}, \min_{1 \leqslant i \leqslant m}\{g_{ij}^m\}, \min_{1 \leqslant i \leqslant m}\{g_{ij}^r\})$$

and

$$G_j^+ = MAX_{1 \leqslant i \leqslant m}\{G_{ij}\} = (g_j^{l+}, g_j^{m+}, g_j^{r+}) = (\max_{1 \leqslant i \leqslant m}\{g_{ij}^l\}, \max_{1 \leqslant i \leqslant m}\{g_{ij}^m\}, \max_{1 \leqslant i \leqslant m}\{g_{ij}^r\}),$$

for $j = 1, 2, \ldots, n$.

By Lemma 3.2, we know $G_j^+ \succeq G_{ij} \succeq G_j^-$, where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

We assume that d_{ij}^- and d_{ij}^+ are the distance from G_{ij} to G_j^- and G_j^+, respectively, where

$$d_{ij}^- = d(G_{ij}, G_j^-) = \sqrt{\frac{1}{3}[(g_{ij}^l - g_j^{l-})^2 + (g_{ij}^m - g_j^{m-})^2 + (g_{ij}^r - g_j^{r-})^2}$$

and

$$d_{ij}^+ = d(G_{ij}, G_j^+) = \sqrt{\frac{1}{3}[(g_{ij}^l - g_j^{l+})^2 + (g_{ij}^m - g_j^{m+})^2 + (g_{ij}^r - g_j^{r+})^2},$$

for $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Let $W_{jk} = (w_{jk}^l, w_{jk}^m, w_{jk}^r)$ be a triangular fuzzy number among which the weight is expressed with linguistic term from expert E_k on criterion C_j and then set into a fuzzy number, where $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, p$. Assume W_j to be the average weight of criterion C_j, i.e.,

$$W_j = (w_j^l, w_j^m, w_j^r) = (1/p) \otimes (W_{j1} \oplus W_{j2} \oplus W_{j3} \oplus \cdots \oplus W_{jp}),$$

for $j = 1, 2, \ldots, n$,

where $\otimes$ and $\oplus$ are the extended multiplication and addition on fuzzy numbers.

With extension principle, we have

$$w_j^l = \sum_{k=1}^{p} w_{jk}^l / p, \quad w_j^m = \sum_{k=1}^{p} w_{jk}^m / p, \quad w_j^r = \sum_{k=1}^{p} w_{jk}^r / p.$$

Let D_i^- and D_i^+ express the weighted distance from alternative A_i to the anti-ideal solution A^- and ideal solution A^+, respectively.

$$D_i^- = \sum_{j=1}^{n} W_j \otimes d_{ij}^-$$

and

$$D_i^+ = \sum_{j=1}^{n} W_j \otimes d_{ij}^+,$$

where $i = 1, 2, \ldots, m$.

The distance from A_i to A^- and A^+can be merged into the vector $[D_i^-, D_i^+]$, where $i = 1, 2, \ldots, m$.

Let

$$ND^- = MIN_{1 \leqslant i \leqslant m}\{D_i^-\},$$
$$ND^+ = MAX_{1 \leqslant i \leqslant m}\{D_i^-\},$$
$$PD^- = MIN_{1 \leqslant i \leqslant m}\{D_i^+\}$$

and

$$PD^+ = MAX_{1 \leqslant i \leqslant m}\{D_i^+\}.$$

To the distance vector $[D_i^-, D_i^+]$, the anti-ideal solution is $[ND^-, PD^+]$ and the ideal solution is $[ND^+, PD^-]$, $i = 1, 2, \ldots, m$. Let A_i^- and A_i^+ denote the distance summary from $[D_i^-, D_i^+]$ to $[ND^-, PD^+]$ and $[ND^+, PD^-]$ respectively, thus

$$A_i^- = d(D_i^-, ND^-) + d(D_i^+, PD^+)$$

and

$$A_i^+ = d(D_i^-, ND^+) + d(D_i^+, PD^-),$$
$$i = 1, 2, \ldots, m.$$

Finally, the closeness coefficient A_i^* of alternative A_i is defined:

$$A_i^* = \frac{A_i^-}{A_i^- + A_i^+},$$
$$i = 1, 2, \ldots, m.$$

Obviously, $0 \leqslant A_i^* \leqslant 1, i = 1, 2, \ldots, m$. As $A_i^* = 1$, alternative A_i is the ideal solution. Oppositely, A_i is the anti-ideal solution as $A_i^* = 0$. An alternative A_i is closer to the ideal solution and farther from the anti-ideal solution as A_i^* approaches to 1. Thus we can determine the ranking order of a set of alternatives according to their closeness coefficients and then the best alternative is determined. The FMCDM problem is solved.

5. Empirical study of Taiwan airlines

To describe the evaluation work of financial performance clearly, we illustrate an empirical study of three major domestic airlines in Taiwan form 2001 to 2005. We express the normalized values of financial ratios of 3 airlines indicated with A_1, A_2 and A_3 in Table 2.

Based on the normalized values of Table 2 and $\zeta = 0.5$, we construct the grey relation matrices on 4 categories. These matrices are presented with R_1, R_2, R_3, R_4 respectively, where

$$R_1 = \begin{bmatrix} 1 & 0.844 & 0.803 & 0.872 & 0.929 & 0.430 \\ 0.839 & 1 & 0.939 & 0.929 & 0.874 & 0.419 \\ 0.767 & 0.895 & 1 & 0.859 & 0.804 & 0.439 \\ 0.867 & 0.930 & 0.905 & 1 & 0.898 & 0.411 \\ 0.925 & 0.873 & 0.831 & 0.899 & 1 & 0.417 \\ 0.693 & 0.685 & 0.685 & 0.680 & 0.690 & 1 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1 & 0.933 & 0.687 & 0.676 & 0.394 \\ 0.934 & 1 & 0.710 & 0.697 & 0.398 \\ 0.711 & 0.728 & 1 & 0.924 & 0.495 \\ 0.718 & 0.735 & 0.768 & 1 & 0.489 \\ 0.624 & 0.625 & 0.685 & 0.685 & 1 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 1 & 0.466 & 0.658 & 0.704 & 0.683 \\ 0.535 & 1 & 0.632 & 0.614 & 0.674 \\ 0.717 & 0.641 & 1 & 0.811 & 0.732 \\ 0.635 & 0.459 & 0.774 & 1 & 0.729 \\ 0.589 & 0.478 & 0.589 & 0.708 & 1 \end{bmatrix}$$

and

$$R_4 = \begin{bmatrix} 1 & 0.724 & 0.622 & 0.632 & 0.640 \\ 0.641 & 1 & 0.699 & 0.718 & 0.699 \\ 0.621 & 0.708 & 1 & 0.909 & 0.889 \\ 0.610 & 0.763 & 0.962 & 1 & 0.919 \\ 0.622 & 0.752 & 0.928 & 0.922 & 1 \end{bmatrix}.$$

From these above matrices, we can find representative indicators on four categories with the threshold value $r = 0.75$. The partitioned result and representative indicators are shown in Table 3 with the operation of Definitions 2.1–2.5.

Then, the important degrees of 12 criteria weights are given to linguistic terms, VL, L, M, H and VH from four financial experts E_1, E_2, E_3 and E_4 shown in Table 4.

The linguistic rating set {VL, L, M, H, VH} are employed to represent five degrees of weights in terms of triangular fuzzy numbers, where VL = $(0,0,0.3)$, L = $(0, 0.3, 0.5)$, M = $(0.3, 0.5, 0.7)$, H = $(0.5, 0.7, 1)$ and VH = $(0.7, 1, 1)$. Thus we can calculate the average weights to 12 criteria from Table 4, that is,

$W_1 = (0.5, 0.725, 0.925)$,
$W_2 = (0.4, 0.6, 0.85)$,
$W_3 = (0.5, 0.725, 0.925)$,
$W_4 = (0.4, 0.6, 0.85)$,
$W_5 = (0.4, 0.6, 0.85)$,
$W_6 = (0.275, 0.5, 0.725)$,
$W_7 = (0.4, 0.6, 0.85)$,
$W_8 = (0.5, 0.7, 1)$,
$W_9 = (0.4, 0.6, 0.85)$,
$W_{10} = (0.5, 0.725, 0.925)$,
$W_{11} = (0.6, 0.85, 1)$
and
$W_{12} = (0.6, 0.85, 1)$.

Since one representative indicator of an airline has five values from 2001 to 2005. The values of these indicators are set into triangular fuzzy numbers and the setting formula is presented as follows.

Let $b_{ij}(e)$ indicate the value of indicator j for airline i on the period e, where $i = 1, 2, \ldots, 3$; $j = 1, 2, \ldots, 12$; $e = 2001, 2002, \ldots, 2005$.

To reserve the messages of the five periods, the representative indicator of airline i on indicator j presented with the triangular fuzzy number $(g_{ij}^l, g_{ij}^m, g_{ij}^r)$ is defined as:

$$g_{ij}^l = \min\{b_{ij}(e) | e = 2001, 2002, \ldots, 2005\},$$

$$g_{ij}^m = \frac{1}{5} \sum_{e=2005}^{2001} b_{ij}(e)$$

and

$$g_{ij}^r = \max\{b_{ij}(e) | e = 2001, 2002, \ldots, 2005\}.$$

Utilizing the setting method of triangular fuzzy numbers, the fuzzy numbers of normalized representative indicators are expressed in Table 5.

From Table 5, the anti-ideal and ideal solutions on 12 criteria are shown in Table 6.

From Tables 5 and 6, we can calculate the distance values from alternatives to anti-ideal/ideal solution on 12 criteria. Table 7 lists the distance $d(G_{ij}, G_j^-)$ and $d(G_{ij}, G_j^+)$ which are from G_{ij} to G_j^- and G_j^+, $i = 1, 2, 3$; $j = 1, 2, \ldots, 12$.

Multiplying these distance values of Table 7 by their related weights, we get the weighted distance summaries presented as follows.

$D_1^- = (1.4618, 2.1129, 2.6526)$,
$D_2^- = (1.6125, 2.3266, 2.8398)$,
$D_3^- = (0.9598, 1.4302, 2.0142)$,
$D_1^+ = (1.2222, 1.7976, 2.4010)$,
$D_2^+ = (1.0287, 1.5244, 2.1454)$,
$D_3^+ = (1.6614, 2.3896, 2.9273)$,
thus

Table 2
The normalized values of financial ratios of three major airlines

Code	A_1					A_2					A_3				
	2001	2002	2003	2004	2005	2001	2002	2003	2004	2005	2001	2002	2003	2004	2005
F1	0.628	0.749	0.799	0.831	0.823	0.505	0.469	0.421	0.348	0.34	0.592	0.468	0.429	0.434	0.456
F2	0.534	0.537	0.58	0.643	0.584	0.68	0.698	0.647	0.603	0.649	0.501	0.474	0.495	0.473	0.488
F3	0.531	0.49	0.489	0.493	0.474	0.67	0.691	0.673	0.683	0.704	0.519	0.532	0.555	0.539	0.528
F4	0.56	0.602	0.631	0.653	0.646	0.592	0.588	0.564	0.546	0.546	0.58	0.54	0.532	0.525	0.534
F5	0.515	0.662	0.758	0.811	0.806	0.626	0.609	0.515	0.449	0.44	0.586	0.437	0.401	0.377	0.396
F6	−0.91	−0.56	−0.64	−0.66	−0.32	−0.41	−0.81	−0.5	−0.48	−0.77	0.021	−0.16	−0.58	−0.58	−0.55
S1	0.497	0.608	0.599	0.622	0.719	0.528	0.433	0.595	0.57	0.436	0.689	0.665	0.536	0.536	0.541
S2	0.442	0.563	0.505	0.558	0.656	0.457	0.378	0.666	0.605	0.532	0.772	0.735	0.55	0.568	0.535
S3	0.241	0.201	0.112	0.103	0.179	0.186	0.333	0.356	0.363	0.224	0.953	0.921	0.928	0.926	0.958
S4	0.177	0.19	0.124	0.118	0.238	0.145	0.225	0.388	0.382	0.18	0.973	0.956	0.913	0.916	0.955
S5	−0.79	−0.33	−0.43	−0.42	−0.16	−0.61	−0.92	−0.45	−0.58	−0.85	0.045	−0.2	−0.78	−0.7	−0.51
T1	0.762	0.646	0.567	0.636	0.626	0.517	0.694	0.753	0.679	0.587	0.39	0.318	0.334	0.367	0.514
T2	0.296	0.281	0.294	0.352	0.341	0.407	0.38	0.3	0.383	0.402	0.864	0.881	0.908	0.854	0.85
T3	0.68	0.752	0.788	0.797	0.756	0.522	0.43	0.38	0.318	0.313	0.515	0.5	0.484	0.513	0.575
T4	0.579	0.651	0.707	0.725	0.692	0.636	0.562	0.493	0.457	0.448	0.51	0.51	0.507	0.516	0.566
T5	0.626	0.58	0.564	0.54	0.507	0.597	0.531	0.517	0.515	0.508	0.502	0.618	0.644	0.666	0.697
P1	0.567	0.601	0.541	0.529	0.45	0.669	0.775	0.839	0.784	0.873	0.481	0.195	0.06	0.326	0.189
P2	0.141	0.749	0.476	0.686	0.08	0.984	0.639	0.463	0.703	0.525	−0.11	−0.18	−0.75	0.188	−0.85
P3	−0.31	0.424	0.355	0.73	0.19	0.107	0.428	0.409	0.681	0.056	−0.95	−0.8	−0.84	0.049	−0.98
P4	−0.36	0.44	0.407	0.684	0.145	0.135	0.39	0.411	0.722	0.057	−0.92	−0.81	−0.82	0.102	−0.99
P5	−0.33	0.532	0.563	0.819	0.239	0.143	0.446	0.429	0.57	0.06	−0.93	−0.72	−0.71	0.071	−0.97

$$ND^- = (0.9598, 1.4302, 2.0142),$$
$$ND^+ = (1.6125, 2.3266, 2.8398),$$
$$PD^- = (1.0287, 1.5244, 2.1454),$$
$$PD^+ = (1.6614, 2.3896, 2.9273).$$

Then A_i^- and A_i^+ $(i = 1, 2, 3)$ can be calculated.

Table 3
The classification of indicator clusters of financial ratios

Category	Cluster	Ratios within the cluster	Representative indicator
Financial structure	C_1	F1, F2, F3, F4, F5	F4 (debt to total assets ratio)
	C_2	F6	F6 (working capital to total assets ratio)
Solvency	C_3	S1, S2	S2 (quick ratio)
	C_4	S3, S4	S4 (cash flow ratio)
	C_5	S5	S5 (working Capital to current assets ratio)
Turnover	C_6	T1	T1 (accounts payable turnover)
	C_7	T2	T2 (accounts receivable turnover)
	C_8	T3, T4	T3 (fixed assets turnover)
	C_9	T5	T5 (net income(loss) turnover)
Profitability	C_{10}	P1	P1 (gross profit ratio)
	C_{11}	P2	P2 (operation profit ratio)
	C_{12}	P3, P4, P5	P4 (net income ratio)

Table 4
The linguistic weights of 12 criteria given from four experts

	E_1	E_2	E_3	E_4
C_1	VH	M	H	H
C_2	M	H	M	H
C_3	VH	M	H	H
C_4	M	H	H	M
C_5	H	M	H	M
C_6	H	L	M	M
C_7	H	M	M	H
C_8	H	H	H	H
C_9	M	H	H	M
C_{10}	VH	M	H	H
C_{11}	VH	H	H	VH
C_{12}	H	VH	H	VH

Table 6
The anti-ideal and ideal solutions of three airlines on 12 criteria

	A^-	A^+
C_1	(0.525, 0.542, 0.58)	(0.56, 0.618, 0.653)
C_2	(−0.91, −0.62, −0.41)	(−0.58, −0.37, 0.021)
C_3	(0.378, 0.528, 0.656)	(0.535, 0.632, 0.772)
C_4	(0.118, 0.169, 0.238)	(0.913, 0.943, 0.973)
C_5	(−0.92, −0.68, −0.45)	(−0.78, −0.43, 0.045)
C_6	(0.318, 0.385, 0.514)	(0.567, 0.647, 0.762)
C_7	(0.281, 0.313, 0.352)	(0.85, 0.871, 0.908)
C_8	(0.313, 0.393, 0.522)	(0.68, 0.755, 0.797)
C_9	(0.502, 0.534, 0.597)	(0.508, 0.625, 0.697)
C_{10}	(0.06, 0.25, 0.481)	(0.669, 0.788, 0.873)
C_{11}	(−0.85, −0.34, 0.188)	(0.463, 0.663, 0.984)
C_{12}	(−0.99, −0.69, 0.102)	(0.057, 0.343, 0.722)

$$A_1^- = d(D_1^-, ND^-) + d(D_1^+, PD^+) = 0.6126 + 0.5229 = 1.1355,$$
$$A_2^- = d(D_2^-, ND^-) + d(D_2^+, PD^+) = 0.7981 + 0.7660 = 1.5641,$$
$$A_3^- = d(D_3^-, ND^-) + d(D_3^+, PD^+) = 0 + 0 = 0,$$
$$A_1^+ = d(D_1^-, ND^+) + d(D_1^+, PD^-) = 0.1856 + 0.2432 = 0.4288,$$
$$A_2^+ = d(D_2^-, ND^+) + d(D_2^+, PD^-) = 0 + 0 = 0,$$
$$A_3^+ = d(D_3^-, ND^+) + d(D_3^+, PD^-) = 0.7981 + 0.7660 = 1.5641.$$

Finally, the closeness coefficients of alternatives are:

$$A_1^* = \frac{1.1355}{1.1355 + 0.4288} = 0.7259,$$
$$A_2^* = \frac{1.5641}{1.5641 + 0} = 1$$

and

$$A_3^* = \frac{0}{0 + 1.5641} = 0,$$

thus the ranking order of the three airlines is A_2, $A_1$and A_3. We know that the one with best financial performance is A_2.

6. Conclusions

In this paper, we utilize the grey relation analysis to find representative indicators from financial ratios and apply fuzzy TOPSIS to evaluate financial performance of airlines.

Table 5
The fuzzy numbers of normalized representative indicators among 5 years

	A_1	A_2	A_3
C_1	(0.56, 0.618, 0.653)	(0.546, 0.567, 0.592)	(0.525, 0.542, 0.58)
C_2	(−0.91, −0.62, −0.32)	(−0.81, −0.59, −0.41)	(−0.58, −0.37, 0.021)
C_3	(0.442, 0.545, 0.656)	(0.378, 0.528, 0.666)	(0.535, 0.632, 0.772)
C_4	(0.118, 0.169, 0.238)	(0.145, 0.264, 0.388)	(0.913, 0.943, 0.973)
C_5	(−0.79, −0.43, −0.16)	(−0.92, −0.68, −0.45)	(−0.78, −0.43, 0.045)
C_6	(0.567, 0.647, 0.762)	(0.517, 0.646, 0.753)	(0.318, 0.385, 0.514)
C_7	(0.281, 0.313, 0.352)	(0.3, 0.374, 0.407)	(0.85, 0.871, 0.908)
C_8	(0.68, 0.755, 0.797)	(0.313, 0.393, 0.522)	(0.484, 0.517, 0.575)
C_9	(0.507, 0.563, 0.626)	(0.508, 0.534, 0.597)	(0.502, 0.625, 0.697)
C_{10}	(0.45, 0.538, 0.601)	(0.669, 0.788, 0.873)	(0.06, 0.25, 0.481)
C_{11}	(0.08, 0.426, 0.749)	(0.463, 0.663, 0.984)	(−0.85, −0.34, 0.188)
C_{12}	(−0.36, 0.263, 0.684)	(0.057, 0.343, 0.722)	(−0.99, −0.69, 0.102)

Table 7
The distance values from alternatives to anti-ideal/ideal solution on 12 criteria

	A_1		A_2		A_3	
	$d(G_{1j}, G_j^-)$	$d(G_{1j}, G_j^+)$	$d(G_{2j}, G_j^-)$	$d(G_{2j}, G_j^+)$	$d(G_{3j}, G_j^-)$	$d(G_{3j}, G_j^+)$
C_1	0.0641	0	0.0201	0.0466	0	0.0641
C_2	0.0520	0.3097	0.0603	0.3093	0.3450	0
C_3	0.0382	0.0995	0.0058	0.1248	0.1277	0
C_4	0	0.7684	0.1037	0.6814	0.7684	0
C_5	0.2335	0.1185	0	0.3302	0.3302	0
C_6	0.2531	0	0.2344	0.0293	0	0.2531
C_7	0	0.5610	0.0487	0.5166	0.5610	0
C_8	0.3373	0	0	0.3373	0.1257	0.2194
C_9	0.0239	0.0544	0.0035	0.0781	0.0781	0.0035
C_{10}	0.2884	0.2480	0.5209	0	0	0.5209
C_{11}	0.7673	0.2933	1.0589	0	0	1.0589
C_{12}	0.7402	0.2461	0.9215	0	0	0.9215

Then, we illustrate an empirical study about the financial performance evaluation of three domestic airlines in Taiwan. On the evaluation procedure, we know that the financial performance of three airlines is evaluated easily. On the other hand, the number of airlines in Taiwan stock market is merely three, but the FMCDM method solves the evaluation problems among which the number of alternatives is large as well. In fact, the financial performance of these airlines can easily be evaluated with the FMCDM method, whether the number of alternatives is large or not. Besides, comparing one airline with others can identify the competitive strength and weakness of itself. It is useful for that the airline can realize the finance competition location on airline market and ready to improve its competitive advantage for enhancing the finance ability in the future.

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