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Theory and Methodology

Financial ratio distribution irregularities: Implications for ratio classification

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Abstract: This paper incorporates studies investigating the distributional characteristics of financial ratios to studies related to empirical classification of financial ratios. Even though the distributional properties of financial ratios have received some attention in prior research, the implications of observed results for financial decision making have not been researched in detail. The empirical analysis of this study is carried out for 10 ratios on Finnish listed firms. The results reveal that the observed distribution irregularities have a significant impact on the results obtained from the empirical classification of financial ratios. A large part of the time-series instability of financial ratio patterns is caused by financial ratio distribution irregularities. In addition, it is discovered that the interpretation of the underlying financial factors of firms may be affected if distribution irregularities are not paid due attention to.

Keywords: Finance; Financial ratios; Classification

1. Introduction

Financial ratios represent an important tool for decision makers. By using financial ratios financial managers as well as interested external parties such as investors and lenders are assumed to be able to evaluate a firm's financial success and ability to meet its financial obligations in future. Ratios have been found to be useful in many decision making contexts, such as auditing, bankruptcy prediction, bond rating, commercial credit scoring, and security analysis. Usually, ratio analysis is performed by comparing the behaviour of a ratio with some benchmark describing the general success of the industry or economy.

Typically, in financial ratio analysis parametric statistical methods have been used. These techniques are, however, problematic since a large literature reveals that financial ratio distributions do not follow the normal distribution, and they contain a lot of observations identified as outliers. A typical analysing method suggested in ratio literature is factor analysis procedure to reduce the information in a huge number of potentially usable ratios to a smaller set of variables connected to the underlying financial structures of firms. This is important since N financial variables can generate up to $N^2 - N$ financial ratios.

The main purpose of this study is to evaluate

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the implications of financial ratio distribution irregularities for financial decision making. This will be carried out by testing how the observed outliers and distributional irregularities affect the results obtained from the empirical classification of financial ratios. Although it has been generally observed that there exist certain irregularities in financial ratio distributions, the effects of these irregularities on results obtained from empirical classification of financial ratios have not been researched carefully enough. We hypothesize that these distribution irregularities cause significant time-series instability as well as cross-sectional variation in financial ratio patterns estimated by factor analysis. By taking into account the empirical irregularities, we aim to produce more meaningful factor structures of financial ratios.

The remainder of this paper is organised as follows. In section two a brief literature review is given. The methodology used here is described in the third section. The data are presented in the fourth section. Section 5 includes the empirical analysis. Finally, conclusions are drawn in the sixth section.

2. Literature review

2.1. Distributional properties of financial ratios

The distributional characteristics of financial ratios have attained considerable attention in the empirical research of finance. Horrigan (1965) investigated the cross-sectional behaviour of 17 ratios for 50 firms in the USA for the period 1948–1957. Horrigan found that the investigated financial ratios were often positively skewed. A potential reason for this skewness was reported to be the fact that most of these ratios have a lower limit of zero, but an indefinite upper limit. O'Connor (1973) analysed the cross-sectional distributions of 10 ratios for 127 US firms and reported that although most of the ratios were skewed, the cross-sectional distributions appeared to be approximately symmetrical in nature.

Deakin (1976), investigating the cross-sectional distributions of 11 ratios for all manufacturing firms on the COMPUSTAT data file, found that skewness in ratio distributions was reduced by applying logarithm and square root transformations to the raw financial ratio distributions. However, no general guidelines concerning the most appropriate transformation could be given. His results were further extended by Frecka and Hopwood (1983). They reported that after removing some outliers identified by skewness as well as kurtosis tests, and by applying square root transformations to the 'raw' data, normality was achieved in most of the cross-sectional ratio distributions. In this context it should be noted, however, that it is often impossible to determine the correct transformation to be applied in 'raw' financial data (Ezzamel, Mar-Molinero and Beecher, 1987, see also Lee, 1985).

A part of financial ratio non-normality is obviously caused by changes in underlying economic conditions, such as changes in inflation, interest rates, and competitiveness of firms (Mensah, 1984; Wood and Piesse, 1987; Virtanen and Yli-Olli, 1991). Recently, Martikainen (1991) investigated the cross-sectional distributional properties of financial ratios using accounting index models. By eliminating the effects of macroeconomic changes on financial ratios, normality was significantly improved in ratio distributions. Almost all work has concentrated on cross-sectional ratio distributions, while the time-series distributions have received much less attention. An exception is the study by Martikainen (1992). His findings suggest that the results do not markedly differ in this context.

2.2. Classification patterns of financial ratios

A large empirical literature has aimed to create classification patterns of financial ratios to study how these ratios are classified empirically. The main objective in these studies has been to express the maximum amount of information in the original financial ratios by a reduced set of factors. The basic study of this area was carried out by Pinches, Mingo and Caruthers (1973) who classified 48 ratios of 221 firms into seven empirical categories: return on investment, capital intensiveness, inventory intensiveness, financial leverage, receivables intensiveness, short-term liquidity and cash position. Their seminal paper revealed significant time-series instability in the obtained results, and was further supported by Pinches et al. (1975) by establishing a hierarchical classification of financial ratios.

Furthermore, Johnson (1978), using principal component factor analysis and an orthogonal rotation method, reported nine empirical factors for 41 ratios. His data consisted of 306 primary manufacturing and 159 retail firms from the COMPUSTAT data file. Laurent (1979) again applying principal component factor analysis reported 10 factors concerning 63 firms in Hong Kong. Gombola and Ketz (1983) found eight typical factors for 119 industrial COMPUSTAT firms when 58 financial ratios were used in the empirical analysis. Ezzamel, Brodie and Mar-Molinero (1987), and Richardson and Davidson (1984) further supported the observation that the empirical classification patterns may be significantly unstable over time, and Martikainen and Ankelo (1991) discovered that the classifications may also differ across different groups of firms. Recently, Kanto and Martikainen (1991) applied confirmatory factor analysis on financial ratios reporting significant differences between a priori and empirical classifications of financial ratios.

3. Methodology

The empirical analysis performed in this paper is divided into two sections. First, the distributional characteristics of financial ratios are investigated, and second the implications of distribution irregularities to the results obtained from the empirical classification of financial ratios are studied. A critical issue in these kinds of studies is naturally the identification of outliers. In the absence of substantical theories of financial ratio outliers, this identification is based on purely statistical aspects. Thus, we follow the guidelines offered, for instance, by Frecka and Hopwood (1983), and Virtanen and Yli-Olli (1991) in this context.

The first empirical phase includes investigating the distributional characteristics of financial ratios. For non-normally distributed ratios two alternative approaches are used. First, for the ratios that have a lower limit of zero, but an indefinite upper limit, a square root transformation is applied. This is because it has been observed that these type of ratios typically follow a gamma distribution (see, e.g., Frecka and Hopwood, 1983, and Buijink and Jegers, 1986). By applying a square root transformation to a gamma distributed variable, an approximately normally distributed variable is obtained. From the distributions, out of which normality can not be achieved by transformations, outliers are removed. For this purpose, the procedure used by Frecka and Hopwood (1983) is used. In this procedure, outliers are identified using skewness statistics, i.e. according to the third moment of the distribution. The outliers are removed either from the right or left tail of the distribution, if the ratios are positively or negatively skewed, respectively.

The normality of ratio distributions is investigated by applying the Shapiro–Wilk test. The W-statistics by Shapiro and Wilk is the best estimator of the scaled variance (based on the square of a linear combination of the order statistics) to the usual corrected sum of squares estimator of the variance. The statistics is scale and origin invariant and hence supplies a test of the composite null hypothesis of normality. This test is effective for normality even in connection with small samples, and is thus used here. Further details of the Shapiro–Wilk test can be found, for instance, in Shapiro and Wilk (1965) and Royston (1982).

After identifying the outliers, and carrying out suitable ratio transformations, the implications of the observed non-normalities are analysed in the context of results obtained from classification patterns of financial ratios. Traditionally, the cross-sectional similarity and time-series stability of factor patterns have been measured by simple correlation and congruency coefficients. These two methods are, however, unable to characterise the reason for the non-invariant part prevailing in these factor solutions. In terms of explanatory power, particularly of the dissimilar part of the comparable factor solutions, transformation analysis offers a more efficient method to study the stability of factor patterns over time or even across different samples. Transformation analysis was originally applied in Finnish sociological studies. In the financial accounting literature, transformation analysis was first applied by Yli-Olli (1983) in order to study the degree and nature of medium-term stability exhibited by the factor patterns of financial ratios. In the classification of financial ratios, the use of this method was further deepened by Yli-Olli and Virtanen (1989), who applied it to cross-sectional samples over different countries, and Martikainen (1993) to study the stability of economic factors generating stock returns. Yli-Olli and Virtanen (1990) offer a detailed description of this statistical method.

4. The data

The financial statement data is based on annual cross-industry multivariate time-series data. The firms selected to the research effort are from the Helsinki Stock Exchange. The data consists of all firms continuously listed for the 1974–1987 period (34 firms). The ratios are calculated separately for each year in the period 1981–1987. The ratios are calculated according to the recommendations of the Finnish Credit Analysis Commission (see Yritystutkimusneuvottelukunta, 1983). Corporate earnings are determined using the maximum level of depreciations allowed in the Business Income Tax in Finland. The depreciation charges allowed on buildings are 5–10% and those on equipment 30% of the undepreciated balance. The use of rather long period (1974–1987) when-determining the earnings and other accounting items makes it possible to obtain reliable accounting figures.

There are 10 ratios investigated in this study. They are selected from the following five categories: (i) profitability, (ii) financial leverage, (iii) liquidity, (iv) working capital, and (v) cash-flow. The ten ratios representing these categories are ROI, ROE, Debt to Equity, Equity to Capital, Quick Ratio, Current Ratio, Working Capital to Sales, Working Assets to Invested Capital, Cash-flow I to Sales, and Cash-Flow II to Sales. The definitions of the ratios are given in the Appendix.

5. Empirical results

5.1. Distributional characteristics of financial ratios

Table 1 summarises the results concerning the normality tests of financial ratios used. The probability levels in which the normality hypothesis holds for each ratio and each year are presented. The results indicate a high level of non-normality in ratio distributions. This is the case especially for both financial leverage ratios and the Current Ratio for which normality is rejected every single year. With regard to profitability ratios, the ROI ratio seems to follow the normal distribution better than its counterpart, ROE. This may well be because of (i) the problems of firms to obtain equity from the thin Finnish capital markets, and (ii) the adjustments made to the ratios, making the denominator of ROE very low for many firms. These kind of ratios with small denominators are problematic since they easily produce extreme values to ratio distributions. This also increases the estimated variance in ratio distributions. Concerning liquidity ratios, the Quick Ratio seems to follow the normal distribution much better than the Current Ratio does. This finding confirms the earlier results by Virtanen and Yli-Olli (1991) and Perttunen and Martikainen (1989).

To remove outliers with the intent to obtain normally distributed ratios, we follow the procedure described above. For ratios with a lower limit of zero, we employ square root transformations. For the other ratios, outliers are removed from either the right or left tail of the distribution based on the results from skewness tests. The process of removing outliers is continued until

Table 1
Normality of ratio distributions. Probability levels of Shapiro–Wilk W-statistics

	ROI	ROE	DTE	ETC	QR	CR	WCS	WAC	C1S	C2S
1981	0.426	0.129	* [a]	*	0.494	*	*	0.391	0.300	*
1982	0.147	*	*	*	0.114	*	*	0.329	0.273	0.722
1983	0.231	*	*	*	0.687	*	*	0.060	0.547	0.333
1984	0.197	*	*	*	0.935	*	0.177	0.151	0.753	0.607
1985	*	0.930	*	*	0.991	*	0.626	0.731	0.368	0.998
1986	0.782	*	*	*	0.806	*	0.876	0.215	0.129	*
1987	*	*	*	*	0.191	*	*	*	0.808	*

H0: ratios follow the normal distribution.
[a],* H0 is rejected (prob. < 0.05).

normality cannot be rejected by the Shapiro–Wilk W-test ($p = 0.05$).

Table 2 shows the normality tests of the ratio distributions where square root transformation has been applied and outliers have been removed. Thus, distributions which were already normal in their raw form are now excluded in the table. The results indicate that for each ratio normality is achieved either by using the transformation or removing the outliers identified by skewness and Shapiro–Wilk W-statistics. The values in parentheses indicate that the largest number of removed outliers is six in the context of Current Ratio in 1981, while in most cases a much lower number of outliers is sufficient. It should be noted that the number of observations varies between 32, 26, 28, 30, 32, 30 and 26 for the seven investigated years. Thus, a firm is often observed to be an outlier in more than one ratio simultaneously. Moreover, it is relevant to emphasise that the group of outliers is not stable, i.e. the same firms are not typically identified as outliers in successive years.

5.2. Comparison of factor structures based on 'raw' and transformed financial ratios

To study the implications of the above observed results for the empirical classification of firm-specific financial variables, we classify the 'raw' financial ratios and the transformed ratios for each year in the period 1981–1987. The similarity of the varimax-rotated principal component factor analysis (see, e.g., Green, 1978) based financial factors is then compared using transformation analysis. Varimax rotation is selected because it is the most common rotation method used in earlier financial ratio research. The four-factor solutions were selected based on the eigenvalue criterion. In the factor model we have 10 original financial ratios $x_1, x_2, \ldots, x_{10}$ with mean values $\mu_1, \mu_2, \ldots, \mu_{10}$ and variances $\sigma_1, \sigma_2, \ldots, \sigma_{10}$, respectively. In matrix notation the factor model estimated can thus be given as follows (see, e.g., Johnson and Wichern, 1982, p.402–405):

$$\boldsymbol{x} - \boldsymbol{\mu} = \boldsymbol{Lf} + \boldsymbol{u}, \tag{1}$$

where

$$x' = (x_1, x_2, \ldots, x_{10}),$$

$$u' = (\mu_1, \mu_2, \ldots, \mu_{10}),$$

$$\boldsymbol{L} = (l_{ij})_{10 \times 4},$$

$$f' = (x_1, x_2, x_3, x_4)$$

and

$$u' = (u_1, u_2, u_3, u_4).$$

The coefficient of l_{ij} is called the loading of the i-th variable on the j-th factor, so $\boldsymbol{L}$ becomes the

Table 2
Normality of transformed ratio distributions. Probability levels of Shapiro–Wilk W-statistics (number of removed outliers in parentheses)

	ROI	ROE	DTE	ETC	QR	CR	WCS	WAC	C1S	C2S
1981			0.393	0.737	0.792	0.125 (1)	0.311	0.453		0.125 (1)
1982		0.087 (5)	0.634 (2)	0.248 (1)	0.540	0.326 (3)	0.278	0.957		
1983	0.246	0.956 (3)	0.167 (2)	0.977 (1)	0.064	0.338 (3)	0.275			
1984	0.594	0.085 (1)	0.061 (1)	0.907	0.071	0.785 (2)	0.216			
1985	0.153 (1)		0.155 (1)	0.101	0.966	0.321	0.926	0.470		
1986		0.913 (2)	0.144 (1)	0.287 (1)	0.902	0.098 (1)	0.390	0.796		0.151 (1)
1987	0.253 (2)	0.092 (4)	0.229 (2)	0.076 (1)	0.334	0.078 (6)	0.490 (1)	0.373	0.187	(1)
Transformation	–	–	sqrt	sqrt	sqrt	sqrt	sqrt	sqrt	–	–

Table 3
Factor pattern of 'raw' financial ratios. Varimax-rotation, 1981

Variable	Factor 1	Factor 2	Factor 3	Factor 4	h^2
ROI	0.021	−0.007	−0.149	0.967	0.958
ROE	0.023	0.064	−0.163	0.970	0.972
DTE	−0.228	−0.009	0.890	−0.254	0.908
ETC	0.298	−0.012	−0.909	0.173	0.944
QR	0.595	0.468	−0.243	0.101	0.642
CR	0.942	0.067	−0.192	0.050	0.932
WCS	0.473	0.365	0.581	0.049	0.696
WAC	0.938	0.019	−0.091	−0.040	0.891
C1S	0.101	0.954	0.145	0.036	0.943
C2S	0.063	0.968	−0.002	−0.001	0.940
Eigenvalue	2.502	2.208	2.128	1.989	
Cumulative proportion of total variance	0.250	0.471	0.684	0.883	

Table 4
Factor pattern of transformed ratios. Varimax-rotation, 1981

Variable	Factor 1	Factor 2	Factor 3	Factor 4	h^2
ROI	0.870	−0.081	0.229	−0.060	0.820
ROE	0.969	−0.042	0.087	0.128	0.965
DTE	−0.910	−0.267	−0.072	0.140	0.924
ETC	0.769	0.470	0.048	−0.316	0.914
QR	0.046	0.768	0.423	−0.023	0.771
CR	0.156	0.914	0.072	0.275	0.941
WCS	−0.132	0.400	0.050	0.876	0.948
WAC	0.066	0.892	0.047	0.197	0.841
C1S	0.020	0.124	0.959	0.122	0.951
C2S	0.430	0.203	0.849	−0.107	0.959
Eigenvalue	3.349	2.739	1.890	1.048	
Cumulative proportion of total variance	0.349	0.609	0.798	0.903	

matrix of factor loadings. As illustrative examples, Tables 3 and 4 present the factor analysis results for 'raw' financial ratios and transformed ratios based on 1981 data, respectively.

The factors in Tables 3 and 4 appear in the decreasing order of variance explained by the factors, i.e. according to the eigenvalues of the factors. The explanatory power of the factor model appears to be high. The results reveal that the four-factor solutions explain as much as 88.3% and 90.3% of the total variance of the original 10 financial ratios and transformed ratios, respectively. The interpretation of the factors is rather straightforward. In the 'raw' ratio case, the first factor, explaining 25.0% of the total variance, includes high loadings of short-term solvency (liquidity). The second factor mainly represents the cash flow position, while the third factor can be interpreted as the factor of long-term solvency. The fourth factor is the factor of profitabilty. The working capital ratios, however, have their highest loadings on separate factors. In the case of transformed ratios, the first factor (with 34.9% contribution to the variance) includes high loadings of long-term solvency (financial leverage) as well as profitability ratios. The second factor mainly represents short-term solvency, while the third factor can be interpreted as the factor of cash-flow position. The fourth factor has a high loading of WCS. The other working capital ratio, WAC, has a high loading on the liquidity factor, however. Thus, even the preliminary visual analysis detects some differences between these factor solutions. Interestingly, the cash-flow and accrual rules based profitability ratios seem to measure somewhat different characteristics of a firm. This finding is consistent with the results obtained by Gombola and Ketz (1983) on US data. The results for the other years were basically similar to those reported here (these results are obtainable from the authors upon request).

The figures representing the cumulative proportions of the total variance explained by the four factor solutions in each year between 1981 and 1987 are presented in Table 5. An interesting finding is the rather high level of time-series stability in the cumulative proportion figures. These cumulative figures do not, however, indicate that the contents of the four factors esti-

Table 5
Proportion of total variance explained; four-factor solutions

	1981	1982	1983	1984	1985	1986	1987	Average
'Raw' ratios	0.883	0.867	0.887	0.869	0.886	0.889	0.833	0.873
Transf. ratios	0.903	0.881	0.899	0.870	0.899	0.910	0.910	0.896

mated would necessarily be the same. The cumulative proportion figures are in average slightly higher for transformed ratio series, where single outliers do not bias the results.

To further investigate the similarity of the factors, transformation analysis is applied next. The transformation matrix relates two factor structures as follows (for details, see Yli-Olli and Virtanen, 1989):

$$L_2 = L_1 T_{12}. \tag{2}$$

Depending on the transformation matrix, T_{12}, the formation of factors from the variables and thereby the interpretation of the factors either is preserved, or it changes. If the interpretation of the factor is the same, the transformation matrix is an identity matrix I. The transformation matrices between the factor solutions obtained using 'raw' data and transformed data in each year are given in Table 6. The elements above 0.95 are written in bold text. These factors can be seen to be those where distribution irregularities have not caused significant effects on the classification results. It is relevant to notice that solely the absolute values in transformation matrices matter when investigating the structural invariance between factor patterns.

The results indicate obvious differences between the factor solutions based on the two different set of financial statement variables. Typically, the factor contents do not remain unchanged, and the factors are often been produced in a different order across the two patterns of financial ratios. By the 0.95 criterion, only one similar factor can be found in the two factor solutions in 1981. In 1982, 1986 and 1987 two factors appar to be about the same. In 1983–1985 the factor structures are more similar to each other. The variation in results over time is in accordance with earlier studies (see, e.g., Virtanen and Yli-Olli, 1991, and Martikainen, 1992) reporting that the underlying economic conditions have significant impact on the distribution characteristics of financial ratios.

The residual matrix, E_{12}, offers a goodness-of-fit criterion for model (2):

$$E_{12} = L_1 T_{12} - L_2. \tag{3}$$

Non-zero elements in the residual matrix indicate that the empirical meaning of the variables analysed has changed, and it is called abnormal transformation. This can be appointed to specific factors or to specific variables. As an illustrative example, Table 7 offers the residual matrix when comparing the factor structures of 'raw' and transformed financial ratios in 1981. It shows that the highest interpretative changes in ratios appear in the context of DTE and C2S. For the most part, their abnormal transformation can be tied to factors 1 and 4.

Table 8 summarises the abnormal transformation appointed to specific ratios in different years. The results indicate that the highest abnormal transformation has occurred in years 1987, 1982 and 1981. The smallest abnormal transformation is observed in 1985 reflecting that the empirical meaning of variables in that year is about the same whether transformations are used or not. In the context of single ratios, it appears that no systematic pattern appears to be in that which of

Table 6
Transformation matrices of factor patterns, 1981–1987

	Transformed ratios	'Raw' ratios			
		Factor 1	Factor 2	Factor 3	Factor 4
1981	Factor 1	0.073	0.908	−0.088	0.403
	Factor 2	−0.010	0.098	**0.995** [a]	−0.002
	Factor 3	−0.496	−0.316	0.028	0.808
	Factor 4	0.865	−0.257	0.035	0.429
1982	Factor 1	−0.495	0.036	0.867	−0.036
	Factor 2	−0.193	**0.967**	−0.148	0.074
	Factor 3	0.085	−0.045	0.091	**0.994**
	Factor 4	0.843	0.248	0.466	−0.104
1983	Factor 1	**0.990**	0.058	0.102	0.082
	Factor 2	−0.062	**0.998**	0.009	0.030
	Factor 3	0.077	0.035	0.030	**−0.996**
	Factor 4	−0.104	−0.017	**0.994**	0.022
1984	Factor 1	**0.995**	0.102	0.000	−0.004
	Factor 2	−0.102	**0.994**	0.025	−0.043
	Factor 3	0.003	−0.025	**1.000**	−0.006
	Factor 4	−0.001	0.043	0.007	**0.999**
1985	Factor 1	**0.998**	0.054	−0.008	0.018
	Factor 2	0.054	**−0.998**	0.003	−0.012
	Factor 3	0.008	0.003	**1.000**	−0.014
	Factor 4	−0.017	−0.012	0.014	**1.000**
1986	Factor 1	**1.000**	−0.012	−0.005	−0.003
	Factor 2	0.008	0.158	**0.985**	0.077
	Factor 3	0.005	0.256	−0.116	0.940
	Factor 4	0.010	0.944	−0.132	−0.271
1987	Factor 1	**0.972**	−0.023	−0.192	0.131
	Factor 2	−0.029	**−0.999**	−0.001	0.043
	Factor 3	−0.092	0.045	0.194	0.946
	Factor 4	0.213	−0.015	0.942	−0.171

[a] Elements above 0.95 in bold text.

Table 7
Residual matrix between 'raw' and transformed ratio patterns. Four-factor solutions, 1981

Variable	Factor 1	Factor 2	Factor 3	Factor 4	Abnormal transf. to variables
ROI	0.042	−0.102	−0.209	0.363	0.188
ROE	−0.048	−0.128	0.005	0.166	0.046
DTE	0.232	−0.157	0.099	0.379	**0.232**
ETC	−0.147	0.042	−0.106	−0.224	0.085
QR	0.201	−0.131	−0.013	0.108	0.069
CR	0.052	−0.004	−0.091	−0.029	0.012
WCS	−0.083	−0.131	0.290	−0.196	0.147
WAC	0.013	0.001	−0.114	0.089	0.021
C1S	−0.063	0.007	−0.013	0.050	0.007
C2S	−0.435	−0.049	0.108	0.128	**0.220**
Abnormal transf. to factors	0.323	0.090	0.182	0.431	
				Cumulative:	1.026

the ratios is mostly affected by transformations. Further, it seems that all a priori ratio categories have ratios which have high abnormal transformation in at least some years. Altogether, the results indicate that there have occurred interpretative changes both in factors (as shown in transformation matrices) and ratios (as shown in residual matrices).

5.3. *Time-series stability of factor patterns*

In this section the effects of distribution irregularities on the time-series stability of factor patterns are studied. Table 9 first presents the transformation matrices analysing the time-series stability of factor structures based on 'raw' ratio data. The results indicate that the factors are not stable over time, when the 0.95 criterion is used. Between years 1981 and 1982 no stable factors are found. Between 1982 and 1983 only one factor appears to be stable. The same is the case between years 1986 and 1987. The other three transformation matrices indicate more stable factors.

Table 10 offers the time-series analysis between transformed ratio patterns. The results are clear-cut. All matrices indicate that four stable factors can be found, while the results in Table 9 indicated that the factor solutions based on 'raw' data were considerably more instable over time. Thus, a large part in the time-series instability of financial ratio patterns observed in 'raw' ratio data appears to be caused by distribution irregularities.

Table 8
Cumulative abnormal transformation between raw and transformed factor patterns. Four−factor solutions− 1981–1987

Variable	1981	1982	1983	1984	1985	1986	1987
ROI	0.188	0.046	0.031	0.006	0.007	0.036	0.102
ROE	0.046	0.035	0.051	0.135	0.023	**0.212** [a]	0.135
DTE	**0.232**	**0.222**	0.046	0.053	0.014	0.131	0.074
ETC	0.085	0.079	0.055	0.025	0.005	0.030	0.038
QR	0.069	**0.329**	0.017	0.066	0.018	0.028	0.117
CR	0.012	0.051	0.036	0.014	0.005	0.027	0.126
WCS	−0.147	**0.200**	0.107	0.028	0.005	0.013	**0.292**
WAC	0.021	0.031	0.008	0.010	0.012	0.031	0.117
C1S	0.007	0.028	0.009	0.013	0.002	0.004	**0.620**
C2S	**0.220**	0.045	0.009	0.017	0.008	0.030	0.185
Total	1.026	1.065	0.369	0.368	0.098	0.542	1.805

[a] Elements above 0.20 in bold text.

Table 9
Time-series transformation matrices of factor patterns; 'Raw' ratios

		Factor 1	Factor 2	Factor 3	Factor 4
		1981			
1982	Factor 1	0.814	0.908	0.286	0.116
	Factor 2	−0.577	0.098	0.317	0.114
	Factor 3	−0.064	−0.316	0.803	0.385
	Factor 4	−0.004	−0.257	−0.416	0.909
		1982			
1983	Factor 1	−0.021	−0.045	0.575	0.817
	Factor 2	0.879	−0.017	0.399	−0.260
	Factor 3	−0.053	**0.989** [a]	0.133	−0.040
	Factor 4	0.473	0.141	−0.702	0.514
		1983			
1984	Factor 1	**0.993**	−0.067	−0.038	−0.085
	Factor 2	0.090	0.095	−0.041	**0.991**
	Factor 3	−0.057	**−0.993**	0.020	0.101
	Factor 4	0.042	0.022	**0.998**	0.035
		1984			
1985	Factor 1	**1.000**	−0.002	0.022	−0.010
	Factor 2	−0.003	**−0.999**	0.040	−0.001
	Factor 3	−0.022	0.040	**0.999**	−0.023
	Factor 4	0.009	0.000	0.085	**1.000**
		1985			
1986	Factor 1	**0.999**	−0.005	0.046	0.000
	Factor 2	0.007	**0.992**	−0.033	−0.125
	Factor 3	−0.001	0.126	0.024	**0.992**
	Factor 4	−0.046	0.030	**0.998**	−0.028
		1986			
1987	Factor 1	0.938	−0.019	−0.180	0.297
	Factor 2	−0.005	**−0.992**	0.120	0.025
	Factor 3	−0.230	0.062	0.312	0.920
	Factor 4	0.261	0.104	0.925	−0.256

[a] Elements above 0.95 in bold text.

Table 10
Time-series transformation matrices of factor patterns; transformed ratios

		Factor 1	Factor 2	Factor 3	Factor 4
		1981			
1982	Factor 1	0.062	**0.994** [a]	0.075	−0.047
	Factor 2	**0.998**	−0.063	−0.012	−0.018
	Factor 3	0.022	0.040	0.070	**0.996**
	Factor 4	0.006	−0.078	**0.995**	−0.067
		1982			
1983	Factor 1	0.123	0.176	0.018	**0.977**
	Factor 2	**0.991**	0.036	−0.002	−0.131
	Factor 3	0.002	−0.041	**0.999**	−0.011
	Factor 4	−0.058	**0.983**	0.039	−0.171
		1983			
1984	Factor 1	**0.990**	−0.048	−0.093	−0.098
	Factor 2	0.099	0.110	−0.049	**0.988**
	Factor 3	0.097	−0.002	**0.994**	0.040
	Factor 4	0.037	**0.993**	0.003	−0.115
		1984			
1985	Factor 1	**0.998**	−0.058	−0.010	−0.006
	Factor 2	0.059	**0.998**	0.000	0.015
	Factor 3	0.009	0.000	**0.999**	−0.045
	Factor 4	0.006	−0.015	0.045	**0.999**
		1985			
1986	Factor 1	**0.998**	−0.004	0.036	0.052
	Factor 2	0.034	−0.035	**−0.998**	0.047
	Factor 3	0.015	**0.982**	−0.043	−0.183
	Factor 4	−0.052	0.185	0.038	**0.981**
		1986			
1987	Factor 1	**0.996**	−0.032	0.067	−0.048
	Factor 2	0.040	0.069	0.144	**0.986**
	Factor 3	0.026	**0.997**	0.025	−0.075
	Factor 4	−0.074	−0.033	**0.987**	−0.139

[a] Elements above 0.95 in bold text.

Finally, Table 11 presents the cumulative abnormal transformation between successive periods for the 'raw' financial ratio distribution, as well as for the transformed ratios. The results indicate that the largest part of the time-series instability of factor patterns is removed in the factor patterns based on transformed ratio series. This can be seen from the average abnormal transformation figures, 1.680 vs. 0.761. This further suggests that by using transformed ratios

Table 11
Cumulative abnormal transformation between successive years. Four-factor solutions, 1981–1987

	81–82	82–83	83–84	84–85	85–86	86–87	Average
'Raw' ratios	1.026	2.176	2.248	0.889	1.202	2.541	1.680
Transf. ratios	0.431	0.987	1.080	0.545	0.729	0.794	0.761

series, more meaningful factor patterns of financial ratios are indeed obtained. As a conclusion, the use of transformed ratios seems to produce factors (see the transformation matrices) and variables which are interpretatively (see the abnormal transformation figures) rather stable over time.

6. Summary and conclusions

This study aimed to extend the evidence relating to the implications of distribution irregularities for financial statement analysis. This was done by investigating the distribution characteristics of 10 financial ratios of Finnish listed firms, and examining how the observed irregularities affect the results concerning the empirical classification of firm-specific financial characteristics. The empirical classification results were significantly affected by these irregularities. In addition, it was discovered that a large part of the time-series instability of factor patterns is removed when transformed ratios are used.

In general, the results suggest that the distribution irregularities have to be carefully taken into account in financial ratio analysis. Financial analysts and other decision makers should be careful when analysing ratios with small, close-to-zero denominators, which may lead to extreme values in ratio distributions. Due to the small sample analysed here, this analysis should be repeated on more extensive samples in the future. However, the existing evidence suggests that financial analysts should be extremely careful when analysing small samples using financial statement data.

Appendix. Ratios used in the study

Return on Investment

$= (\text{Net income after tax} + \text{Interest expenses})$

$\times (\text{Total assets})^{-1}.$

Return on Equity

$= (\text{Net operating income})$

$\times (\text{Stockholders' equity} + \text{reserves})^{-1}.$

Debt to Equity

$= (\text{Long-term debt} + \text{Current liabilities})$

$\times (\text{Shareholders' equity}$

$+ \text{Reserves} + \text{Cum. appreciations})^{-1}.$

Equity to Capital

$= (\text{Shareholders' equity} + \text{Reserves}$

$+ \text{Cum. appreciations}) \times (\text{Liabilities})^{-1}.$

Current Ratio

$= (\text{Current assets})$

$\times (\text{Current liabilities})^{-1}.$

Quick Ratio

$= (\text{Current assets} - \text{Current inventories})$

$\times (\text{Current liabilities})^{-1}.$

Working Capital to Sales

$= (\text{Working capital}) \times (\text{Net sales})^{-1}.$

Working Assets to Invested Capital

$= (\text{Current assets} - \text{Current liabilities})$

$\times (\text{Liabilities})^{-1}.$

Cash Flow I to Sales

$= (\text{Cash receipts}$

$- \text{Variable and fixed (short-term) cash}$

$\text{disbursements} - \text{extra expenses})$

$\times (\text{Sales (Cash receipts)})^{-1}.$

Cash Flow II to Sales

$= (\text{Cash receipts}$

$- \text{Variable and fixed (short-term)}$

$\text{cash disbursements}$

$- \text{extra expenses} - \text{profit distribution})$

$\times (\text{Sales (Cash receipts)})^{-1}.$

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