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Theory and Methodology

A fuzzy set approach to financial ratio analysis

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Abstract: Ratio analysis is a useful tool of financial analysis. Nevertheless, the traditional ratio analysis is under several constraints: over-empiricist, certainty, standard of reference not useful in all circumstances, etc. Recent researches have pointed out that to overcome those constraints formal decision models can be applied. In this article, fuzzy set theory is applied to ratio analysis with respect to one of the major management problems: liquidity. This approach enables the decision maker to include his own experience and any other type of information to that obtained by the ratio. If all the possible decisions are uniform in time, it is possible to adopt them by the decision maker in each period of analysis in a programmed form through a simple model inputs combination. The approach provided in this article can be extended to other ratio or ratio sets.

Keywords: Fuzzy sets, decision, finance

1. Introduction

The use of ratios as a tool of financial statement analysis is a common practice in financial accounting and management accounting. The wide range of users of the accounting information knows about its importance as a way to analyze the enterprises in order to conclude on its current state and/or its future evolution.

In the field of management accounting, ratio analysis contributes to the development of management control, but before this, a standard value has to be established in order to compare the variation between this value and the actual value so as to make decisions on the variables which are related by the ratio.

These standards are an essential part of ratio analysis, Foulke (1968) states a Detroit banker as saying in 1919 that the correct value of current ratio (current assets/current liability) was 2. Usually the mean ratio industry has been considered the ideal ratio, the standard of reference. Empirical evidence has pointed out that the comparison between this ratio and the one of a specific business has implied a correction of the behavior of such business in the following period to that of the comparison (Lev, 1969). Empirical evidence is not available on the behavior of a firm when there are variations between its ratio and the one of the top firm industry.

The comparison of actual and standard values (i.e. standard industry values) is one of the most criticized aspects of ratio analysis because it is viewed as an over-empiricist practice, so it is questionable whether in all cases and circumstances the industry standard for similar sized

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firms should be recommended, finally it is criticized that the information provided by actual and standard ratios is expressed in terms of certainty because such figures have been collected from financial statements. These aspects could imply that the decisions to correct the variables related by the ratio firm with respect to the standard industry cannot always be optimal ones (i.e. due to a different environment which advice the maintenance of the current state).

In addition, this traditional ratio analysis is critized for do not including any other information as the expressed in certainty terms. For instance, current ratio does not specify anything about the different possibilities of collecting each one of the accounts receivables or the possibility of selling inventory items. That other relevant information, not expressed in certainty terms, never is collected by the traditional ratio analysis.

An approach that enables us to overcome this rigid analysis is to consider the actual ratio value acceptable in a wide range, trying to complement the dichotomy 'correct value vs. incorrect value', allowing the possibility of accepting ratios that have different values, that is, values 'more o less' acceptable. More precisely, having ratio range as a fuzzy and not as a conventional set.

An approach like this places ratio analysis into the formal decision models. The relevance of these models with respect to ratio analysis has been stated by Lev (1974, p. 5) when saying that "recent researches in this field points out the beginning of a new methodology characterized basically for the usage of tools developed in the context of formal decision models".

In this article an application of ratio analysis into one of these formal decision models—fuzzy set theory—will be developed. The choice of this approach is justified by the fact that decision problems on ratio analysis can be very well developed by such theory. In recent years several applications with respect to management problems have been published: PERT Method (Chanas and Kamburowski, 1981), to investment problems (Tanaka, Okuda and Asai, 1976, 1979), to personnel management (Van Velthoven, 1977), to security selection (Hammerbacher and Yager, 1981). Recently an application to accounting has been published: investigation of cost variances (Zebda, 1984).

This application to ratio analysis will be referred to as liquidity decisions. As it is well known, liquidity and profits are the two major problems in management today. Liquidity has been chosen for its relevance in management decisions and also because it allows concise and precise logic schemes, and therefore the understanding will not be reduced by accounting conventions and rules on asset valuation, allocation, etc.

For all these reasons, the following ratio can be considered as an immediate liquidity decision tool:

$$\frac{\text{cash}}{\text{immediate liability}}.$$

This ratio has been chosen for the simplicity of its two components. However, the following analysis could be also easily extended to current ratio (current assest/current liability), quick ratio (cash + account receivables + marketable securities/ current liability) or any other type of ratio or ratio sets associated with liquidity decisions but taking into account that in these cases it is neccessary a basic knowledge of accounting rules and conventions.

2. A review of the fuzzy set theory

Since 1965, Zadeh has been carrying out a line of research which states that fuzziness is the biggest source of imprecision in human systems, that is, in those systems existing human interactions. Because of this, Zadeh states that conventional techniques applied to human systems to model imprecisions are frequently inadequate. These techniques require an accuracy level in the estimates which is very difficult to reach.

To explain the distinction between fuzziness and randomnes and therefore to reduce the necessity of processing numerical inputs in decision analysis models, Zadeh (1965) introduces the concept of fuzzy set:

"Let X be a space of points (objects), with a generic element denoted by x. Thus, $X=\{x\}$. A fuzzy set (class) A in X is characterized by a membership (characteristic) function $\mu_A(x)$ which associates with each point in X a real number in the interval [0,1], with the value of $\mu_A(x)$ at x representing the 'grade of membership' of x in A".

Due to this, given a fuzzy set A, any member of E can belong to it 'a little', 'a lot', 'intensely', etc.

So, to define a fuzzy set correctly it is necessary to use pairs of values where the first value x stands for an element of the considered set and the second one, the intensity with which that element belongs to the set:

$$A = \{[x, \mu_A(x)]\}. \tag{1}$$

The membership function is a basic concept in fuzzy set theory. So, while in conventional or ordinary sets the characteristic function can only admit 0 and 1 as values in the membership function of fuzzy sets the admitted values belong to the cosed interval [0, 1].

As a result of this different operations and techniques in fuzzy sets theory differ from ordinary sets.

Basic operations among fuzzy sets are union, intersection and complementation.

If E is the universal set and A and B are two fuzzy subsets of E, with membership functions $\mu_A(x)$ and $\mu_B(x)$:

– We define the union between A and B, $A \cup B$ as the fuzzy set C, such as:

$$\mu_C(x) = \mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x), \tag{2}$$

with $\vee$ meaning the maximum

– The intersection between A and B, $A \cap B$, will be another fuzzy set D with the following membership function:

$$\mu_D(x) = \mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x), \tag{3}$$

$\wedge$ being the minimum.

– The complementation of a fuzzy set A will be another fuzzy set $\bar{A}$ with the following membership function:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{4}$$

Now we will use fuzzy set theory in the immediate liquidity ratio analysis problem.

3. A fuzzy model for liquidity decisions

A good management accounting system has to be able to provide timely reports concerning the evolution of cash. The ratio evolution lets us establish conclusions called state of performance.

The state of performance set can be represented by

$$X_t = \{x_1, \dots, x_j, \dots x_m\}. \tag{5}$$

Each state $x_j \in X_t$, can be defined as a fuzzy set

$$x_j = \{r_t, \mu_{x_j}(r_t)\}. \tag{6}$$

The membership function $\mu_j(r_t)$ associates each ratio value r_t with a value that belongs to the closed interval [0,1] which shows the compatibility with the state x_j.

For example, if we consider the state of performance x_1: "state o situation in control". The membership function $\mu_{x_1}(r)$ associates each value of the ratio with the universe R = [0, ..., 3].

For example, the state in control can be described by

$$\mu_{x_1}(r) = \begin{cases} 0, & 0 \leqslant r \leqslant 0.85, \\ (r-0.85)/0.4, & 0.85 \leqslant r \leqslant 1.25, \\ (1.6-r)/0.35, & 1.25 \leqslant r \leqslant 1.6, \\ 0, & r \geqslant 1.6, \end{cases} \tag{7}$$

which graph is shown in Figure 1.

Hence the compatibility of a 1.3 ratio with the state 'in control' is 0.857.

On the other hand, with every periodic liquidity report the manager can make several decisions. The set of possible decisions can be described by

$$D_t = \{d_1, d_2, \dots, d_i, \dots, d_n\}. \tag{8}$$

Related to each decision $d_i \in D$ and each state $x_j \in X$, will be a performance or benefit derived from this combination. The performance set is expressed in an $n \times m$ matrix where the elements are defined as fuzzy sets, as shown in Table 1.

$$B_{ij} = \{[b_k, \mu_{B_{ij}}(b_k)]\}, \quad \mu_{B_{ij}}(b_k) \in [0, 1]. \tag{9}$$

In our example, net benefits will be expressed in terms of linguistic variables. The membership function related to every net benefit b_k in the universe $B = \{0, 1, \dots\}$ indicates its membership

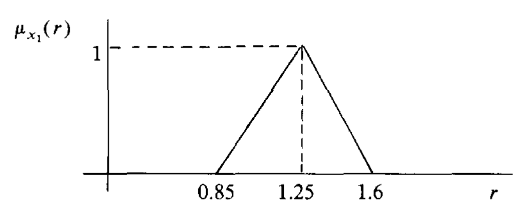

Figure 1. Graphical representation of the example (equation (7))

Table 1
Matrix of benefits B_{ij}

Decisions	States				
	x_1	...	x_j	...	x_m
d_1	B_{11}		...		B_{1m}
$\vdots$		$\ddots$			
d_i	$\vdots$		B_{ij}		$\vdots$
$\vdots$				$\ddots$	
d_n	B_{n1}		...		B_{nm}

to the set B_{ij}. For example, if the net benefit universe is

$$B = \{100, 200, \ldots, 1000\},$$

then the very low benefits of the fuzzy set can subjectively represented by

$$\mathrm{VL} = \{(100, 1), (200, 0.3)\}. \tag{10}$$

4. The resolution procedure

We will assume that the decision maker's goal is to maximize benefits derived from his decision in liquidity management; the solutions will be expressed by the following fuzzy set:

$$D_0 = \{[d_i, \mu_{D_0}(d_i)]\}, \tag{11}$$

which represents the optimum decision fuzzy set. The grade of membership of the elements in this set, $\mu_D(d_i)$, is a measurement of the compatibility of each possible decision with respect to the optimum, that is, the relative merit of the decision d_i. The optimum alternative will be the one with the highest grade of membership to D_0 (it is to say, the one that has the highest relative merit).

The calculation of the relative merit of the decisions it is necessary to determine the related net benefits, in order to discover if the state of performance is fuzzy with each decision.

According to the extension principle (Zadeh, 1975), the net benefit related to a decision is

$$B_i = \{[B_{ij}, \mu_{B_i}(B_{ij})]\} \tag{12}$$

where

$$\mu_{B_i}(B_{ij}) = \mu_{x_j}(r_t) \tag{13}$$

and $\mu_{x_j}(r_t)$ is the compatibility of the liquidity ratio value with X_j state.

It is interesting to point out that B_{ij} is a fuzzy net benefit set. This set of fuzzy sets can be reduced to the fuzzy set of non-fuzzy net benefits through the fuzzification operation (Zadeh, 1972).

The reduced set (called B_{ir}) will be

$$B_{ir} = \{[b_k, \mu_{B_{ir}}(b_k)]\}, \tag{14}$$

where $b_k \in B_{ij}$, and

$$\mu_{B_{ir}}(b_k) = \mu_{B_i}(B_{ij}) \wedge \mu_{B_{ij}}(b_k). \tag{15}$$

The problem will be to choose an optimum decision based on fuzzy benefits related to each decision. Therefore the decision maker has two possible choices:

(1) To choose the alternative which has the highest benefit in the set B_{ir}.

(2) To choose the decision which has the largest grade of membership to the set B_{ir}.

Nevertheless, both ways can lead us to a non-satisfactory decision. It is clear that if we choose the alternative with the highest associated benefit, the grade of membership to B_{ir} may be very small, while other decisions may have a higher one. On the other hand, if we base the selection on the benefits maximum grade of membership to B_{ir} of the decision, we could even obtain a very low benefit.

To interrelate both aspects Jain (1976) uses the maximizing set concept. The maximizing set of a set Y, called $M(Y)$, is a fuzzy subset so that the grade of membership of an element $y \in Y$ in the set $M(Y)$ represents the grade in which y approaches the supreme of Y in any given example.

In this context, Y represents all the possible net benefits sets. Mathematically,

$$Y = \bigcup_{i=1}^{m} S(B_{ir}), \tag{16}$$

$S(B_{ir})$ being the support set of B_{ir}, that is, $b_t \in B_{ir}$.

We then have to look for the maximum sets for $d_i \in D_t$, that is, B_{im}:

$$B_{im} = \{[b_k, \mu_{B_{im}}(b_k)]\}, \tag{17}$$

where

$$\mu_{B_{im}}(b_k) = [b_k / b_{max}]^n, \quad b_{max} = \sup Y. \tag{18}$$

n is a number which depends on the application and sup Y is the maximum or supremum.

Clearly $\mu_{B_{im}}(b_k)$ represents the degree with which b_k approximates the maximum net benefit b_{max}. On the other hand, $\mu_{b_{ir}}(b_k)$ expresses a fuzzy knowledge about the performance state. Both types of information have to be mixed in order to obtain a new fuzzy set, B_{i0}, through the intersection of the two sets B_{im} and B_{ir}. This set is

$$B_{i0} = \{[b_k, \mu_{B_{i0}}(b_k)]\}, \tag{19}$$

where

$$\mu_{B_{i0}}(b_k) = \mu_{B_{ir}}(b_k) \wedge \mu_{B_{im}}(b_k). \tag{20}$$

When we define the intersection by the connective 'and', $\mu_{B_{i0}}(b_k)$ gives us the degree with which b_k approximates the maximum net benefit possible, measured by $\mu_{B_{im}}(b_k)$ and the fuzzy knowledge about the state of the system, represented by $\mu_{B_{ir}}(b_k)$.

The fuzzy set

$$D_0 = \{[d_i, \mu_{D_0}(d_i)]\} \tag{21}$$

represents the optimum fuzzy decision space. The grade of membership $\mu_{D_0}(d_i)$ (which is the relative merit of each d_i), for each decision d_i in D_0, is

$$\mu_{D_0}(d_i) = \bigvee_k \mu_{B_{i0}}(b_k) \tag{22}$$

The optimum decision d_0 is the one possessing the highest grade of membership with the fuzzy set D_0, that is:

$$\mu_{D_0}(d_0) = \bigvee_i \mu_{D_0}(d_i). \tag{23}$$

We have to point out that sometimes an element b_p can appear several times in a fuzzy set with the same or different grade of membership. In this case the reduction rule proposed by Jain (1977) will be used. If b_p appears L times with a grade of membership $\mu_1(b_p)$, $\mu_2(b_p)$, $\ldots \mu_L(b_p)$, then the grade of membership of b_p will be:

$$\mu(b_p) = \mu_1(b_p) + \mu_2(b_p) + \cdots + \mu_L(b_p) \tag{24}$$

where

$$\mu_i(b_p) + \mu_j(b_p) = \mu_i(b_p) + \mu_j(b_p) - \mu_i(b_p) \cdot \mu_j(b_p). \tag{25}$$

5. Model inputs

Model inputs include:
(1) monthly liquidity ratio;
(2) the definition of state of performance and with each state a membership function;
(3) the number of possible decisions.
(4) the estimation of the net benefits and the membership function for each net benefit.

As an output of the model we have the optimum decision.

Immediate liquidity ratio is obtained through reports of the accounting department to the management but the rest of the inputs have to be obtained by the decision maker.

The most notorious and contradictory aspect of the model is the necessity to estimate the different values of the membership function. Membership functions are usually calculated in a very subjective way: studying an element's degree to satisfy the set. Nevertheless, some authors, like Watson, Weiss and Donnell (1979), argue the use of subjectivity in the membership function estimation and measure it through empirical derivation from them. Due to this, a lot of studies have suggested different forms of membership function derivation (Saaty, 1974, 1978).

This criticism can be overcome by stating that subjectivity is the essence of fuzziness and that other decision making techniques such as utility theory or bayesian analysis also imply subjectivity. Both ways of defining membership functions (subjectivity estimation and empirical derivation) are acceptable, nevertheless we will follow Zadeh's subjectivity estimation.

6. Application

Consider the following states set:

$$S_t = \{S_1, S_2, S_3\}$$

where

S_1 represents the 'not enough cash' state;
S_2 'in control cash state; and

Table 2
The net benefit matrix

Decisions	States		
	S_1	S_2	S_3
d_1	L	H	L
d_2	VL	M	VH
d_3	H	L	VL
d_4	VH	M	L

S_3 'over cash' state.

Also, all possible decisions related to immediate liquidity ratio evolution are:

d_1: Do nothing.
d_2: Purchase new raw material (reducing cash).
d_3: Ask for a short run credit (obtain cash).
d_4: Negotiate with suppliers (increase cash).

The net benefit matrix is shown in Table 2, where the benefits VH (very high), H (high), M (medium), L (low) and VL (very low) are defined by the following fuzzy sets:

VL = {(100, 1), (200, 0.3)};
L = {(100, 0.6), (200, 1), (300, 0.4)};
M = {(100, 0.1), (200, 0.5), (500, 1), (600, 0.7), (700, 0.2) };
H = {(300, 0.5), (400, 0.7), (600, 0.8), (700, 1), (900, 0.2) };
VH= {(900, 0.7), (1000, 1)}.

Moreover, once we have received the financial report we must see if the ratio reaches a value 1.45. It is necessary to point out that in different periods of time the same ratio can have different membership values. This is very common in cash management. For example, a ratio 1.45 can be considered low if the company has cash purchases of machinery, raw materials, etc. in mind in the short run. Meanwhile, the same ratio can also be considered high during a period where short run profitable investments suggest that the company should invest as much as possible. This decision maker in each analysis period. Also to assume the former ratio as to have compatibility of 0.4, 0.6 and 0.7 for each of the states S_1, S_2 and S_3 respectively.

Using equation [12] and [13] we can obtain the following sets B_1, B_2, B_3, and B_4.

$$B_1 = \{(\mathrm{L}, 0.4), (\mathrm{H}, 0.6), (\mathrm{L}, 0.7)\}$$

Because in B_1 fuzzy benefit L has two membership values, using reduction rules [24] and [25], we will assign the unique value $0.4 + 0.7 - 0.4 \cdot 0.7 = 0.82$:

$$B_1 = \{(\mathrm{L}, 0.82), (\mathrm{H}, 0.60)\},$$
$$B_2 = \{(\mathrm{VL}, 0.4), (\mathrm{M}, 0.6), (\mathrm{VH}, 0.7)\},$$
$$B_3 = \{(\mathrm{H}, 0.4), (\mathrm{L}, 0.6), (\mathrm{VL}, 0.7)\},$$
$$B_4 = \{(\mathrm{VH}, 0.4), (\mathrm{M}, 0.6), (\mathrm{L}, 0.7)\}.$$

B_1, B_2, B_3, and B_4 are net benefit fuzzy sets that can be transformed into the following sets B_{1r}, B_{2r}, B_{3r}, and B_{4r} described in [14].

B_{1r} = {[(100, 0.6), (200, 1), (300, 0.4)/0.82], [(300, 0.5), (400, 0.7), (600, 0.8), (700, 1), (900, 0.2)/0.6]},

B_{2r} = {[(100, 1), (200, 0.3)/0.4], [(100, 0.1), (200, 0.5), (500, 1), (600, 0.7), (700, 0.2)/0.6], [(900, 0.7), (100, 1)/0.7]},

B_{3r} = {[(300, 0.5), (400, 0.7), (600, 0.8), (700, 1), (900, 0.2)/0.4], [(100, 0.6), (200, 1) (300, 0.4)/0.6], [(100, 1), (200, 0.3)/0.7]}.,

B_{4r} = {[(900, 0.7), (1000, 1)/0.4], [(100, 0.1), (200, 0.5), (500, 1), (600, 0.7), (700, 0.2)/0.6], [(100, 0.6), (200, 1) (300, 0.4)/0.70]}.

Using [16] and the reduction procedure [25] we will have

B_{1r} = {(100, 0.6), (200, 0.82), (300, 0.7), (400, 0.6), (600, 0.6), (700, 0.6), (900, 0.2)},
B_{2r} = {(100, 0.46), (200, 0.65), (500, 0.6), (600, 0.6), (700, 0.2), (900, 0.7), (1000, 0.7)},
B_{3r} = {(400, 0.4), (600, 0.4), (700, 0.4), (900, 0.2), (100, 0.88), (200, 0.72), (300, 0.64)},
B_{3r} = {(900, 0.4), (1000, 0.4), (500, 0.6), (600, 0.6), (700, 0.2), (300, 0.4), (100, 0.84)}.

According to [16] we can obtain Y set where:

$$Y = \{100, 200, 300, 400, 500, 600, 700, 900, 2000\}$$
$$\sup Y = 1000.$$

If $n = 2$, the maximizing sets described in [17] and [18] will be

B_{1m} = {(100, 0.1), (200, 0.04), (300, 0.09), (400, 0.16), (600, 0.36), (700, 0.49), (900, 0.81)},

B_{2m} = {(100, 0.01), (200, 0.04), (500, 0.25), (600, 0.36), (700, 0.49), (900, 0.81), (1000, 1)},

B_{3m} = {(100, 0.01), (200, 0.04), (300, 0.09), (400, 0.16), (600, 0.36), (700, 0.49), (900, 0.81)},

B_{4m} = {(100, 0.01), (200, 0.04), (300, 0.09), (500, 0.25), (600, 0.36), (700, 0.49), (900, 0.81), (1000, 1)}.

At this point and with expressions [19] and [20] we will obtain the sets B_{i0}:

B_{10} = {(100, 0.01), (200, 0.04), (300, 0.09), (400, 0.16), (600, 0.36), (700, 0.49), (900, 0.2),

$B_{20} = \{(100,\ 0.01),\ (200,\ 0.04),\ (500,\ 0.25),\ (600,\ 0.36),\ (700,\ 0.02),\ (900,\ 0.7),\ (1000,\ 1)\},$

$B_{30} = \{(100,\ 0.01),\ (200,\ 0.04),\ (300,\ 0.09),\ (400,\ 0.16),\ (700,\ 0.4),\ (900,\ 0.2)\},$

$B_{40} = \{(100,\ 0.01),\ (200,\ 0.04),\ (300,\ 0.09),\ (500,\ 0.25),\ (600,\ 0.36),\ (900,\ 0.4),\ (1000,\ 0.4)\}.$

Using equation [22] we can determine d_1, d_2, d_3 and d_4 compatibilities with the optimal decision space, hence

$$D_0 = \{(d_1, 0.49,), (d_2, 0.7), (d_3, 0.36), (d_4, 0.36)\}$$

Therefore the optimal decision will be d_2.

If we normalize D_0—dividing by the highest value of the membership function—we will have

$$D_0 = \{(d_1, 0.7), (d_2, 1), (d_3, 0.51), (d_4, 0.51)\}.$$

Therefore the membership values in D_0 reveal the distance between non-optimal decisions and d_2 following a hierarchy of alternatives.

7. Concluding remarks

The use of fuzzy set theory for financial ratio analysis allows us to elude some of the typical constraints to these types of analyses: we overcome the good vs. bad dichothomy related to the ratio value and if, for example, we analyze a liquidity problem we can find its membership degree to fuzzy sets as: 'no liquidity', 'in control', 'excess liquidity'.

The definition of states as fuzzy sets allows us to introduce decision maker's subjectiveness in membership value assignment using his experience or complementary available information, i.e. from other ratios.

If the possible decisions are uniform in time, we could adopt them in a programmed form through a simple input combination by the decision maker in each period of time of the analysis.

This framework can be easily extended to the analyses of other ratios or ratio sets.

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