Research Reports

A Discriminant Analysis of Predictors of Business Failure

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The failure of a business firm is an event which can produce substantial losses to creditors and stockholders. Therefore, a model which predicts potential business failures as early as possible would serve to reduce such losses by providing ample warning to these interested parties. This was a sufficient motivation for Beaver (1967, 1968) and Altman (1968) to develop models for predicting failure based on the financial reports of firms. The purpose of this paper is to propose an alternative model for predicting failure.

Beaver used a dichotomous classification test to determine the error rates a potential creditor would experience if he classified firms on the basis of their financial ratios as failed or nonfailed. Beaver was able to accurately classify 78% of his sample of firms five years before failure.

Altman used discriminant analysis to rank firms on the basis of a weighted combination of five ratios. His results were 95% effective in selecting future bankrupts in the year prior to bankruptcy. The firms he examined went bankrupt on an average of seven and one-half months after the close of the last fiscal year for which reports were prepared. However, the predictive ability of the model declined rapidly as the number of years prior to failure increased. In the second through fifth years prior to failure, the discriminant model led to more misclassifications than did Beaver's dichotomous test using only a cash flow/total debt ratio. The correct classification rates from the two studies are summarized in Table 1.

Although Beaver's empirical results suggest that his method has greater predictive ability, the method used by Altman has more intuitive appeal. In this study, I will first replicate the Beaver study, using the same ratios he used. Next I will search for the linear combination of the 14 ratios used

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TABLE 1

Classification Error Rates for Predicting Future Bankruptcies

Year before failure	Beaver cash flow/total debt	Altman discriminant function
1	13	5
2	21	28
3	23	52
4	24	71
5	22	64

Source: Beaver (1968), Altman (1968).

by Beaver which best predicts potential failure in each of five years prior to failure. Finally, I will devise a decision rule which will be validated over a cross-sectional sample of firms.

The Dichotomous Classification Test

The dichotomous classification test used by Beaver was based on two samples of firms. The first consisted of 79 firms which went bankrupt between 1954 and 1964. The second sample contained a nonfailed firm of approximately the same size and industry group for each of the failed firms. Fourteen financial ratios were then calculated for each firm. The ratios were selected on the basis of their appearance in the literature as indicators of the ability of a firm to avoid failure. The overall set of firms was then divided into two subsamples, each consisting of about half of the pairs of firms. The firms in each subsample were then ranked by the values of their ratios. That value of each ratio in one subsample which showed the smallest number of misclassifications was then used as the critical value of the ratio for classifying the firms in the second subsample. The procedure was then reversed by deriving a critical value of each ratio in the second subsample and using that value to classify the firms in the first subsample. The number of errors resulting from the use of each of the ratios in each of the five years before failure was calculated, and the better predictors were deemed to be those which showed the smallest classification error rate.

The same general procedure was followed in this study, but with one major difference. Thirty-two failed firms were selected from a population which experienced failure between 1964 and 1970. However, the term failure was defined here to include only those firms which experienced bankruptcy, insolvency, or were otherwise liquidated for the benefit of creditors. Beaver included firms which defaulted on loan obligations or missed preferred dividend payments. But unless the nonbankrupt firms were matched by debt structure as well as by size and industry, there could be a potential bias in certain of the ratios. Each of my failed firms was matched with a nonfailed firm on the basis of industry classification, year of the financial information provided and asset size.

The results of applying the dichotomous classification test to this second set of data are summarized in Table 2 along with the results obtained by Beaver. The data for this study were not sufficient to determine if the differences in the error rates are significant. However, the Spearman rank-order correlation coefficient can be used to indicate the order of the predictive power of the ratios in the two studies. The value of the coefficient (r_s) is indicated in Table 2.

A review of the table indicates a rather high correlation of relative predictive ability of the various ratios. The rank-order correlation coefficients are very high in four of the five years. Considering that differences could arise from the use of independent samples and from the later time period of the second sample, the results would tend to confirm Beaver's observations.

However, the correlation coefficient in the third year before failure is only .56. Although this is still a significant value, it is considerably lower than the coefficients in the other four years. A comparison was made of the means of 13 financial statement items which were used in the calculation of the financial ratios in the hope of giving some insight into why this coefficient is relatively less. These are shown in Table 3. Consider the means of total assets for both sets of firms for the five years. Apparently,

TABLE 2

Percentage Error for 14 Ratios on Dichotomous Classification Test

Ratio	Year before failure				
	5	4	3	2	1
Non-liquid asset group:					
cash flow/total debt	27 (22)	24 (24)	28 (23)	16 (21)	20 (13)
net income/total assets	38 (28)	43 (29)	30 (23)	20 (21)	27 (13)
total debt/total assets	33 (28)	33 (27)	22 (34)	22 (25)	20 (19)
Liquid asset to total asset group:					
current assets/total assets	47 (49)	45 (47)	41 (48)	47 (48)	36 (38)
quick assets/total assets	52 (40)	52 (48)	47 (36)	53 (42)	34 (38)
working capital/total assets	34 (41)	39 (45)	34 (33)	28 (34)	30 (24)
cash/total assets	47 (38)	44 (36)	47 (30)	42 (29)	33 (28)
Liquid asset to current debt group:					
current assets/current liabilities	41 (45)	36 (38)	25 (36)	27 (32)	28 (20)
quick assets/current liabilities	44 (37)	39 (34)	41 (40)	28 (32)	30 (24)
cash/current liabilities	41 (38)	45 (38)	36 (36)	28 (28)	36 (22)
Liquid asset turnover group:					
current assets/sales	50 (51)	56 (49)	52 (48)	61 (51)	48 (44)
quick assets/sales	53 (44)	59 (52)	55 (45)	59 (47)	52 (46)
working capital/sales	47 (40)	52 (46)	45 (42)	31 (33)	28 (26)
cash/sales	48 (45)	53 (43)	52 (36)	42 (24)	47 (34)
r_s	.76	.80	.56	.82	.88

Data in parentheses are from Beaver (1968, p. 118).

TABLE 3

Comparison of Means of 13 Financial Statement Items for Nonfailed (NF) and Failed (F) Firms

Item	Year before failure 5	4	3	2	1
Sales:					
NF	16989.	16395.	17610.	20445.	22426.
F	16508.	15425.	18359.	16656.	16938.
Diff.	481.	970.	−749.	3789.	5488.
Net income:					
NF	206.	46.	211.	426.	480.
F	65.	−29.	−664.	−858.	−1309.
Diff.	141.	75.	875.	1284.	1789.
Cash flow:					
NF	523.	381.	557.	865.	941.
F	145.	308.	−244.	−518.	−909.
Diff.	378.	73.	801.	1383.	1850.
Cash & mkt. sec.:					
NF	981.	1086.	1107.	1304.	1365.
F	753.	1020.	730.	511.	329.
Diff.	228.	66.	377.	793.	1036.
Receivables:					
NF	1716.	1730.	1837.	2146.	2358.
F	1845.	3164.	3563.	2359.	2090.
Diff.	−129.	−1434.	−1726.	−213.	268.
Quick assets:					
NF	2697.	2816.	2944.	3449.	3723.
F	2598.	4184.	4292.	2871.	2419.
Diff.	99.	−1368.	−1348.	578.	1304.
Inventory:					
NF	3022.	3000.	3006.	3479.	3974.
F	2827.	6064.	7390.	2525.	2378.
Diff.	195.	−3064.	−4384.	954.	1596.
Working capital:					
NF	3695.	3565.	3579.	4009.	4392.
F	1646.	5428.	3591.	−706.	2063.
Diff.	2049.	−1863.	−12.	4715.	2329.

TABLE 3. *Continued*

Item	Year Before Failure				
	5	4	3	2	1
Total assets:					
NF	9397.	9811.	10175.	12172.	13262.
F	9362.	17266.	20015.	10502.	10026.
Diff.	35.	−7455.	−9840.	1670.	3236.
Current liabilities:					
NF	2023.	2251.	2370.	2920.	3305.
F	3780.	4820.	8092.	6102.	6860.
Diff.	−1757.	−2569.	−5722.	−3182.	−3555.
All liabilities & preferred stock:					
NF	4192.	4683.	5568.	6573.	7105.
F	5929.	13327.	19876.	9639.	10268.
Diff.	−1737.	−8644.	−14308.	−3066.	−3163.
Net worth:					
NF	5205.	5128.	5326.	6426.	6157.
F	3433.	3829.	2139.	863.	−242.
Diff.	1772.	1299.	3187.	5563.	6399.

the failed firms tended to expand rapidly in the third and fourth years prior to failure. If we look at the capital structure, it seems that the expansion was financed by increased debt and preferred stock rather than common stock or retained earnings. Therefore, funds raised were invested in plant and equipment rather than in liquid assets. This phenomenon did not appear in the Beaver study. These firms were unable later to generate the sales and net income to support their heavier debt, and so they lost their assets rather rapidly after the third year prior to failure. At that point, their asset ratios and debt ratios tended to fall back in line with the ratios shown in the earlier study.

It should also be noted that in the three years before failure there was a marked difference in the predictive ability of the cash/sales ratio. While most of the other ratios tended to be consistent with those observed by Beaver, this ratio was consistently and significantly different. One possible explanation is that corporations tended to invest more of their cash reserves during the late 1960's when interest rates were high. Thus a low cash/sales ratio may have been due to good money management rather than to general company mismanagement.

Although the results of this method provide a fairly high degree of accuracy, it should be possible to improve on the 20% error in a misclassification in the year prior to bankruptcy.[1] That is the purpose of the discriminant analysis described below.

The Discriminant Analysis

The purpose of discriminant analysis is to find the linear combination of ratios which best discriminates between the groups which are being classified. The methodology is too complex for detailed discussion here.[2] Briefly, the distributions of the scores on various variables for two or more groups are projected onto the axis on which there is the least overlap of the distributions. The two variables, two group case is shown in Figure 1. Obviously, D_1 will provide a greater discriminating ability than the combination of scores devised on the D_2 axis. The axis upon which the minimum overlap occurs is found by a differential calculus procedure (see Tatsuoka, 1971).

One assumption of the procedure is that each of the groups is drawn at random from independent samples. If the two samples are not drawn at random, such as with the sample of nonfailed firms, more complex procedures are needed to use the method.[3] To avoid this, a second sample of 32 nonfailed firms was drawn at random from the 1962 to 1966 Moody's Industrial Manual. The data from these firms were obtained for the same five-year period used in the first sample. The 14 financial ratios used by Beaver were input to the discriminant analysis program. The output from the program consisted of a set of discriminant weights which indicates that linear combination of the variables which maximizes the differences between the groups as well as a scaled vector which indicates the relative contribution of each variable. Table 4 presents the scaled vectors from the discriminant functions.

The relative magnitude of the contribution of each variable to the discriminant function may be observed by comparing the element in the scaled vector for that variable with the elements in the scaled vector for all other variables. For example, in the scaled vector for the fifth year prior to failure, the working capital/total assets element has a value of 1.163. Since this is the largest value in the vector, it indicates that this ratio makes a significant contribution to the discriminating ability of the function. Similarly, the value of .036 associated with the current assets/current liabilities ratio indicates that this ratio contributes little. If the

[1] Tamari (1966) applied an arbitrary weighting system to various ratios to reduce the likelihood of misclassification. An alternative would be to use a statistical method that would assign weights to the various ratios in a nonarbitrary fashion such that the differences between potential bankrupts and nonbankrupts would be maximized.

[2] A more extensive exposition may be found in Tatsuoka (1970 and 1971) and Rao (1952).

[3] While Altman used a matched pair sample, he did not perform this step in preparing his analysis.

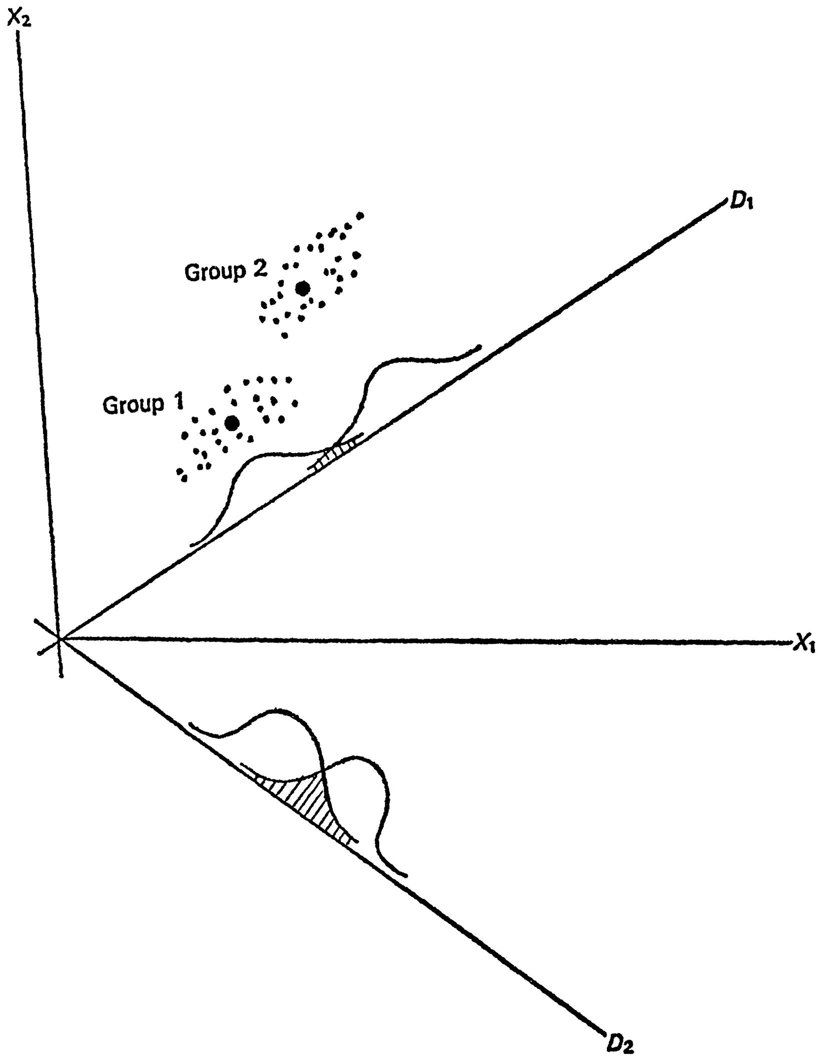

FIG. 1. Rationale of the Discriminant Function

contribution is small, it is often possible to decrease the number of variables included in the functions without reducing the discriminating value of the matrix. However, when this was done in the present study, the number of classification errors increased substantially. This would tend to support the use of many of the variables considered important in the literature for the prediction of business failure. It is also interesting to note the changes in the relative importance of these variables over the five years prior to failure. Virtually all of the variables contribute significantly to the discriminating ability of the function. However the significance of the variables changes over the five-year period.

TABLE 4
Scaled Vectors

Ratio	Year before failure 5	4	3	2	1
Cash flow/total debt	−.562	.222	.271	−.085	.009
Net income/total assets	.139	.294	−.450	.256	.097
Total debt/total assets	−.956	−.197	.417	−.327	−.401
Current assets/total assets	.721	−.030	.804	−.772	−.198
Quick assets/total assets	−.245	−.073	−.608	.484	.252
Working capital/total assets	1.163	−.084	.172	.191	−.462
Cash/total assets	.440	−.245	−.098	−.209	−.213
Current assets/current liabilities	.036	−.004	−.617	.346	.426
Quick assets/current liabilities	.694	.085	.226	−.996	−.375
Cash/current liabilities	−.727	.364	.298	.392	.112
Current assets/sales	−.139	1.008	1.874	−.290	−.084
Quick assets/sales	.915	.326	.925	−.064	−.312
Working capital/sales	−.061	.669	−2.615	.625	.316
Cash/sales	−.592	−.486	−.027	.215	.202

RESULTS OF THE DISCRIMINANT ANALYSIS

The discriminant analysis program produces a vector of weights such that the summation of the products of each element of the vector times the associated ratio will produce a score which maximizes the distinctions between the two groups. The vectors of weights for each of the five years are shown in Table 5.

The significance of each of the discriminant functions is measured using Wilks' lambda (see Tatsuoka, 1971, pp. 164–70). This statistic is used to test the hypothesis that the mean of the ratio vectors for each group are equal. Wilks' lambda can be converted to an F value.[4] The F ratio is then used to indicate the probability of a significant separation between the scores of failed and nonfailed firms. These F values are also shown in Table 5 with the level of significance indicated. These F's are significant beyond the .001 α level for the first two years and are still significant at $\alpha = .01$ in the fourth and at $\alpha = .05$ in the fifth year prior to failure. The extremely high confidence levels in the first three years prior to failure suggests that it should be possible to correctly identify a large number of potential failures as far as three years before the firm files for bankruptcy.

Since the scores for the various firms will be distributed over a fairly broad range, some method must be used to determine the group membership of each firm. It would be possible to simply observe the scores of failed and nonfailed firms and classify the firms on the basis of a critical score value which correctly classifies the maximum number of firms. This

[4] See Rao (1952).

TABLE 5

Coefficients of Discriminant Functions and Significance Tests

Ratio	Year before failure 5	4	3	2	1
Cash flow/total debt	−.250	.094	.104	−.046	.005
Net income/total assets	.122	.219	−.585	.378	.083
Total debt/total assets	.220	−.133	.287	−.225	−.184
Current assets/total assets	.406	−.017	.436	−.410	−.101
Quick assets/total assets	.230	−.062	−.479	.394	.212
Working capital/total assets	.487	−.054	.106	.102	−.176
Cash/total assets	.621	−.701	−.205	−.626	−.900
Current assets/current liabilities	.003	−.001	−.069	.020	.052
Quick assets/current liabilities	*.068*	*.017*	*.034*	*−.065*	*−.068*
Cash/current liabilities	−.077	.165	.151	.111	.096
Current assets/sales	−.018	.283	.057	−.060	−.020
Quick assets/sales	.123	.138	.176	−.014	−.074
Working capital/sales	−.009	.243	−.159	.132	.069
Cash/sales	−.084	.492	−.055	−.203	.209
Wilks' lambda	.585	.571	.344	.252	.300
F^{14}_{49}	2.48	2.63	6.66	10.40	8.18
Significant at =	.05	.011	<.001	<.001	<.001

method was used by Altman (1968), Frishkoff (1970) and Frank and Weygandt (1971). However, this approach fails to take the relative scores into account. As Frank and Weygandt (p. 123) showed, the greatest number of classification errors occur close to the critical value of the score.

If the assumption can be made that the vectors of the scores follow a p-variate normal distribution and that the variance covariance matrix of the groups used matches the population variance covariance matrix, a probability of group membership can be assigned based on the multivariate extension of the univariate Z test as follows:

$$\underset{\sim}{d}' \underset{\sim}{\Sigma}^{-1} \underset{\sim}{d} \sim \chi_p^2$$

where $\underset{\sim}{d}'$ = the row vector of deviation scores

$\underset{\sim}{d}$ = the column vector of deviation scores

$\underset{\sim}{\Sigma}$ = the population variance-covariance matrix, and

p = the degrees of freedom of the chi-square distribution and equals the number of elements in the deviation score vector.[5]

Each firm used in deriving the discriminant functions was classified in this manner. The results of the classifications are summarized in Table 6.

[5] For a further explanation and proof of the derivation of the chi-square test, see, for example, Tatsuoka (1971).

TABLE 6

Classification Errors for Firms Used in Deriving the Discriminant Functions

Incorrectly classified as:	Probability					Total	%
1st year before failure:							
Not failed	.92					1	3
Failed	.83					1	3
2nd year before failure:							
Not failed	.70					1	3
Failed	.52	.80				2	6
3rd year before failure:							
Not failed	.79	.86				2	6
Failed	.55					1	3
4th year before failure:							
Not failed	.50	.60	.72	.72	.95	5	16
Failed	.51	.51	.55	.56	.57		
	.60	.76	.93			8	25
5th year before failure:							
Not failed	.52	.64	.67	.70	.70		
	.72	.90	.91			8	25
Failed	.62	.95	.96			3	9

The results in Table 6 indicate that misclassification errors averaged 3%, 4½%, and 4½% for the first, second and third years respectively. Compared to the classification results shown in Table 1, the multiple-years test gave consistently better results than either the best predictor variable in the dichotomous classification test or the single-year discriminant analysis. Notice, however, that the error rates increase markedly in the fourth and fifth years rising to 21% and 17% respectively. While such error rates are probably too high for decision-making purposes, they are still lower than the rates in Table 1. The number of classification errors also tends to follow very closely the F values for the significance test of differences between the two population centroids.

This model was tested on an independent sample consisting of 11 failed and 23 nonfailed firms selected at random from the 1964 and 1963 Moody's Industrial Manual. The results of applying the discriminant functions to the cross-validation sample are shown in Table 7. Error rates of 22%, 6%, 12%, 23%, and 15% were observed for each of the five years prior to failure. We would expect some deterioration in applying a statistical test to sample populations other than the population from which the model was drawn. However, the deterioration of the first year is rather severe and cannot be explained by the presence of any unusual events peculiar to the sample used.

CONSIDERATIONS IN APPLYING THE MODEL

While this model can be derived rather readily ex post, there are problems in applying the model in a practical situation. In the first place, a loan investment portfolio will consist of many firms that will experience

TABLE 7

Errors in Classification on Cross-Validation Sample

Incorrectly classified as:	Probability	Total	%
1st year before failure:			
Not failed	.83 .97	2	18
Failed	.52 .68 .86 .88 .97 .99	6	23
2nd year before failure:			
Not failed	.99	1	08
Failed	.96	1	04
3rd year before failure:			
Not failed	.80 .99	2	18
Failed	.80 .81	2	06
4th year before failure:			
Not failed	.50 .54 .72	3	33
Failed	.56 .61 .87	3	09
5th year before failure:			
Not failed	.61 .67	2	22
Failed	.56 .67	2	13

failure anywhere from less than one year to an almost infinite number of years in the future. From an ex ante viewpoint, it is only possible to apply these functions to obtain probability statements that the firm will fail in year $t + 1, t + 2, \ldots$ into the future. Second, the constraints of discriminant analysis prohibit the derivation of discriminant functions from populations where a firm could possibly belong to more than one group over time. For example, a firm could be a potential failure at one point in time but is able to reverse the trend before failure occurs.

In light of the above, a test of the previously derived functions was made on the groups of firms from the above sample which were either known to be nonfailures in the next five years or were known to have failed anywhere from one to five years in the future. The functions derived for the fourth and fifth years prior to failure were excluded because of their large error rates.

The classification errors from this approach are summarized in Table 8. It is not surprising that the middle year provides the greatest classification ability. The middle year is the central function and the significance of the discriminating power of its assigned scores was far greater than for the other functions. Thus, application of the second year's discriminant

TABLE 8

Error Rates Using Discriminant Functions on Mixed Data

Year prior to failure from which function was derived	% misclassified as: Failed	Nonfailed	Total error
1	14	13	13
2	10	9	10
3	19	17	18

function correctly classified 90% of all firms that failed or did not fail in the next one to three years.

Conclusion

The application of statistical techniques, particularly disciminant analysis, can be used to predict business failure from accounting data as far as three years in advance with a fairly high accuracy. Since such a long lead time period can be discovered with this method, it should be possible for the management of potentially failing firms to take steps to avert such an occurrence.

It must be realized, however, that the model was derived from a rather small population and while the results are encouraging on the cross-validation sample, subsequent observations should be made to extend the external validity of the model. Furthermore, with a 10% error rate, these probabilities of group membership should be used only as further evidence of probable failure rather than as conclusive proof in themselves.

APPENDIX

Assumptions of the Chi-Square Classification Method

As noted above, firms were classified as failed or nonfailed by a test of the relationship of the discriminant score of the classified firm with the discriminant scores observed in the two groups from which the discriminant functions were derived. The procedure assumes that the population is normal and that the observed $\underset{\sim}{\sum}$ equals the population variance-covariance matrix. To the extent that these assumptions are not true, the probabilities will be incorrect. To examine the outcomes of the classification tests, 1,260 classifications were observed and the assigned probabilities of group membership noted. The classifications were arrayed in the order of the membership probability assigned, and the number of errors for each probability class was noted. The result of this test is summarized in Table A1. The expected error rates were derived by taking the mean of the

TABLE A1

Classification Errors Separated into Assigned Probabilities of Group Membership for 1,260 Selected Firm Classifications

Membership probability	Number of cases	Error rates	
		Observed	Expected
.95–1.00	305	7.2%	2.5%
.90–.949	98	11.2	7.5
.80–.899	195	14.4	15.0
.70–.799	228	27.2	25.0
.60–.699	233	30.0	35.0
.50–.599	201	45.7	45.0

classification interval, and the observed error rates include all the misclassifications that occurred where the assigned probability of group membership was within the interval.

The results of this analysis tend to confirm the assumptions, although within some region of tolerance. It appears that the scores of these firms tend to be more heavily concentrated in the tails of the density function than is the case with a normally distributed population. However, the error rate for all firms within the probability class of .90 to 1.00 would still be about 91.8%, which is not radically different from the interval mean of 95%. Therefore, the difference between the given probability and the actual probability is probably small. We can conclude that the discriminant classification is sufficiently robust to be used for distributions of financial data.

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