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ZETA$^{\text{TM}*}$ ANALYSIS

A new model to identify bankruptcy risk of corporations

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The paper explores the development of a bankruptcy classification model which incorporates comprehensive inputs with respect to discriminant analysis and utilizes a sample of bankrupt firms essentially covering the period 1969–1975. Financial statement data and market related measures are transformed along guidelines suggested by traditional security analysis to promote comparability of companies and to reflect the most recent reporting standards so as to make the model relevant to future analysis. The results of the study are compared with alternative bankruptcy classification strategies via the explicit introduction of prior probabilities of group membership, observed accuracies, and estimates of costs of errors in misclassification. The latter is based on cost estimates derived from commercial bank lending errors. The results of the study indicate potential significant application to credit worthiness assessment, portfolio management, and to external and internal performance analysis.

1. Introduction and purposes of the study

The purposes of this study are to construct, analyse and test a new bankruptcy classification model which considers explicitly recent developments with respect to business failures. The study also incorporates current refinements in the utilization of discriminant statistical techniques. Several reasons for building a new model, despite the availability of several fairly impressive 'old' models, are presented below and the empirical results seem to substantiate the effort. The new

*Henceforth simply referred to as ZETA. This paper is an elaboration of a model presently being applied to investment analysis and statistical service by Wood, Struthers & Winthrop (WSW), hence the trademark designation.

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model, which we call ZETA (not from the people who brought you Beta), is effective in classifying bankrupt companies up to five years prior to failure on a sample of corporations consisting of manufacturers and retailers.

2. Reasons for attempting to construct a new model

There are at least five valid reasons why a new bankruptcy classification model can improve and extend upon those statistical models which have been published in the literature in the last decade.[1] These include:

(1) The change in the size, and perhaps the financial profile, of business failures in recent years. The average size of bankrupt firms has increased dramatically with the consequent greater visibility and concern from financial institutions, regulatory agencies and the public at large. Most of the past studies used relatively small firms in their samples with the exception of Altman's (1973) railroad study and the commercial bank studies. Any new model should be as relevant as possible to the population to which it will eventually be applied. This present study utilizes a bankrupt firm sample where the average asset size two annual reporting periods prior to failure was approximately $100 million. No firm had less than $20 million in assets.[2]

(2) Following (1) above, a new model should be as current as possible with respect to the temporal nature of the data. With the exception of 3 (out of 53) firms, every bankrupt firm in our sample failed in the last 7 years. The entire sample of both bankrupt and non-bankrupt firms is listed in appendix A.

(3) Past failure models concentrated either on the broad classification of manufacturers or on specific industries. We feel that with the appropriate analytical adjustments, retailing companies, a particularly vulnerable group, could be analysed on an equal basis with manufacturers.

(4) An important feature of this study is that the data and footnotes have been scrupulously analyzed to include the most recent changes in financial reporting standards and accepted accounting practices. Indeed, in at least one instance, a change which will be implemented in a very short time was applied. The purpose of these operations is to make the model not only relevant to past failures, but to the data that will appear in the future. The predictive as well as the classification accuracy of the ZETA model is implicit in our efforts. The major modifications are discussed in the next section.

[1]These studies include models for manufacturers by Beaver (1967), Altman (1968), Wilcox (1971, 1976), Deakin (1972, 1977) and Edmister (1972), among others, and models for specific industries such as Altman on railroads (1973), Sinkey on commercial banks (1975), Korobow and Stuhr (1975) and with Martin (1976), also on commercial banks, Altman and Lorris on broker/dealers (1976) and Altman on savings and loan associations (1977a).

[2]This is in contrast to one of the authors past efforts on bankrupt manufacturers [Altman (1968)], where the largest firm had assets of less than $25 million.

(5) To test and assess several of the recent advances and still controversial aspects of discriminant analysis. Recent articles in the literature indicate that this statistical technique is being utilized with increasing frequency but not without controversy.[3]

3. Principal findings

We conclude in this paper that the new ZETA model for bankruptcy classification appears to be quite accurate for up to five years prior to failure with successful classification of well over 90% of our sample one year prior and 70% accuracy up to five years. We also observe that the inclusion of retailing firms in the same model as manufacturers does not seem to affect our results negatively. This is probably true due to the adjustments to our data based on recent and anticipated financial reporting changes – primarily the capitalization of leases.

We also find that the ZETA model outperforms alternative bankruptcy classification strategies in terms of expected cost criteria utilizing prior probabilities and explicit cost of error estimates. In our investigation we were surprised to observe that, despite the statistical properties of the data which indicate that a quadratic structure is appropriate, the linear structure of the same model outperforms the quadratic in tests of model validity. This was especially evident regarding the long-term accuracy of the model.

4. Sample and data characteristics and statistical methodology

4.1. Sample characteristics

Our two samples of firms consist of 53 bankrupt firms and a matched sample of 58 non-bankrupt entities. The latter are matched to the failed group by industry and year of the data.[4] Table 1 lists the bankrupt firms by type, size and year of bankruptcy petition. Note that our sample is almost equally divided into manufacturers and retailer groups and that 94% (50 of 53) of the firms failed during the period 1969–1975. As mentioned earlier, the average asset size of our failed group is almost $100 million, indicative of the increasing size of failures.[5] The bankrupt firms represent all publicly held industrial failures which had at least $20 million in assets, with no known fraud involved and where sufficient

[3]For example, see Joy and Tollefson (1975) and the consequent comment by Altman and Eisenbeis (1976) as well as the forthcoming article by Eisenbeis (1977).

[4]We have 5 more non-failed observations than failed because 5 of the failed firms in our original sample did not have sufficient data for our purposes.

[5]Dun and Bradstreet (1976) reports that the percentage of business failures with short-term liabilities in excess of $1 million has risen from 1.1% in 1970 to 4.5% in 1976. No longer is the large billion dollar enterprise immune to failure. In fact, three (3) S&P 500 firms have failed since 1970 (Penn Central, Arlans Department Stores, and W.T. Grant).

data was available. Five non-bankruptcy petition companies were included due to either substantial government support (1), a forced merger (1), or where the banks took over the business (3) (see appendix A).

Table 1
Sample characteristics

	Bankrupt	Non-bankrupt
Number of firms	53	58
Type of firm		
Manufacturer	29	32
Retailer	24	26
Average size (tangible assets)	$96 million	$167 million

Number of firms by years of bankruptcy	
1975	9
1974	9
1973	14
1972	3
1971	5
1970	5
1969	5
1967	1
1962	2
Total	53

4.2. *Variables analyzed*

A number of financial ratios and other measures have been found in other studies to be helpful in providing statistical evidence of impending failures. We have assembled data to calculate these variables and in addition have included several 'new' measures that were thought to be potentially helpful as well. The 27 variables are listed in appendix B along with certain relevant statistics which will be discussed shortly. Note that in a few cases – e.g. nos. 7 and 9, tangible assets and interest coverage – the variables are expressed in logarithmic form in order to reduce outlier possibilities and to adhere to statistical assumptions. The variables can be classified as profitability (1–6), coverage and other earnings relative to leverage measures (8–14), liquidity (15–18), capitalization ratios (19–23), earnings variability (24–26) and a few miscellaneous measures (7, 27).

4.3. *Reporting adjustments*

As noted earlier, we have adjusted the basic data of our sample to consider explicitly several of the most recent and, in our opinion, the most important accounting modifications. These adjustments include the following:

(1) *Capitalization of leases.* Without doubt, the most important and pervasive adjustment made was to capitalize all non-cancellable operating and finance leases. The resulting capitalized lease amount was added to the firms' assets and liabilities and also we have imputed an interest cost to the 'new' liability. The procedure involved preparation of schedules of current and expected lease payment obligations from information found in footnotes to the financial statements. The discount rate used to capitalize leases was the average interest rate for new issue, high grade corporate bonds in the year being analyzed plus a risk premium of 10 % of the interest rate. For example, if a firm had lease payments of \$100,000 a year for the next 10 years and the current interest rate was 7.3 %, the capitalized lease equals \$671,000. Symbolically,

$$CL=\sum_{t=1}^{N} \frac{L_t}{(1+r+0.1r)^t},$$

CL = capitalized lease,
L_t = lease payment in period t,
r = average interest rate for new issue high grade corporate bonds,
N = the number of years of leasehold rights and obligations.

An amount equal to the interest rate used in the capitalization process times the capitalized lease amount is added to interest costs.

(2) *Reserves.* If the firms' reserves were of a contingency nature, they were included in equity and income was adjusted for the net change in the reserve for the year. If the reserve was related to the valuation of certain assets, it was netted against those assets.

(3) *Minority interests and other liabilities on the balance sheet.* These items were netted against other assets. This allows for a truer comparison of earnings with the assets generating the earnings.

(4) *Captive finance companies and other non-consolidated subsidiaries.* These were consolidated with the parent company accounts as well as the information would allow. The pooling of interest method was used.

(5) *Goodwill and intangibles.* Deducted from assets and equity because of the difficulty in assigning economic value to them.

(6) *Capitalized research and development costs, capitalized interest and certain other deferred charges.* These costs were expensed rather than capitalized. This is done to improve comparability and to give a better picture of actual funds flows.

4.4. Statistical methodology

Bankruptcy classification is attempted via the use of a multivariate statistical technique known as discriminant analysis. In this study, the results using both linear and quadratic structures are analyzed. Since 1968, there have been several studies devoted to the failure classification and prediction problem, most of these which utilised discriminant analysis are referenced in footnotes 1 and 3 above. It is now fairly well documented that the test for assessing whether a linear or quadratic structure is appropriate – sometimes referred to as the H_1 test, derived from Box (1949)[6] – will provide the proper guidance when analyzing a particular sample's classification characteristics. Essentially, if it is assessed that the variance–covariance matrices of the G groups are statistically identical, then the linear format which pools all observations is appropriate. If, however, the dispersion matrices are not identical, then the quadratic structure will provide the more efficient model since each group's characteristics can be assessed independently as well as between groups. Efficiency will result in more significant multivariate measures of group differences and greater classification accuracy of that *particular sample.* What has not been assessed up to this point, is the relative efficiency of the linear vs. quadratic structures when the sample data are not the same as that used to construct the model, i.e. holdout or secondary samples. We will analyze this point in the next section.

5. Empirical results

5.1. The 7-variable model

After an iterative process of reducing the number of variables, we selected a 7-variable model which not only classified our test sample well, but also proved the most reliable in various validation procedures. That is, we could not significantly improve upon our results by adding more variables, and no model with fewer variables performed as well.

X_1 *Return on assets*, measured by the earnings before interest and taxes/total assets, variable no. 1 in appendix B. This variable has proven to be extremely helpful in assessing firm performance in several past multivariate studies including two by Altman (1968, 1973) and by the leading univariate study [see Beaver (1967)].

X_2 *Stability of earnings*, measured by a normalized measure of the standard error of estimate around a ten-year trend in X_1; no. 24 in appendix B. Business risk is often expressed in terms of earnings fluctuations and this measure proved to be particularly effective. We did assess the information content of several similar variables which attempted to measure the potential susceptibility of a firm's earnings level to declines which could

[6]This test, as well as the actual quadratic algorithm, is incorporated into a computer program known as MULDIS, developed by Eisenbeis and Avery (1972). We have utilized a revised version of their original program in this study.

jeopardize its ability to meet its financial committments. These variables, nos. 25 and 26 (EBIT drop and margin drop) in appendix B, were quite significant on a univariate level but did not enter into our final multivariate model.[7]

X_3 *Debt service* (no. 9 in appendix B), measured by the familiar interest coverage ratio, i.e. earnings before interest and taxes/total interest payments (including that amount imputed from the capitalized lease liability). We have transposed this measure by taking the log 10 in order to improve the normality and homoscedasticity of this measure.

X_4 *Cumulative profitability*, measured by the firm's retained earnings (balance sheet)/total assets; no. 19 in appendix B. This ratio, which imputes such factors as the age of the firm and dividend policy as well as its profitability record over time, was found to be quite helpful in past studies including one, Altman (1968), whose results will be directly compared to the results of this study. As our results will show, this cumulative profitability measure is unquestionably the most important variable – measured univariately and multivariately (table 2 below).

X_5 *Liquidity*, measured by the familiar current ratio; no. 16 in appendix B. Despite previous findings that the current ratio was not as effective in identifying failures as some other liquidity measures, we find it slightly more informative than others, such as the working capital/total assets ratio.

X_6 *Capitalization*, measured by common equity/total capital; no. 22 in appendix B. In both the numerator and the denominator the common equity is measured by a five-year average of the total market value rather than book value. The denominator also includes preferred stock at liquidating value, long-term debt and capitalized leases. We have utilized a 5-year average to smooth out possible severe, temporary market fluctuations and to add a trend component (along with X_2 above) to the study.[8]

X_7 *Size*, measured by the firms' total assets. This variable, as is the case with the others, is adjusted for recent financial reporting changes. No doubt, the capitalization of leasehold rights has added to the average asset size of both the bankrupt and nonbankrupt groups. We have also transformed the size variable to help normalize the distribution of the variable due to outlier observations. Again a logarithmic transformation was applied.

[7]Variable no. 25 measures the potential drop in EBIT based on the worst single year's rate of change in EBIT in the past 10 years. Variable no. 26 measures a similar fall in EBIT with respect to the ratio of EBIT/sales over a like period. These earnings 'declines' are measured against the current year's interest and debt respectively.

[8]Conceptually, the most recent market value should best portray the future earning power of a firm, and we have seen its powerful failure information content before [see Beaver (1968) and Altman (1968)]. Due to a certain amount of caution for practical and application purposes, we have selected the 5-year average. Note that only the most recent market value is measured in V_{21} (appendix B) and its univariate F-test is approximately identical to the ratio using the 5-year average (V_{22}).

5.2. Relative importance of discriminant variables

The procedure of reducing a variable set to an acceptable number is closely related to an attempt to determine the relative importance within a given variable set. Several of the prescribed procedures for attaining the 'best' set of variables, e.g. stepwise analysis, can also be used as a criterion for ranking importance. Unfortunately, there is no one best method for establishing a relative ranking of variable importance. Hence, we have assessed this characteristic by analyzing the ranks suggested by six different tests (table 2). In several studies that we have observed, the rankings across these tests are not very consistent and the researcher is left with a somewhat ambiguous answer. This was definitely not the case in our study.

As noted in table 2, regardless of which test statistic is observed, the most important variable is the cumulative profitability ratio, X_4.[9] In fact, our scaled vector analysis indicates that this single ratio contributes 25% of the total discrimination. Second in importance is the stability of earnings ratio (X_2) and, except for the univariate test of significance, it too has a consistent ranking across tests. Similarly, our capitalization variable (X_6) is almost totally consistent across tests. The least important variable appears to be the overall profitability ratio (X_1), but it still is an important contributor to the model's success.

5.3. Linear vs. quadratic analysis

The H_1 test of the original sample characteristics equals 6.20 and clearly rejects the hypothesis that the group dispersion matrices are equal. Therefore, the linear structure classification rule (excluding error costs),

$$X'\Sigma^{-1}(\mathbf{U}_1-\mathbf{U}_2)-\tfrac{1}{2}(\mathbf{U}_1+\mathbf{U}_2)'\Sigma^{-1}(\mathbf{U}_1-\mathbf{U}_2)\geqq \ln P,$$

is not appropriate and the quadratic structure with a classification rule: assign a firm to one group, e.g. non-bankrupt,

$$X'(\Sigma_1^{-1}-\Sigma_2^{-1})X-2(\mathbf{U}_1'\Sigma_1^{-1}-\mathbf{U}_2'\Sigma_2^{-1})X+\mathbf{U}_1'\Sigma_1^{-1}\mathbf{U}_1-\mathbf{U}_2'\Sigma_2^{-1}\mathbf{U}_2$$

$$\geqq \ln|\Sigma_2^{-1}\cdot\Sigma_1^{-1}|-2\ln P,^{10}$$

[9]These tests include: (1) forwards stepwise, (2) backwards stepwise, (3) scaled vector (multiplication of the discriminant coefficient by the appropriate variance–covariance matrix item) (4) separation of means test [suggested by Mosteller and Wallace (1963) and supported by Joy and Tollefson (1975)], (5) the conditional deletion test which measures the additional contribution of the variable to the multivariate F-test given that the other variables have already been included [supported by Altman and Eisenbeis (1976)] and (6) the univariate F-test.

[10]Essentially these equations are cut off scores for classification in a two-group analysis. We will introduce costs of errors into the actual cutoff score when we assume unequal priors and unequal costs.

Table 2

Influence of each variable in the ZETA model, order of importance.[a]

Variable	Variable number	Forward stepwise discriminant analysis	Backward stepwise discriminant analysis	Scaled vector test (relative contribution)	Separation of means test (relative contribution)	Conditional deletion test	Univariate F-statistic
Overall profitability	1	7	7	7 (5%)	5	7	2
Stability of earnings	2	2	2	2 (20%)	2	2	4
Debt service	3	6	6	6 (6%)	7	6	6
Cumulative profitability	4	1	1	1 (25%)	1	1	1
Liquidity	5	5	5	5 (11%)	4	5	3
Capitalization	6	3	3	3 (18%)	3	3	5
Asset size	7	4	4	4 (15%)	5	4	7

[a]The actual coefficients and covariance terms for the seven variables cannot be reported due to the proprietary nature of the ZETA™ model to the firm of WSW.

where X = variable vector, $\mathbf{U}_1$, $\mathbf{U}_2$ = mean vectors of groups 1 and 2, Σ_1, Σ_2 = dispersion matrices of groups 1 and 2, and P = prior probability of an observation being drawn from one group $\div$ prior probability of being drawn from the other group.

As we will show in the next section, the quadratic and linear models yield essentially equal overall accuracy results for the original sample classifications, but the holdout sample tests indicate a clear superiority for the linear framework. This creates a dilemma and we have chosen to concentrate on the linear test due to (1) the possible high sensitivity to individual sample observations of the quadratic parameters (that is, we observe 35 different parameters in the quadratic model compared with only 7 in the linear case, not including the intercept), and (2) the fact that all of the relative tests of importance discussed in section 5.2 above, are based on the linear model.

5.4 Classification accuracy – Original and holdout samples

Table 3 presents classification accuracy of the original sample based on data from one year prior to bankruptcy. Lachenbruch validation tests[11] and years 2–5 'holdout' sample results are also presented.[12] These results are listed for both the linear and quadratic structures of the seven variable model.

Table 3
Overall classification accuracy (in percent).

Years prior to bankruptcy	Bankrupt firms		Non-bankrupt firms		Total	
	Linear	Quadratic	Linear	Quadratic	Linear	Quadratic
1 Original sample	96.2	94.3	89.7	91.4	92.8	92.8
1 (Lachenbruch validation test)	(92.5)	(85.0)	(89.7)	(87.9)	(91.0)	(86.5)
2 Holdout	84.9	77.4	93.1	91.9	89.0	84.7
3 Holdout	74.5	62.7	91.4	92.1	83.5	78.9
4 Holdout	68.1	57.4	89.5	87.8	79.8	74.0
5 Holdout	69.8	46.5	82.1	87.5	76.8	69.7

The linear model's accuracy, based on one year prior data, is 96.2% for the bankrupt group and 89.7% for the non-bankrupt. The upward bias[13] in these results appears to be slight since the Lachenbruch results are only 3% less for the

[11] Lachenbruch (1967) suggests an almost unbiased validation test of original sample results by means of a type of jackknife, or one isolated observation at a time, approach. The individual observations' classification accuracy is then cumulated over the entire sample.

[12] Data from the original sample's financial statements 2–5 years prior to failure are applied to the parameters established from one-year prior data and the results observed.

[13] Mainly due to the classification of the same observations used to construct the model; see Frank, Massy and Morrison (1965) for a discussion of discriminant analysis classification bias.

failed group and identical for the non-failed group. As expected, the failed group's classification accuracy is lower as the data become more remote from bankruptcy, but still quite high.[14] In fact, we observe 70% accuracy as far back as five years prior to failure. This compares very favorably to the results recorded by Altman (1968), where the accuracy dropped precipitously after two years prior. For a comparison of these two studies' results, see table 5 below.

An interesting result was outlined by comparing the quadratic structure's results for that of the linear (table 3). As noted earlier, the *total* samples' classification accuracy is identical for the two structures in period 1, with the linear showing a slight edge in the bankrupt group and the quadratic in the non-bankrupt group. The most obvious and important differences, however, are in the validation and 'holdout' tests of the bankrupt group. Here, the linear model is clearly superior, with the quadratic misclassifying over fifty percent of the future bankrupts five years prior. The Lachenbruch validation test also shows a large difference (over 7% favoring the linear model). Subsequent analysis will report only the linear results.

Table 4

Random sample validation test results: Classification accuracy (in percent).

Replication no. 1			Replication no. 2		
Year	Bankrupt	Non-bankrupt	Year	Bankrupt	Non-bankrupt
1	92.5	91.4	1	96.2	79.6
2	84.9	91.4	2	86.8	81.0
3	76.5	91.4	3	80.4	79.3
4	61.7	93.0	4	74.5	82.5
5	62.8	84.0	5	69.8	76.8

One further type of validation test was run involving the random selection of approximately one-half of the observations in order to ascertain the model's parameters with the remaining observations as 'holdouts.' The holdout observations were examined not only the first year prior to bankruptcy but for all five years. We made two replications of this test, with the results reported in table 4. Note that the accuracy is still quite impressive for independent observations

[14]Alternative temporal-type bankruptcy modelling strategies would include either completely separate models in each of the years (5) of analysis *or* the same variables for each of the five years, only the parameters would change to reflect the differences in data as bankruptcy becomes more remote. The latter analysis was first presented by Deakin (1972) using Beaver's (1967) 14 variables. We think this latter approach is of some interest but for application purposes the analyst is left somewhat confused as to which model to apply to new data. We did in fact, however, experiment with this approach and found that the year 1 model was, overall (5 years combined), more accurate than the other years' alternative models. Of course, each individual year's model was the most accurate in reclassifying that particular year's observations. These results, too lengthy to report in full, are available from the authors.

and for the five years of analysis. There was some difference, however, in the type I and type II errors from each of the replications.

5.5. Comparison with Altman's 1968 model

The 1968 model of one of the authors has received a good deal of exposure in leading finance texts, e.g. Weston and Brigham (1975), Van Horne (1974), in non-academic publications [Dun's Review (1975) and Metz (1976)], as well as immediate [Johnson (1970)] and delayed [Joy and Tollefson (1975)] criticism. To some extent, it has become a standard of comparison for subsequent bankruptcy classification studies. We have compared the ZETA model developed in this paper with the earlier (1968) model in several ways. First, we compare the 5-year accuracy of each model using the particular sample of firms of each study. These results are reported in columns 2–3 and 4–5 of table 5. Note that the newer ZETA model is far more accurate in bankruptcy classification in years 2–5 with the initial year's accuracy about equal. The older model showed slightly more accurate non-bankruptcy classification in the two years when direct comparison is possible.

Second, we have utilized the 1968 model's five variables and parameters,[15] calculated the five variables for the new sample of firms, arrived at each of these firm's Z-scores and classified them as bankrupt if $Z \leqq 2.675$ (the 1968 model's cutoff score) and as non-bankrupt if $Z > 2.675$. The observed accuracy of applying the new sample to the old model is illustrated in columns 6–7 of table 5. In every year (with the exception of year 5, non-bankrupts) the ZETA model dominates the 1968 model applied to the ZETA sample, especially in years 1 and 5.[16] Finally, we selected the five variables utilized in the 1968 model and calculated the parameters based on the 111-firm ZETA sample. Columns 8–9 list the five-year classification accuracy of the new sample based on the old variables, with year 1 as the standard and years 2–5 as 'holdouts'. Once again the ZETA model dominates in every year, but notice that the new 7-variable model is, in some years, only slightly more accurate than the 'old' 5-variable model when the data is comparable, i.e. adjusted for more meaningful evaluation.

[15] These variables are: the working capital/total assets; retained earnings/assets; earnings before interest and taxes/total assets; market value of equity/total debt; and sales/total assets.

[16] This type of test is more akin to an assessment of the 'predictive' accuracy of the 'old' model as opposed to the 'classification' and 'validation' accuracy involved with observations from the same periods as that of the model's sample. The bankrupt sample's predictive accuracy (column 6, table 5) is higher for the ZETA sample than we found in a prior study [Altman and McGough (1974)], i.e. 86.8 % and 83.0 % for year's 1 and 2 respectively vs. 82.0 % and 58.0 % in that 1974 study. We would be remiss not to point out that this type of test is not completely valid since the ZETA sample's data has been adjusted before being applied to the 1968 model. Hence, all firms will look worse and the type I error (13.2 %) is probably lower than if no adjustments were made, with the opposite effect on the type II error. These adjustments help to explain the fact that the 1968 model applied to the ZETA sample outperforms the same model applied to the 1968 sample in years 2–5.

Table 5

Comparative classification accuracy between the ZETA model and various forms of a prior bankruptcy model (in percent).

Years prior to bankruptcy	ZETA model		Altman's 1968 model[a]		1968 model, ZETA sample		1968 variables, ZETA para.	
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	96.2	89.7	93.9	97.0	86.8	82.4	92.5	84.5
2	84.9	93.1	71.9	93.9	83.0	89.3	83.0	86.2
3	74.5	91.4	48.3	n.a.	70.6	91.4	72.7	89.7
4	68.1	89.5	28.6	n.a.	61.7	86.0	57.5	83.0
5	69.8	82.1	36.0	n.a.	55.8	86.2	44.2	82.1

[a] *Source:* Altman (1968). n.a. = not available.

Unfortunately, direct comparison of the parameters of the 5-variable 1968 model for that earlier model's sample and the more recent sample did not make sense since the latest sample's data is adjusted along the lines as outlined in section 4.3. above, while the earlier data was not. One can observe the difference in average ratios for the two samples in table 6. Note that since the total assets and liabilities of all firms are 'inflated' by lease capitalization (for example), the ratios of both bankrupts and non-bankrupts are quite different ('higher' negatives and lower positives) for the new sample vis-à-vis the older one. This will cause the group means to be closer when the bankrupt average is negative and the non-bankrupt average positive, as is the case for the second and third variables.

Table 6
Comparison of group means and significance tests, 1968 model variables.

Variable	Average bankrupt group ratio		Average non-bankrupt group ratio		Univariate F-test	
	1968	ZETA	1968	ZETA	1968	ZETA
Working capital/ total assets	(0.06)	0.15	0.41	0.31	32.4	40.6
Retained earnings/ total assets	(0.62)	(0.001)	0.36	0.29	58.9	114.6
Earnings before interest and taxes/ total assets	(0.13)	(0.01)	0.15	0.11	26.6	54.3
Market value equity/total debt	0.40	0.61	2.47	1.84	33.3	11.6
Sales/total assets	1.50X	1.31X	1.90X	1.62X	2.8	3.3

The higher asset totals will also partly explain the lower capital turnover ratios (sales/assets) in the ZETA sample vs. the 1968 model's average. Another item of some interest is that, despite the 'inflated' total assets (deriving from higher *fixed assets*) in the ZETA sample, the working capital/total asset ratio of the bankrupt group is significantly larger in the more recent sample vis-à-vis the earlier one. The expected ranking is observed, however, for the non-bankrupt sample.

A comparison of univariate F-tests for the five variables shows that in 4 of the 5 cases (excepting the market value of equity/total debt), the more recent study has a higher test statistic. This is observed despite the greater mean differentials for the 1968 samples – no doubt due to the considerably smaller within-group variance in the recent samples.

Finally, we note that two of the seven variables in the ZETA model are also found in the 1968 model, and a third 1968 measure (market value of equity/total debt) is very similar to X_6 of the ZETA model.

5.6. Group prior probabilities, error costs and model efficiency

In section 5.3 above, we showed the classification rules for both linear and quadratic analyses. Note that if one sets equal prior probabilities of group membership, the linear model will result in a cutoff or critical score of zero. All firms scoring above zero are classified as having characteristics similar to the non-bankrupt group and those with negative scores similar to bankrupts. The same zero cutoff score will result if one desired to minimize the total cost of misclassification. That is, assuming multinormal populations and a common covariance matrix, the optimal cutoff score $ZETA_c$ is equal to:[17]

$$ZETA_c = \ln \frac{q_1 C_{\mathrm{I}}}{q_2 C_{\mathrm{II}}},$$

where q_1, q_2 = prior probability of bankrupt (q_1) or non-bankrupt (q_2), and C_{I}, C_{II} = costs of type I and type II errors, respectively.

Further, if one wanted to compare the efficiency of the ZETA bankruptcy classification model with alternative strategies, the following is appropriate for the expected cost of ZETA (EC_{ZETA}),

$$EC_{\mathrm{ZETA}} = q_1 (M_{12}/N_1) C_{\mathrm{I}} + q_2 (M_{21}/N_2) C_{\mathrm{II}},$$

where M_{12}, M_{21} = observed type I and type II errors (misses) respectively, and N_1, N_2 = number of observations in the bankrupt (N_1) and non-bankrupt (N_2) groups.

In our tests, we have implicitly assumed equal prior probabilities and equal costs of errors, resulting in a zero cutoff score. We are acutely aware, however, of the potential bias involved in doing so. Instead of attempting earlier to integrate probability priors and error costs, we have assumed equal estimates for each parameter, because to a great extent the two parameters neutralize each other, and it was much easier than attempting to state them precisely.[18] The following is our reasoning.

The 'correct' estimate of q_1/q_2 is probably in the 0.01–0.05 range. That is, the prior probability that a firm will go bankrupt in the future is probably in this

[17] See Joy and Tollefson (1975) and Altman and Eisenbeis (1976) for a full discussion of the optimal cutoff score and efficiency tests. Note that if the assumption of multinormality and common dispersion matrices is violated, the cutoff score derived from this formula may not be optimal. We will examine this in section 5.8.

[18] We could have easily adjusted $ZETA_c$ for prior probability estimates, but instead we have deferred this (and also error cost inputs) to a later section.

0.01–0.05 range. Dun and Bradstreet estimate that, in the last decade, between 0.4 and 0.5% of the firm population failed *in any given year*. Although the ZETA model's parameters are based on data from one year prior to bankruptcy, it is not specifically a one-year prediction model. The procedure in this sense is atemporal.[19] It is, in our opinion, incorrect to base one's prior probability estimates on a single year's reported statistics. In addition, there are many definitions of business failure which, economically, approximate bankruptcy. These include non-judicial arrangements, extreme liquidity problems which require the firm's creditors or other external forces to take over the business,[20] bond default, etc. In the final analysis, we simply do not know the precise estimate of bankruptcy priors, but at the same time assert that one must assume the estimate is greater than a single year's reported data. Hence, we believe the prior probability estimate is in the 2–5% range and in subsequent analysis we utilize the 2% figure.

5.7. *Cost of classification errors*

Another input that is imperative to the specification of an alternative to the zero cutoff score is the cost of errors in classification. No prior study of the type attempted here has explicitly included this element of analysis. In order to attempt to precise the cost component into an analysis of model efficiency, it is necessary to specify the decision-maker's role. In this study we utilize the *commercial bank loan* function as the framework of analysis. The type I bankruptcy classification error is analogous to that of an accepted loan that defaults and the type II error to a rejected loan that would have resulted in a successful payoff. Many of the conceptual factors involved in assessing these error costs were first noted in an excellent discussion [following Beaver's (1967) paper] by Neter and Beaver.

An empirical study was performed to assess the costs of these lending errors with the following specification for the equivalent type I (C_{I}) and type II (C_{II}) error costs:[21]

$$C_{\mathrm{I}} = 1 - \frac{LLR}{GLL}, \qquad C_{\mathrm{II}} = r - i,$$

where:

LLR = amount of loan losses recovered,
GLL = gross loan losses (charged-off),
r = effective interest rate on the loan,
i = effective opportunity cost for the bank.

[19] For an attempt to add a temporal aspect to failure prediction see Wilcox (1971) and Santomero and Vinso (1977).

[20] Several large firm problems of this type come to mind, including LTV Corp., Memorex and, in a different context, Lockheed Aircraft Corp.

[21] For a more detailed discussion of this investigation see Altman (1977b).

The commercial bank takes the risk of losing all or a portion of the loan should the applicant eventually default. The exact amount is a function of the success the bank has in recovering the loan principal.[22] This recovery aspect is rarely considered in conceptual, as well as practical, analysis.

We have measured C_I based on annual report data from 26 of the largest U.S. commercial banks and questionnaire returns from a sample of smaller, regional banks in the Southeast U.S.[23] Both data sources encompass a five year period, 1971–1975 inclusive, and we measure the average loan loss recovery statistics on a contemporary and a one-year lag (recoveries lagging charge-offs) basis. The results of this investigation are illustrated in table 7 and show that the average C_I on a contemporary basis is in the 76.7–83.0% range; when measured on a one-year lag basis, the averages are lower (68.6–72.2%). The year 1975 was an abnormally high loan charge-off year in the U.S. banking system and since this

Table 7

Net loan loss experience, 1971–1975 averages, two samples.

Net loan loss (C_I) (percentages)[a]	Major banks No.	Major banks Percent	Southeast regional banks No.	Southeast regional banks Percent
0–20	0	0.0	0	0.0
20–30	1	3.8	0	0.0
30–40	0	0.0	0	0.0
40–50	0	0.0	1	3.0
50–60	0	0.0	2	6.1
60–70	1	3.8	4	12.1
70–80	4	15.4	12	36.4
80–90	13	50.0	11	33.3
90–100	7	27.0	3	9.1
Total	26	100.0	33	100.0
Average (contemporary)	—	83.0	—	76.7
Average (one-year lag)	—	72.2	—	68.6

[a]Recoveries and loan loss measured on a contemporary basis.

[22]We are quite aware that there are additional costs involved in the recovery process, including legal, transaction, and loan charge-off officer opportunity costs. These costs are not reported but obviously would increase the type I error cost. In addition, if the type II error (C_{II})is positive, i.e. $r > i$, then there will be an added cost element in C_I. This added element involves the lost interest on that remaining part of the loan which is not recovered ($GLL - LLR$) for the duration of the defaulted loan. We will examine C_{II} below, but will not include this added element in our calculation of C_I. Again, however, it is clear that we are understating C_I somewhat.

[23]A questionnaire was sent to approximately 100 Southeast banks with 33 usable responses. The range of commercial bank asset sizes in this small-bank sample was between \$12 million and \$3 billion, with the average equal to \$311 million and the median equal to \$110 million. The large-bank sample's asset size averaged \$13.4 billion with a \$10 billion median

data is included in the contemporary statistics but not in the one-year lag data, we believe the more representative result for C_{I} is in the vicinity of 70 %. We use this statistic for C_{I}.

The simple formula for C_{II} specifies that the decision not to lend to an account that would have repaid successfully foregoes the return on that loan, but the loss is mitigated by the alternative use of loanable funds. In its strictest sense, the bank's opportunity cost implies another loan at the same risk which is assumed to pay off. In this case, C_{II} is probably zero or extremely small. Conservatively speaking, however, an account is rejected due to its high risk characteristics and alternative uses probably will carry lower risk attributes. Hence, $r-i$ will be positive but still quite low. Carried to the other extreme, the alternative use would be an investment in a riskless asset, i.e. government securities of the same maturity as the loan, and $r-i$ will be somewhat higher – perhaps 2–4 %. The relationship between $r-i$ will vary over time and is particularly sensitive to the demand and supply equilibrium relationship for loanable funds.[24] As an approximation, we specify $C_{\mathrm{II}}=2\%$, hence $C_{\mathrm{I}}/C_{\mathrm{II}}$ is equal to 35 times.

5.8. *Revised cutoff score and model efficiency tests*

With respect now to the calculation of the critical or cutoff score ZETA$_c$, we have,

$$ZETA_c = \ln \frac{q_1 C_{\mathrm{I}}}{q_2 C_{\mathrm{II}}} = \frac{0.02 \cdot 0.70}{0.98 \cdot 0.02} = \ln 0.714,$$

$$ZETA_c = -0.337.$$

Before comparing the efficiency of the various alternative bankruptcy classification strategies, it should be noted that the observed classification accuracy of a model such as ZETA will change with the new cutoff score.[25] For example, with the cutoff score of -0.337, the number of type I errors increases from two (3.8 %) to four (7.6 %) while the type II errors decrease from 6 (10.3 %) to 4 (7.0 %). These new estimates will form the basis of comparison along with the more realistic priors and measures of error costs.

[24]Various abstractions of the C_{II} calculation are not considered here – such as the loss of the customer's future interest payments on loans. Assuming $r-i>0$, we might approximate this loss as the perpetuity of $r-i$ discounted by the bank's cost of capital. Note that if r equals i ($C_{\mathrm{II}}=0$), the critical cutoff score will equal positive infinity (see the next section) and all loans will be rejected by the model.

[25]This point is often overlooked in assessing various strategy performance, e.g. Joy and Tollefson (1975) overlook this aspect in their critique of the 1968 Altman model.

The following calculations represent our efficiency comparison tests:

$$EC_{\text{ZETA}} = q_1(M_{12}/N_1)C_{\text{I}} + q_2(M_{21}/N_2)C_{\text{II}}$$
$$= 0.02(0.076)0.70 + 0.98(0.07)0.02 = 0.00243,$$

$$EC_{\max} = q_1 C_{\text{I}} = 0.02(0.70) = 0.0140,$$

$$EC_{\text{prop}} = q_1 q_2 C_{\text{I}} + q_2 q_1 C_{\text{II}}$$
$$= 0.02(0.98)0.70 + 0.98(0.02)0.02 = 0.01411,$$

where $EC_{\max}$ is based on the naive strategy that all firms are classified as non-bankrupt, e.g. all loan applications are accepted, and EC_{prop} is a proportional chance strategy based on observed error rates equalling a priori probabilities.

Therefore, the best estimate, or most likely result, shows that $EC_{\text{ZETA}} < 5.7 \, X EC_{\max} \approx EC_{\text{prop}}$. That is, both naive strategies are almost six times less efficient than the ZETA model. Rather than leave the comparison analysis with just one set of estimates of prior probabilities and error costs, we have specified the likely limits of each parameter and calculated the resulting ZETA cutoff score, type I and type II errors, and the expected cost of each of the two naive models plus the ZETA model under each assumption. That is, we can expect that the true value of q_1/q_2 is between 0.01/0.99 and 0.05/0.95, the values of C_{I} could vary between 0.6 and 0.8 and C_{II} between 0.01 and 0.05. We therefore can specify four new comparison tests and their results as listed in table 8.[26]

Table 8

Model efficiency comparisons under various input assumptions.

Assumptions	ZETA results: Cutoff score	Type I error	Type II error	EC_{ZETA}	$EC_{\max}$	EC_{prop}
(1) $q_1=0.02$; $q_2=0.98$ $C_{\text{I}}=0.70$; $C_{\text{II}}=0.02$	−0.33	0.076	0.070	0.0024	0.0140	0.0141
(2) $q_1=0.01$; $q_2=0.99$ $C_{\text{I}}=0.60$; $C_{\text{II}}=0.05$	−2.11	0.226	0.000	0.0014	0.0060	0.0064
(3) $q_1=0.01$; $q_2=0.99$ $C_{\text{I}}=0.80$; $C_{\text{II}}=0.01$	−0.21	0.057	0.070	0.0011	0.0080	0.0080
(4) $q_1=0.05$; $q_2=0.95$ $C_{\text{I}}=0.60$; $C_{\text{II}}=0.05$	−0.46	0.076	0.070	0.0056	0.0300	0.0309
(5) $q_1=0.05$; $q_2=0.95$ $C_{\text{I}}=0.80$; $C_{\text{II}}=0.01$	1.43	0.000	0.225	0.0021	0.0400	0.0385

[26]In fact, one can specify an alternative equally naive strategy, i.e. all firms will go bankrupt, and arrive at a more efficient result in case number 5 (table 8) than the $EC_{\max}$ strategy. ZETA would, of course, dominate this strategy as well.

We observe that regardless of the assumptions, the ZETA model is considerably more efficient, and this efficiency differential ranges from 4.3 to 19.0 times.[27] Of course, this is not a mathematical proof of the ZETA superiority but one can feel confident that under reasonable assumptions this will be the case.

5.9. Adjustments to the cutoff score and practical applications

In addition to the utilization of prior probabilities of group membership and cost estimates of classification errors for comparative model efficiency assessment, these inputs could prove valuable for practical application purposes. For instance, the bank lending-officer or loan-review analyst may wish to be able to logically adjust the critical cutoff score to consider his own estimates of group priors and error costs and/or to reflect current economic conditions in his environment. One could imagine the cutoff score falling (thereby lowering the acceptance criterion) as business conditions improve and the banker's prior probability of bankruptcy estimate falls from say 0.02 to 0.015. Or, a rise in cutoff scores could result from a change (rise) in the estimate of the type I error cost vis-à-vis the type II error cost. The latter condition possibly will occur for different decision-makers. For instance, the cost to a portfolio manager of not selling a security destined for failure is likely to be extremely high relative to his cost of not investing in a stock (which does not fail) due to its relatively low ZETA. The portfolio manager may indeed want to raise the cutoff or threshold level to reduce the possibility of intangible (law suit costs) as well as tangible (lower prices) costs involved with holding a failed company's stock.

Another example of a practical application of cutoff score adjustment is the case of an accounting auditor. He might wish to use the model to decide whether a 'going concern' qualified opinion should be applied. His expected cost for doing so is likely to be quite high (loss of client) relative to the expected cost of a stockholder law suit. This might lead to a fairly low cutoff score. On the other hand, the environment may be such that the law suit expected cost is prohibitive.

5.10. Distribution of ZETA

Fig. 1 illustrates the mean ZETA score for the two groups from five years prior to bankruptcy to one year prior. While the average ZETA for bankrupts diminishes as bankruptcy approaches, the standard deviation for each period remains fairly stable, i.e. 2.7, 2.5, 2.5, 2.7 and 3.0 for years 1–5. The non-bankrupt group's variance is also quite stable although slightly higher, averaging 3.0. Fig. 2 shows the actual ZETA distribution for the 111 firms (see appendix A). Note that

[27] Since the population distributions are not perfectly multinormal, it is possible to search by an iterative process and establish a cutoff score which results in slightly lower overall expected costs. For example, cases 1 and 3 in table 8 could be very slightly improved in terms of EC_{ZETA}.

the overlap area (the interval of ZETA where errors in classification are observed) is relatively small, i.e. from −1.45 to +0.87. As expected, the overlap range widens (except in year 5) as the time prior to bankruptcy is more remote.[28]

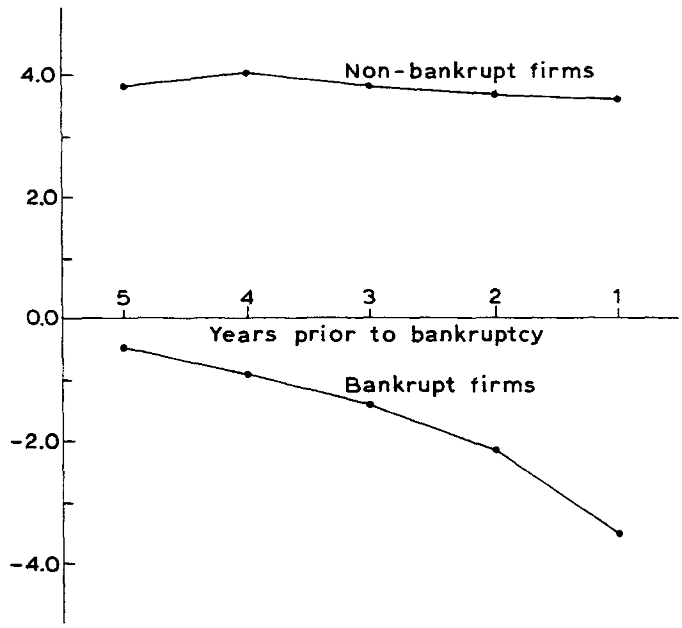

Fig. 1. Mean ZETA score.

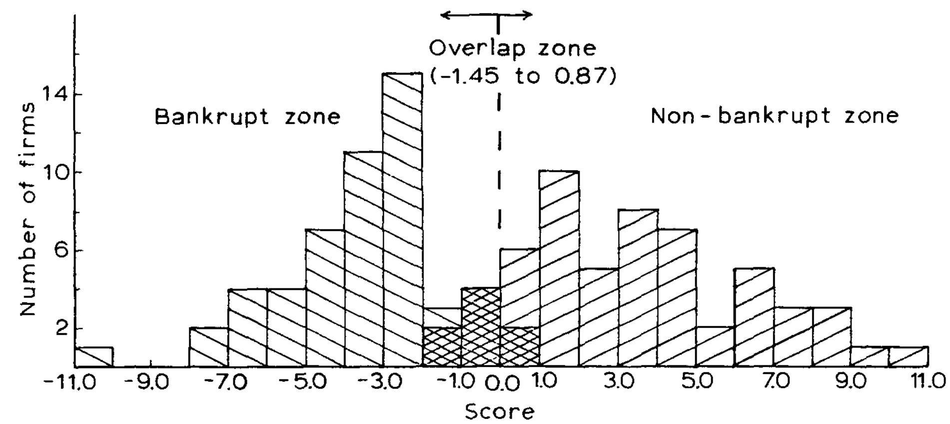

Fig. 2. Distribution of ZETA scores 1 year prior to bankruptcy

6. Conclusions

The ZETA model for assessing bankruptcy risk of corporations developed in this paper demonstrates significantly improved accuracy over an existing failure classification model and, perhaps more importantly, is based on data most

[28]Based on data from 2 years prior, the overlap interval is from −2.4 to +1.6; 3 years prior, from −1.9 to +3.1; 4 years prior, from −3.3 to +5.5; and 5 years prior, from −3.1 to +4.6.

relevant to current conditions. We are concerned with refining existing bankruptcy classification techniques by the use of the most relevant data combined with the most recent developments in the application of discriminant analysis to finance. The model's bankruptcy classification accuracy ranges from over 96 (93% holdout) percent one period prior to bankruptcy to 70% five annual reporting periods prior. We have assessed the effect of several elements involved with the application of discriminant analysis to financial problems. These include linear vs. quadratic analysis for the original and holdout samples, introduction of prior probabilities of group membership and costs of error estimates into the classification rule, and comparison of the model's results with naive bankruptcy classification strategies.

The potential applications of the ZETA bankruptcy identification model are in the same spirit as previously developed models. These include credit worthiness analysis of firms for financial and non-financial institutions, identification of undesirable investment risk for portfolio managers and individual investors and to aid in more effective internal and external audits of firms with respect to going-concern considerations, among others.

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Appendix A

Sample failed firms and year of failure.

Company	Year of failure
American Beef Packers (M)	1975
American Book – Stratford Press (M)	1973
Ancorp National Services (R)	1973
Arlans Department Stores (R)	1973
Atlas Sewing Centers (R)	1962
Beck Industries (R)	1970
Bishop Industries (M)	1970
Bohack Stores (R)	1975
Botany Industries (R)	1971
Bowmar Instruments (M)[e, r]	1974
Coit International (R)	1975
Commodore Corp. (M)	1974
Daylin (R)	1975
Diversa (M)	1969
Dolly Madison Ind. (M)	1970
Douglas Aircraft (M)*	1967
Dynamics Corp. (M)	1972
Ecological Science (M)	1970
Electrospace (M)[r]	1973
Esgro Corp. (M)	1973
Farrington Mfg. (M)	1969
Federals Corp. (R)	1973
Fishman, M.H. (R)	1974
Giant Stores (R)	1973
Grant, W. T. (R)	1975
Gray Mfg. (M)	1975
Grayson Robinson Stores (R)	1962
Hartfield–Zody's (R)	1974
Harvard Industries (M)	1972
Hoe, R. (M)	1969
Horn & Hardart Baking (R)	1971
Interstate Dept. Stores (R)	1974
Kenton Corp. (R)	1974
Ling–Tempco–Vought (M)*	1971
Lockheed Aircraft (M)*	1971
Mangel Stores (R)	1975
Meister Brau (M)	1971
Memorex Corp. (M)*	1973
Miller Wohl (R)	1973
Mohawk Data Sciences (M)*	1975
National Bellas Hess (R)	1974
National Video (M)	1969
Omega–Alpha (M)	1973
Parkview-General (R)	1973
Penn Fruit (R)	1975
Photon (M)[e]	1972
Potter Instruments (M)	1975
Roberts Co. (M)	1969

Failed firms (*continued*)

Company	Year of failure
Scottex (M)	1973
Sequoyah Industries (M)	1973
Simon Stores (R)	1970
Unishops (R)	1973
Westgate California (M)	1974

[a](M) indicates manufacturer,
(R) indicates retailer,
*indicates the firm remained a non-bankrupt only due extraordinary external support,
e = zero cutoff-point error,
r = revised cutoff error,
−0.33 score.

Sample non-failed firms[a]

Company	Company
Airco (M)	Hewlett Packard (M)
Alexanders (R)	High Voltage Eng. (M)
American Furniture (M)	Hoffman Electronics (M)
Ampex (M)	House of Fabrics (R)
Armada (M)[e]	Ideal Toy (M)
Associated Dry Goods (R)	Itek (M)
Automatic Service (R)	Jamesway (R)[e, r]
Buffalo Forge (M)	Kings Dept. Stores (R)
Bunker Ramo (M)	Kroger (R)
Caldor (R)	Kuhns Big K Stores (R)
Chesebrough Ponds (M)	Lane Bryant (R)
Cone Mills (M)	Lucky Stores (R)
Cooper Industries (M)	Mays. J.W. (R)
Curtiss-Wright (M)	New Process (R)
Digital Equipment (M)	Outlet Co. (R)
Eagle Clothes (R)[e, r]	Pay N'Save (R)
Emporium Capwell (R)	Penn Traffic (R)
Esmark (M)	Phillips Van-Heusen (M)
Ford Motor Co. (M)	Reliable Stores (R)
Foxboro Corp. (M)	Richton Int'l (R)[e, r]
Franklin Stores (R)	Scovill Mfg (M)
General Dynamics (M)	Sterling Precision (M)
Genesco (R)	Supermarkets General (R)
Ginos (R)[e]	Varian Associates (M)
Grace, W.R. (M)	Walgreen (R)
Grumman Corp. (M)	Wallace Business Forms (M)
Gulf Resources (M)	West Point–Pepperell (M)
Harris Intertype (M)	White Consolidated (M)[e, r]
	Winnebago (M)
	Zayre (R)

[a](M) indicates manufacturer, (R) retailer, [e] = zero cutoff point error, [r] = revised cutoff error, −0.33 score.

Appendix B

Listing of all variables, group means, and F-tests based on one period prior to bankruptcy data.[a]

Variable		Population means		Univariate
No.	Name	Failed	Non-failed	F-test
(1)	EBIT/TA	−0.00555	0.11176	54.3
(2)	NATC/TC	−0.02977	0.0742	36.6
(3)	Sales/TA	1.312	1.620	3.3
(4)	Sales/TC	2.107	2.160	0.0
(5)	EBIT/Sales	0.00209	0.07709	30.2
(6)	NATC/Sales	−0.01535	0.04002	33.1
(7)	Log tang. assets	1.985	2.222	5.5
(8)	Interest coverage	−0.5995	5.341	26.1
(9)	Log no. (8) & 15	0.9625	1.162	26.1
(10)	Fixed charge coverage	0.2992	2.1839	15.7
(11)	Earnings/debt	−0.0792	0.1806	32.8
(12)	Earnings/5 yr. mats	−0.1491	0.6976	8.8
(13)	Cash flow/fixed charges	0.1513	2.9512	20.9
(14)	Cash flow/TD	−0.0173	0.3136	31.4
(15)	WC/LTD	0.3532	2.4433	6.0
(16)	Current ratio	1.5757	2.6040	38.2
(17)	WC/total assets	0.1498	0.3086	40.6
(18)	WC/cash expenses	0.1640	0.2467	5.2
(19)	Ret. earn./total assets	−0.00066	0.2935	114.6
(20)	Book equity/TC	0.202	0.526	64.5
(21)	MV equity/TC	0.3423	0.6022	32.1
(22)	5 yr. MV equity/TC	0.4063	0.6210	31.0
(23)	MV equity/total liabilites	0.6113	1.8449	11.6
(24)	Standard error of estimate of EBIT/TA (norm)	1.687	5.784	33.8
(25)	EBIT drop	−3.227	3.179	9.9
(26)	Margin drop	−0.217	0.179	15.6
(27)	Capital lease/total assets	0.251	0.178	4.2
(28)	Sales/fixed assets	3.172	4.179	3.5

[a]*Notation:*

EBIT = earnings before interest and taxes,
NATC = net available for total capital,
TA = total tangible assets,
LTD = long-term debt,
MV = market value
TC = total capital,
TD = total debt,
WC = working capital.