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Combining grey relation analysis with FMCGDM to evaluate financial performance of Taiwan container lines

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Abstract

In this paper, we combine grey relation analysis with fuzzy multi-criteria group decision-making (FMCGDM) to evaluate financial performance of Taiwan container lines. In the evaluating process, we apply grey relation analysis to partition financial ratios into several clusters, and find representative indices from the clusters. These representative indices are considered as evaluation criteria on financial performance assessments. Then an FMCGDM method is utilized to evaluate the financial performance of Taiwan container lines. By the evaluation, one container line can realize the finance competitive strength on container shipping market.

Keywords: Container lines; Financial performance; FMCGDM; Grey relation analysis; Representative indices

1. Introduction

The evaluation of financial performance is essential for container lines because they commonly need large capitals. In Taiwan, the domestic container lines includes Evergreen, Yang-Ming, Wan-Hai, etc. These container lines have lots of container ships and related equipments for cargo transportation. The machines and tools usually take them lots of capitals. Based on the concept, a container line has to evaluate the financial performance which directly influences the company's survival. To evaluate financial performance of container lines, evaluation criteria are firstly grasped from their financial ratios (Walter & Robert, 1988). Financial ratios are commonly from balance sheet, income statement, cash flow, etc. However, some financial ratios are similar on identified patterns. To avoid repeated evaluation on similar financial ratios, financial ratios will be partitioned into several clusters, and then one representative index is found from a cluster to be an evaluation criterion (Deogun, Kratsch, & Steiner, 1997; Dubes, 1988; Duda & Hart, 1973; Eom, 1999; Feng & Wang, 2000; Hirano, Sun, & Tsumoto, 2004; Kaufman & Rousseeuw, 1990; Krishnapuram & Keller, 1993; Lee, 1999; Miyamoto, 2003; Pedrycz & Vukovich, 2002). Finally, an evaluation method is utilized to evaluate the financial performance of container lines.

Further, the number of major container lines is merely three in Taiwan. The number is very small and the distribution is unknown, so the classic clustering methods (Johnson & Wichern, 1992) are not suitable for these situations. To solve the situations of scarce data and unknown distribution, we utilize grey relation analysis (Deng, 1989) to cluster financial ratios and then find representative indices to be evaluation criteria. Since there will be several criteria on the evaluation problem, financial performance evaluation is one of multi-criteria decision-making (MCDM) problems (Hwang & Yoon, 1981; Keeney & Raiffa, 1976). On the other hand, we evaluate the financial performance of container lines in five periods. These performance values expressed in the five periods are aggregated into fuzzy numbers, so the evaluation problem belongs to fuzzy multi-criteria decision-making (FMCDM) problems (Boender, de Graan, & Lootsma, 1989; Chang & Yeh, 2002; Chen, 2000; Chen & Hwang, 1992; Hsu & Chen, 1997; Jain, 1978; Lee, 2005; Liang, 1999; Ostrowski, O'Brien, & Gordon, 1993; Parasurman, Zeithaml, & Berry,

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1985; Truitt & Haynes, 1994; Tsaur, Chang, & Yen, 2002; Wang & Lee, 2007; Wang, Lee, & Lin, 2003). In this paper, we apply a FMCDM method combining experts' opinions to evaluate the financial performance of Taiwan container lines, so the evaluating method is a fuzzy multi-criteria group decision-making (FMCGDM) method (Wang & Lee, 2007). With the FMCGDM method, the evaluation problem of financial performance can be easily solved.

For the sake of clarity, representative indices found by grey relation analysis are expressed in Section 2. The notions of fuzzy sets and fuzzy numbers are introduced in Section 3. The FMCGDM method is presented in Section 4. Finally, an empirical study of three container lines in Taiwan is illustrated in Section 5.

2. Finding the representative indices of financial ratios by grey relation analysis

In accounting aspect, financial ratios are commonly classified into several categories (Feng & Wang, 2000; Walter & Robert, 1988), because experts suppose that financial ratios are partially similar in the same one category. Thus the financial ratios of container lines are originally divided into four categories shown in Table 1.

In Table 1, fixed assets to stockholder's equity ratio, debt to total assets ratio and accounts payable turnover belong to cost items, whereas the other ratios belong to benefit items.

According to the Table 1, grey relation analysis is applied to partition the financial ratios into several clusters and then find representative indices as evaluation criteria form clusters. Grey relation analysis is one technique of grey theory. Grey theory was first introduced by Deng (1989). The fundamental definition of greyness is the information being incomplete or unknown, so an element of the incomplete message is a grey element. Grey relation analysis is the method to measure the relations between the grey elements. Further, the definition and application of grey relation analysis in mathematics are stated as follows.

Assume that there are m container liners (companies) evaluated on s financial ratios. Let $x_i = \{x_i(k)|k = 1,2,\ldots,m\} \in X$ denote the sequence of financial ratio i on m companies, where $i = 1,2,\ldots,s$. Thus X is the set consisting of all financial ratio sequences. Then the elements will be normalized according to two following situations.

As $x_i(k)$ is a benefit item

$$y_i(k) = \frac{x_i(k)}{\sqrt{\sum_{t=1}^{m}[x_i(t)]^2}}.$$

Otherwise, $x_i(k)$ is a cost item, then

$$y_i(k) = \frac{1/x_i(k)}{\sqrt{\sum_{t=1}^{m}[1/x_i(t)]^2}}.$$

Table 1
The financial ratios on four categories

Category	Code	Formula	Ratio
Financial structure	F1	Fixed assets/total stockholder's equity	Fixed assets to stockholder's equity ratio
	F2	Fixed assets/long-term liabilities	Fixed assets to long-term liabilities ratio
	F3	Fixed assets/long term capital	Fixed assets to long term capital ratio
	F4	Total liabilities/total assets	Debt to total assets ratio
	F5	Total stockholder's equity/total liabilities	Stockholder's equity to total liabilities ratio
	F6	Working capital/total assets	Working capital to total assets ratio
Solvency	S1	Current assets/current liabilities	Current ratio
	S2	Quick assets/current liabilities	Quick ratio
	S3	Cash and cash equivalent/current assets	Cash ratio
	S4	Net cash provided by operating activities/current liabilities	Cash flow ratio
	S5	Working capital/current assets	Working Capital to current assets ratio
Turnover	T1	Operation cost/accounts payable	Accounts payable turnover
	T2	Operation cost/accounts receivable	Accounts receivable turnover
	T3	Operation revenue/fixed assets	Fixed assets turnover
	T4	Operation revenue/total assets	Total assets turnover
	T5	Net income (loss)/operation revenue	Net income (loss) turnover
Profitability	P1	(Operation revenue–operation cost)/operation revenue	Gross profit ratio
	P2	Operation income (loss)/operation revenue	Operation profit ratio
	P3	Income (loss) before tax/operation revenue	Income before tax ratio
	P4	Net income (loss)/operation revenue	Net income ratio
	P5	Net income (loss)/total assets	Return on total assets

In these above situations, $y_i(k)$ is the normalized value of the financial ratio i on the company k, $i = 1,2,\ldots,s$; $k = 1,2,\ldots,m$.

Let $y_i = \{y_i(k)|k = 1,2,\ldots,m\} \in Y$ indicate the sequences of normalized financial ratio i on m companies, where $i = 1,2,\ldots,s$. Y is a set composed of all the normalized financial ratios. Assume Y to be a factor set of grey relation. Let $y_0 \in Y$ represent the referential sequence, and $y_i \in Y$ represent the comparative sequence. $y_0(k)$ and $y_i(k)$ denote the financial ratio values of y_0 and y_i on company k, respectively. As average relation value $r(y_0, y_i)$ of $\{r(y_0(k), y_i(k))|k = 1,2,\ldots,m\}$ is a real number, the value can be defined by grey relation.

Let

$$r(y_0, y_i) = \frac{1}{m}\sum_{k=1}^{m} r(y_0(k), y_i(k)) = r_{0i}, \tag{3}$$

where

$$r(y_0(k), y_i(k)) = \frac{\min_{y_i(\neq y_0)\in Y}\min_k|y_0(k) - y_i(k)| + \zeta\max_{y_i(\neq y_0)\in Y}\max_k|y_0(k) - y_i(k)|}{|y_0(k) - y_i(k)| + \zeta\max_{y_i(\neq y_0)\in Y}\max_k|y_0(k) - y_i(k)|},$$

where ζ is the distinguished coefficient ($\zeta \in [0,1]$).

Grey relation matrix $R = (r_{ij})$ is derived by grey relation analysis, where $i = 1,2,\ldots,s$, $j = 1,2,\ldots,s$. The definition of clustering financial ratios based on the entries of the grey relation matrix is presented as follows.

Definition 2.1. As $r_{ij} \geqslant r$ and $r_{ji} \geqslant r$, Y_i and belong to the same one cluster, where r is a threshold value of clustering.

Definition 2.2. When $r_{ij} \geqslant r$, $r_{ji} \geqslant r$, $r_{ik} \geqslant r$ and $r_{ki} \geqslant r$, but $r_{jk} < r$ or $r_{kj} < r$. If $\min\{r_{ij}, r_{ji}\} \geqslant \min\{r_{ik}, r_{ki}\}$, then Y_i and belong to the same one cluster.

As financial ratios can be partitioned into several clusters, the finding of representative indices of clusters is stated as follows.

Definition 2.3. As Y_i and Y_j belong to the same one cluster, the representative index of the cluster is found according to the maximum value of r_{ij} and r_{ji}. As $r_{ij} \geqslant r_{ji}$, the representative index of the cluster is financial ratio i.

Definition 2.4. As Y_i, Y_j and Y_k are in the same one cluster, the representative index of the cluster is found according to the maximum value of $r_{ij} + r_{ik}$,$r_{ji} + r_{jk}$ and $r_{ki} + r_{kj}$. If $r_{ij} + r_{ik}$ is the maximum value, then the representative index of the cluster is financial ratio i.

Definition 2.5. As Y_i belongs to cluster T, and the element number of T is more than 3. The representative index of the cluster is financial ratio i as $\sum_{j(\neq i)\in T} r_{ij} > \sum_{j(\neq k)\in T} r_{kj}$, for all $k \in T$, but $k \neq i$. Thus we can find the representative indices of financial ratios by these above definitions.

3. Fuzzy sets and fuzzy numbers

We review some notions of fuzzy sets and fuzzy numbers (Zadeh, 1965; Zimmermann, 1987, 1991) in this section. These notions are expressed as follows.

Definition 3.1. Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \to [0,1]$, where $\mu_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 3.2. A fuzzy set A of the universe set U is normal iff $\sup_{x\in U}\mu_A(x) = 1$.

Definition 3.3. A fuzzy set A of the universe set U is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geqslant (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U$, $\forall \lambda \in [0,1]$, where $\wedge$ denotes the minimum operator.

Definition 3.4. A fuzzy set A of the universe set U is a fuzzy number iff A is normal and convex on U.

Definition 3.5. A triangular fuzzy number A is a fuzzy number with piecewise linear membership function μ_A defined by

$$\mu_A(x) = \begin{cases} (x - a^l)/(a^m - a^l), & a^l \leqslant x < a^m, \\ 1, & x = a^m, \\ (a^r - x)/(a^r - a^m), & a^m < x \leqslant a^r, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a triplet (a^l, a^m, a^r) shown in Fig. 1.

Definition 3.6. Let $A = (a^l, a^m, a^r)$ and $B = (b^l, b^m, b^r)$ be two triangular fuzzy numbers. A distance measure function $d(A, B)$ can be defined (Chen, 2000):

$$d(A,B) = \sqrt{\frac{1}{3}[(a^l - b^l)^2 + (a^m - b^m)^2 + (a^r - b^r)^2]}.$$

Definition 3.7. Let A be a fuzzy number. A^L_α and A^U_α are defined as

$$A^L_\alpha = \inf_{\mu_A(z)\geqslant\alpha}(z)$$

and

$$A^U_\alpha = \sup_{\mu_A(z)\geqslant\alpha}(z),$$

respectively.

Definition 3.8. Let S be a set composed of fuzzy numbers $X_1, X_2, \ldots, X_n$, i.e. $S = \{X_1, X_2, \ldots, X_n\}$. Assume $L(S)$ and $U(S)$ to be two boundaries of the set S. Define

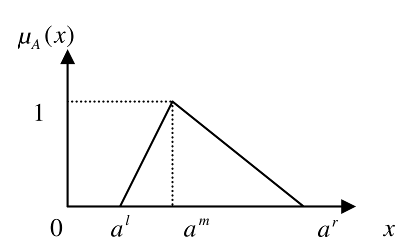

Fig. 1. The membership function of triangular fuzzy number A.

$$L(S) = \min\{x_j^l | j = 1, 2, \ldots, n\}$$

and

$$U(S) = \max\{x_j^r | j = 1, 2, \ldots, n\},$$

where X_j is in the interval $[x_j^l, x_j^r], j = 1, 2, \ldots, n$.

Definition 3.9. Let $R_{(S)}(X_j)$ denote the relation of X_j between $L(S)$ and $U(S)$, where $S = \{X_1, X_2, \ldots, X_n\}$. Define

$$R_{(S)}(X_j) = \frac{\int_0^1 ((X_j)_\alpha^L - L(S))\mathrm{d}\alpha}{\int_0^1 ((X_j)_\alpha^L - L(S))\mathrm{d}\alpha + \int_0^1 (U(S) - (X_j)_\alpha^U)\mathrm{d}\alpha}.$$

Lemma 3.1. *Let $A = (a^l, a^m, a^r)$ be a triangular fuzzy number on S. Thus*

$$R_{(S)}(A) = \frac{a^l + a^m - 2L(S)}{a^l - a^r + 2(U(S) - L(S))}.$$

Definition 3.10. Let $\succ$ be a binary relation on fuzzy numbers. Assume A and B to be two fuzzy numbers on S. $A \succ B$ iff $R_{(S)}(A) \geqslant R_{(S)}(B)$, then A is said to be bigger than or equal to B.

Lemma 3.2. *$\succ$ is a partial ordering relation* (Wang & Lee, 2007) *on fuzzy numbers.*

Definition 3.11. Let S indicate a set composed of $X_1, X_2, \ldots, X_n$. Define

$$\begin{aligned} X^+ &= Up(S) \\ &= Up\{X_1, X_2, \ldots, X_n\} \text{ to be the fuzzy maximum value on } S \end{aligned}$$

and

$$X^- = Lo(S) = Lo\{X_1, X_2, \ldots, X_n\} \text{ to be the } \times \text{ fuzzy minimum value on } S,$$

where

$$X^+ = X_i \quad \text{if } X_i \succ X_t \forall X_t \in S,$$
$$\text{i.e. } \max\{R_{(S)}(X_t) | t = 1, 2, \ldots, n\} = R_{(S)}(X_i)$$

and

$$X^- = X_j \quad \text{if } X_t \succ X_j \ \forall X_t \in S,$$
$$\text{i.e. } \min\{R_{(S)}(X_t) | t = 1, 2, \ldots, n\} = R_{(S)}(X_j)$$

for $t = 1, 2, \ldots, n$.

Lemma 3.3. *For $S = \{X_1, X_2, \ldots, X_n\}$, $Up(S)$ and $Lo(S)$ satisfy the partial ordering relation on S.*

4. FMCGDM applied in financial performance evaluation

Commonly, performance ratings and criteria weights are known precisely in the classical MCDM problems. As the ratings and weights are uncertain or expressed in several periods, the FMCDM problems are still solved by classical MCDM methods. However, the ratings and weights may be transformed into crisp values, and then some messages are lost. To avoid the losing of messages, the classical MCDM methods will be generalized under uncertain environment. In real world, the evaluation of container lines' financial performance is commonly one of FMCDM problems. Thus we apply FMCGDM method (Wang & Lee, 2007) to solve the FMCDM problem for reserving the messages. The FMCGDM method is the extension of TOPSIS (Hwang & Yoon, 1981) under fuzzy environment, and TOPSIS is one of famous MCDM methods. Generally, most steps of TOPSIS can be easily extended to uncertain environment, except max and min operations in finding ideal and anti-ideal solutions. In Section 3, we express two operators, *Up* and *Lo*, to be the extension of max and min on fuzzy environment. Based on *Up* and *Lo*, the FMCGDM method can extend TOPSIS under fuzzy environment. To evaluate financial performance of container lines, the ratings are expressed with fuzzy numbers in a given period. Moreover, the importance of weights are expressed with linguistic terms by the experts' opinions, and then transformed and aggregated into fuzzy numbers. Thus the FMCGDM method can be applied to evaluate the above problem. To demonstrate the evaluating procedure more clearly, we will present the FMCGDM as below.

We first formulate a FMCDM problem about the comparative evaluation of financial performance of container lines. The FMCDM problem involves m alternatives evaluated on n financial indices. Thus the problem can be modeled:

$$G = [G_{ij}]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ G_{m1} & G_{m2} & \cdots & G_{mn} \end{bmatrix} \end{array},$$

and

$$W = [W_1, W_2, \ldots, W_n],$$

where $A_1, A_2, \ldots, A_m$ are the possible alternatives which are evaluated, $C_1, C_2, \ldots, C_n$ are the criteria against performance of alternatives are measured, G_{ij} is the financial performance rating of alternative A_i on criteria C_j, and W_j is the related weight of C_j.

In the evaluating process, the weights expressed with the linguistic terms (Delgado, Verdegay, & Vila, 1992; Herrera, Herrera-Viedma, & Verdegay, 1996) represent the important degrees of criteria from finance experts via surveys on subjective assessments. These linguistic terms are categorized into very low (VL), low (L), medium (M), high (H) and very high (VH). Assume that all linguistic terms can be transferred into triangular fuzzy numbers, and these fuzzy numbers are limited in $[0,1]$. Thus the fuzzy numbers will not be normalized. On the other hand, the financial performance ratings are grasped easily as representative indices are determined. The values of these representative indices are set into performance ratings and the setting method is expressed as follows.

Let $b_{ij}(e)$ indicate the value of representative index j of alternative i on the period e, where $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$; $e = 1,2,\ldots,t$. Define

$$G_{ij} = (g_{ij}^l, g_{ij}^m, g_{ij}^r),$$

where

$$g_{ij}^l = \min\{b_{ij}(e)|e = 1,2,\ldots,t\},$$
$$g_{ij}^m = \frac{1}{t}\sum_{e=1}^{t} b_{ij}(e)$$

and

$$g_{ij}^r = \max\{b_{ij}(e)|e = 1,2,\ldots,t\}.$$

Let $[G_{i1}, G_{i2},\ldots,G_{in}]$ denote the performance ratings of alternative A_i on all criteria. By *Up* and *Lo*, anti-ideal and ideal solutions of alternatives are found.

Let

$$A^- = [G_1^-, G_2^-,\ldots,G_n^-]$$

and

$$A^+ = [G_1^+, G_2^+,\ldots,G_n^+]$$

be the anti-ideal solution and ideal solution respectively, where

$$G_j^- = Lo\{G_{ij}|i = 1,2,\ldots,m\} = (g_j^{l-}, g_j^{m-}, g_j^{r-})$$

and

$$G_j^+ = Up\{G_{ij}|i = 1,2,\ldots,m\} = (g_j^{l+}, g_j^{m+}, g_j^{r+}),$$

for $j = 1,2,\ldots,n$.

By Lemma 3.2, $G_j^+ \succ G_{ij} \succ G_j^-$, where $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$.

Assume that d_{ij}^- and d_{ij}^+ indicate the distance from G_{ij} to G_j^- and G_j^+, respectively, where

$$d_{ij}^- = d(G_{ij}, G_j^-) = \sqrt{\frac{1}{3}[(g_{ij}^l - g_j^{l-})^2 + (g_{ij}^m - g_j^{m-})^2 + (g_{ij}^r - g_j^{r-})^2]}$$

and

$$d_{ij}^+ = d(G_{ij}, G_j^+) = \sqrt{\frac{1}{3}[(g_{ij}^l - g_j^{l+})^2 + (g_{ij}^m - g_j^{m+})^2 + (g_{ij}^r - g_j^{r+})^2]}$$

for $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$.

Let $W_{jk} = (w_{jk}^l, w_{jk}^m, w_{jk}^r)$ be a triangular fuzzy number among which the weight of criterion C_j is expressed with linguistic term by expert E_k, and then set into the fuzzy number, where $j = 1,2,\ldots,n$; $k = 1,2,\ldots,p$. Assume W_j to be the average weight of criterion C_j, so

$$W_j = (w_j^l, w_j^m, w_j^r) = (1/p) \otimes (W_{j1} \oplus W_{j2} \oplus W_{j3} \oplus \ldots \oplus W_{jp}), \quad j = 1,2,\ldots,n,$$

where $\otimes$ and $\oplus$ are the extended multiplication and addition on fuzzy numbers.

By extension principle, we have

$$w_j^l = \sum_{k=1}^{p} w_{jk}^l / p,$$
$$w_j^m = \sum_{k=1}^{p} w_{jk}^m / p$$

and

$$w_j^r = \sum_{k=1}^{p} w_{jk}^r / p.$$

Let D_i^- and D_i^+ indicate the weighted distance from alternative A_i to anti-ideal solution A^- and ideal solution A^+ respectively. Define

$$D_i^- = \sum_{j=1}^{n} W_j \otimes d_{ij}^-$$

and

$$D_i^+ = \sum_{j=1}^{n} W_j \otimes d_{ij}^+,$$

where $i = 1,2,\ldots,m$.

The weighted distance from A_i to A^- and A^+ can be integrated into a vector $[D_i^-, D_i^+]$, where $i = 1,2,\ldots,m$.

Let

$$ND^- = Lo\{D_i^-|i = 1,2,\ldots,m\},$$
$$ND^+ = Up\{D_i^-|i = 1,2,\ldots,m\},$$
$$PD^- = Lo\{D_i^+|i = 1,2,\ldots,m\}$$

and

$$PD^+ = Up\{D_i^+|i = 1,2,\ldots,m\}.$$

Thus we assume that anti-ideal and ideal solutions of $[D_i^-, D_i^+]$ are $[ND^-, PD^+]$ and $[ND^+, PD^-]$, respectively, $i = 1,2,\ldots,m$. Let A_i^- and A_i^+ denote the distance summaries from $[D_i^-, D_i^+]$ to $[ND^-, PD^+]$ and $[ND^+, PD^-]$, respectively, so

$$A_i^- = d(D_i^-, ND^-) + d(D_i^+, PD^+)$$

and

$$A_i^+ = d(D_i^-, ND^+) + d(D_i^+, PD^-), \quad i = 1,2,\ldots,m.$$

Finally, the closeness coefficient A_i^* of alternative A_i is defined as

$$A_i^* = \frac{A_i^-}{A_i^- + A_i^+}, \quad i = 1,2,\ldots,m.$$

Obviously, $0 \leqslant A_i^* \leqslant 1, i = 1,2,\ldots,m$. As $A_i^* = 1$, alternative A_i is ideal solution. Oppositely, A_i will be anti-ideal solution as $A_i^* = 0$. Thus alternative A_i is closer to ideal solution and farther from anti-ideal solution as A_i^* approaches to 1. We can determine the ranking order of alternatives by their closeness coefficients, and then the best alternative is found.

5. Empirical study of Taiwan container lines

To present the evaluation of financial performance more clearly, we illustrate an empirical study of 3 Taiwan container lines in five periods. First, we express the values of financial ratios of three container lines indicated with A_1,A_2 and A_3 in Table 2.

After normalizing the values of Table 2, we can partition financial ratios into several clusters on the four categories by $\zeta = 0.5$ and a threshold value $r = 0.75$, and the classification result is shown in Table 3. Then the representative indices are found in this table as evaluation criteria to evaluate the financial performance of Taiwan container lines.

$$R_1 = \begin{bmatrix} 1 & 0.633 & 0.831 & 0.592 & 0.613 & 0.642 \\ 0.650 & 1 & 0.723 & 0.455 & 0.548 & 0.632 \\ 0.850 & 0.786 & 1 & 0.693 & 0.721 & 0.734 \\ 0.614 & 0.459 & 0.581 & 1 & 0.670 & 0.612 \\ 0.586 & 0.502 & 0.583 & 0.619 & 1 & 0.706 \\ 0.642 & 0.615 & 0.650 & 0.591 & 0.728 & 1 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1 & 0.989 & 0.537 & 0.686 & 0.825 \\ 0.985 & 1 & 0.534 & 0.684 & 0.816 \\ 0.553 & 0.553 & 1 & 0.647 & 0.504 \\ 0.746 & 0.767 & 0.752 & 1 & 0.772 \\ 0.835 & 0.830 & 0.491 & 0.675 & 1 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 1 & 0.580 & 0.733 & 0.740 & 0.738 \\ 0.594 & 1 & 0.480 & 0.668 & 0.657 \\ 0.817 & 0.636 & 1 & 0.757 & 0.769 \\ 0.670 & 0.574 & 0.544 & 1 & 0.813 \\ 0.657 & 0.554 & 0.572 & 0.810 & 1 \end{bmatrix},$$

and

$$R_4 = \begin{bmatrix} 1 & 0.746 & 0.767 & 0.752 & 0.772 \\ 0.794 & 1 & 0.711 & 0.689 & 0.716 \\ 0.751 & 0.668 & 1 & 0.888 & 0.707 \\ 0.804 & 0.689 & 0.959 & 1 & 0.715 \\ 0.757 & 0.637 & 0.652 & 0.628 & 1 \end{bmatrix}.$$

According to these above matrices, we can partition financial ratios into several clusters on the four categories by threshold value $r = 0.75$, and the classification result is shown in Table 3. Then the representative indices are found in this table as evaluation criteria to evaluate the financial performance of Taiwan container lines.

On the other hand, the important degrees of fifteen criteria weights are given with linguistic terms, i.e. VL, L, M, H, VH, employed by four financial experts E_1, E_2, E_3, E_4 shown in Table 4.

The linguistic rating set $\{\mathrm{VL}, \mathrm{L}, \mathrm{M}, \mathrm{H}, \mathrm{VH}\}$ is employed to present five states of weights in terms of triangular fuzzy numbers, where $\mathrm{VL} = (0, 0, 0.3)$, $\mathrm{L} = (0, 0.3, 0.5)$, $\mathrm{M} = (0.3, 0.5, 0.7)$, $\mathrm{H} = (0.5, 0.7, 1)$ and $\mathrm{VH} = (0.7, 1, 1)$. Then the average criteria weights are derived from Table 4, i.e.

Table 2
The values of financial ratios of three container lines in five periods

Period	A_1					A_2					A_3				
	1th	2th	3th	4th	5th	1th	2th	3th	4th	5th	1th	2th	3th	4th	5th
F1	27.84	26.69	25.47	23.67	19.13	41.86	39.68	34.22	47.79	40.91	60.05	57.1	56.87	54.36	55.33
F2	66.52	61.05	50.26	44.7	35.57	62.1	81.79	97.61	115.87	129.04	201.7	203.33	218.06	68.04	73.31
F3	19.63	18.57	16.9	15.47	12.44	25.01	26.72	25.34	33.84	31.06	46.27	44.58	45.11	30.22	31.53
F4	42.91	48.19	49	48.34	45.61	51.27	45.32	40.3	47.38	43.64	39.65	39.99	41.6	57.94	52.85
F5	133.05	107.51	104.07	106.87	119.25	95.04	120.64	148.15	111.06	129.14	152.2	150.05	140.38	72.6	89.21
F6	238.92	228.73	197.32	188.85	185.91	148.34	206.15	285.22	242.46	315.47	335.88	356.11	383.43	125.16	132.49
S1	104.02	94.65	118.86	128.23	196.13	336.43	333.38	342.18	221.46	213.69	130	125.48	124.23	172.43	194.11
S2	96.05	90.39	114.79	121.62	189.5	314.37	315.62	317.21	207.74	202.04	121.73	119.23	117.86	166.62	185.76
S3	9.54	14.58	44.92	42.85	6.7	21.05	22.12	17.79	14.22	19.55	15.89	20.57	23.69	16.61	33.85
S4	9.93	13.8	53.39	54.95	13.15	70.82	73.75	60.88	31.5	41.78	20.66	25.81	29.43	28.65	65.7
S5	3.86	−5.65	15.87	22.01	49.01	70.28	70	70.78	54.85	53.2	23.08	20.31	19.5	42	48.48
T1	1.83	0.89	0.95	0.79	0.98	3.1	2.68	2.63	1.54	1.66	1.84	1.55	1.43	1.22	1.96
T2	0.94	1.03	1	1.09	1.4	2.58	2.19	1.63	1.72	2.22	3	3.22	3.68	3.41	3.65
T3	88.96	87.26	86.84	98.08	121.47	147.68	128.87	125.12	111.81	132.78	76.96	78.98	71.82	97.71	96.57
T4	14.14	12.07	11.28	11.99	12.64	30.13	27.96	25.56	28.12	30.61	27.89	27.06	23.85	22.34	25.19
T5	24.77	23.29	22.12	23.21	23.24	61.82	51.13	42.82	53.43	54.31	46.21	45.1	40.85	53.12	53.43
P1	3.58	8	11.23	14.59	25.28	12.4	9.62	5.43	12.02	13.95	10.09	6.46	5.86	14.06	14.1
P2	−3.19	−0.9	3.92	8.13	19.28	9.8	6.06	2.92	9.59	10.89	6.36	0.83	1.36	10.47	11.14
P3	17.42	22.52	23.88	28.44	45.19	18.12	13.37	12.5	14.61	18.16	17.16	9	13.37	13.99	20.39
P4	17.05	18.67	21.42	26.43	38.21	13.92	10.8	9.73	12.77	13.69	14.81	8.61	12.13	11.73	17.28
P5	2.41	2.25	2.42	3.17	4.83	4.19	3.02	2.49	3.59	4.19	4.13	2.33	2.89	2.62	4.35

Table 3
The classification of financial ratios and the found of representative indices

Category	Cluster	Ratios within each cluster	Representative indices of each cluster
Financial structure	C_1	F1, F3	F3 (fixed assets to long term capital ratio)
	C_2	F2	F2 (fixed assets to long term liabilities ratio)
	C_3	F4	F4 (debt to total assets ratio)
	C_4	F5	F5 (Stockholder's equity to total liabilities ratio)
	C_5	F6	F6 (working capital to total assets ratio)
Solvency	C_6	S1, S2, S5	S1 (current ratio)
	C_7	S3	S3 (cash ratio)
	C_8	S4	S4 (cash flow ratio)
Turnover	C_9	T1	T1 (accounts payable turnover)
	C_{10}	T2	T2 (accounts receivable turnover)
	C_{11}	T3	T3 (fixed assets turnover)
	C_{12}	T4, T5	T4 (total assets turnover)
Profitability	C_{13}	P1, P5	P1 (gross profit ratio)
	C_{14}	P2	P2 (operation profit ratio)
	C_{15}	P3, P4	P4 (net income ratio)

$$W_1 = (0.6, 0.85, 1), \quad W_2 = (0.5, 0.725, 0.925),$$
$$W_3 = (0.5, 0.725, 0.925), \quad W_4 = (0.4, 0.625, 0.775),$$
$$W_5 = (0.4, 0.6, 0.85), \quad W_6 = (0.5, 0.725, 0.925),$$
$$W_7 = (0.4, 0.6, 0.85), \quad W_8 = (0.4, 0.6, 0.85),$$
$$W_9 = (0.275, 0.5, 0.725), \quad W_{10} = (0.4, 0.6, 0.85),$$
$$W_{11} = (0.5, 0.7, 1), \quad W_{12} = (0.4, 0.6, 0.85),$$
$$W_{13} = (0.5, 0.725, 0.925), \quad W_{14} = (0.6, 0.85, 1),$$
$$W_{15} = (0.6, 0.85, 1).$$

Since one representative index on a container line has five values from 1th period to 5th period, the values of representative indices will be set into triangular fuzzy numbers and the setting method is presented as below. Let $b_{ij}(e)$ be the value of index j of container line i on the period e, where $i = 1, 2, \ldots, 3$, $j = 1, 2, \ldots, 15$ and $e = 1, 2, \ldots, 5$. The value of container line i on index j presented with triangular fuzzy number $(g_{1ij}, g_{2ij}, g_{3ij})$ is defined as

Table 4
The linguistic weights of fifteen criteria given by four experts

	E_1	E_2	E_3	E_4
C_1	H	VH	H	VH
C_2	M	H	VH	H
C_3	H	M	VH	H
C_4	VH	M	M	M
C_5	M	H	H	M
C_6	H	M	H	VH
C_7	M	H	H	M
C_8	H	H	M	M
C_9	L	H	M	M
C_{10}	H	M	H	M
C_{11}	H	H	H	H
C_{12}	H	M	M	H
C_{13}	M	VH	H	H
C_{14}	H	VH	H	VH
C_{15}	H	H	VH	VH

Table 5
The fuzzy numbers of normalized representative indices among five periods

	A_1	A_2	A_3
C_1	(0.271, 0.318, 0.350)	(0.445, 0.555, 0.706)	(0.630, 0.755, 0.829)
C_2	(0.206, 0.265, 0.316)	(0.281, 0.541, 0.846)	(0.480, 0.732, 0.912)
C_3	(0.543, 0.584, 0.646)	(0.530, 0.568, 0.660)	(0.510, 0.573, 0.650)
C_4	(0.454, 0.554, 0.627)	(0.425, 0.585, 0.655)	(0.426, 0.571, 0.681)
C_5	(0.382, 0.492, 0.569)	(0.339, 0.574, 0.810)	(0.340, 0.596, 0.767)
C_6	(0.257, 0.364, 0.562)	(0.612, 0.805, 0.905)	(0.324, 0.425, 0.559)
C_7	(0.169, 0.534, 0.891)	(0.296, 0.506, 0.751)	(0.345, 0.564, 0.854)
C_8	(0.133, 0.377, 0.790)	(0.453, 0.714, 0.951)	(0.278, 0.438, 0.832)
C_9	(0.276, 0.352, 0.453)	(0.604, 0.753, 0.837)	(0.455, 0.536, 0.713)
C_{10}	(0.231, 0.263, 0.311)	(0.393, 0.500, 0.634)	(0.738, 0.819, 0.887)
C_{11}	(0.471, 0.527, 0.595)	(0.628, 0.708, 0.782)	(0.408, 0.462, 0.549)
C_{12}	(0.296, 0.310, 0.326)	(0.686, 0.711, 0.743)	(0.590, 0.630, 0.664)
C_{13}	(0.219, 0.602, 0.815)	(0.394, 0.556, 0.757)	(0.425, 0.507, 0.616)
C_{14}	(−0.260, 0.328, 0.778)	(0.439, 0.678, 0.980)	(0.134, 0.403, 0.640)
C_{15}	(0.643, 0.792, 0.866)	(0.310, 0.414, 0.525)	(0.371, 0.430, 0.558)

Table 6
The anti-ideal and ideal solutions of three alternatives on fifteen criteria

	A^-	A^+
C_1	(0.271, 0.318, 0.350)	(0.630, 0.755, 0.829)
C_2	(0.206, 0.265, 0.316)	(0.480, 0.732, 0.912)
C_3	(0.510, 0.573, 0.650)	(0.543, 0.584, 0.646)
C_4	(0.454, 0.554, 0.627)	(0.426, 0.571, 0.681)
C_5	(0.382, 0.492, 0.569)	(0.340, 0.596, 0.767)
C_6	(0.257, 0.364, 0.562)	(0.612, 0.805, 0.905)
C_7	(0.296, 0.506, 0.751)	(0.345, 0.564, 0.854)
C_8	(0.133, 0.377, 0.790)	(0.453, 0.714, 0.951)
C_9	(0.276, 0.352, 0.453)	(0.604, 0.753, 0.837)
C_{10}	(0.231, 0.263, 0.311)	(0.738, 0.819, 0.887)
C_{11}	(0.408, 0.462, 0.549)	(0.628, 0.708, 0.782)
C_{12}	(0.296, 0.310, 0.326)	(0.686, 0.711, 0.743)
C_{13}	(0.425, 0.507, 0.616)	(0.219, 0.602, 0.815)
C_{14}	(−0.260, 0.328, 0.778)	(0.439, 0.678, 0.980)
C_{15}	(0.310, 0.414, 0.525)	(0.643, 0.792, 0.866)

Table 7
The distance values from alternatives to anti-ideal/ideal solution on criteria

	A_1		A_2		A_3	
	$d(G_{1j}, G_j^-)$	$d(G_{1j}, G_j^+)$	$d(G_{2j}, G_j^-)$	$d(G_{2j}, G_j^+)$	$d(G_{3j}, G_j^-)$	$d(G_{3j}, G_j^+)$
C_1	0	0.4279	0.2666	0.1726	0.4279	0
C_2	0	0.4649	0.3477	0.1637	0.4649	0
C_3	0.0202	0	0.0132	0.0144	0	0.0202
C_4	0	0.0365	0.0294	0.0171	0.0365	0
C_5	0	0.1314	0.1491	0.0279	0.1314	0
C_6	0	0.3822	0.3822	0	0.0523	0.3401
C_7	0.1103	0.1053	0	0.0739	0.0739	0
C_8	0	0.284	0.284	0	0.094	0.2008
C_9	0	0.3723	0.3723	0	0.2109	0.168
C_{10}	0	0.5471	0.2495	0.3081	0.5471	0
C_{11}	0.0586	0.1755	0.2332	0	0	0.2332
C_{12}	0	0.4028	0.4028	0	0.3178	0.0857
C_{13}	0.1742	0	0.088	0.1097	0	0.1742
C_{14}	0	0.4662	0.4662	0	0.2449	0.3078
C_{15}	0.3512	0	0	0.3512	0.0411	0.3162

$$g_{1ij} = \min\{b_{ij}(e) | e = 1, 2, \ldots, 5\},$$
$$g_{2ij} = \frac{1}{5}\sum\nolimits_{e=1}^{5} b_{ij}(e)$$

and

$$g_{3ij} = \max\{b_{ij}(e) | e = 1, 2, \ldots, 5\}.$$

By the above method, the normalized representative indices presented with fuzzy numbers are shown in Table 5.

According to Table 5, the anti-ideal and ideal solutions on fifteen criteria derived by Definition 3.11 are shown in Table 6.

Then the distance values from three alternatives to anti-ideal/ideal solution on fifteen criteria are derived. Table 7 lists the distance $d(G_{ij}, G_j^-)$ and $d(G_{ij}, G_j^+)$ from G_{ij} to G_j^- and G_j^+, respectively, where $i = 1, 2, 3$; $j = 1, 2, \ldots, 15$.

As these distance values of Table 7 multiplies by their related weights, the weighted distance summaries are derived:

$$D_1^- = (0.3814, 0.5467, 0.6835),$$
$$D_2^- = (1.5201, 2.2444, 2.9500),$$
$$D_3^- = (1.2252, 1.8086, 2.3631),$$
$$D_1^+ = (1.7529, 2.5882, 3.4012),$$
$$D_2^+ = (0.6290, 0.9105, 1.1517)$$

and

$$D_3^+ = (0.9191, 1.3371, 1.717).$$

Thus

$$ND^- = (0.3814, 0.5467, 0.6835),$$
$$ND^+ = (1.5201, 2.2444, 2.9500),$$
$$PD^- = (0.6290, 0.9105, 1.1517)$$

and

$$PD^+ = (1.7529, 2.5882, 3.4012).$$

Then A_i^- and A_i^+ $(i = 1, 2, 3)$ can be derived as below.

$$A_1^- = d(D_1^-, ND^-) + d(D_1^+, PD^+) = 0 + 0 = 0,$$
$$A_2^- = d(D_2^-, ND^-) + d(D_2^+, PD^+) = 1.7622 + 1.7453 = 3.5075,$$
$$A_3^- = d(D_3^-, ND^-) + d(D_3^+, PD^+) = 1.3071 + 1.3035 = 2.6106,$$
$$A_1^+ = d(D_1^-, ND^+) + d(D_1^+, PD^-) = 1.7622 + 1.7453 = 3.5075,$$
$$A_2^+ = d(D_2^-, ND^+) + d(D_2^+, PD^-) = 0 + 0 = 0,$$
$$A_3^+ = d(D_3^-, ND^+) + d(D_3^+, PD^-) = 0.4551 + 0.4419 = 0.8970.$$

The closeness coefficients of the three container lines are

$$A_1^* = \frac{0}{0 + 3.5075} = 0,$$
$$A_2^* = \frac{3.5075}{3.5075 + 0} = 1$$

and

$$A_3^* = \frac{2.6106}{2.6106 + 0.8970} = 0.7443.$$

Thus the ranking order of the three container liners is $A_2(1) > A_3(0.7443) > A_1(0)$.

6. Conclusions

In this paper, we combine grey relation analysis with FMCGDM to evaluate the financial performance of container lines, and then illustrate an empirical study about the financial performance evaluation of three container lines in Taiwan. According to the computation procedure, it is obvious that the financial performance of three container lines is evaluated easy and simple. Although the number of Taiwan major container lines is merely three, the FMCGDM can solve the evaluation problems among which the number of alternatives is large as well. On the other hand, the ranking order of financial performance identifies the competitive location of a container line on finance. It is useful for that the container line can realize the finance competition strength and weakness on market, and then ready for improving the competitive ability to enhance finance strength in the future.

References

Boender, C. G. E., de Graan, J. G., & Lootsma, F. A. (1989). Multi-attribute decision analysis with fuzzy pairwise comparisons. *Fuzzy Sets and Systems, 29*, 133–143.

Chang, Y. H., & Yeh, C. H. (2002). A survey analysis of service quality for domestic airlines. *European Journal of Operational Research, 139*, 166–177.

Chen, C. T. (2000). Extensions to the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems, 114*, 1–9.

Chen, S. J., & Hwang, C. L. (1992). Fuzzy multiple attribute decision making methods and application. *Lecture notes in economics and mathematical systems*. New York: Springer.

Delgado, M., Verdegay, J. L., & Vila, M. A. (1992). Linguistic decision-making models. *International Journal of Intelligent System, 7*, 479–492.

Deng, J. (1989). Introduction to grey system theory. *The Journal of Grey System, 1*, 1–24.

Deogun, J. S., Kratsch, D., & Steiner, G. (1997). An approximation algorithm for clustering graphs with dominating diametral path. *Information Processing Letters, 61*, 121–127.

Dubes, R., & Jains, A. (1988). *Algorithm that cluster data*. Englewood Cliffs, NJ: Prentice-Hall.

Duda, R. O., & Hart, P. E. (1973). *Pattern classification and scene analysis*. New York: Wiley.

Eom, K. B. (1999). Fuzzy clustering approach in supervised sea-ice classification. *Neurocomputing, 25*, 149–166.

Feng, C. M., & Wang, R. T. (2000). Performance evaluation for airlines including the consideration of financial ratios. *Journal of Air Transport Management, 6*, 133–142.

Herrera, F., Herrera-Viedma, E., & Verdegay, J. L. (1996). A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets and Systems, 78*, 73–87.

Hirano, S., Sun, X., & Tsumoto, S. (2004). Comparison of clustering methods for clinical databases. *Information Sciences, 159*, 155–165.

Hsu, H. M., & Chen, C. T. (1997). Fuzzy credibility relation method for multiple criteria decision-making problems. *Information Sciences, 96*, 79–91.

Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and application*. New York: Springer.

Jain, R. (1978). A procedure for multi-aspect decision making using fuzzy sets. *The International Journal of Systems Sciences, 8*, 1–7.

Johnson, R. A., & Wichern, D. W. (1992). *Applied multivariate statistical analysis*. Englewood Cliffs, NJ: Prentice-Hall.

Kaufman, L., & Rousseeuw, P. J. (1990). *Finding groups in data: An introduction to cluster analysis*. New York: Wiley.

Keeney, R., & Raiffa, H. (1976). *Decision with multiple objective: Preference and value tradeoffs*. New Work: Wiley.

Krishnapuram, R., & Keller, J. M. (1993). A possibilistic approach to clustering. *IEEE Transactions on Fuzzy Systems, 1*, 98–110.

Lee, H. S. (1999). Automatic clustering of business processes in business systems. *European Journal of Operational Research, 114*, 354–362.

Lee, H. S. (2005). A fuzzy multi-criteria decision making model for the selection of the distribution center. *Lecture notes in artificial intelligence, 3612*, 1290–1299.

Liang, G. S. (1999). FMCDM based on ideal and anti-ideal concepts. *European Journal of Operational Research, 112*, 682–691.

Miyamoto, S. (2003). Information clustering based on fuzzy multisets. *Information Processing and Management, 39*, 195–213.

Ostrowski, P. L., O'Brien, T. V., & Gordon, G. L. (1993). Service quality and customerloyalty in the commercial airline industry. *Journal of Travel Research, 32*, 16–24.

Parasurman, A., Zeithaml, V. A., & Berry, L. L. (1985). SERVQUAL: A multi-item scale for measuring consumer perceptions of service quality. *Journal of Retailing, 64*, 38–39.

Pedrycz, W., & Vukovich, G. (2002). Logic-oriented fuzzy clustering. *Pattern Recognition Letters, 23*, 1515–1527.

Truitt, L. J., & Haynes, R. (1994). Evaluating service quality and productivity in the regional airline industry. *Transportation Journal, 33*, 21–32.

Tsaur, S. H., Chang, T. Y., & Yen, C. H. (2002). The evaluation of airline service quality by FMCDM. *Tourism Management, 23*, 107–115.

Walter, B. M., & Robert, F. M. (1988). *Accounting: The basis for business decisions*. New York: McGraw-Hill.

Wang, Y. J., & Lee, H. S. (2007). Generalizing the TOPSIS for fuzzy multiple-criteria group decision-making. *Computers and Mathematics with Applications, 53*, 1762–1772.

Wang, Y. J., Lee, H. S., & Lin, K. (2003). Fuzzy TOPSIS for multi-criteria decision-making. *International Mathematical Journal, 3*, 367–379.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.

Zimmermann, H. J. (1987). *Fuzzy Set, decision making and expert system*. Boston: Kluwer.

Zimmermann, H. J. (1991). *Fuzzy set theory – And its application* (2nd ed.). Boston: Kluwer.