Unit - 1 PDE

1) Formation of PDE
- (a) By elimination of arbitrary const.
- (b) By elimination of arbitrary functⁿ

Linear PDE of first order :

1) Lagrange's eqⁿ : $\boxed{Pp + Qq = R}$ —(1) $\boxed{\phi(u,v) = 0}$ — (2)

→ Working Rule : To ~~from~~ solve eqⁿ $[Pp + Qq = R]$

(1) Form AE $\left[\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}\right]$

(2) Solve AE by the method of grouping or the method of multiplier or both to get independent solⁿ $u = a$ & $v = b$, where a & b arbitrary const.

3) Then $\phi(u,v) = 0$ or $v = f(u)$ is general solⁿ.

Non-linear PDE of first order :

(1) Charpit's Method :- Charpit's Auxiliary eqⁿ :

$$\boxed{\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-\left(p\frac{\partial f}{\partial p} + q\frac{\partial f}{\partial q}\right)} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dF}{0}}$$

→ Working Rule :

1) Write given eqⁿ in form of $f(x, y, z, p, q) = 0$

2) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial p}$ and $\frac{\partial f}{\partial q}$.

3) consider Charpit's Auxiliary equation.

4) Find p (or q) from step (3) and use the given eqⁿ to find q (or p).

5) Use $\boxed{dz = p\,dx + q\,dy}$ and integrate to find z.

Chauchy's Method:

Ex: $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x + y$; $u(x,0) = 0$

Soln: The system of eqn is $\frac{dx}{1} = \frac{dy}{1} = \frac{du}{x+y}$ —— (1)

Taking the first two, we get

$x - y = c$ —— (2)

where c is const.

Now taking, $\frac{dy}{1} = \frac{du}{x+y} \Rightarrow \int \frac{dy}{1} = \int \frac{du}{2y+c}$

We get, $u(x,y) = y^2 + cy + c_1$ —— (3)

let $c_1 = g(c)$ then,

from (3) $u(x,y) = y^2 + cy + g(c)$

$= y^2 + y(x-y) + g(x-y)$ —— (4)

where $g(x-y)$ is an arbitrary functn.

Now apply initial condn:

$u(x,0) = 0 + 0 + g(x) = 0$

$g(x-y) = 0$

$u(x,y) = y^2 + y(x-y)$

$= y^2 + xy - y^2$

$u(x,y) = xy$ Ans

Homogeneous Linear Partial diff. eqn with const. coeff.:

Rules for finding CF:

→ Consider $\frac{\partial^2 z}{\partial x^2} + a_1 \frac{\partial^2 z}{\partial x \partial y} + a_2 \frac{\partial^2 z}{\partial y^2} = 0$

which in symbolic form: $D^2 + a_1 DD' + a_2 D'^2 = 0$ → also called AE

A.E :- $m^2 + a_1 m + a_2 = 0$.

find the roots, m_1, m_2.

Case (1): When AE has distinct roots:-

$$\left[CF = f_1(y + m_1 x) + f_2(y + m_2 x) \right]$$

Case (2): When A.E has equal roots:

$$\left[CF = f_1(y + mx) + x f_2(y + mx) \right]$$

Rules for finding PI:

Case (i): when $\phi(x, y)$ is fn of $ax + by$

$$\left[PI = \frac{1}{F(a,b)} \int \phi(u)\,du \right]$$

Case (ii) # For finding CF on Non-homogeneous pdE:

Ex: $(D + D' - 1)(D + 2D' - 2)z = 0$

$CF = e^{x} f_1(y - x) + e^{2x} f_2(y - 2x)$ A.

PI for non-homogeneous PDE:

Case (i): $\left[\frac{1}{F(D, D')} e^{ax+by} = \frac{1}{F(a,b)} e^{ax+by} \right]$, $F(a,b) \neq 0$

Unit-4

Probability $= \dfrac{\text{no. of favourable cases}}{\text{Total no. of cases}}$

Addition law of Probability :- If A & B are two events associated with an experiment then,

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Theorem : $\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)}$ ***

Multiplication : $\boxed{P(AB) = P(A) \times P(B)}$ **

Random variable : (i) Discrete Random variable
(ii) Continuous Random Variable.

Probability distribution :
(1) Discrete Prob. distribution :
(a) $[P(x) \geq 0$ for any value of $x]$
b) $\left[\sum_{i=0}^{n} = P[x_i] = 1\right]$

Mean :- $\boxed{\mu = \sum x_i p_i}$ other name of mean is avg or expected value E(X)

Expectation :- $[E(X) = \mu = \sum x_i p_i]$

Variance :- $\boxed{V(x) = \sum x_i^2 p_i - \mu^2}$ ***

Properties : (I) $E(X) = \mu$
(II) $E(a) = a$; where a is const.
(iii) $E(kX) = k\,E(X)$
IV) $E(X + k) = E(X) + k$
V) $E(aX \pm b) = aE(X) \pm b$

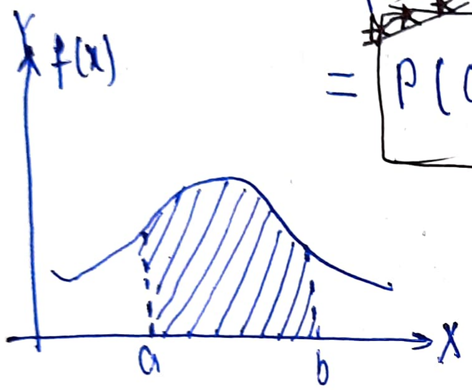
(2) Continuous Prob. Distribution :-

A fns $f(x)$ is said to be Prob. desity fns of x if.

(a) $f(x) \geq 0$, for $\forall\, x$

b) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

c) Area under $f(x)$ b/w ordinates $x=a$ & $x=b$



$= \boxed{P(a < X < b) = \int_a^b f(x)\, dx}$ ★★★

Note: (i) $P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b)$

(ii) Probability at a pt, $\boxed{P(X=a) = \int_{a-\Delta x}^{a+\Delta x} f(x)\, dx}$ ★★

* Mean : $\boxed{\mu = \int_{-\infty}^{\infty} x f(x)\, dx}$ ★★ * Expectation : $\boxed{E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx}$ ★★★

* Variance : $Var(X) = \boxed{V(x) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2}$ ★★★

* Cumulative distribution fn $F(x)$:

$\left[F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx \right]$; $\left[f(x) = \frac{d}{dx} F(x) = F'(x) \geq 0 \right]$ ★★

Properties : $F(-\infty) = 0$, $F(+\infty) = 1$

$0 \leq F(x) \leq 1$ for $-\infty < x < +\infty$

★★★ $\left[P(a < X < b) = F(b) - F(a) \right]$

Binomial Probability Distribution :

$$\boxed{P(r) = P(X=r) = {}^{n}C_r \, p^r q^{n-r}}$$

; n = no. of trials
r = no. of success
p = prob. of success
q = prob. of failure

$\boxed{\text{Mean } (\mu) = np}$, $\left[\text{Variance } (\sigma^2) = npq\right]$; $\boxed{p+q=1}$

* Condⁿ applicable for Binomial Distributions :
1) No. of trials must be finite (n is finite)
2) The trails are Independent.
3) Prob. of success in each trail remains const.

* Binomial distribution Fit : $\boxed{N(p+q)^n}$.

Poissons Distribution : [when $n \to \infty$ & $p \to 0$) Apply Poisson

$$\boxed{P(r) = \frac{e^{-\lambda}\lambda^r}{r!}}$$; $[\lambda = np]$

Normal Distribution (continuous) :-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(z)^2}$$

Working :
① convert $x \to z$
② z - area from table

Standard : $\boxed{Z = \frac{X-\mu}{\sigma}}$; μ = mean $(\bar{x})$, σ = standard deviation

→ The normal distribution is symmetrical about its mean.
→ It is a unimodal distribution. The mean, mode & median of this distribution coincide.

Baye's Theorem :-

$$\boxed{P(A_i/B) = \frac{P[A_i] \times P[B/A_i]}{\sum_{i=1}^{n} (P[A_i] \times P[B/A_i])}}$$

unit - 5

Procedure of Hypothesis testing :

(1) State the null & Alternate Hypothesis

(2) State a significance level (1%, 5%, 10% etc).

(3) Decide a test, t / f / chi-square / ANOVA etc.

(4) Calculate the value of test statistics.

(5) Find tabulated value from table.

(6) Compare calculated value with tabulated value

If, Cal. value < Tab. value
$\Rightarrow$ H_0 is accepted

If, cal. value > Tab. value
H_0 is rejected

d.f = n - 1

Unit - 5

(1) (a) t- test : [For sample size ≤ 30 & S is unknown].

$$\boxed{t = \frac{\bar{x} - \mu}{S/\sqrt{n}}}$$

where, $S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

μ = population mean, n = sample size

$\bar{x}$ = sample mean.

S = standard deviation of population.

→ If standard deviation of sample 's' is given,

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}}$$

; Note : $\boxed{ns^2 = (n-1)S^2}$

(b) t - Test for difference of mean of two small sample :

$$\boxed{t = \frac{(\bar{x} - \bar{y})}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

$[dof = n_1 + n_2 - 2]$

Note (i) $\boxed{S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$

or (ii) $\boxed{S^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$

(2) CHI - SQUARE (χ^2) test : (i) Test of goodness of fit

(ii) test of independence of attributes.

$$\boxed{\chi^2 = \sum_{i=n}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right]}$$

where, ; $\Sigma O_i = \Sigma E_i = N$ [total freq.]

$dof = n - 1$

(i) if $\chi^2 = 0$, the observed & theoretical freq. agree exactly

(ii) if $\chi^2 > 0$, do not agree exactly

O_i = Observed freq

E_i = Expected freq.

(ii) Test of independence :

→ Take Null Hypothesis independent.

$$\left[E_i = \frac{\text{Row sum} \times \text{Column sum}}{\text{Total sum.}} \right]$$

$\boxed{dof = (\text{Row}-1) * (\text{Column}-1)}$

F-test:

$$\boxed{F = \frac{S_1^2}{S_2^2}} \quad (S_1 > S_2)$$

Anova test :- Find (0) Let H_0, then

- Steps (1): Grand Total (GT) = $\sum A + \sum B + \sum C$

(2) Correction factor (C.F) = $\frac{(GT)^2}{N}$

(3) Sum of ~~Sample~~ squares b/w samples : $\boxed{(SSC) = \frac{(\sum A)^2}{n_1} + \frac{(\sum B)^2}{n_2} + \frac{(\sum C)^2}{n_3} - C.F}$

(4) Total sum of squares : $\boxed{SST = \sum A^2 + \sum B^2 + \sum C^2 - CF}$

5) Sum of squares within sample : $\boxed{SSE = SST - SSC}$

6) Now Make Table :

Source of variance	Sum of square	Dof	Mean square	
B/w samples (column means)	SSC	$C-1$	$MSC = \frac{SSC}{C-1}$	$F = \frac{MSC}{MSE}$
Within samples	SSE	$C(r-1)$	$MSE = \frac{SSE}{C(r-1)}$	
Total	SST	$Cr-1$		

7) Compare $F_{cal} < F_{tab}$.

Control chart :-

(1) Control chart for variables :

(i) For $\bar{X}$ - chart : Control Line (CL) = $\boxed{\bar{\bar{x}}}$ or y [when tolerance limit is given]

& $\boxed{y = \frac{1}{2}[LCL + UCL]}$

→ LCL (Lower control limit) = $\bar{\bar{x}} - A_2\bar{R}$ or $y - A_2\bar{R}$

→ (Upper control limit) UCL : $\bar{\bar{x}} + A_2\bar{R}$ or $y + A_2\bar{R}$

(ii) For R chart : $\boxed{CL = \bar{R}}$; where D_3 & D_4 depend on sample size & found from table.

→ LCL (for R chart) = $\boxed{D_3\bar{R}}$

→ UCL (for R-chart) = $\boxed{D_4\bar{R}}$

2) Control chart for Attributes :

(i) Control ~~CL~~ limits (3σ limit) on p-chart :- [When to used. Subgroup size is variable/ const.]

→ $\boxed{CL = \bar{p}}$; $\bar{p} = \frac{\text{no. of defective article found in any inspection}}{\text{Total no. of articles actually inspected}}$

→ $UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \bar{p} + 3\sigma_p$

→ $LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \bar{p} - 3\sigma_p$

(ii) Control limit for np - chart : (Subgroup size is const) $\boxed{CL = n\bar{p}}$; n = sample size

→ $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = n\bar{p} + 3\sigma_{np}$; $LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = n\bar{p} - 3\sigma_{np}$

(iii) Control limit for c - chart : (no. of deffects / used instead of defective)

$$CL = \bar{c} = \frac{\text{no. of defects in all sample}}{\text{Total no. of samples}}$$

→ $\boxed{UCL_c = \bar{c} + 3\sqrt{\bar{c}}}$; → $\boxed{LCL_c = \bar{c} - 3\sqrt{\bar{c}}}$

Unit-2:

Method of separation of variable:

1) let $u = XT$ — (1)
2) separate the variable, then equate with K $\left(\frac{aX'}{X} = \frac{bT'}{T} = K\right) = (-p^2)$
3) compare the separated variables with K.
4) Apply initial condⁿ after putting separated variable in (1)

~~1-D wave~~: Rules for finding CF:

(1) If n roots of AE are real & distinct say $m_1, m_2, \dots m_n$

$$\boxed{CF = C_1 e^{m_1 x} + C_2 e^{m_2 x} + \dots + C_n e^{m_n x}}$$

(2) If two or more roots are equal i.e. $m_1 = m_2 = \dots m_k$, $k \le n$

$$\boxed{CF = (C_1 + C_2 x + C_3 x^2 + \dots + C_k x^{k-1}) e^{m_1 x} + \dots + C_n e^{m_n x}}$$

(3) If A.E has a pair of imaginary roots i.e. $m_1 = \alpha + i\beta$, $m_2 = \alpha - i\beta$

$$\boxed{C.F = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x) + C_3 e^{m_3 x} + \dots + C_n e^{m_n x}}$$

1-D wave: $\boxed{\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}}$; where $c^2 = \frac{T}{\rho}$

→ The Boundary condⁿ:

(i) $y(0,t) = 0$ (ii) $y(l,t) = 0$

→ Initial condⁿ:

(i) $\frac{\partial y}{\partial t} = 0$ (when $t = 0$) (ii) $y = f(x)$, when $t = 0$

Fourier series:

(1) Half range cosine series:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}, \text{ where, } a_n = \frac{2}{L}\int_0^l f(x)\cos\frac{n\pi x}{L}\,dx$$

Half-range sine series :-

$$\left[f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \ , \quad \text{where } b_n = \frac{2}{L} \int_0^l f(x) \sin \frac{n\pi x}{l} \, dx \right]$$

One-dimensional Heat flow :

$$\boxed{\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}} \ , \quad c^2 = \frac{k^2}{sp} \text{ is known as}$$

diffusivity of material of the bar.

Let $u(x,t)$ be temp. in the bar

→ The boundary cond^n(s) are :-

(i) $u(0,t) = 0$, (ii) $u(l,t) = 0$

→ Initial cond^n :

(x) $[u = f(x)$ or $u(x,0) = f(x)]$

: Transient sol^n : if $u \downarrow \Rightarrow t \uparrow$

; steady state cond^n : variables are independent of the time 't'.

* Inhomogeneous Boundary cond^n(s) : $\alpha = 30°$, $80° = \beta$

→ initial temp. distribution : $\boxed{u_1(x,t) = \left[\alpha + \left(\frac{\beta - \alpha}{L}\right)x\right]}$

→ final temp. distribution : $\boxed{u_2(x,t) = \left[\alpha + \left(\frac{\beta - \alpha}{L}\right)x\right]}$

To get u in the intermediate period : $\boxed{u = u_1(x,t) + u_2(x)}$

→ u_2 does not depend on t, steady state temp. distrib. in rod

& $u_1(x,t)$ is transient temp distribution which tends to 0 as t increases.

∴ $u_1(x,t)$ satisfies 1-D heat flow eq^n : $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

$$\left[u = u_2 + \sum_{n=1}^{\infty} (c_1 \cos px + c_2 \sin px) e^{-c^2 p^2 t} \right]$$

2-D heat flow :

$$\boxed{c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{\partial u}{\partial t}}$$

→ In steady state, u is independent of t, so that $\frac{\partial u}{\partial t}=0$

→ Laplace' eqn in 2D : $\boxed{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0}$ —— (1)

→ Again $u = XY$ —— (2) Putting (2) in (1)

→ equating with $-p^2$ and find X and Y.

→ Multiply X and Y and apply boundary & initial condn

Trick :

$$\int f(x)\cdot g(y) = f(x)\int g(y) - d f(x)\iint g(y) + d^2 f(x)\iiint g(y)$$

Transmission Line eqn :-

$$\frac{\partial^2 V}{\partial x^2} = LC\cdot\frac{\partial^2 V}{\partial t^2} + (RC+LG)\frac{\partial V}{\partial t} + RGV \quad \text{—— (1)}$$

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} + (RC+LG)\frac{\partial I}{\partial t} + RGI \quad \text{—— (2)}$$

a) If $L=G=0$; $\boxed{\frac{\partial^2 V}{\partial x^2} = RC\frac{\partial^2 V}{\partial t^2} \ \& \ \frac{\partial^2 I}{\partial x^2} = RC\frac{\partial^2 I}{\partial t^2}}$ → telegraph eqn

b) If $R=G=0$; $\boxed{\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} \ \& \ \frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2}}$ → radio eqn

c) If R & G are negligible ; $\boxed{\frac{\partial V}{\partial x} = -L\frac{\partial i}{\partial t} \ \& \ \frac{\partial i}{\partial x} = -C\frac{\partial V}{\partial t}}$

d) If $L=C=0$; $\boxed{\frac{\partial^2 V}{\partial x^2} = RGV \ \& \ \frac{\partial^2 I}{\partial x^2} = RGI}$ → eqn for sub-marine cable.

Unit-3 :

★ Mean

1) $$\boxed{AM = \frac{\Sigma x}{n} = \frac{\Sigma fx}{\Sigma f} = a + \frac{\Sigma fd}{\Sigma f} = a + \frac{\Sigma fD}{\Sigma f} \times i}$$

Where, a = assumed mean, $\left[D = \frac{x-a}{i}\right]$, $[d = x-a]$

2) Median :- (a) Discrete data with freq.,

b) Continuous data with freq :-

N = total freq.

★ $$\boxed{\text{Median} = l + \frac{\frac{1}{2}N - C_f}{f} \times i}$$

where,
l = lower limit in median class
f = freq. of class.
i = class-length.

C_f = cumulative freq. of class preceding the median class

→ Median is decided by ~~N/2 freq~~; CF just greater than N/2

3) Mode : value of variable that occurs most frequently.

(b) continuous data with freq :

★ $$\boxed{\text{Mode} = l + \left(\frac{f - f_{-1}}{2f - f_{-1} - f_{+1}}\right) \times i}$$

→ For symmetrical distribution :

$$\boxed{\text{Mean} = \text{Mode} = \text{Median}}$$

Curve fitting : (Method of least square or Best fit)

Trick : eqn $y = a + bx$; coeff $a = (1)$, coeff of $b = x$

(1) multiply coeff. of a both side and take summation :

$\Sigma y = a\Sigma 1 + b\Sigma x$: $[\Sigma y = an + b\Sigma x]$

(2) Similarly multiply coeff of b both side of eqn : —"—

$[\Sigma xy = a\Sigma x + b\Sigma x^2]$ → Normal eqns

* Some eqn(s): (i) straight line: $y = a + bx$

(ii) Parabola: $y = a + bx + cy^2$

Fitting an exponential curve: $y = ae^{bx}$:

1) Taking log both side: $\log_{10} y = \log_{10} a + bx \log_{10} e$

$$\boxed{Y = A + BX}$$

Where, $Y = \log y$, $A = \log a$, $B = b \log_{10} e$

normal eqn:
$$\begin{cases} ① \ \Sigma Y = nA + B\Sigma X \\ ② \ \Sigma XY = \cancel{nA} A\Sigma X + B\Sigma X^2 \end{cases}$$

2) Solve to get A & B by making table

3) Take Antilog: $a = \text{antilog } A$, $b = \text{antilog} \dfrac{B}{\log_{10} e}$

4) Put value of a & b in main eqn to get best fit.

Correlation :- (1) Karl Pearson's coeff. of correlation: (r_{xy} or $r(x,y)$):

(i)*
$$\boxed{r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n\Sigma y^2 - (\Sigma y)^2}}} \rightarrow \text{(main)}$$

(or) (ii)

(iii)

2)' Rank correlation : (Spearman's Rank Correlation) :

$$\boxed{r = 1 - \frac{6 \sum D_i^2}{n(n^2-1)}}$$, D_i = diff in rank of i^{th} individual
n = no. of pairs.

Note : $[\sum D_i = \sum (x_i - y_i) = \sum x_i - \sum y_i = 0] \rightarrow$ always

* Tide Rank :

$$\boxed{r = 1 - \frac{6\left[\sum D_i^2 + \frac{1}{12} m_1(m_1^2-1) + \frac{1}{12} m_2(m_2^2-1) \dots\right]}{n(n^2-1)}}$$

Imp

Regression eqn / Line of regression :

Imp Line of Regression of y on x ; Line of regression of x on y :

$y = a + bx$

$\bar{y} = a + b\bar{x}$

$$\boxed{y - \bar{y} = b_{yx}(x - \bar{x})}$$

$x = a + by$

$\bar{x} = a + b\bar{y}$

$$[x - \bar{x} = b_{xy}(y - \bar{y})]$$

$\rightarrow [r = \sqrt{b_{yx} \cdot b_{xy}}$ or $\sqrt{b_1 \cdot b_2}]$

$\rightarrow$ $$\boxed{b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}}$$; $$b_{xy} = \frac{n \sum xy - \sum x \sum y}{n \sum y^2 - (\sum y)^2}$$

$\rightarrow [b_{yx} = r \frac{\sigma_y}{\sigma_x}]$