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Assignment - 3

Q-1 Explain chomsky classification of Languages.

Ans-1 Chomsky classification of Languages:

In the definition of a grammer (V_N, Σ, P, S), V_N and Σ are the sets of symbols and $S \in V_N$. So if we want to classify grammers, we have to do it only by considering the form of productions.

Chomsky classified the grammers into four types in terms of productions.

1 Type -0 grammer (Unrestricted grammer):

a) Type - 0 grammer are constructed with no restrictions on replacement rule, except that a non- terminal.

b) A non- terminal must appear in the string on the left side. The languages generated is called recursive enumerable language.

c) Thus, a type - 0 grammer is:

i) An alphabet Σ of terminal symbols.

ii) An alphabet V of non-terminals, including a start symbol.

iii) A set of production rule $\alpha \rightarrow \beta$, where α and β are from $(\Sigma \cup V)^*$, 'α' Contains at least one non-terminal and there is no restriction on 'β'.

d) The type-0 grammer is recognized by turing machines.

2) Type-1 grammer (Context sensitive grammer):-

a) A grammer is said to be type-1 grammer or context sensitive grammer if it follows the following conditions:

i) Each production in the form $\alpha \rightarrow \beta$, and length of 'α' is less than or equal to length of β i.e., there are non empty production, those in which right side is empty string C.

ii) Each production of the form $\alpha_i A a_2 \rightarrow \alpha_i \beta a_2$, ~~wth~~ with $\beta \neq \varepsilon$.

b) The turing machine can be constructed to recognize the Context-sensitive language generated by Context-sensitive grammer (CSG).

3) Type-2 grammer (Context Free grammer):-

a) A grammer is said to be type 2 grammer, if the production is in the form of

$A \rightarrow \alpha$, where A is a non-terminal and 'α' is a sentential form i.e., $a \in (V \cup T)^*$ i.e. α can be ε.

b) The left hand side of production must contain only one non-terminal.

c) The type-2 grammer can be recognized by push down automata.

4) Type-3 grammer (Regular grammer):-

a) A grammer is said to be type-3 grammer if the production is in the form $A \rightarrow a$ or $A \rightarrow aB$, i.e., the left hand side of each production should contain only one non-terminal or first symbol on right hand side must be a terminal and may be followed by a non-terminal.

b) The language generated by this grammer is recognized by finite state machine.

c) These regular languages can also be expressed by simpler expressions called regular expression.

Q-2 Convert the grammer $S \to AB$, $A \to BS|b$, $B \to SA|a$ into GNF.

Sol2- As the given grammer is in GNF, we can omit step 1 and proceed to step 2 after renaming S, A, B as A_1, A_2, A_3, respectively. The productions are

$A_1 \to A_2 A_3$, $A_2 \to A_3 A_1 | b$

$A_3 \to A_1 A_2 | a$

Step2:- i) The A_1-production $A_1 \to A_2 A_3$ is in the required form.

ii) The A_2- production $A_2 \to A_3 A_1 | b$ are in required form.

iii) $A_3 \to a$ is in the required form.

Check whether $A_i \to A_j$

if $i < j$ then ok

if $i \geq j$, then problem

Apply Lemma 1 to $A_3 \to A_1 A_2$. The resulting productions are $A_3 \to A_2 A_3 A_2$. Applying the Lemma once again to $A_3 \to A_2 A_3 A_2$, we get

$A_3 \to A_3 A_1 A_3 A_2 | b A_3 A_2$.

Step3:- The A_3- productions are $A_3 \to a | b A_3 A_2$ and $A_3 \to A_3 A_1 A_3 A_2$. As we have $A_3 \to A_3 A_1 A_3 A_2$. we have to apply Lemma 2 to A_3- productions. Let

Z_3 be the new variable. The resulting productions are

$A_3 \to a \mid bA_3A_2$, $A_3 \to aZ_3 \mid bA_3A_2Z_3$

$Z_3 \to A_1A_3A_2$, $Z_3 \to A_1A_3A_2Z_3$

Step 4:- i) The A_3- productions are

$$A_3 \to a \mid bA_3A_2 \mid aZ_3 \mid bA_3A_2Z_3 \quad -(1)$$

ii) Among the A_2- productions, we retain $A_2 \to b$ and eliminate $A_2 \to A_3A_1$ using Lemma I. The resulting production are

$A_2 \to aA_1 \mid bA_3A_2A_1 \mid aZ_3A \mid bA_3A_2Z_3A_1$

The modified A_2- productions are

$$A_2 \to b \mid aA_1 \mid bA_3A_2A_1 \mid aZ_3A_1 \mid bA_3A_2Z_3A_1 \quad -(2)$$

iii) We apply Lemma I to $A_1 \to A_2A_3$ to get

$$A_1 \to bA_3 \mid aA_1A_3 \mid bA_3A_2A_1A_3 \mid aZ_3A_1A_3 \mid bA_3A_2Z_3A_1A_3 \quad -(3)$$

Step 5:- The Z_3- productions to be modified are

$$Z_3 \to A_1A_3A_2 \mid A_1A_3A_2Z_3$$

We apply Lemma I and get

$Z_3 \to bA_3A_3A_2 \mid bA_3A_2Z_3$

$Z_3 \to aA_1A_3A_3A_2 \mid aA_1A_3A_3A_2Z_3$

$Z_3 \to bA_3A_2A_1A_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2Z_3 \quad -(4)$

$Z_3 \to aZ_3A_1A_3A_3A_2 \mid aZ_3A_1A_3A_3A_2Z_3$

$Z_3 \to bA_3A_2Z_3A_1A_3A_3A_2 \mid bA_3A_2Z_3A_1A_3A_3A_2Z_3$

is equivalent to GNF

The required grammer in GNF is given by (1-4).
The following example uses the Remark appearing after Theorem 1. In this example we retain productions of the form $A \to a\alpha$ and replace the terminals only when they appear as the second or subsequent symbol on R.H.S.

Q-3 Find a grammer in GNF form equivalent to $S \to aAbB$, $A \to aA/a$, $B \to bB/b$.

Sol3:- As there are no unit productions or null productions, we need not carry out step 1. We proceed to step 2.

Step 2:- Let $G_1 = (V_N' \{a, b\}, P_1, S)$, where P_1 and V_N' are constructed as follows:-

i) $A \to a$, $B \to b$ are added to P_1.
ii) $S \to aAbB$, $A \to aA$, $B \to bB$ yield
$S \to C_a A C_b B$, $A \to C_a A$, $B \to C_b B$,
$C_a \to a$, $C_b \to b$.

$$V_N' = \{S, A, B, C_a, C_b\}$$

Step 3:- P_1 consists of $S \to C_a C_b B$, $A \to C_a A$
$B \to C_b B$, $C_a \to a$, $C_b \to b$, $A \to a$, $B \to b$.

$S \to C_a A C_b B$ is replaced by $S \to C_a C_1$,
$C_1 \to A C_2$, $C_2 \to C_b B$.
The remaining productions in P_1 are added to P_2. Let
$G_2 = (\{S, A, B, C_a, C_b, C_1, C_2\}, \{a, b\},$
$P_2, S)$,
where P_2 consists of $S \to C_a C_1$,
$C_1 \to A C_2$, $C_2 \to C_b B$, $A \to C_a A$,
$B \to C_b B$, $C_a \to a$, $C_b \to b$, $A \to a$ and
$B \to b$.
G_2 is in CNF and equivalent to the given grammer.

Q-4 Construct a reduced grammer equivalent to the grammer
$S \to aAa$, $A \to Sb \mid bCC \mid DaA$,
$C \to abb \mid DD$, $E \to aC$, $D \to aDA$.

Sol-4 Step 1:- $W_1 = \{C\}$ as $C \to abb$ is the only production with a terminal string on the R.H.S.
$W_2 = \{C\} \cup \{E, A\}$
as $E \to aC$ and $A \to bCC$ are productions with R.H.S in $(\Sigma \cup \{C\})^*$
$W_3 = \{C, E, A\} \cup \{S\}$.

as $S \to aAa$ and aAa is in $(\Sigma \cup W_2)^*$.

$$W_4 = W_3 \cup \phi$$

Hence,

$$V_N' = W_3 = \{S, A, C, E\}$$

$$P' = \{A_1 \to \alpha \mid \alpha \in (V_N \cup \Sigma)^*\}$$

$$= \{S \to aAa, A \to Sb \mid bCC, C \to abb, E \to aC\}$$

$$G_1 = (V_N', \{a, b\}, P', S)$$

Step 2: We have to apply theorem to G_1.

we start with

$$W_1 = \{S\}$$

As we have $S \to aAa$,

$$W_2 = \{S\} \cup \{A, a\}$$

As $A \to Sb \mid bCC$,

$$W_3 = \{S, A, a\} \cup \{S, b, C\} = \{S, A, C, a, b\}$$

As we have $C \to abb$,

$$W_4 = W_3 \cup \{a, b\} = W_3$$

Hence,

$$P'' = \{A_1 \to \alpha \mid A_1 \in W_3\}$$

$$= \{S \to aAa, A \to Sb \mid bCC, C \to abb\}$$

Therefore,

$$G' = (\{S, A, C\}, \{a, b\}, P'', S)$$

is the reduced grammer.