

Lecture 7: Function Approximation, SGD, DQN

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Need for Function Approximation

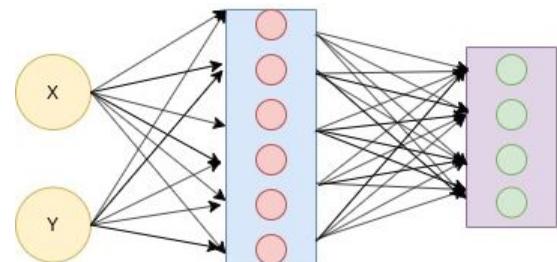
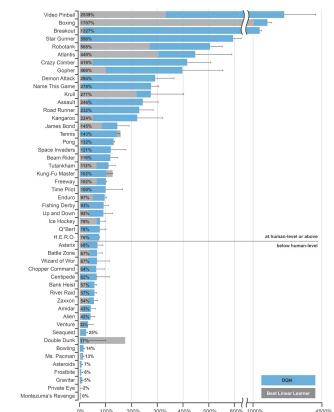
- Issues with large state/action spaces:
 - tabular approaches not memory efficient
 - data sparsity
 - continuous state/action spaces
 - generalization
- Use a parameterized representation
 - Value Functions
 - Policies
 - Models

Non-Linear Function Approximator

- Linear function approximators are very restrictive. Can only model linear functions. Basis expansion does help to generate non-linear functions in the original input space.
- Non-linear approximators can model complex functions and are very powerful.
- The features are learnt on the fly and are not hard-coded as is the case with tile and sparse coding.
- Can generalize to unseen states.

Disadvantage:-

Requires a lot of data and compute.



Gradient Descent

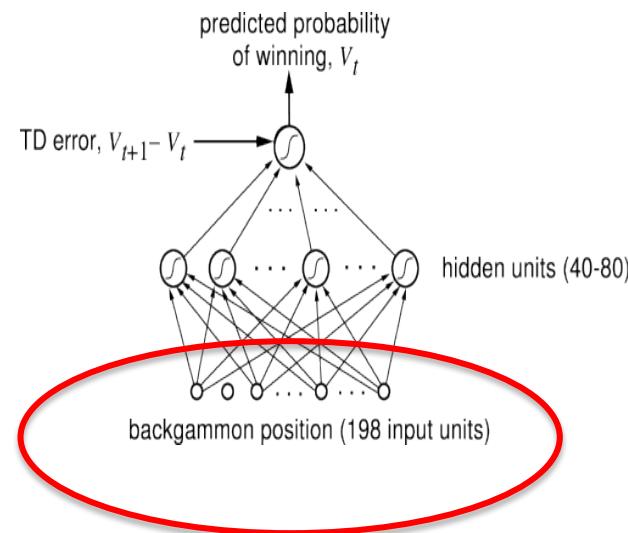
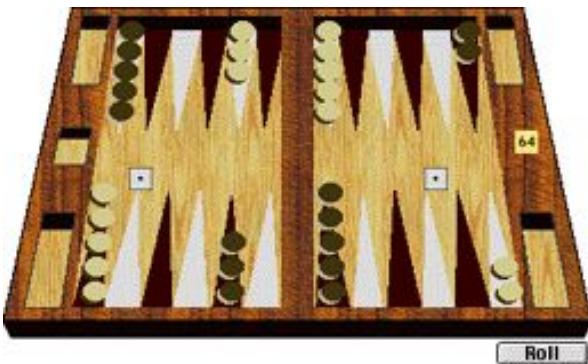
- **Gradient Descent** is a first-order iterative optimization algorithm for finding a local minimum of a differentiable function.
- In Gradient Descent, we compute the gradient of the function at the current point and take a step in the opposite direction. This is the direction of steepest descent.
- The logic behind Gradient Descent can be understood by considering the Taylor Series expansion of the function around the current point, up to first order.

$$f(x_0 + h) \approx f(x_0) + h \cdot f'(x_0)$$

Human Level Backgammon player

TD-Gammon (Tesauro 92, 94, 95)

- Beat the best human player in 1995.
- Learnt completely by *self play*.
- New moves not recorded by humans in centuries of play.



Semi Gradient Methods

While computing the gradient of the TD error in Q-learning, we typically ignore the gradient w.r.t the TD target. Hence, it is a **Semi Gradient** method i.e we are computing an approximation of the true gradient.

$$\hat{q}(s_t, a_t) = \phi^T(s_t, a_t) \times w_t$$

$$\delta_t = r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a) - \hat{q}(s_t, a_t) \quad \text{TD Error}$$

$$\nabla_{w_t} \left[r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a) - \hat{q}(s_t, a_t) \right]^2 = -2 \delta_t \phi(s_t, a_t)$$

$$w_{t+1} = w_t + \alpha \delta_t \phi(s_t, a_t)$$

Stochastic Gradient Descent

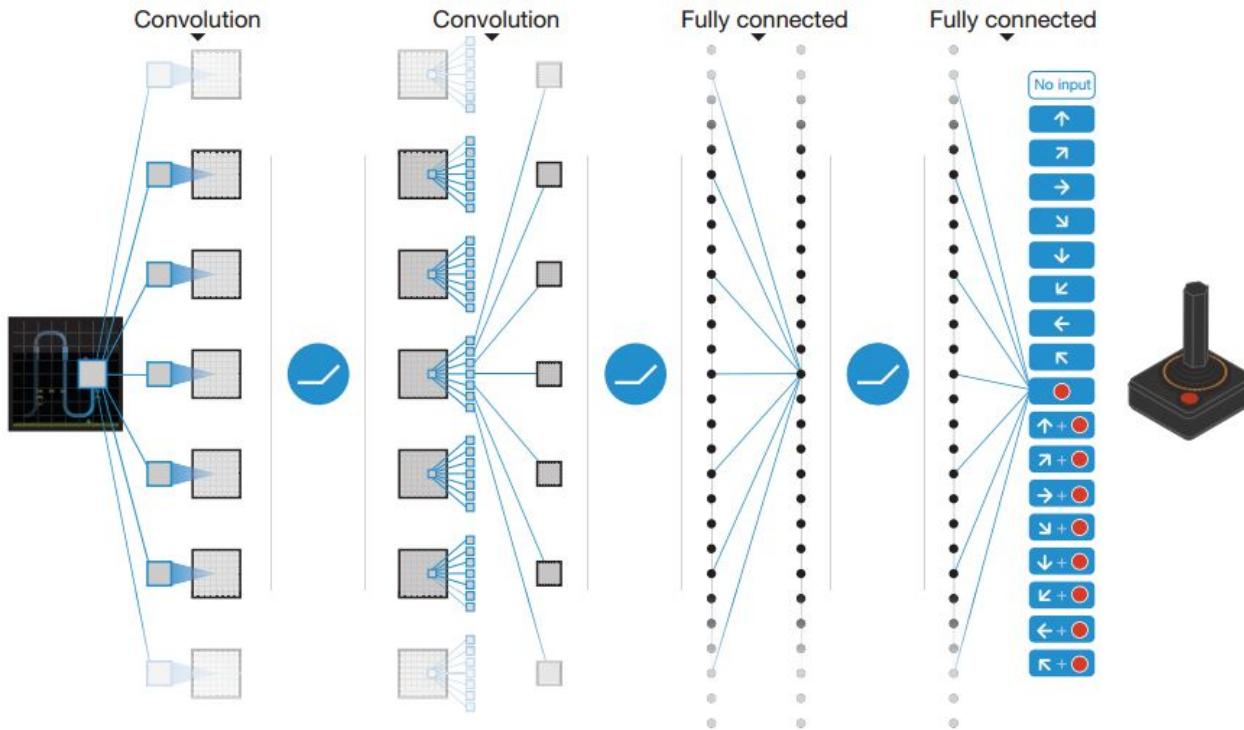
- In **Stochastic Gradient Descent**, the true gradient of the loss function is approximated by the gradient at a single example.
- In practice, we usually perform **Mini Batch Gradient Descent**, where we compute the gradient using a mini batch of examples. This allows for more efficient computation and smoother convergence.

What about the features?

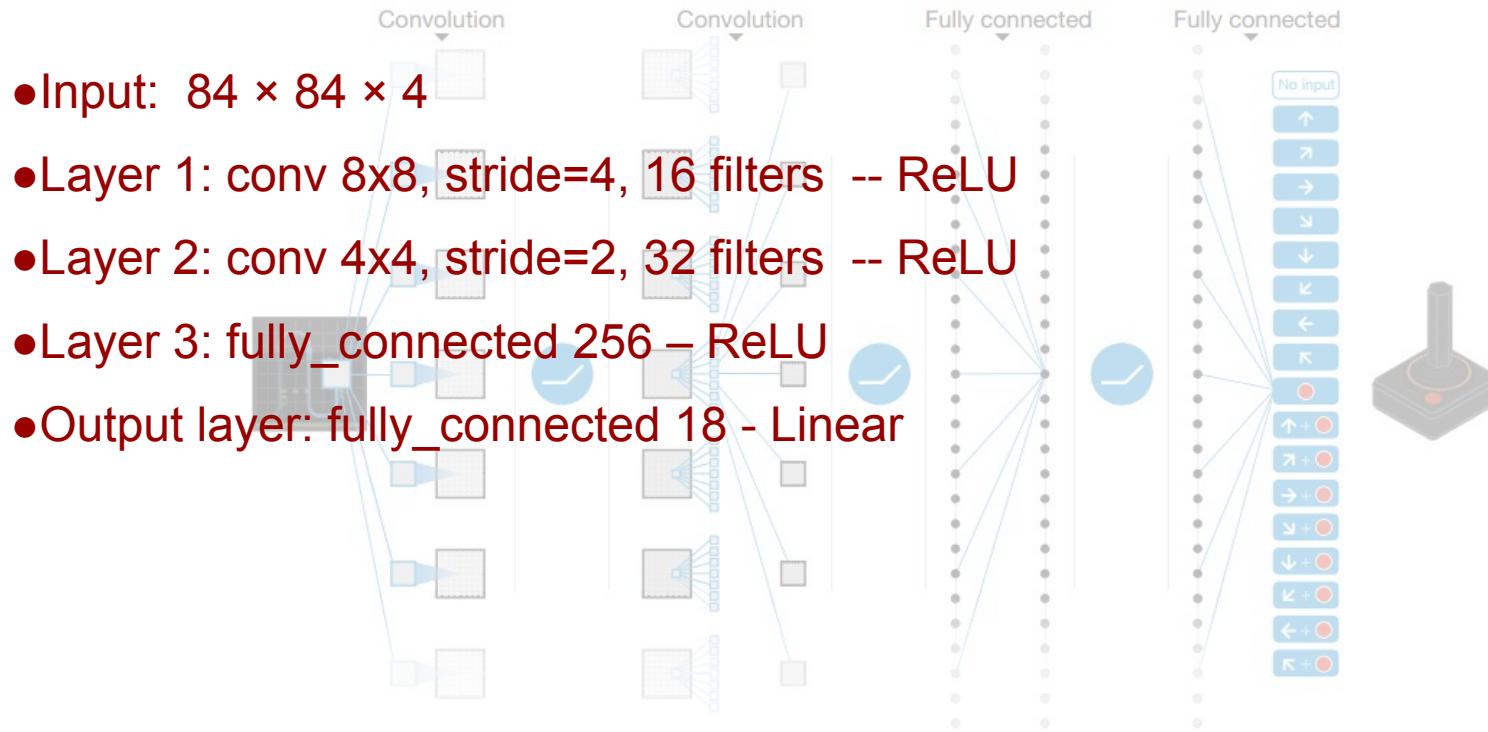
Learnt to play from video input from scratch!



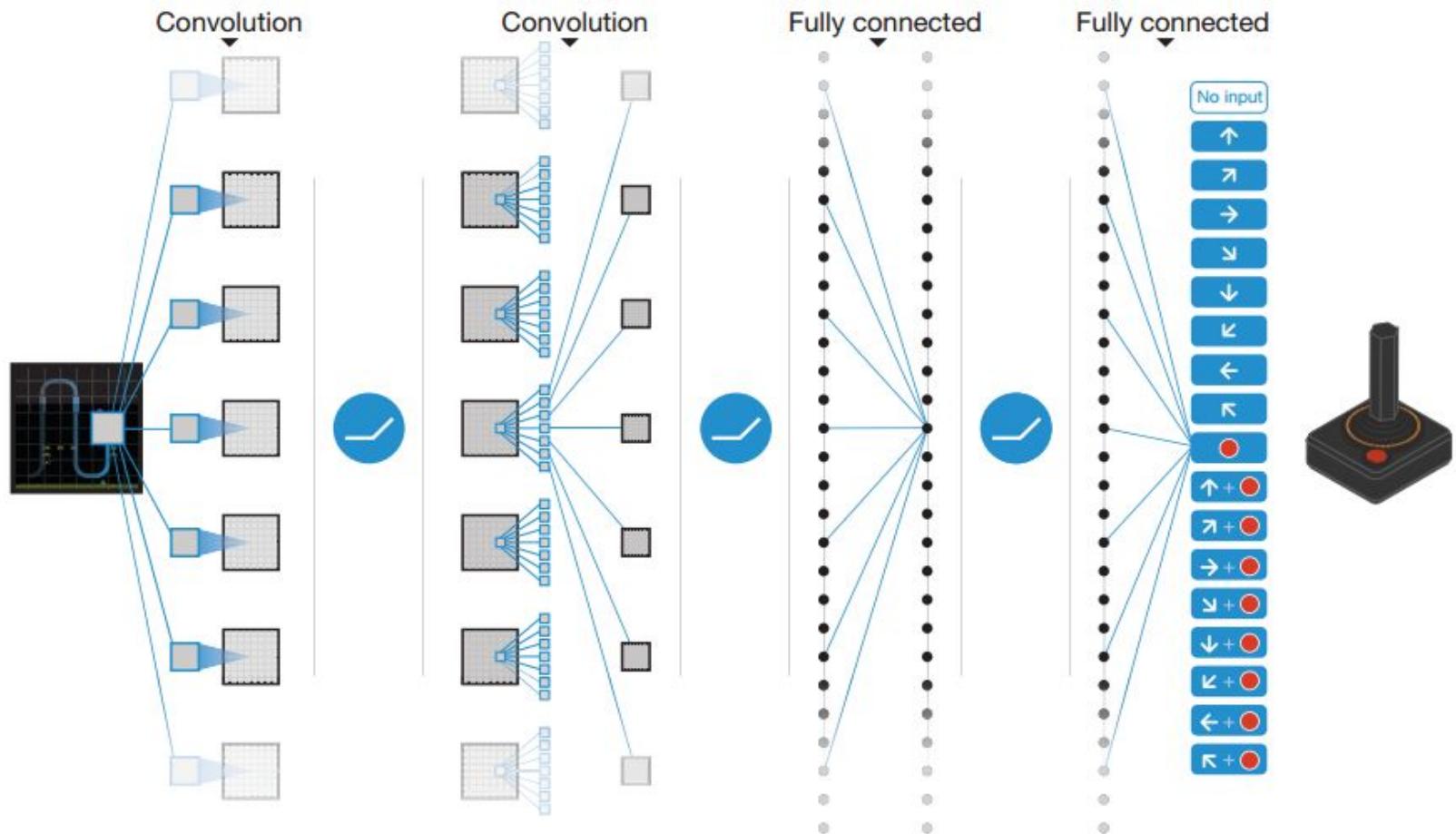
Deep Q-Learning



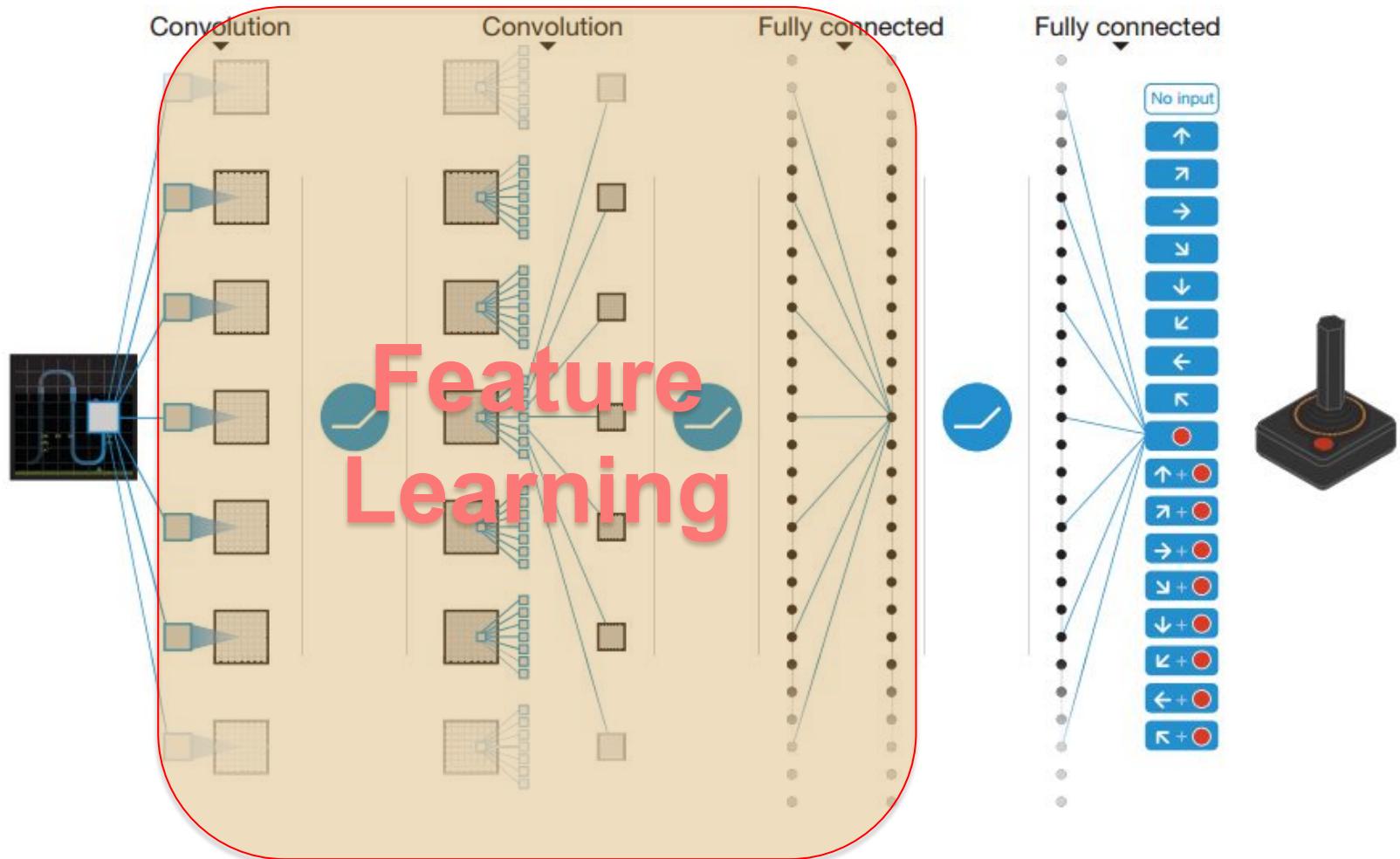
Deep Q-Learning



Deep Q-Learning

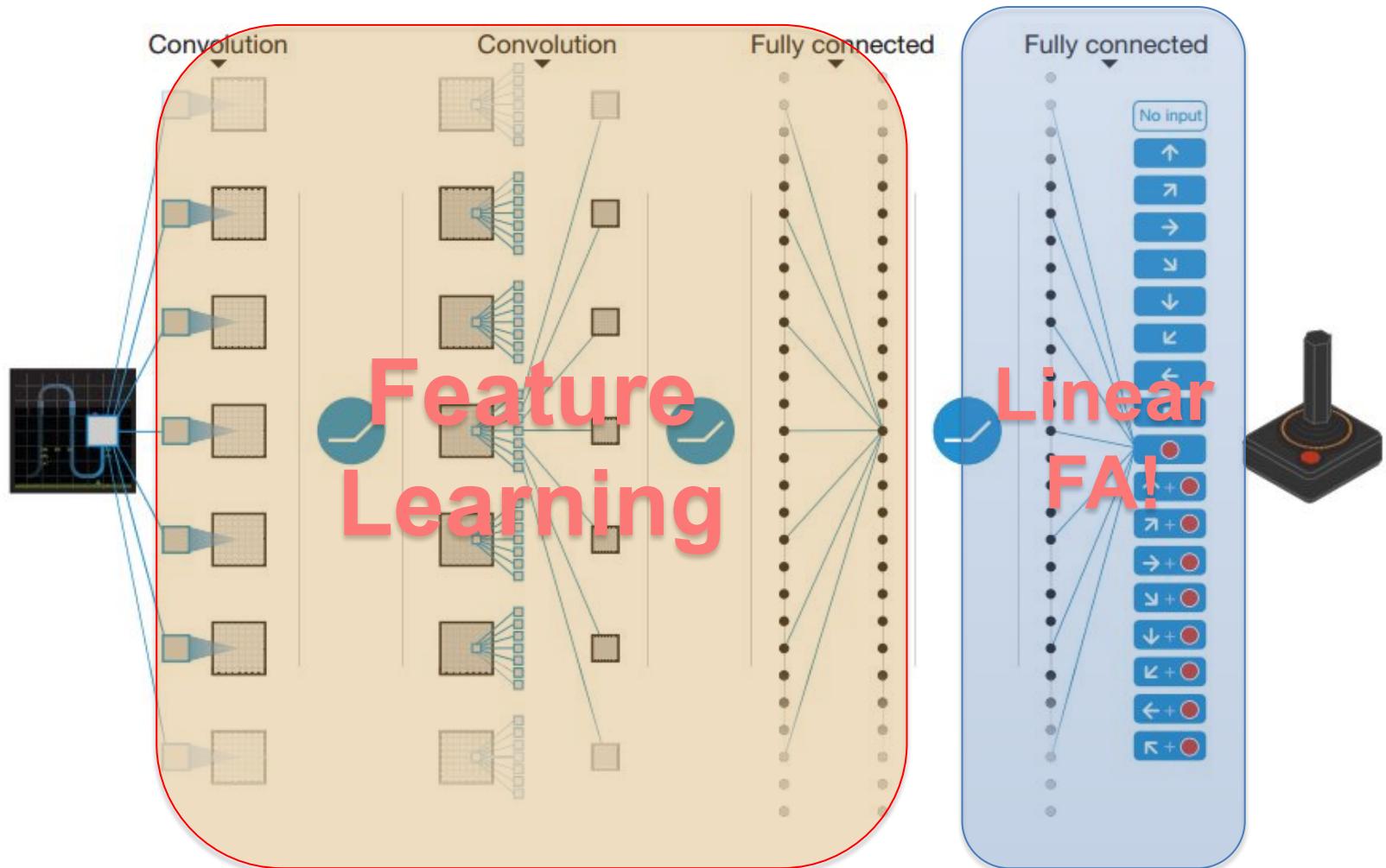


Deep Q-Learning



Source: Deep Q
Networks, Nature 2015
12

Deep Q-Learning



Source: Deep Q
Networks, Nature 2015
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Q-Network Learning

$$w_{t+1} = w_t - \frac{1}{2}\alpha \nabla_{w_t} [r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a) - \hat{q}(s_t, a_t)]^2$$

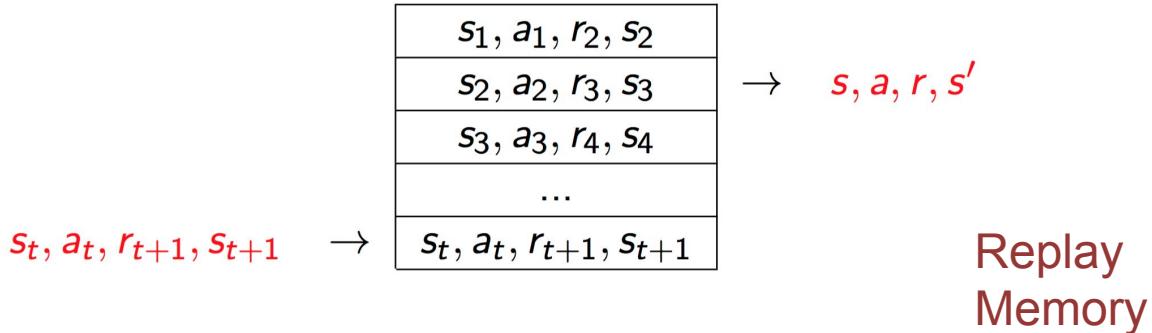
Divergence is an issue since the current network is used decide its own target

- Correlations between samples.
- Non-stationarity of the targets.
- How do we address these issues?

Replay Memory & Freezing target network

Q-Network Learning

To remove correlations, we build data-set from the agent's experience



Sample experiences from dataset; w^- frozen (with periodic updates) to address non-stationarity

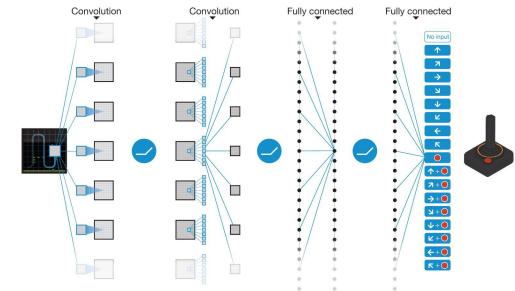
$$\left[\left\{ r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a; w^-) \right\} - \hat{q}(s_t, a_t; w) \right]^2$$

Freeze Target Network

Deep-Q Networks

Architecture:-

- Has a set of convolutional layers which act as feature extractors.
- These features are then passed through a series of fully connected layers.
- The output layer has $|A|$ number of nodes which are used to calculate the Q-value for each action.



The above network is updated using huber loss and not regular least squares loss.

Below is the regular expression for huber loss.

$$L_\delta(a) = \begin{cases} \frac{1}{2}a^2 & \text{for } |a| \leq \delta, \\ \delta(|a| - \frac{1}{2}\delta), & \text{otherwise.} \end{cases}$$

