notebook

February 8, 2024

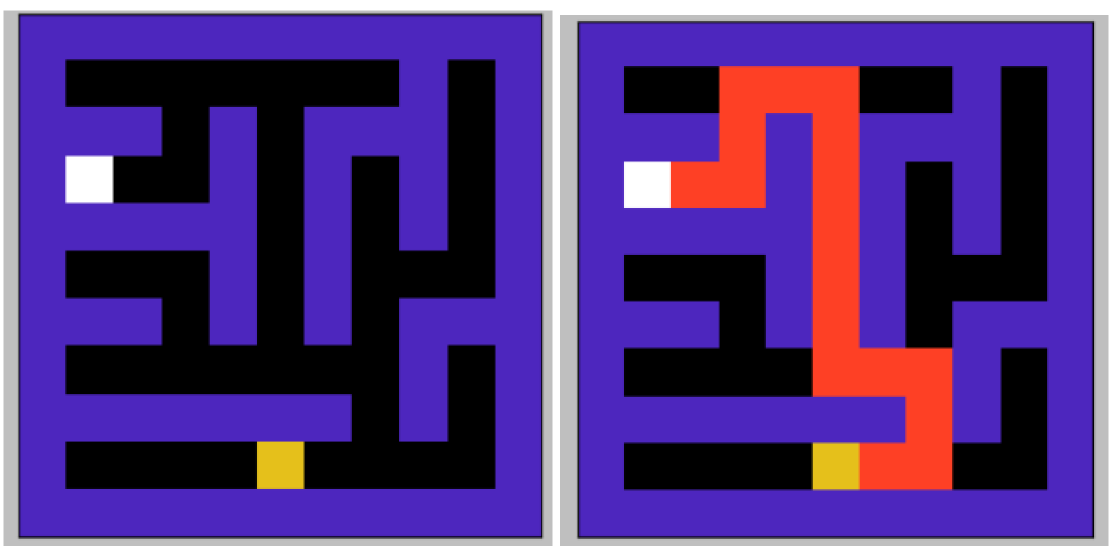
1 CS6700: Tutorial 3 - Policy Iteration

[1]:
```
import numpy as np
from enum import Enum
import copy
```

Consider a standard grid world, where only 4 (up, down, left, right) actions are allowed and the agent deterministically moves accordingly, represented as below. Here yellow is the start state and white is the goal state.

Say, we define our MDP as: - S: 121 (11 x 11) cells - A: 4 actions (up, down, left, right) - P: Deterministic transition probability - R: -1 at every step - gamma: 0.9

Our goal is to find an optimal policy (shown in right).



[2]:
```
# Above grid is defined as below:
#   - 0 denotes an navigable tile
#   - 1 denotes an obstruction/wall
#   - 2 denotes the start state
#   - 3 denotes an goal state
```

```
# Note: Here the upper left corner is defined as (0, 0)
#       and lower right corner as (m-1, n-1)

# Optimal Path: RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT LEFT DOWN␣
↪DOWN LEFT LEFT


GRID_WORLD = np.array([
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1],
    [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1],
    [1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1],
    [1, 3, 0, 0, 1, 0, 1, 0, 1, 0, 1],
    [1, 1, 1, 1, 1, 0, 1, 0, 1, 0, 1],
    [1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 1],
    [1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1],
    [1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1],
    [1, 1, 1, 1, 1, 1, 1, 0, 1, 0, 1],
    [1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1],
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
])
```

1.0.1 Actions

[3]:
```
class Actions(Enum):
  UP    = (0, (-1, 0))  # index = 0, (xaxis_move = -1 and yaxis_move = 0)
  DOWN  = (1, (1, 0))   # index = 1, (xaxis_move = 1 and yaxis_move = 0)
  LEFT  = (2, (0, -1))  # index = 2, (xaxis_move = 0 and yaxis_move = -1)
  RIGHT = (3, (0, 1))   # index = 3, (xaxis_move = 0 and yaxis_move = -1)

  def get_action_dir(self):
    _, direction = self.value
    return direction

  @property
  def index(self):
    indx, _ = self.value
    return indx

  @classmethod
  def from_index(cls, index):
    action_index_map = {a.index: a for a in cls}
    return action_index_map[index]
```

[4]:
```
# How to use Action enum
for a in Actions:
  print(f"name: {a.name}, action_id: {a.index}, direction_to_move: {a.
↪get_action_dir()}")
```

```
print("\n-----------------------------------\n")

# find action enum from index 0
a = Actions.from_index(0)
print(f"0 index action is: {a.name}")
```

```
name: UP, action_id: 0, direction_to_move: (-1, 0)
name: DOWN, action_id: 1, direction_to_move: (1, 0)
name: LEFT, action_id: 2, direction_to_move: (0, -1)
name: RIGHT, action_id: 3, direction_to_move: (0, 1)

-----------------------------------

0 index action is: UP
```

1.0.2 Policy

[5]:
```
class BasePolicy:
  def update(self, *args):
    pass

  def select_action(self, state_id: int) -> int:
    raise NotImplemented


class DeterministicPolicy(BasePolicy):
  def __init__(self, actions: np.ndarray):
    # actions: its a 1d array (|S| size) which contains action for each state
    self.actions = actions

  def update(self, state_id, action_id):
    assert state_id < len(self.actions), f"Invalid state_id {state_id}"
    assert action_id < len(Actions), f"Invalid action_id {action_id}"
    self.actions[state_id] = action_id

  def select_action(self, state_id: int) -> int:
    assert state_id < len(self.actions), f"Invalid state_id {state_id}"
    return self.actions[state_id]
```

1.0.3 Environment

[6]:
```
class Environment:
  def __init__(self, grid):
    self.grid = grid
    m, n = grid.shape
    self.num_states = m*n
```

```python
def xy_to_posid(self, x: int, y: int):
  _, n = self.grid.shape
  return x*n + y

def posid_to_xy(self, posid: int):
  _, n = self.grid.shape
  return (posid // n, posid % n)

def isvalid_move(self, x: int, y: int):
  m, n = self.grid.shape
  return (x >= 0) and (y >= 0) and (x < m) and (y < n) and (self.grid[x, y] !
↪= 1)

def find_start_xy(self) -> int:
  m, n = self.grid.shape
  for x in range(m):
    for y in range(n):
      if self.grid[x, y] == 2:
        return (x, y)
  raise Exception("Start position not found.")

def find_path(self, policy: BasePolicy) -> str:
  max_steps = 50
  steps = 0

  P, R = self.get_transition_prob_and_expected_reward()
  num_actions, num_states = R.shape
  all_possible_state_posids = np.arange(num_states)

  path = ""
  curr_x, curr_y = self.find_start_xy()
  while (self.grid[curr_x, curr_y] != 3) and (steps < max_steps):
    curr_posid = self.xy_to_posid(curr_x, curr_y)
    action_id = policy.select_action(curr_posid)
    next_posid = np.random.choice(
        all_possible_state_posids, p=P[action_id, curr_posid])
    action = Actions.from_index(action_id)
    path += f" {action.name}"
    curr_x, curr_y = self.posid_to_xy(next_posid)
    steps += 1
  return path

def get_transition_prob_and_expected_reward(self):  # P(s_next | s, a), R(s,␣
↪a)
  m, n = self.grid.shape
  num_states = m*n
```

```python
    num_actions = len(Actions)
    P = np.zeros((num_actions, num_states, num_states))
    R = np.zeros((num_actions, num_states))
    for a in Actions:
      for x in range(m):
        for y in range(n):
          xmove_dir, ymove_dir = a.get_action_dir()
          xnew, ynew = x + xmove_dir, y + ymove_dir  # find the new co-ordinate␣
↪after the action a

          posid = self.xy_to_posid(x, y)
          new_posid = self.xy_to_posid(xnew, ynew)


          if self.grid[x, y] == 3:
            # the current state is a goal state
            P[a.index, posid, posid] = 1
            R[a.index, posid] = 0
          elif (self.grid[x, y] == 1) or (not self.isvalid_move(xnew, ynew)):
            # the current state is a block state or the next state is invalid
            P[a.index, posid, posid] = 1
            R[a.index, posid] = -1
          else:
            # action a is valid and goes to a new position
            P[a.index, posid, new_posid] = 1
            R[a.index, posid] = -1
    return P, R
```

1.0.4 Policy Iteration

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$; $V(\textit{terminal}) \doteq 0$

2. Policy Evaluation
   Loop:
   - $\Delta \leftarrow 0$
   - Loop for each $s \in \mathcal{S}$:
     - $v \leftarrow V(s)$
     - $V(s) \leftarrow \sum_{s',r} p(s',r \mid s,\pi(s))\big[r + \gamma V(s')\big]$
     - $\Delta \leftarrow \max(\Delta, |v - V(s)|)$

   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   - *old-action* $\leftarrow \pi(s)$
   - $\pi(s) \leftarrow \operatorname{argmax}_a \sum_{s',r} p(s',r \mid s,a)\big[r + \gamma V(s')\big]$
   - If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*

   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

[7]:
```
def policy_evaluation(P: np.ndarray, R: np.ndarray, gamma: float,
                      policy: BasePolicy, theta: float,
                      init_V: np.ndarray=None):
  _, num_states = R.shape

  # Please try different starting point for V you will find it will always
  # converge to the same V_pi value.

  if init_V is None:
    init_V = np.zeros(num_states)

  V = copy.deepcopy(init_V)

  delta = 100.0

  while delta > theta:
    delta = 0.0

    for state_id in range(num_states):

      action_id = policy.select_action(state_id)
```

```
      # Following equation is a different way of writing the same equation␣
↪given in the slide.
      # Note here R is an expected reward term.
      v = copy.deepcopy(V[state_id])
      V[state_id] = R[action_id, state_id] + gamma * np.dot(P[action_id,␣
↪state_id], V)

      # YOUR CODE HERE
      delta = max(delta, np.abs(v - V[state_id])) # Calculate delta which␣
↪determines when to terminate the evaluation step

  return V


def policy_improvement(P: np.ndarray, R: np.ndarray, gamma: float,
                      policy: BasePolicy, V: np.ndarray):
  _, num_states = R.shape
  policy_stable = True

  for state_id in range(num_states):

    old_action_id = policy.select_action(state_id)

    # YOUR CODE HERE
    new_action_id = np.argmax(R[:, state_id] + gamma * np.dot(P[:, state_id],␣
↪V)) # update new_action_id based on the value function.

    policy.update(state_id, new_action_id)
    if old_action_id != new_action_id:
      policy_stable = False

  return policy_stable


def policy_iteration(P: np.ndarray, R: np.ndarray, gamma: float,
                    theta: float=1e-3, init_policy: BasePolicy = None):
  _, num_states = R.shape

  # Please try exploring different policies you will find it will always
  # converge to the same optimal policy for valid states.
  if init_policy is None:
    # Say initial policy = all up actions.
    init_policy = DeterministicPolicy(actions=np.zeros(num_states, dtype=int))

  # creating a copy of a initial policy
  policy = copy.deepcopy(init_policy)
  policy_stable = False
```

```python
    while not policy_stable:
      V = policy_evaluation(P, R, gamma, policy, theta)
      policy_stable = policy_improvement(P, R, gamma, policy, V)

    return policy, V
```

1.0.5 Experiments

[8]:
```python
def is_same_optimal_value(V1, V2, diff_theta=1e-3):
  diff = np.abs(V1 - V2)
  return np.all(diff < diff_theta)
```

[9]:
```python
seed = 0
np.random.seed(seed)

gamma = 0.9
theta = 1e-5
```

[10]:
```python
env = Environment(GRID_WORLD)
P, R = env.get_transition_prob_and_expected_reward()
```

Exercise 1: Using Policy iteration algorithm find the optimal path from start to goal position

[11]:
```python
# # Start with random choice of init_policy.
# One such choice could be: init_policy = np.ones(env.num_states, dtype=int)
init_policy = DeterministicPolicy(actions=np.ones(env.num_states, dtype=int))

pitr_policy, pitr_V_star = policy_iteration(P, R, gamma, theta=theta,
 ↪init_policy=init_policy)
pitr_path = env.find_path(pitr_policy)
print(pitr_path)
```

```
RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT LEFT DOWN DOWN LEFT LEFT
```

Exercise 2: Using initial guess for V as random values, find the optimal value function using policy evaluation and compare it with the optimal value function

[12]:
```python
# Start with random choice of init_V.
# One such choice could be: init_V = np.random.randn(env.num_states)
# Another choice could be: init_V = 10*np.ones(env.num_states)
init_V = 10*np.ones(env.num_states)

V_star = policy_evaluation(P, R, gamma, pitr_policy, theta, init_V)
is_same_optimal_value(pitr_V_star, V_star)
```

[12]: True

[ ]:

To-do: Repeat Exercise 1 with a random Deterministic policy

[13]:
```
# # Start with random choice of init_policy.
init_policy = DeterministicPolicy(actions=np.random.randint(0, 4, size=env.
 ↪num_states, dtype=int))

pitr_policy, pitr_V_star = policy_iteration(P, R, gamma, theta=theta,␣
 ↪init_policy=init_policy)
pitr_path = env.find_path(pitr_policy)
print(pitr_path)
```

```
RIGHT RIGHT UP UP LEFT LEFT UP UP UP UP UP UP LEFT LEFT DOWN DOWN LEFT LEFT
```

Dynamic Programming in RL for Finite-State Full-Information Deterministic Systems:

- **Policy Iteration:**
  1. **Policy Evaluation:**
     - Iteratively update the value function according to the current policy.
     - Update based on the Bellman expectation equation.
  2. **Policy Improvement:**
     - Once the value function is stable, improve the policy by choosing greedy actions.
- **Value Iteration:**
  - Iteratively update the value function directly based on the maximum expected future reward.
  - Combines policy evaluation and improvement in a single step.

Independence from Initialization: - The statement that the dynamic programming algorithm is not dependent on the initialization of the value function refers to the convergence properties of the algorithm. - The iterative updates and convergence are based on Bellman equations and principles of dynamic programming. - The final results converge to the true value function regardless of the initial values assigned to the states.

Convergence and Error Threshold: - Convergence is determined by a specified error criterion or threshold. - The algorithm iteratively updates the value function until changes between consecutive iterations fall below the specified error threshold. - The choice of the error threshold affects the termination condition of the algorithm. - While initialization is independent of the final results, the overall convergence is dependent on the chosen error threshold.