Actor Critic Methods

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Recap

- Actor-Critic methods learn both a policy and a state-value function simultaneously.
- The policy is referred to as the actor that suggests actions given a state.
- The estimated value function is referred to as the critic. It evaluates actions taken by the actor based on the given policy.

$$
\begin{aligned}
\boldsymbol{\theta}_{t+1} &\doteq \boldsymbol{\theta}_t + \alpha\Big(G_{t:t+1} - \hat{v}(S_t,\mathbf{w})\Big)\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)} \\
&= \boldsymbol{\theta}_t + \alpha\Big(R_{t+1} + \gamma\hat{v}(S_{t+1},\mathbf{w}) - \hat{v}(S_t,\mathbf{w})\Big)\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)} \\
&= \boldsymbol{\theta}_t + \alpha\,\delta_t\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)}.
\end{aligned}
$$

Recap: One Step Actor Critic

One-step Actor–Critic (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
    Initialize S (first state of episode)
    $I \leftarrow 1$
    Loop while S is not terminal (for each time step):
        $A \sim \pi(\cdot|S, \boldsymbol{\theta})$
        Take action A, observe S', R
        $\delta \leftarrow R + \gamma \hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if S' is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S,\mathbf{w})$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I \delta \nabla \ln \pi(A|S, \boldsymbol{\theta})$
        $I \leftarrow \gamma I$
        $S \leftarrow S'$

Comparison to REINFORCE

- Recall the REINFORCE(with baseline) update:

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha\Big(G_t - b(S_t)\Big)\frac{\nabla\pi(A_t|S_t,\boldsymbol{\theta}_t)}{\pi(A_t|S_t,\boldsymbol{\theta}_t)}$$

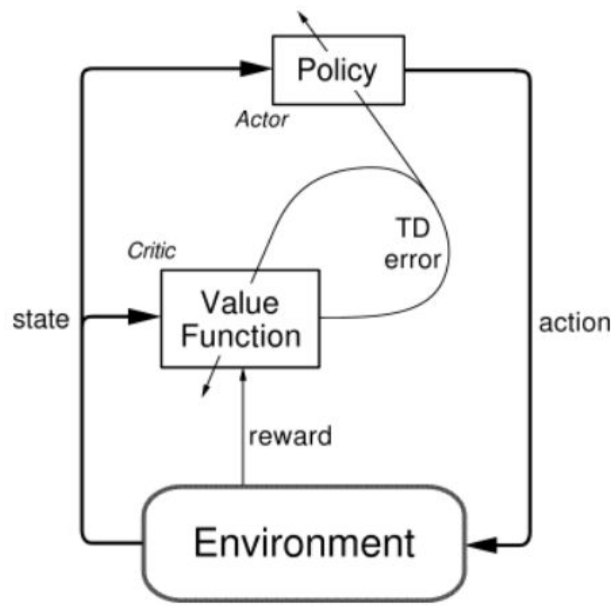
- The G_t term (although unbiased) causes the high variance of the algorithm.
- Recall that $\mathbb{E}_\pi[G_t|S_t,A_t] = q_\pi(S_t,A_t)$.
- If we had an estimate of $q_\pi(S_t,A_t)$ with less variance, then we can use that instead of G_t.

Comparison to REINFORCE

- In the one step AC algorithm, we use $\hat{v}$ for both estimating $q_\pi(S_t, A_t)$ and as the baseline.
- The bootstrapping in the update introduces bias but decreases the variance.
- This reduced variance can accelerate learning.

Common Features of AC Methods

- Actor: Computes the policy π_θ and updates θ .
- Critic: Typically computes an estimate $\hat{v}(\mathbf{s}, \mathbf{w})$ of the state value function . Updates the parameter $\mathbf{w}$.



Basic Actor Critic Algorithm

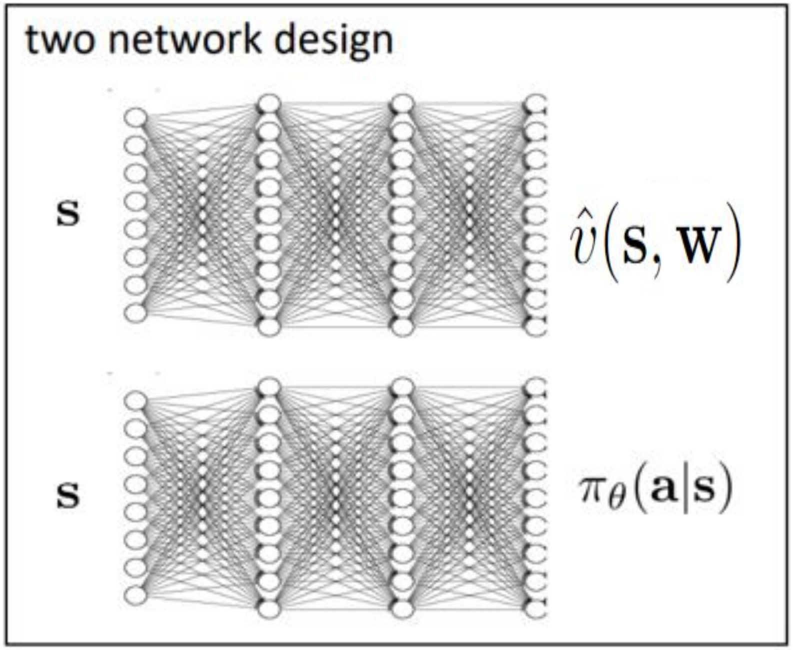
1. Take action $\mathbf{a} \sim \pi_\theta(\mathbf{a} \mid \mathbf{s})$ and receive $(\mathbf{s},\ \mathbf{a},\ \mathbf{s}', r)$
2. Update value parameter $\mathbf{w}$ using data $(\mathbf{s}, r + \gamma \hat{v}(\mathbf{s}', \mathbf{w}))$
3. Compute $\hat{\delta}(\mathbf{s}, \mathbf{a}) = r + \gamma \hat{v}(\mathbf{s}', \mathbf{w}) - \hat{v}(\mathbf{s}, \mathbf{w})$
4. $\theta \leftarrow \theta + \alpha \cdot \hat{\delta}(\mathbf{s},\ \mathbf{a}) \cdot \nabla_\theta \log \pi_\theta(\mathbf{a} \mid \mathbf{s})$
5. Go back to step 1

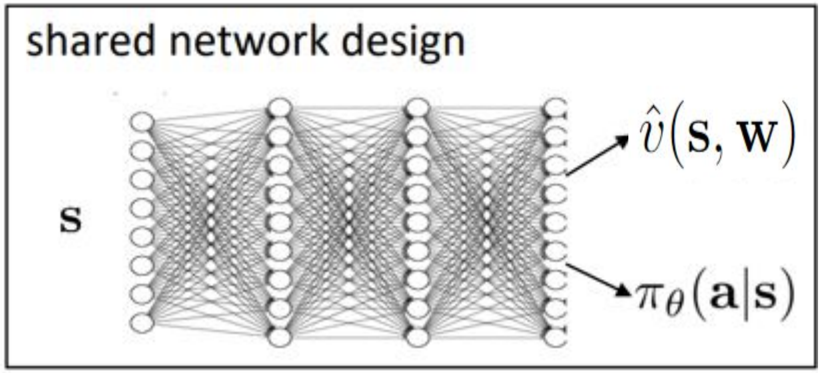
How is the critic updated?

- Step 2 of the previous algorithm usually happens in batches. We get multiple data points of the form $(\mathbf{s}, y)$ from parallel workers.
- Minimize the squared loss:

$$L(\mathbf{w}) = \sum_i \|\hat{v}(\mathbf{s}_i, \mathbf{w}) - y_i\|^2$$

Design Choices



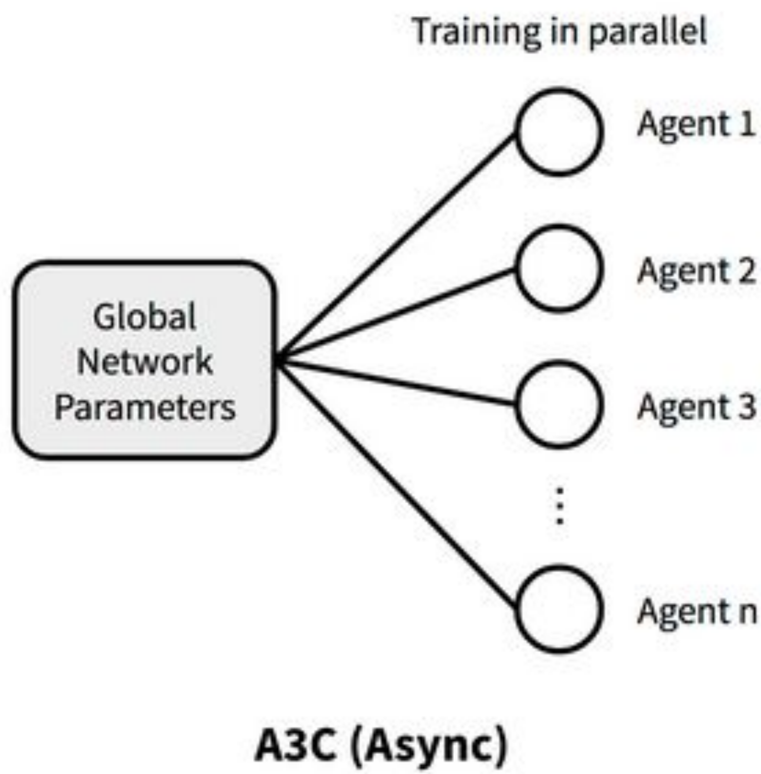


Advantage Function

- The advantage function is the difference between the q-value and the value function.
- It can be interpreted as a measure of the advantage of taking action $\mathbf{a}$ in state $\mathbf{s}$ as compared to following policy π

$$\delta_\pi(\mathbf{s}, \mathbf{a}) = q_\pi(\mathbf{s}, \mathbf{a}) - v_\pi(\mathbf{s})$$

A3C – Asynchronous Advantage Actor Critic



A3C - [*Mnih et. al. 2016*]

Algorithm S3 Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

// Assume global shared parameter vectors θ and w and global shared counter $T = 0$
// Assume thread-specific parameter vectors θ' and w'
Initialize thread step counter $t \leftarrow 1$
repeat
    Reset gradients: $d\theta \leftarrow 0$ and $d\mathrm{w} \leftarrow 0$.
    Synchronize thread-specific parameters $\theta' = \theta$ and $\mathrm{w}' = \mathrm{w}$
    $t_{start} = t$
    Get state s_t
    repeat
        Perform a_t according to policy $\pi(a_t|s_t;\theta')$
        Receive reward r_t and new state s_{t+1}
        $t \leftarrow t + 1$
        $T \leftarrow T + 1$
    until terminal s_t **or** $t - t_{start} == t_{max}$
    $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \mathrm{w}') & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
    for $i \in \{t-1, \ldots, t_{start}\}$ **do**
        $R \leftarrow r_i + \gamma R$
        Accumulate gradients wrt θ': $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i;\theta')(R - V(s_i;\mathrm{w}'))$
        Accumulate gradients wrt w': $d\mathrm{w} \leftarrow d\mathrm{w} + \partial\left(R - V(s_i;\mathrm{w}')\right)^2 / \partial \mathrm{w}'$
    end for
    Perform asynchronous update of θ using $d\theta$ and of w using $d\mathrm{w}$.
until $T > T_{max}$

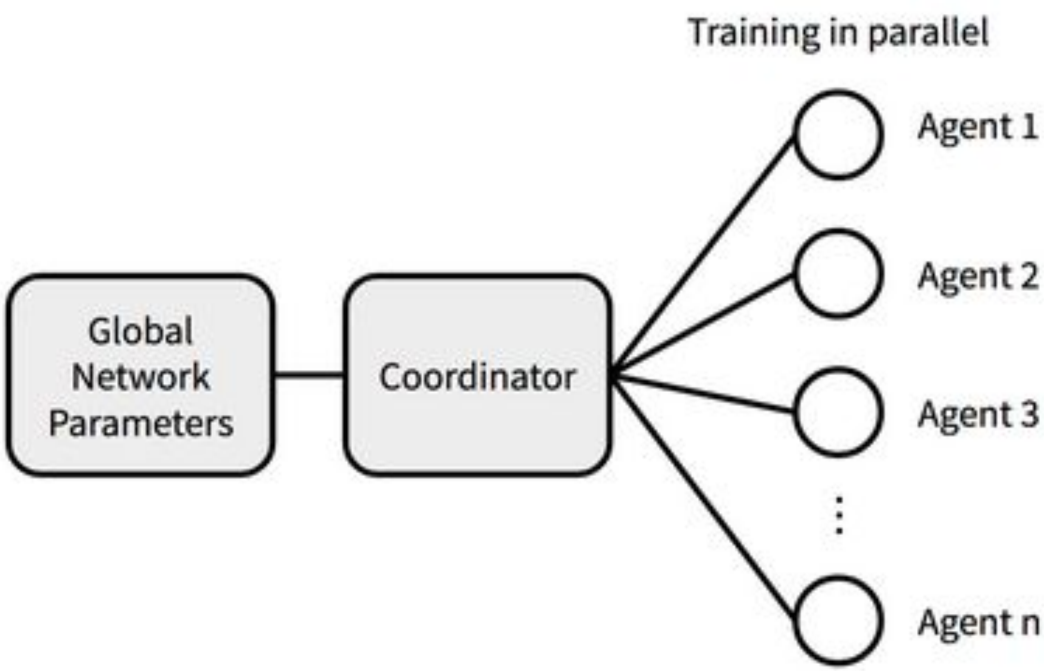
Reset thread params, update local params with global params

Gather experience

Compute the gradients for this thread

Update global params

A2C – Synchronous Advantage Actor Critic

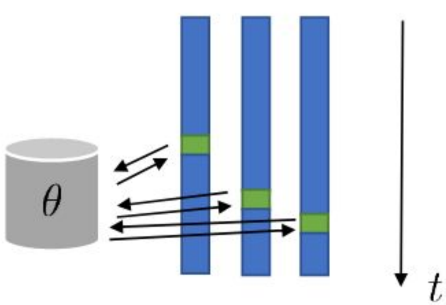


A2C (Sync)

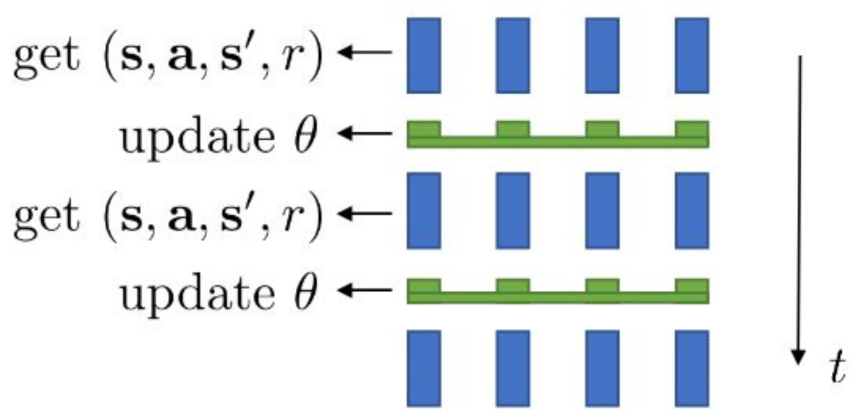
A3C vs A2C

- We remove the "asynchronous" part of A3C.
- The updates to the global parameters are executed only after all the threads have finished their computation.

asynchronous parallel actor-critic



synchronized parallel actor-critic



Can we re-use experience

- Recall the DQN algorithm where we could use reuse experience using a replay buffer. This was possible as it was an off-policy algorithm.
- Can we do something similar for actor critic algorithms?

Attempt 1

1. Take action $\mathbf{a} \sim \pi_\theta(\mathbf{a} \mid \mathbf{s})$, receive $(\mathbf{s},\ \mathbf{a},\ \mathbf{s}', r)$ and store in $\mathcal{R}$
2. Sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}_{1 \le i \le N}$ from buffer $\mathcal{R}$
3. Update value parameter $\mathbf{w}$ using data $\{(\mathbf{s}_i, r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}))\}_{1 \le i \le N}$
4. Compute $\hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) = r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}) - \hat{v}(\mathbf{s}_i, \mathbf{w})$
5. $\theta \leftarrow \theta + \alpha \cdot \left(\frac{1}{N} \sum_{i=1}^{N} \hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) \cdot \nabla_\theta \log \pi_\theta(\mathbf{a}_i \mid \mathbf{s}_i) \right)$
6. Go back to step 1

Problems?

1. Take action $\mathbf{a} \sim \pi_\theta(\mathbf{a} \mid \mathbf{s})$, receive $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ and store in $\mathcal{R}$
2. Sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}_{1 \leq i \leq N}$ from buffer $\mathcal{R}$
3. Update value parameter $\mathbf{w}$ using data $\{(\mathbf{s}_i, r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}))\}_{1 \leq i \leq N}$
4. Compute $\hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) = r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}) - \hat{v}(\mathbf{s}_i, \mathbf{w})$
5. $\theta \leftarrow \theta + \alpha \cdot \left(\frac{1}{N} \sum_{i=1}^{N} \hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) \cdot \nabla_\theta \log \pi_\theta(\mathbf{a}_i \mid \mathbf{s}_i) \right)$
6. Go back to step 1

Not the right target.

Not the action taken by π_θ

Fixes

1. Take action $\mathbf{a} \sim \pi_\theta(\mathbf{a} \mid \mathbf{s})$, receive $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ and store in $\mathcal{R}$
2. Sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}_{1 \leq i \leq N}$ from buffer $\mathcal{R}$
3. Update value parameter $\mathbf{w}$ using data $\{(\mathbf{s}_i, r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}))\}_{1 \leq i \leq N}$
4. Compute $\hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) = r_i + \gamma \hat{v}(\mathbf{s}'_i, \mathbf{w}) - \hat{v}(\mathbf{s}_i, \mathbf{w})$
5. $\theta \leftarrow \theta + \alpha \cdot \left(\frac{1}{N} \sum_{i=1}^N \hat{\delta}(\mathbf{s}_i, \mathbf{a}_i) \cdot \nabla_\theta \log \pi_\theta(\mathbf{a}_i \mid \mathbf{s}_i) \right)$
6. Go back to step 1

Learn q_π instead. Use the fact that $v_\pi(\mathbf{s}) = \mathbb{E}_{\mathbf{a} \sim \pi(\mathbf{a}|\mathbf{s})}[q_\pi(\mathbf{s}, \mathbf{a})]$

Replace $\mathbf{a}_i$ with $\mathbf{a}'_i \sim \pi_\theta(\mathbf{a} \mid \mathbf{s}_i)$

Final Algorithm

1. Take action $\mathbf{a} \sim \pi_\theta(\mathbf{a} \mid \mathbf{s})$, receive $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$ and store in $\mathcal{R}$
2. Sample a batch $\{\mathbf{s}_i, \mathbf{a}_i, r_i, \mathbf{s}'_i\}_{1 \leq i \leq N}$ from buffer $\mathcal{R}$
3. Update action-value parameter $\mathbf{w}$ using data $\{((\mathbf{s}_i, \mathbf{a}_i), r_i + \gamma \hat{q}(\mathbf{s}'_i, \mathbf{a}'_i, \mathbf{w}))\}_{1 \leq i \leq N}$
4. $\theta \leftarrow \theta + \alpha \cdot \left(\frac{1}{N} \sum_{i=1}^{N} \hat{q}(\mathbf{s}_i, \mathbf{a}''_i, \mathbf{w}) \cdot \nabla_\theta \log \pi_\theta(\mathbf{a}''_i \mid \mathbf{s}_i) \right)$
5. Go back to step 1

$\mathbf{a}'_i \sim \pi_\theta(\mathbf{a} \mid \mathbf{s}_i)$

$\mathbf{a}''_i \sim \pi_\theta(\mathbf{a} \mid \mathbf{s}_i)$

Note that we are not subtracting a baseline. This might increase the variance. However this is not an issue as we are averaging the update over a large batch.

Problems?

- The states are not sampled from the steady state distribution of π_θ. This was a requirement for the policy gradient theorem to work.
- Thus, the final policy we get is the optimal policy for a different distribution over the states.
- Similar ideas will be seen when we do soft actor critic.

Compatible Parametrization

- Substituting the approximation $\hat{q}(\mathbf{s}, \mathbf{a}, \mathbf{w})$ instead of the true value of $q_\pi(\mathbf{s}, \mathbf{a})$ may introduce bias.
- It can be proved that there is no bias if the function approximator has a "compatible" parametrization with the policy parametrization.
- Condition 1: $\hat{q}(\mathbf{s}, \mathbf{a}, \mathbf{w}) = \nabla_\theta \log \pi_\theta(\mathbf{a} \mid \mathbf{s})^T \mathbf{w}$
- Condition 2: $\mathbf{w}$ minimizes mean squared error:

$$\mathbf{w} = \arg\min \mathbb{E}_{\mathbf{s} \sim \rho^{\pi_\theta}, \mathbf{a} \sim \pi_\theta} \left[\left(\hat{q}(\mathbf{s}, \mathbf{a}, \mathbf{w}) - q_{\pi_\theta}(\mathbf{s}, \mathbf{a}) \right)^2 \right]$$