

CS7015 (Deep Learning) : Lecture 11

Convolutional Neural Networks, LeNet, AlexNet, ZF-Net, VGGNet, GoogLeNet and ResNet

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Module 11.1 : The convolution operation

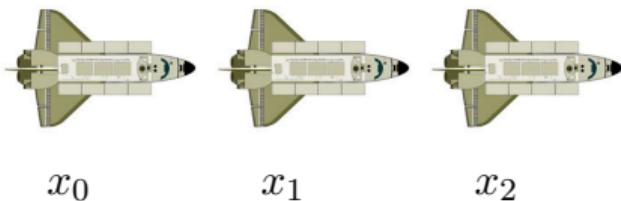


- Suppose we are tracking the position of an aeroplane using a laser sensor at discrete time intervals

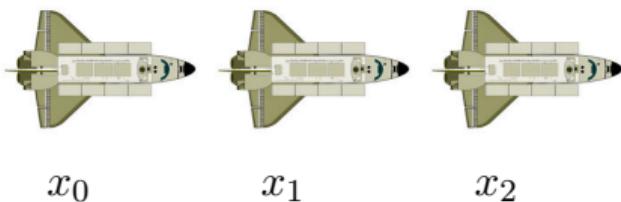
x_0



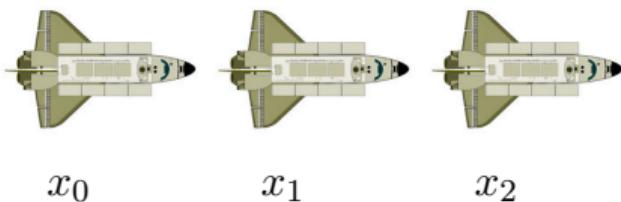
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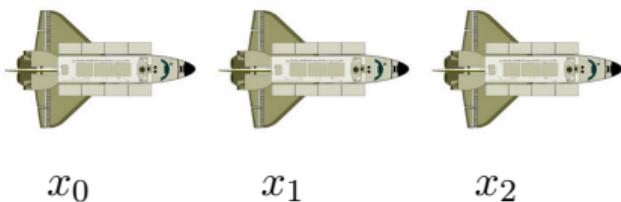
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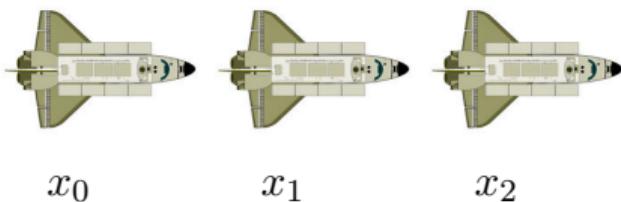
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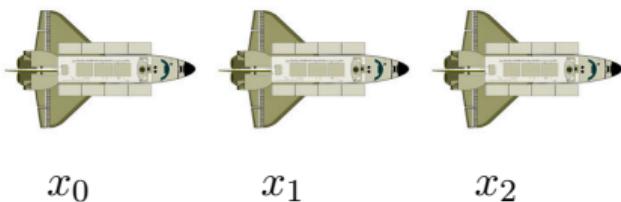


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- More recent measurements are more important so we would like to take a weighted average



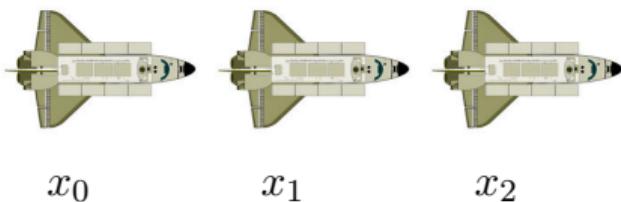
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filter
convolution

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- We just slide the filter over the input and compute the value of s_t based on a window around x_t

	w_{-6}	w_{-5}	w_{-4}	w_{-3}	w_{-2}	w_{-1}	w_0
W	0.01	0.01	0.02	0.02	0.04	0.4	0.5

X	1.00	1.10	1.20	1.40	1.70	1.80	1.90	2.10	2.20	2.40	2.50	2.70
---	------	------	------	------	------	------	------	------	------	------	------	------

S	1.80						
---	------	--	--	--	--	--	--

$$s_6 = x_6 w_0 + x_5 w_{-1} + x_4 w_{-2} + x_3 w_{-3} + x_2 w_{-4} + x_1 w_{-5} + x_0 w_{-6}$$

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S	1.80	1.96				
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S	1.80	1.96	2.11	2.16		
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S

1.80	1.96	2.11	2.16	2.28	
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- Here the input (and the kernel) is one dimensional
- Can we use a convolutional operation on a 2D input also?

W

w_{-6}	w_{-5}	w_{-4}	w_{-3}	w_{-2}	w_{-1}	w_0
0.01	0.01	0.02	0.02	0.04	0.4	0.5

X

1.00	1.10	1.20	1.40	1.70	1.80	1.90	2.10	2.20	2.40	2.50	2.70
------	------	------	------	------	------	------	------	------	------	------	------

S

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- We can think of images as 2D inputs
- We would now like to use a 2D filter ($m \times n$)
- First let us see what the 2D formula looks like
- This formula looks at all the preceding neighbours ($i - a, j - b$)
- In practice, we use the following formula which looks at the succeeding neighbours

- Let us apply this idea to a toy example and see the results

Input

a	b	c	d
e	f	g	h
i	j	k	ℓ

Kernel

w	x
y	z

- Let us apply this idea to a toy example and see the results

Output

$aw + bx + ey + fz$		

Input

a	b	c	d
e	f	g	h
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Kernel

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- Let us apply this idea to a toy example and see the results

Output

$aw + bx + ey + fz$	$bw + cx + fy + gz$	

Input

a	b	c	d
e	f	g	h
i	j	k	ℓ

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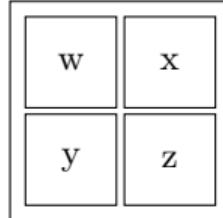
Output

$aw + bx + ey + fz$	$bw + cx + fy + gz$	$cw + dx + gy + hz$

Input

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Kernel



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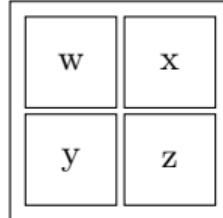
Output

$aw + bx + ey + fz$	$bw + cx + fy + gz$	$cw + dx + gy + hz$
$ew + fx + iy + jz$		

Input

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Kernel



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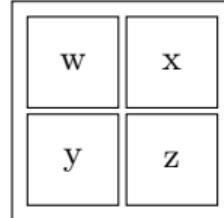
Output

$aw + bx + ey + fz$	$bw + cx + fy + gz$	$cw + dx + gy + hz$
$ew + fx + iy + jz$	$fw + gx + jy + kz$	

Input

a	b	c	d
e	f	g	h
i	j	k	ℓ

Kernel



- Let us apply this idea to a toy example and see the results

Output

$aw + bx + ey + fz$	$bw + cx + fy + gz$	$cw + dx + gy + hz$
$ew + fx + iy + jz$	$fw + gx + jy + kz$	$gw + hx + ky + \ell z$

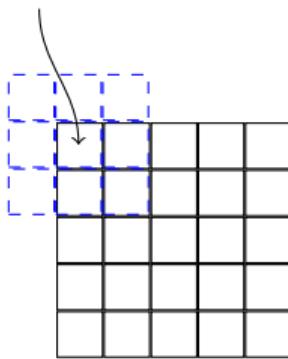
- For the rest of the discussion we will use the following formula for convolution

$$S_{ij} = (I * K)_{ij} = \sum_{a=\left\lfloor -\frac{m}{2} \right\rfloor}^{\left\lfloor \frac{m}{2} \right\rfloor} \sum_{b=\left\lfloor -\frac{n}{2} \right\rfloor}^{\left\lfloor \frac{n}{2} \right\rfloor} I_{i-a,j-b} K_{\frac{m}{2}+a, \frac{n}{2}+b}$$

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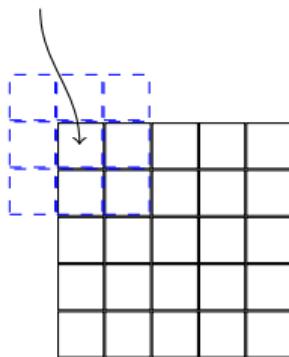
pixel of interest



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- In other words we will assume that the kernel is centered on the pixel of interest

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pixel of interest



- For the rest of the discussion we will use the following formula for convolution
- In other words we will assume that the kernel is centered on the pixel of interest
- So we will be looking at both preceding and succeeding neighbors

Let us see some examples of 2D convolutions applied to images



$$\begin{array}{r} & 1 & 1 & 1 \\ * & 1 & 1 & 1 & = \\ & 1 & 1 & 1 \end{array}$$



$$\begin{matrix} & 1 & 1 & 1 \\ * & 1 & 1 & 1 \\ & 1 & 1 & 1 \end{matrix} =$$



blurs the image



$$\begin{matrix} & 0 & -1 & 0 \\ * & -1 & 5 & -1 & = \\ & 0 & -1 & 0 \end{matrix}$$



$$\begin{matrix} & 0 & -1 & 0 \\ * & -1 & 5 & -1 \\ & 0 & -1 & 0 \end{matrix} =$$



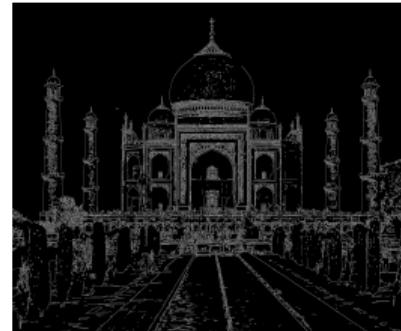
sharpens the image



$$\begin{array}{ccc} 1 & 1 & 1 \\ * & 1 & -8 & 1 & = \\ 1 & 1 & 1 \end{array}$$

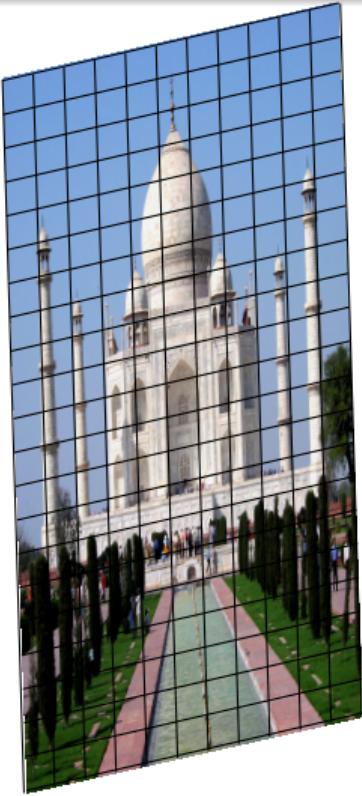


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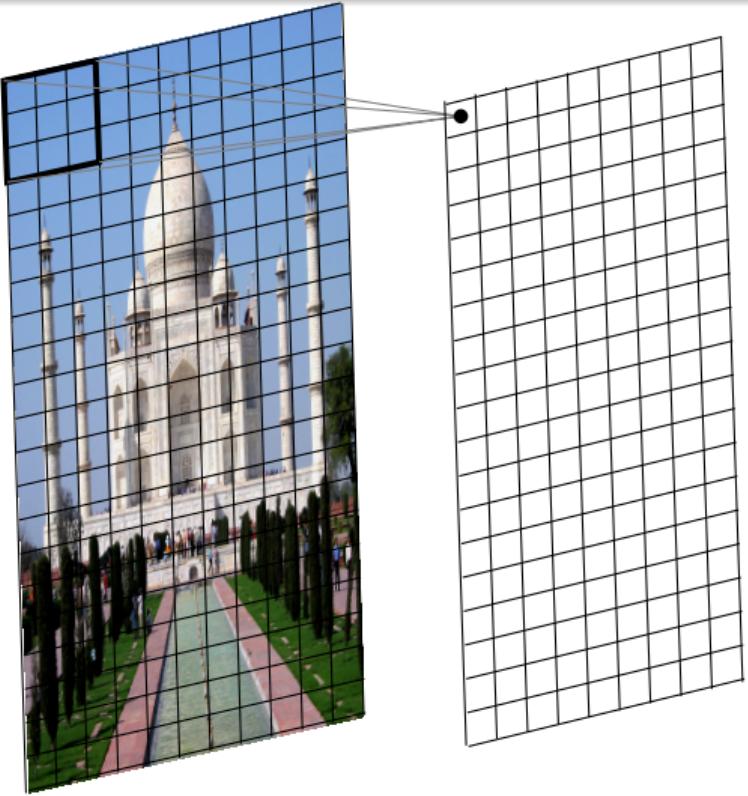


detects the edges

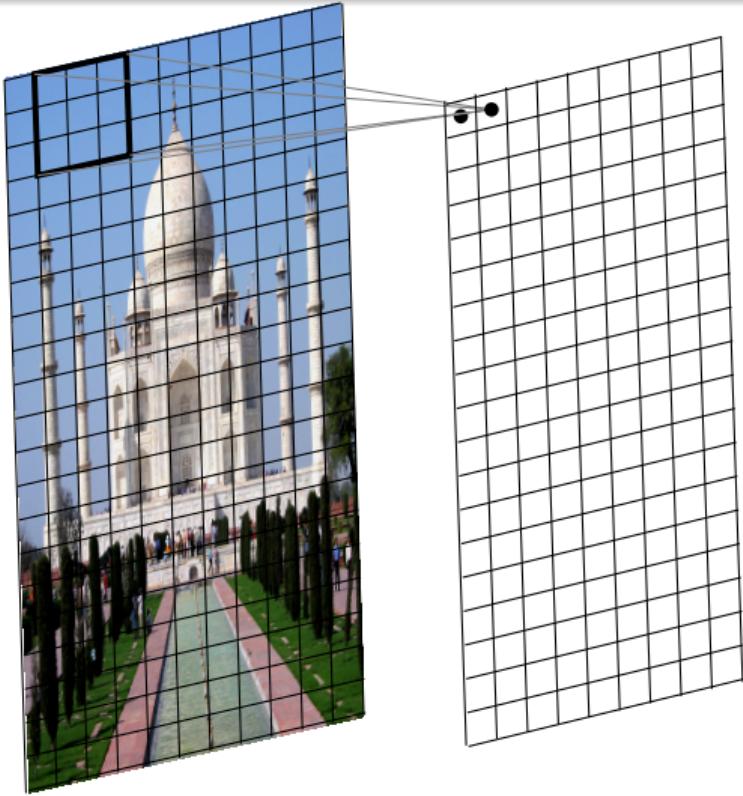
We will now see a working example of 2D convolution.



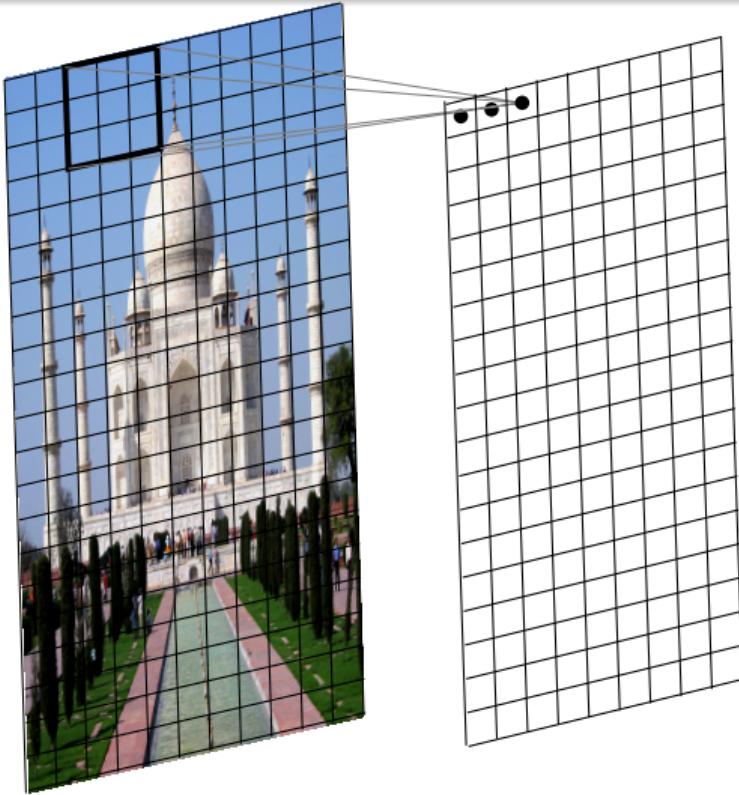
- We just slide the kernel over the input image



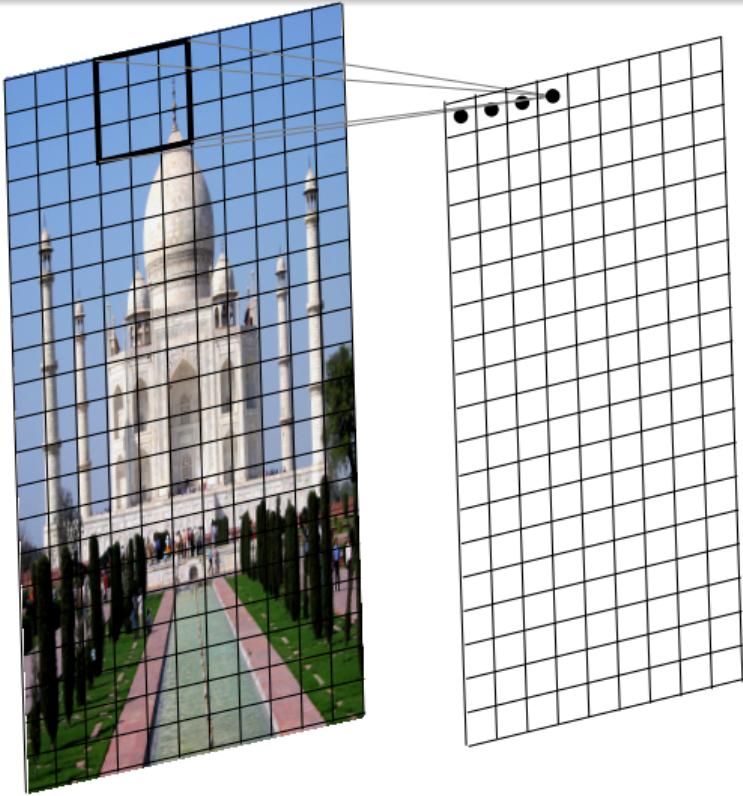
- We just slide the kernel over the input image
- Each time we slide the kernel we get one value in the output



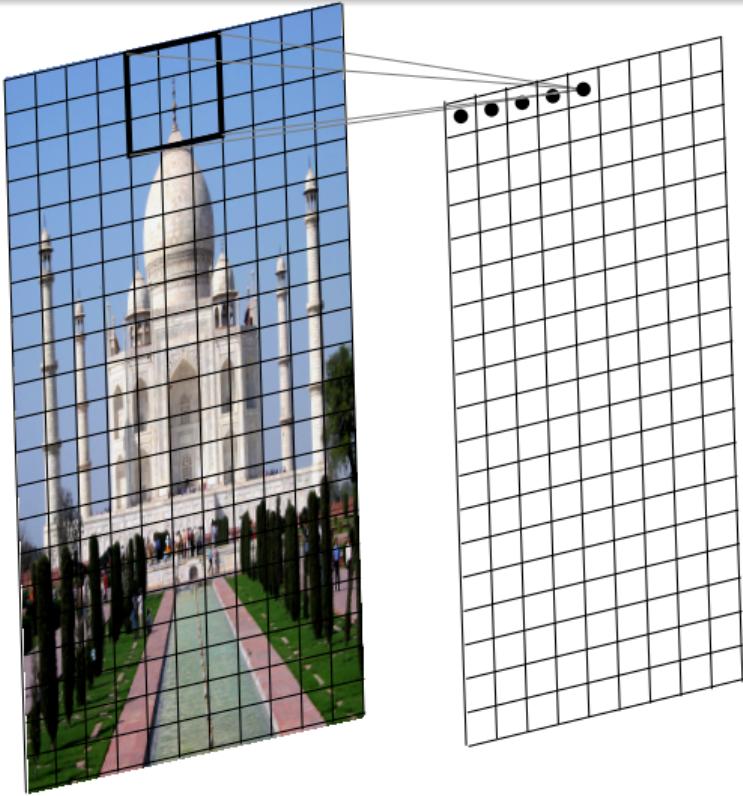
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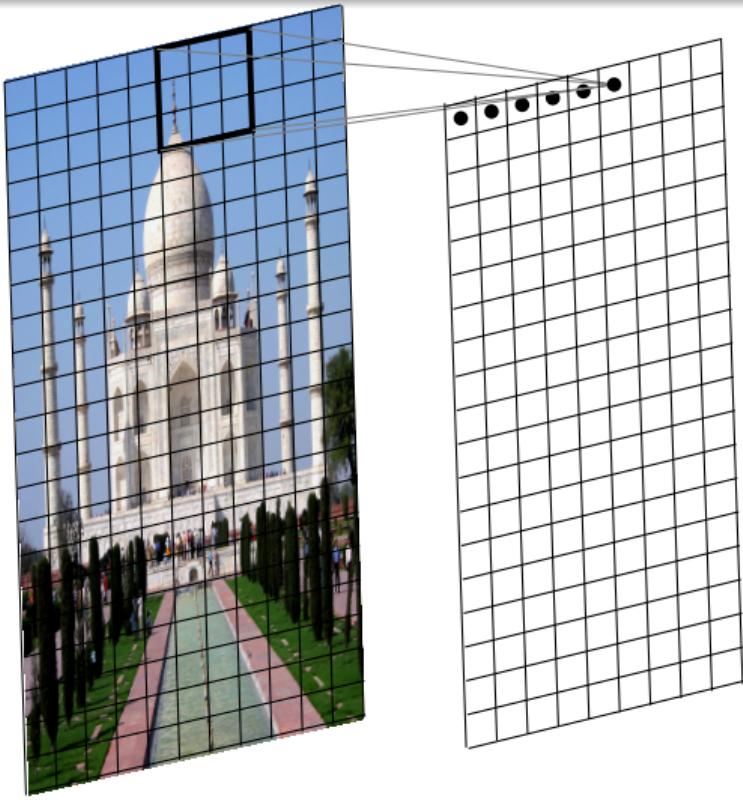
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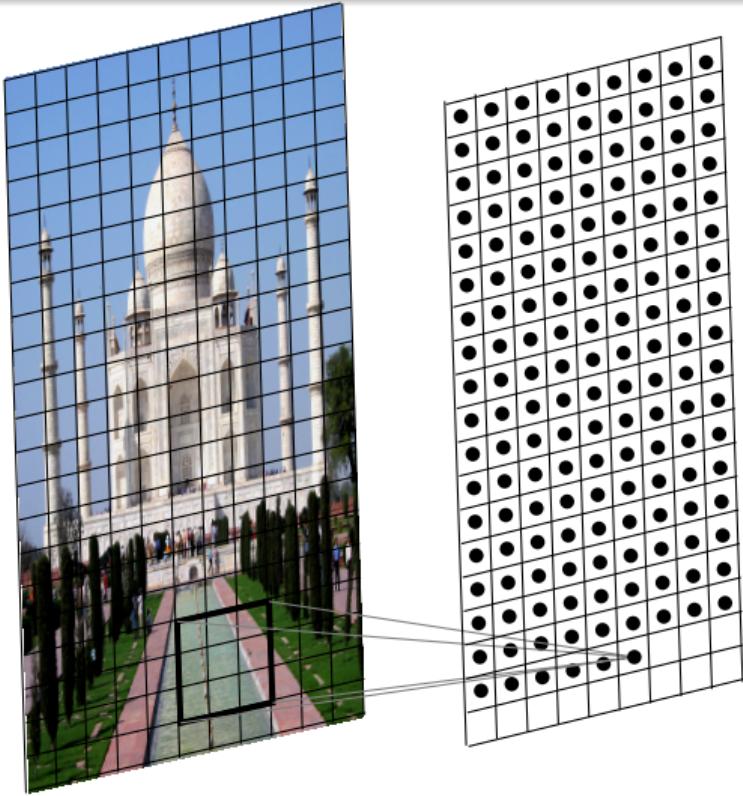
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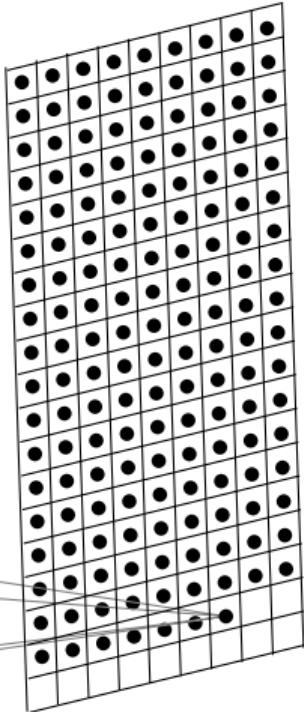
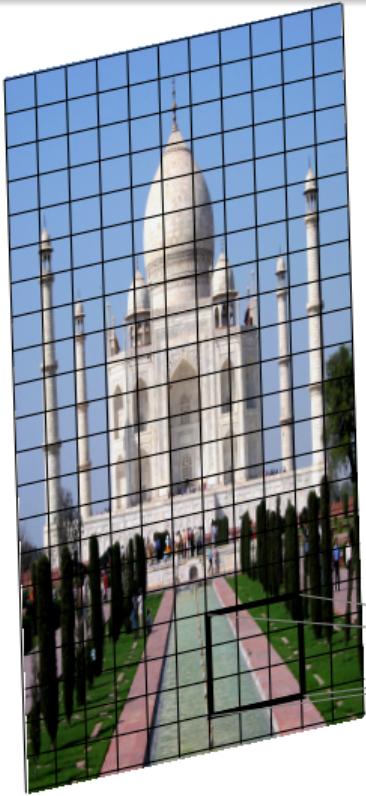
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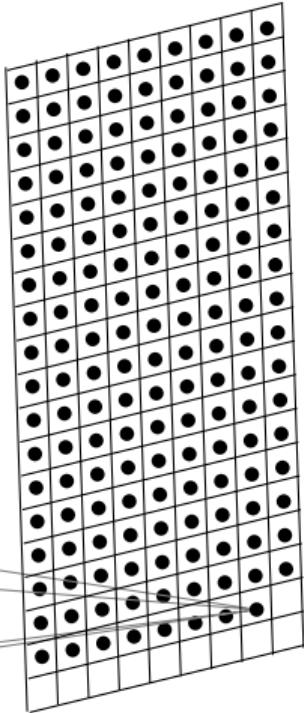
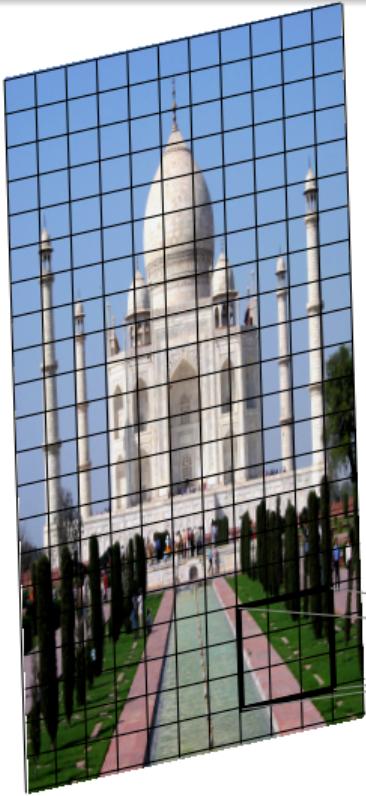
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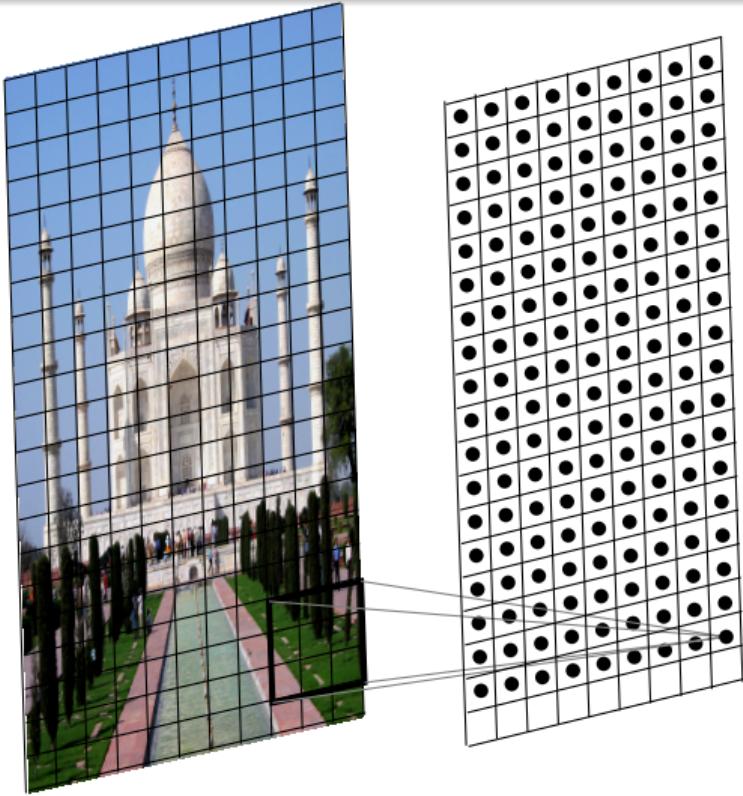
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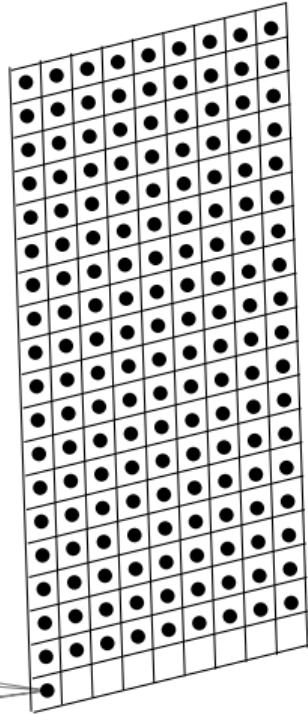
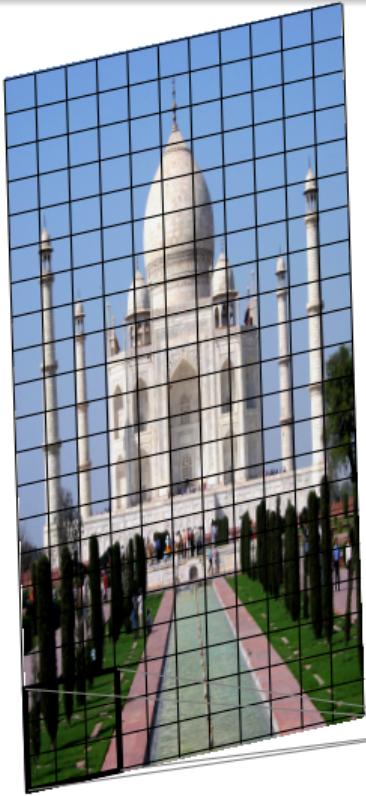
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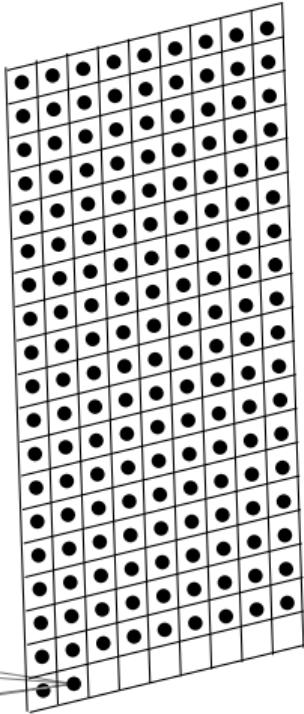
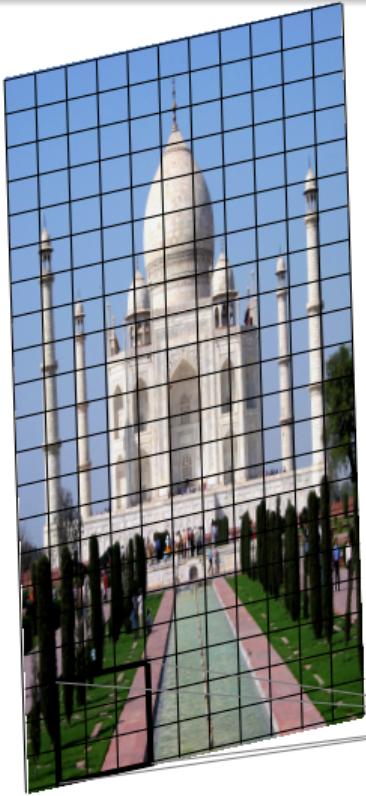
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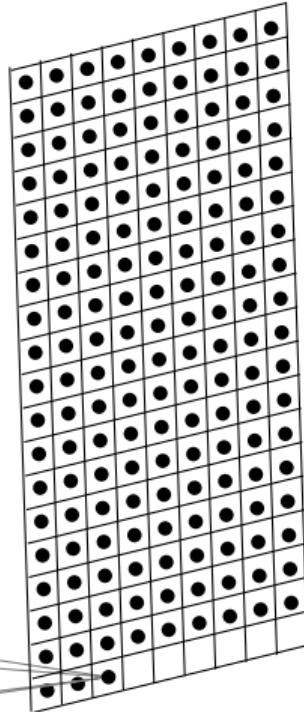
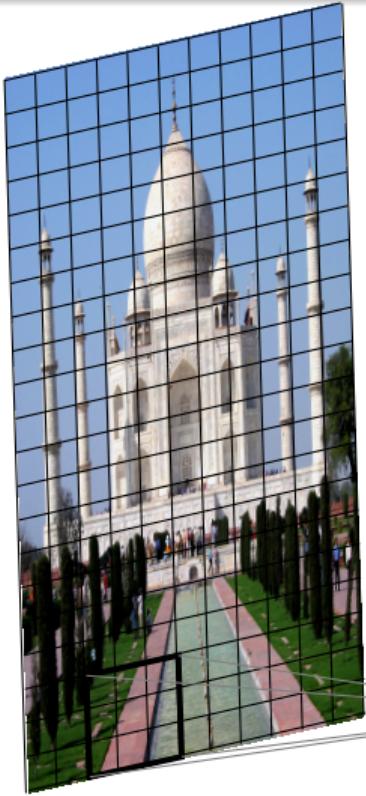
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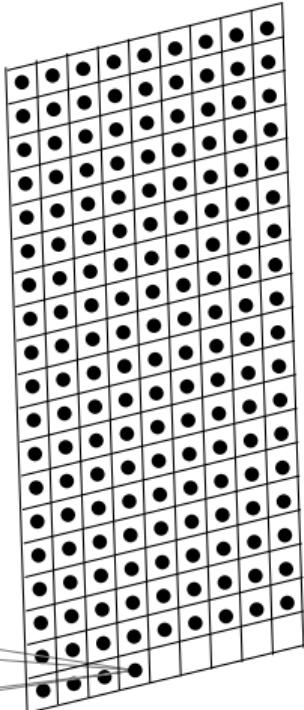
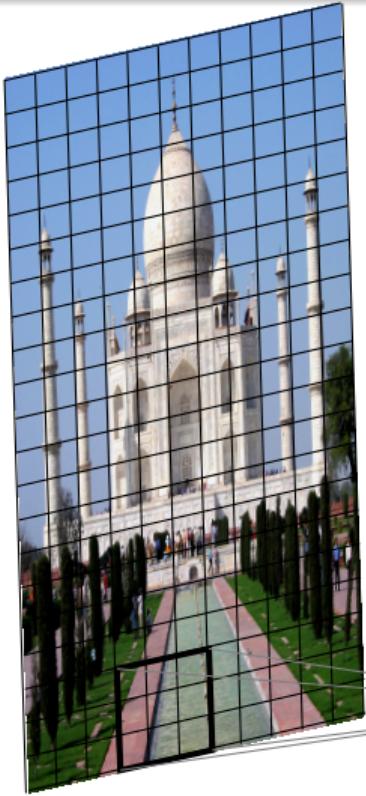
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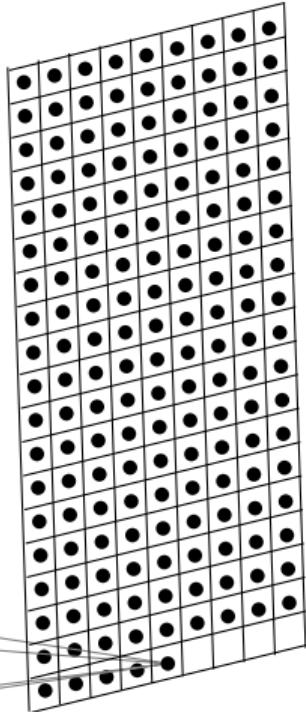
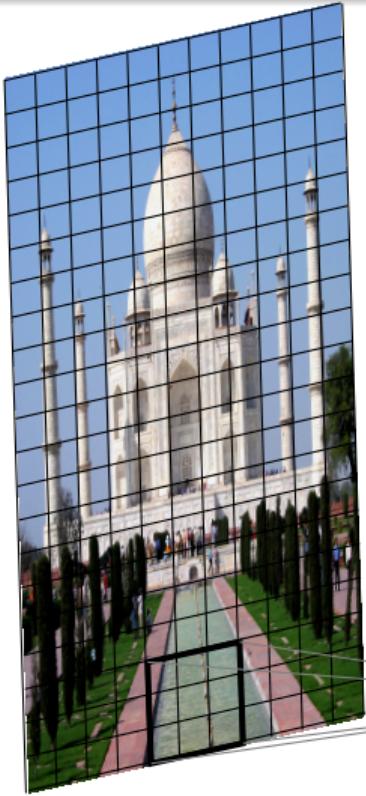
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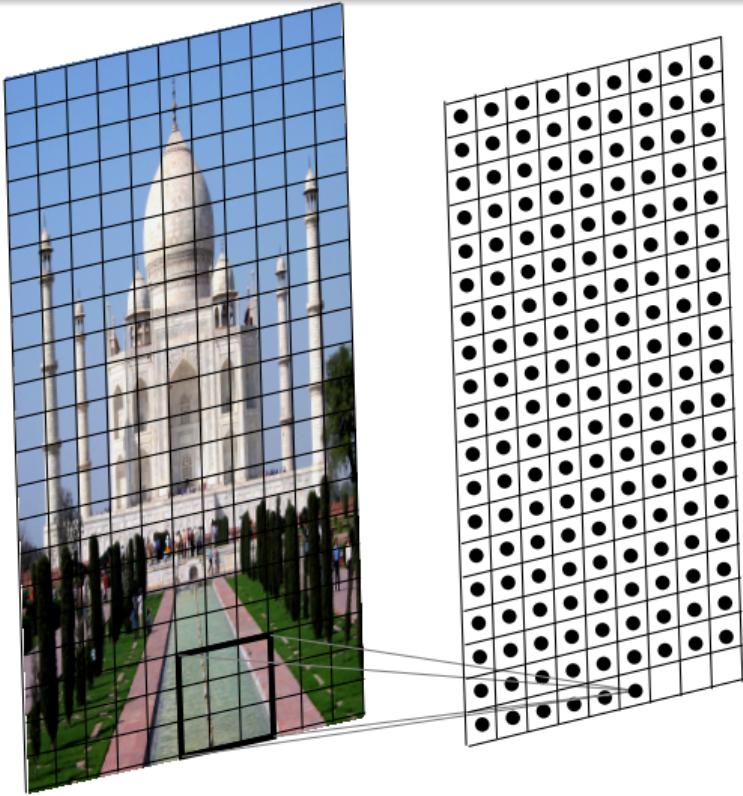
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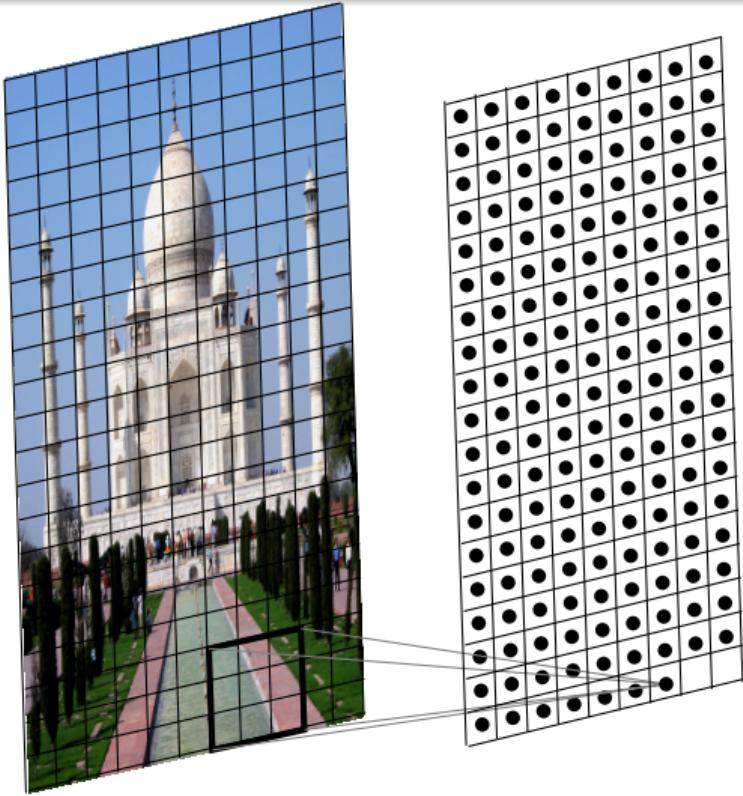
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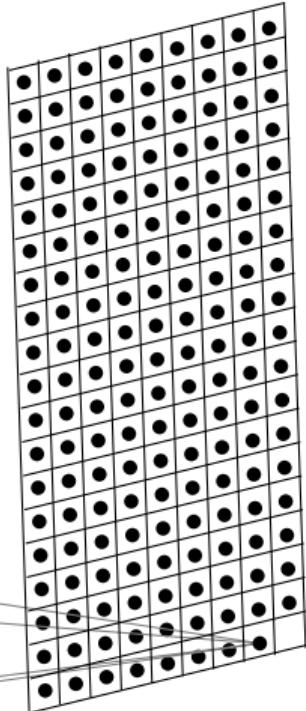
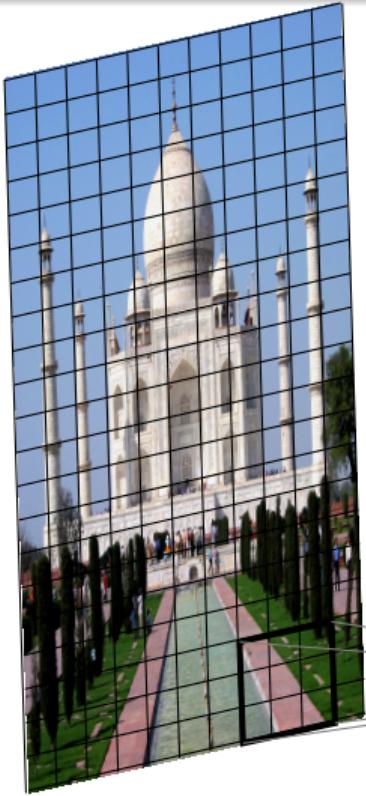
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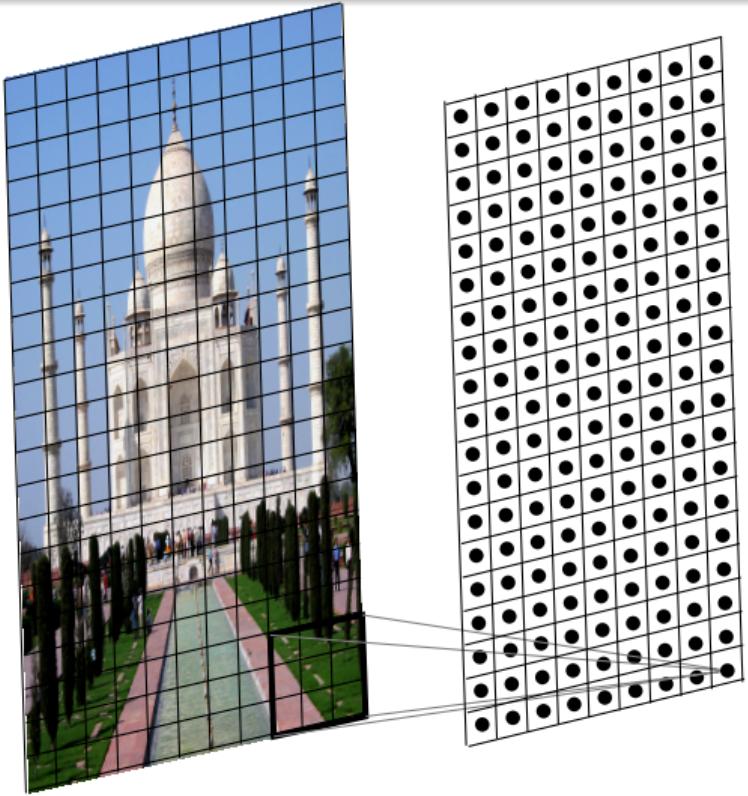
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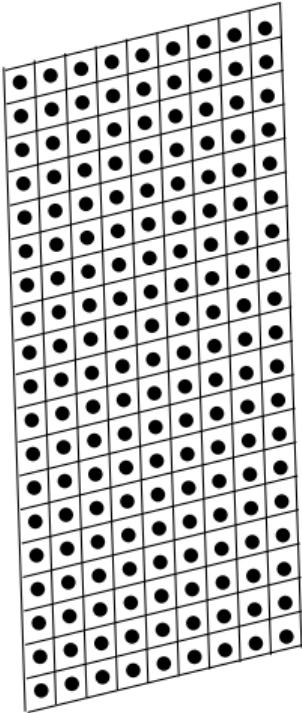
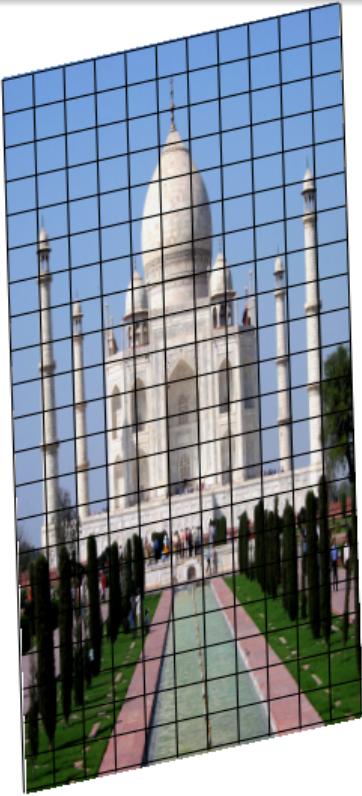
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- We can use multiple filters to get multiple feature maps.

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e	f	g	h
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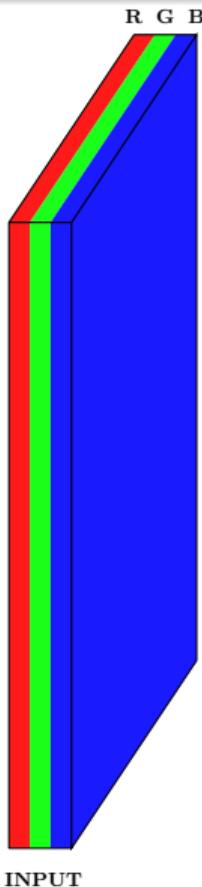
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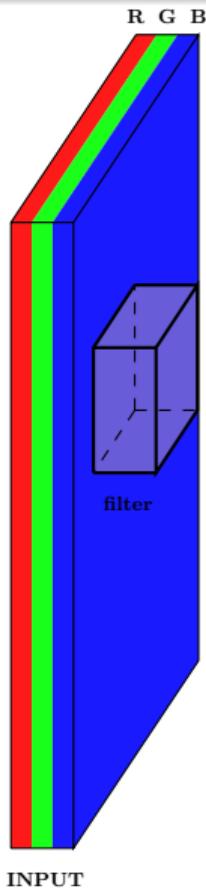
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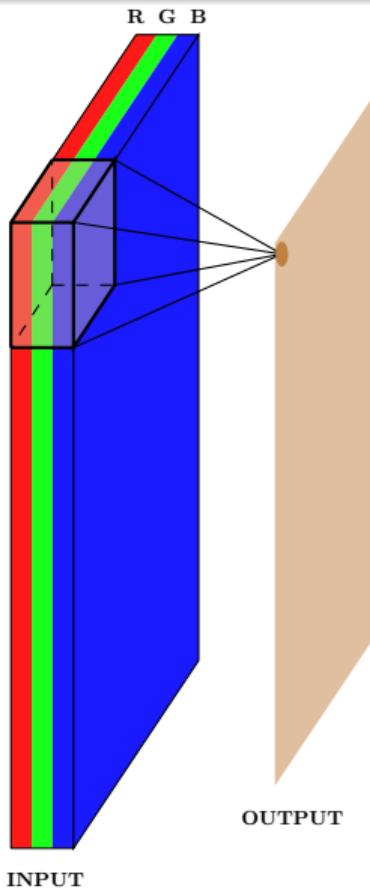
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- What would happen in the 3D case?



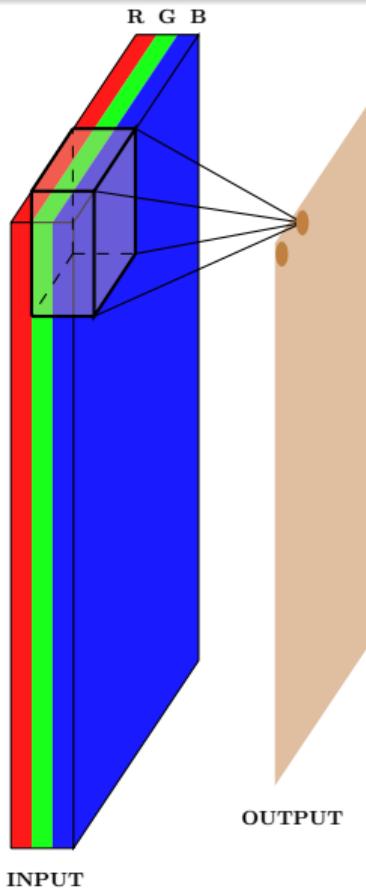
- What would a 3D filter look like?



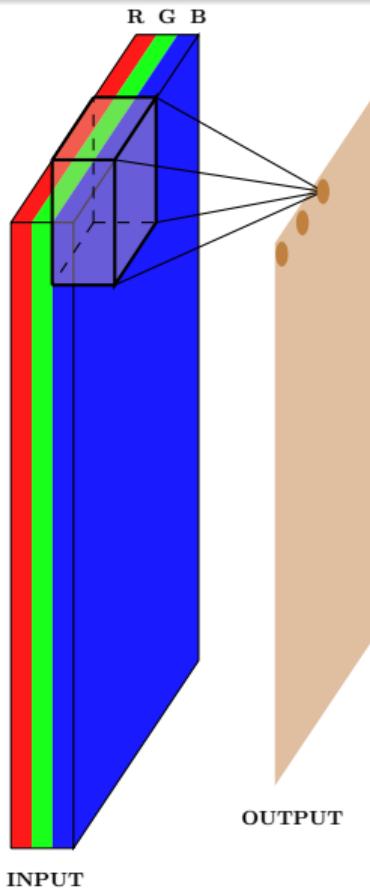
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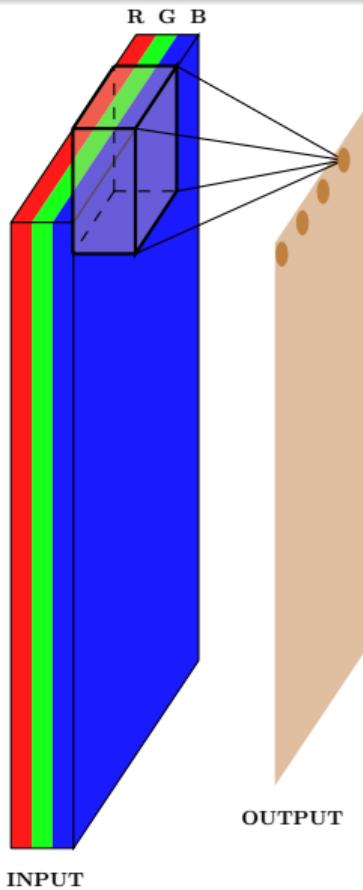
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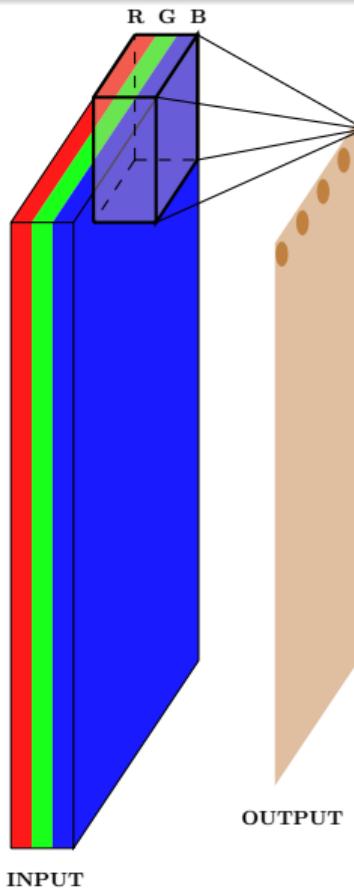
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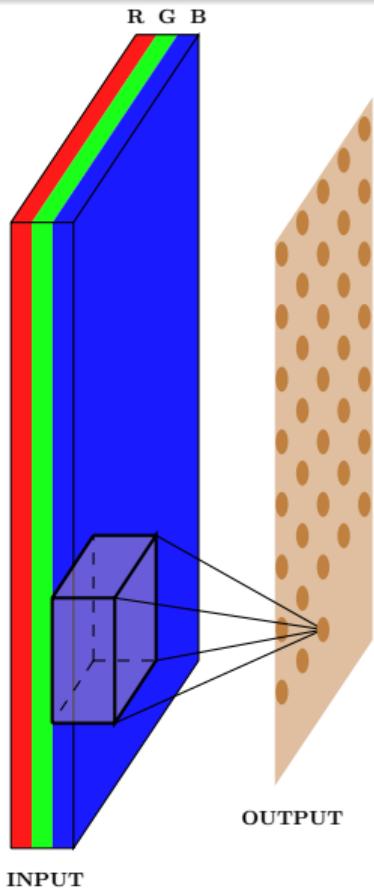
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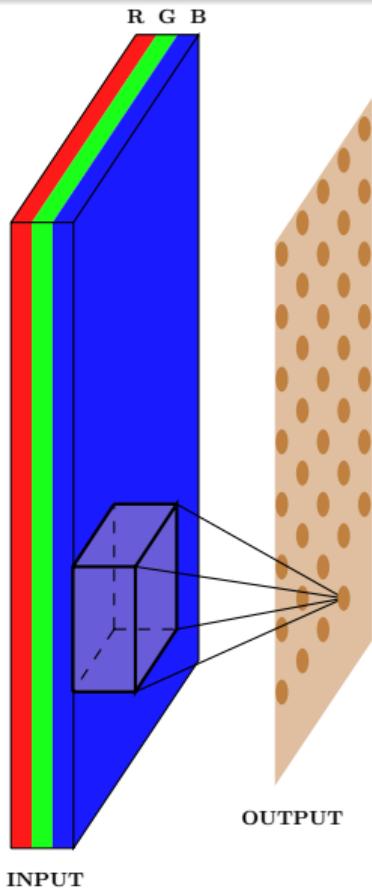
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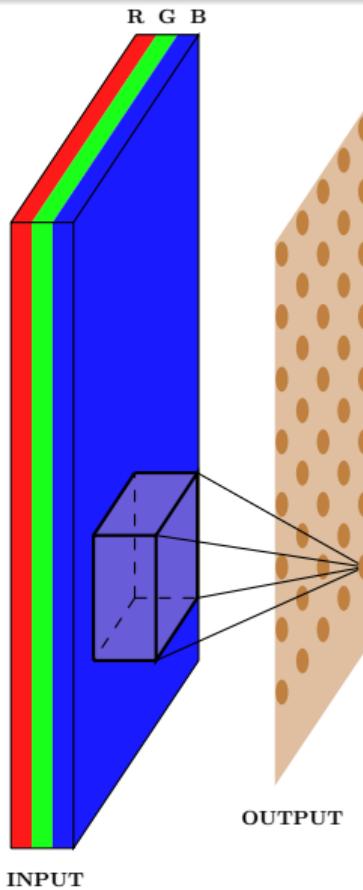
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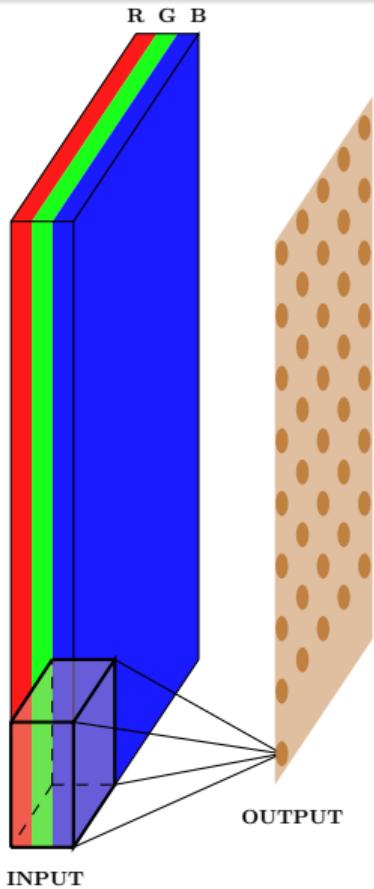
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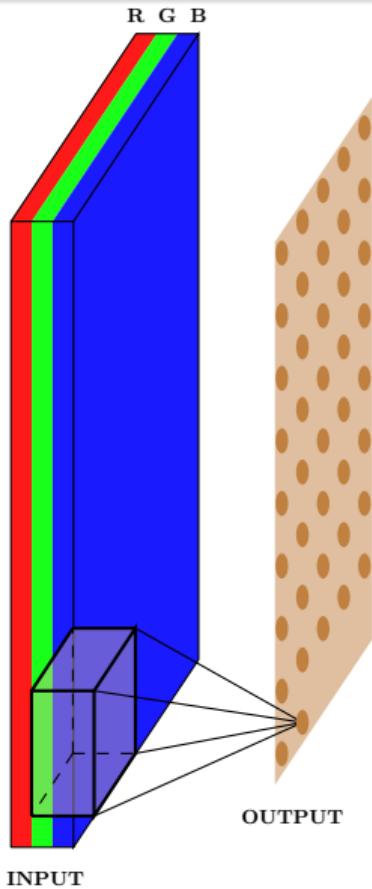
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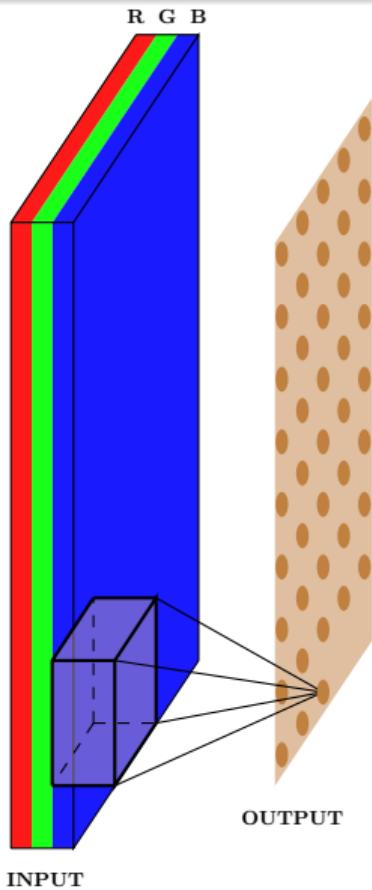
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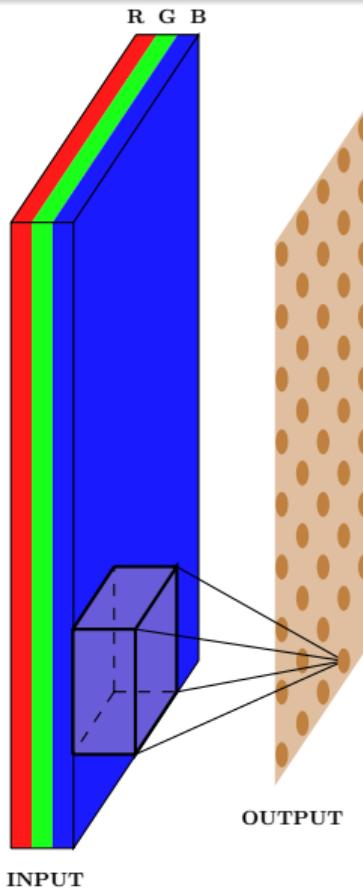
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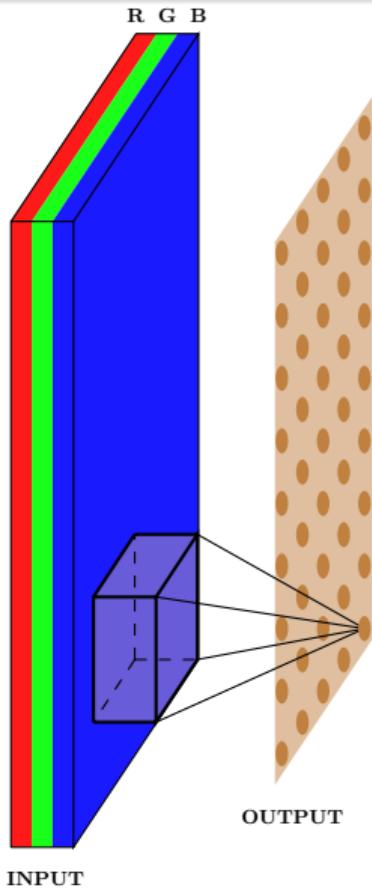
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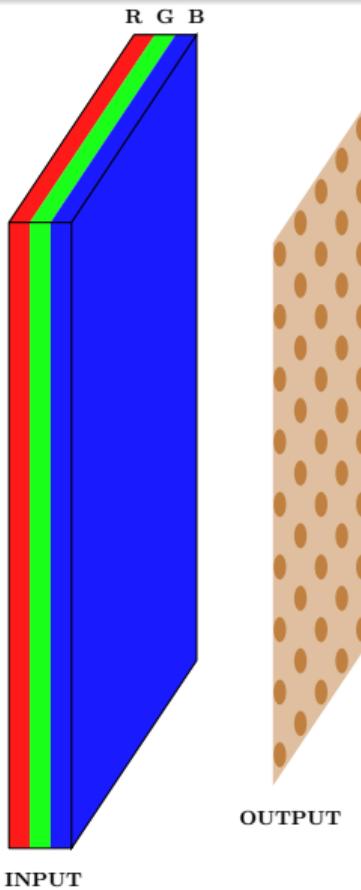
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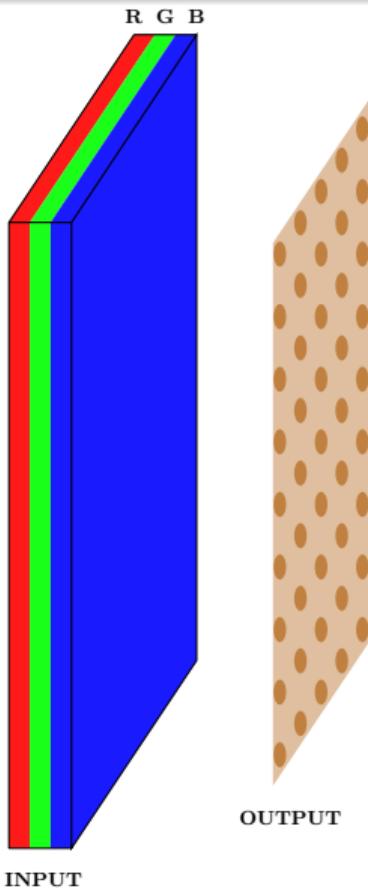
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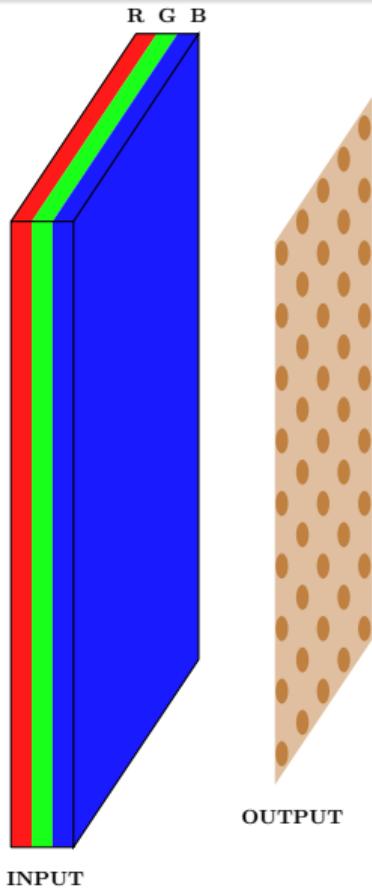
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Module 11.2 : Relation between input size, output size and filter size

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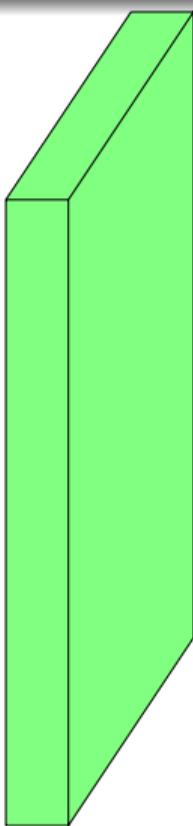
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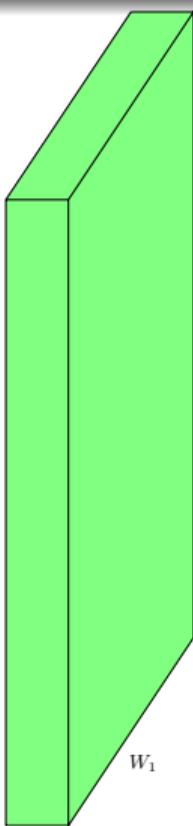
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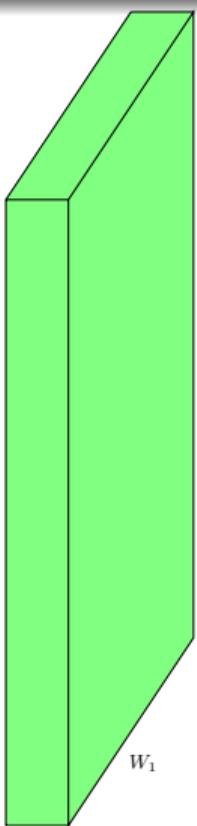
- So far we have not said anything explicit about the dimensions of the
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 - ③ outputsand the relations between them
- We will see how they are related but before that we will define a few quantities



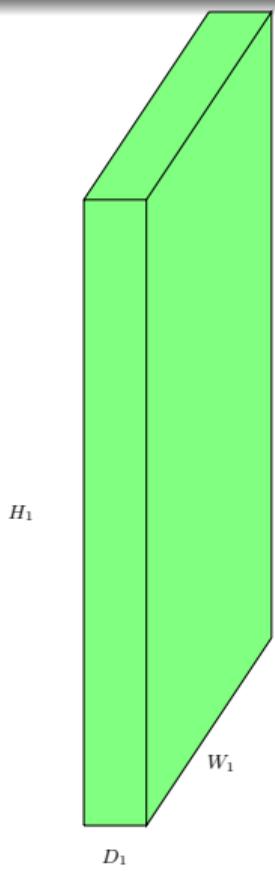
- We first define the following quantities



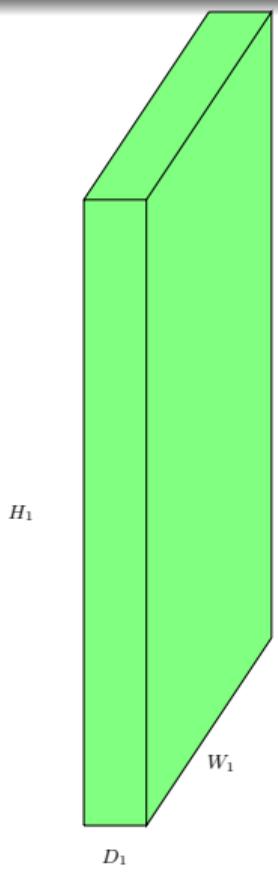
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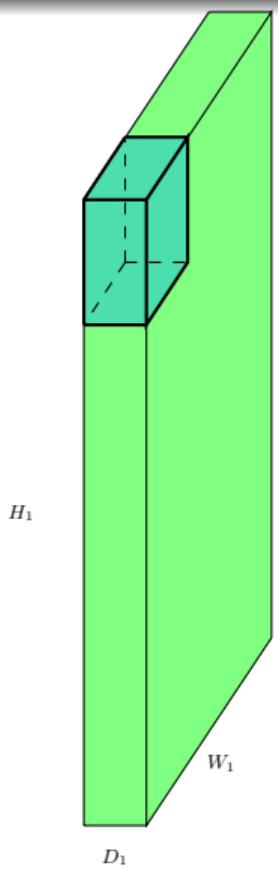
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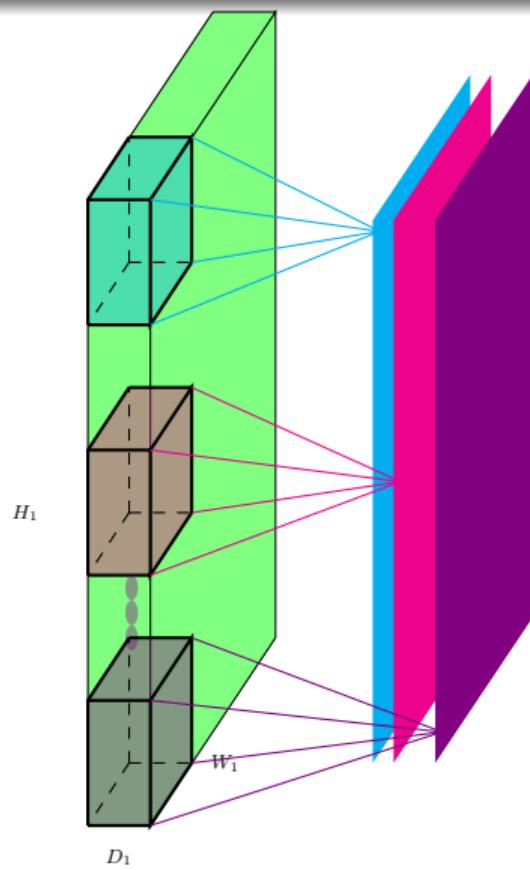
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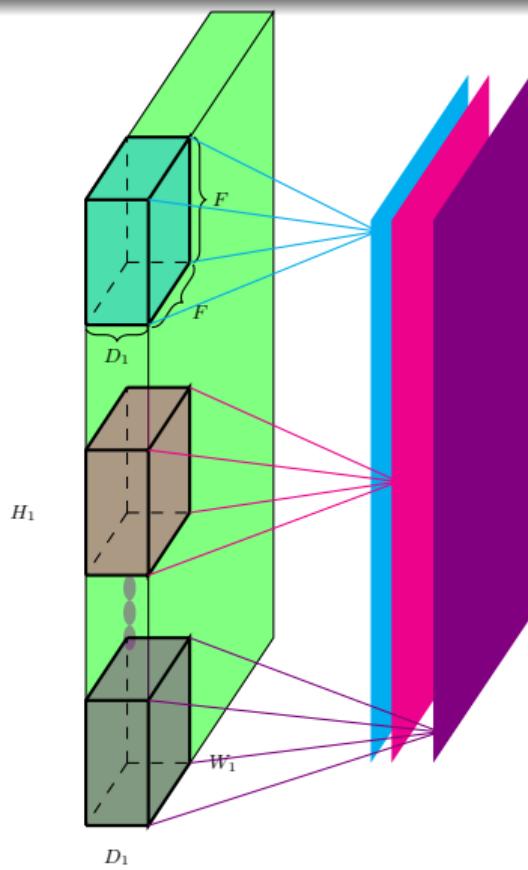
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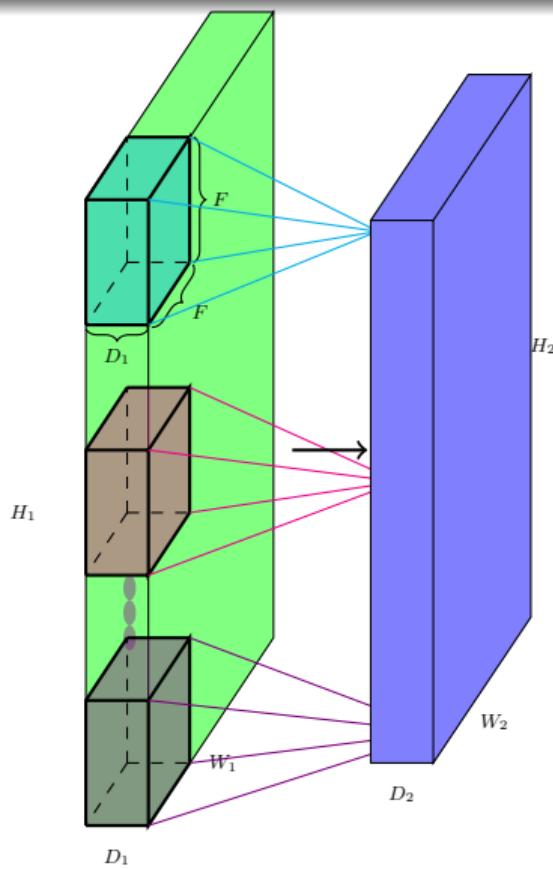
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- The output is $W_2 \times H_2 \times D_2$ (we will soon see a formula for computing W_2 , H_2 and D_2)

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In general, $W_2 = W_1 - F + 1$

$$H_2 = H_1 - F + 1$$

We will refine this formula further

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0	0	0	0	0	0	0	0
0							0
0							0
0							0
0							0
0							0
0							0
0	0	0	0	0	0	0	0

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0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

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0								0
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0	0	0	0	0	0	0	0	0

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- What if we want the output to be of same size as the input?
- We can use something known as padding
- Pad the inputs with appropriate number of 0 inputs so that you can now apply the kernel at the corners
- Let us use pad $P = 1$ with a 3×3 kernel
- This means we will add one row and one column of 0 inputs at the top, bottom, left and right

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & & & & & & & & & 0 \\ \hline
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline
 \end{array} =
 \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline
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 \end{array}$$

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We now have,

$$W_2 = W_1 - F + 2P + 1$$

$$H_2 = H_1 - F + 2P + 1$$

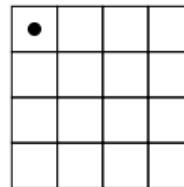
We will refine this formula further

- What does the stride S do?

- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)

0	0	0	0	0	0	0	0	0
0								0
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0	0	0	0	0	0	0	0	0

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- What does the stride S do?
- It defines the intervals at which the filter is applied (here $S = 2$)
- Here, we are essentially skipping every 2nd pixel which will again result in an output which is of smaller dimensions

0	0	0	0	0	0	0	0	0
0								0
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0								0
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0	0	0	0	0	0	0	0	0

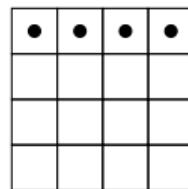
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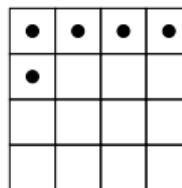
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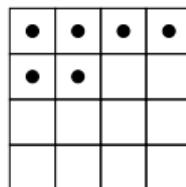
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So what should our final formula look like,

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
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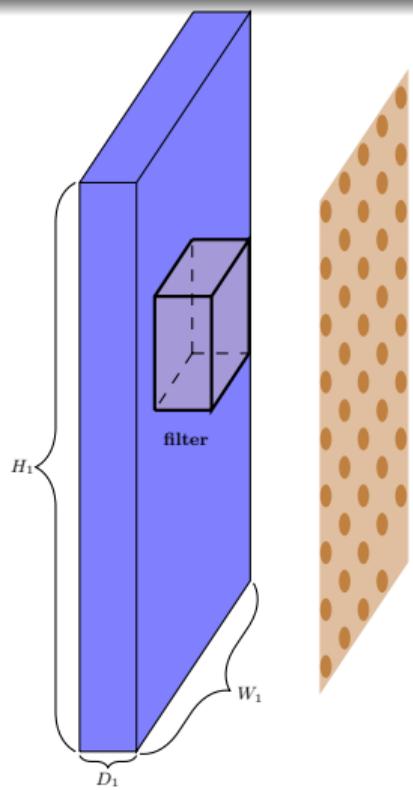
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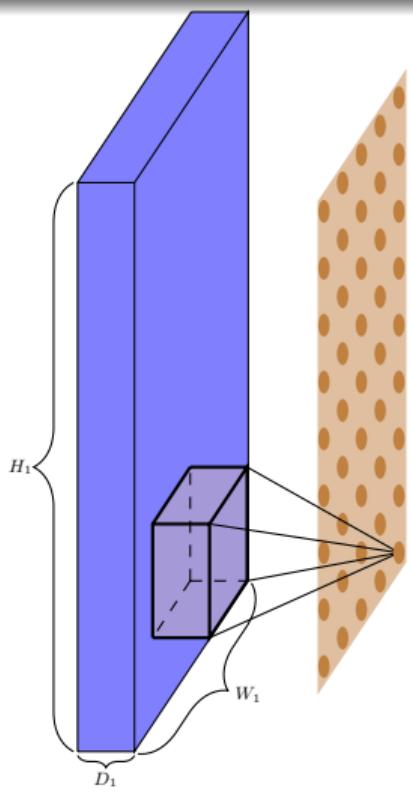
So what should our final formula look like,

$$W_2 = \frac{W_1 - F + 2P}{S} + 1$$

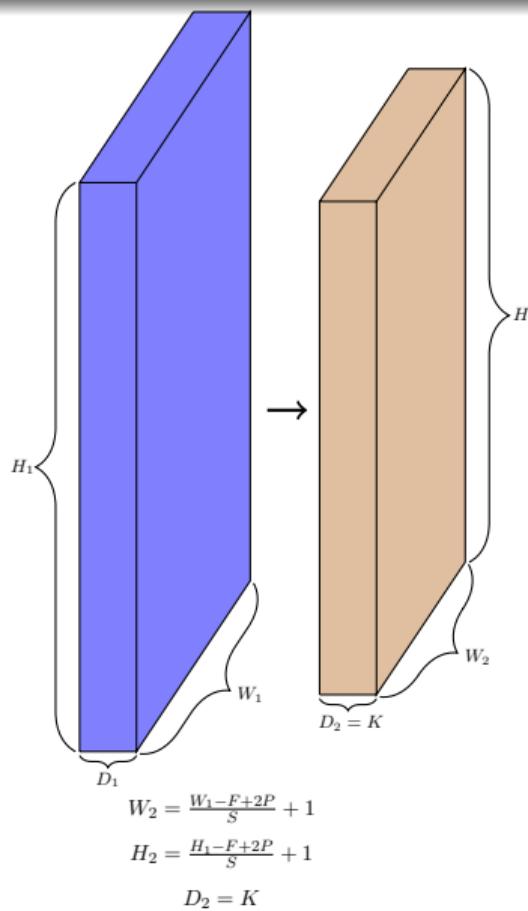
$$H_2 = \frac{H_1 - F + 2P}{S} + 1$$

- Finally, coming to the depth of the output.

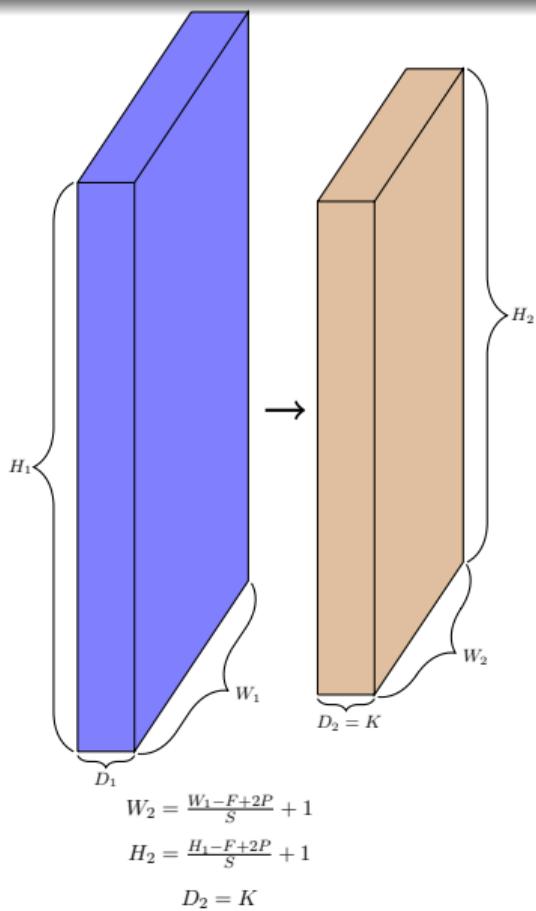




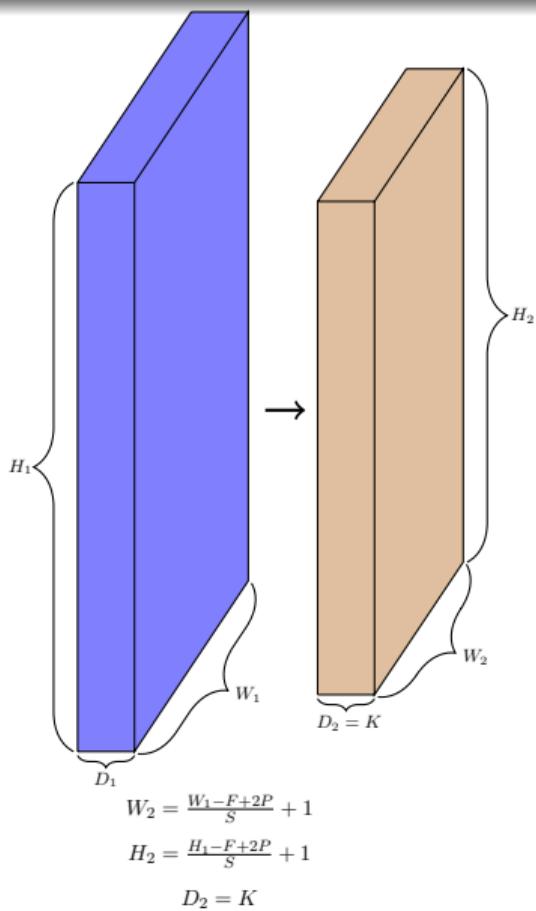
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- Each filter gives us one 2D output.



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- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs

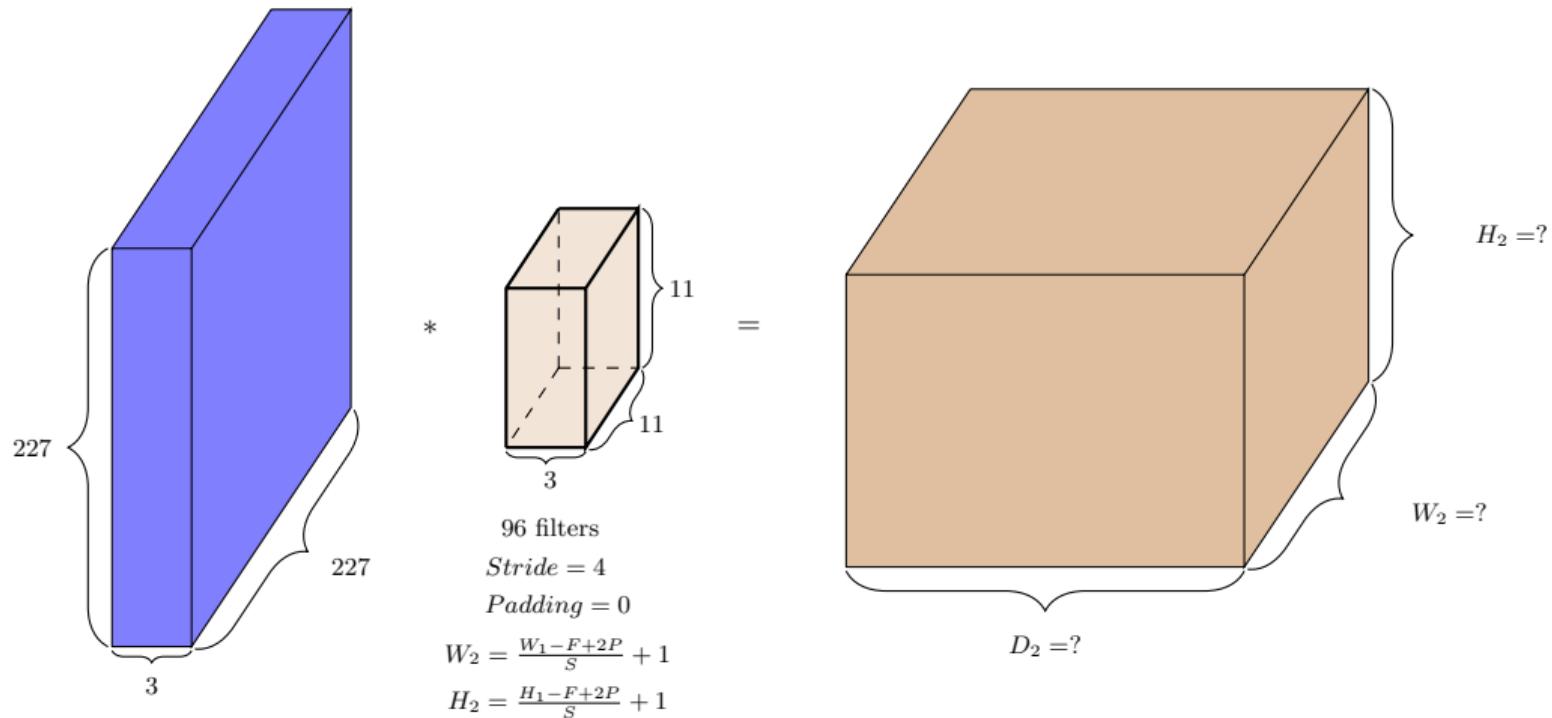


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- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs
- We can think of the resulting output as $K \times W_2 \times H_2$ volume

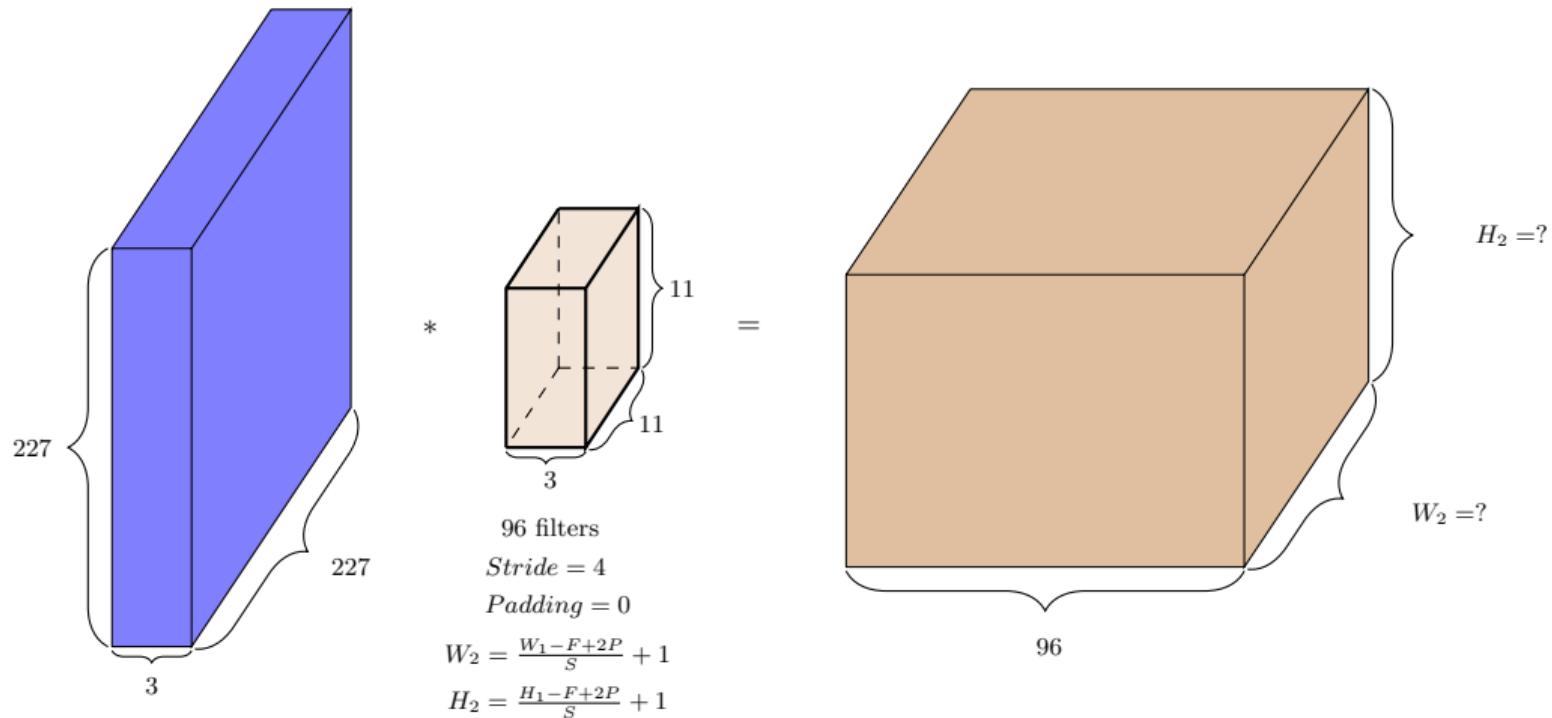


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- Each filter gives us one 2D output.
- K filters will give us K such 2D outputs
- We can think of the resulting output as $K \times W_2 \times H_2$ volume
- Thus $D_2 = K$

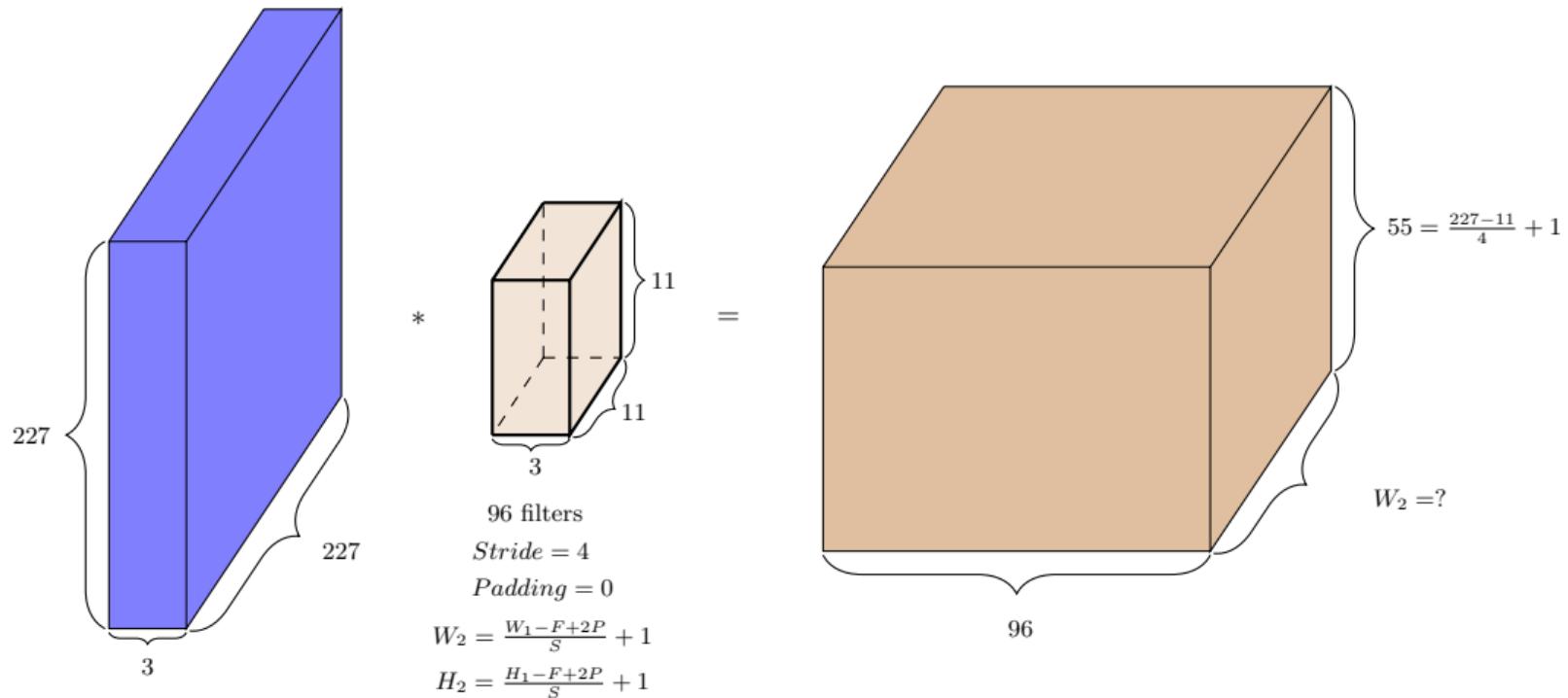
Let us do a few exercises



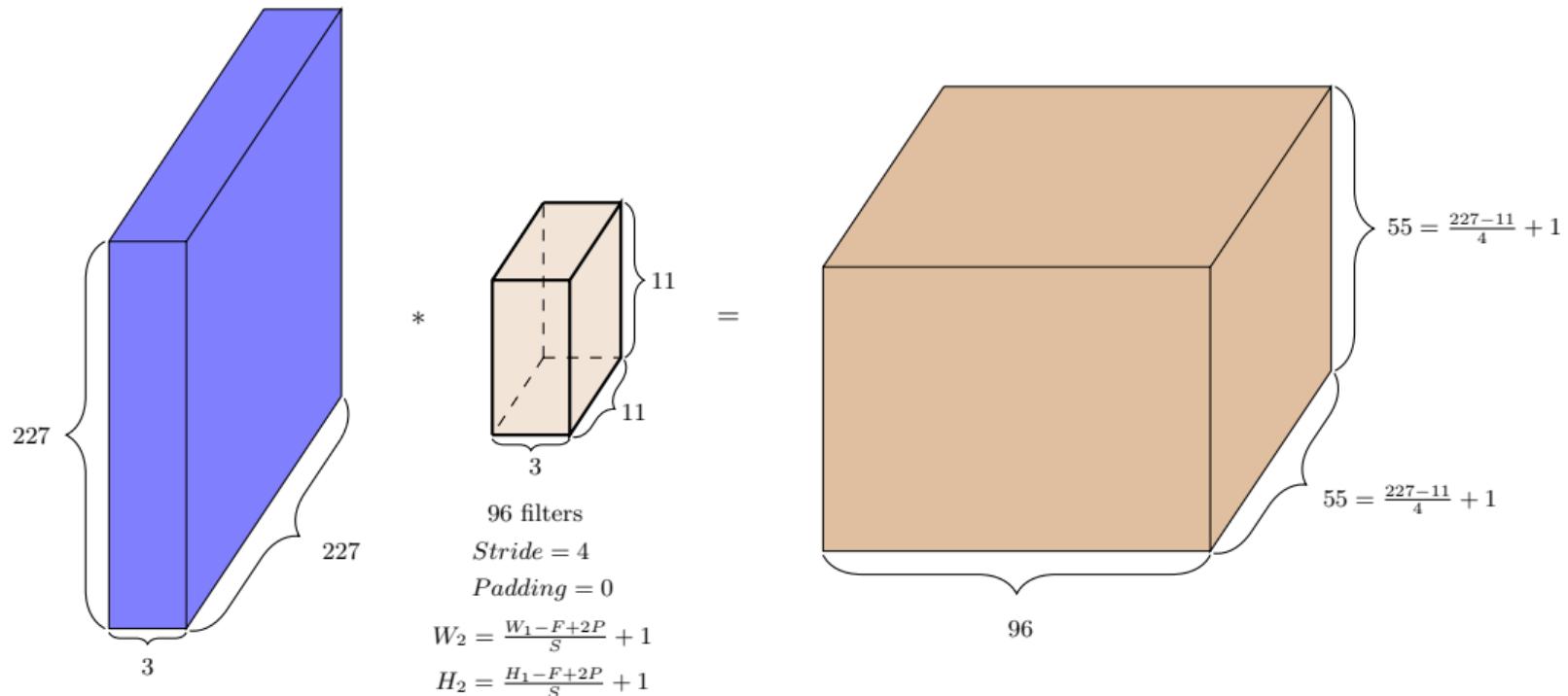
Let us do a few exercises



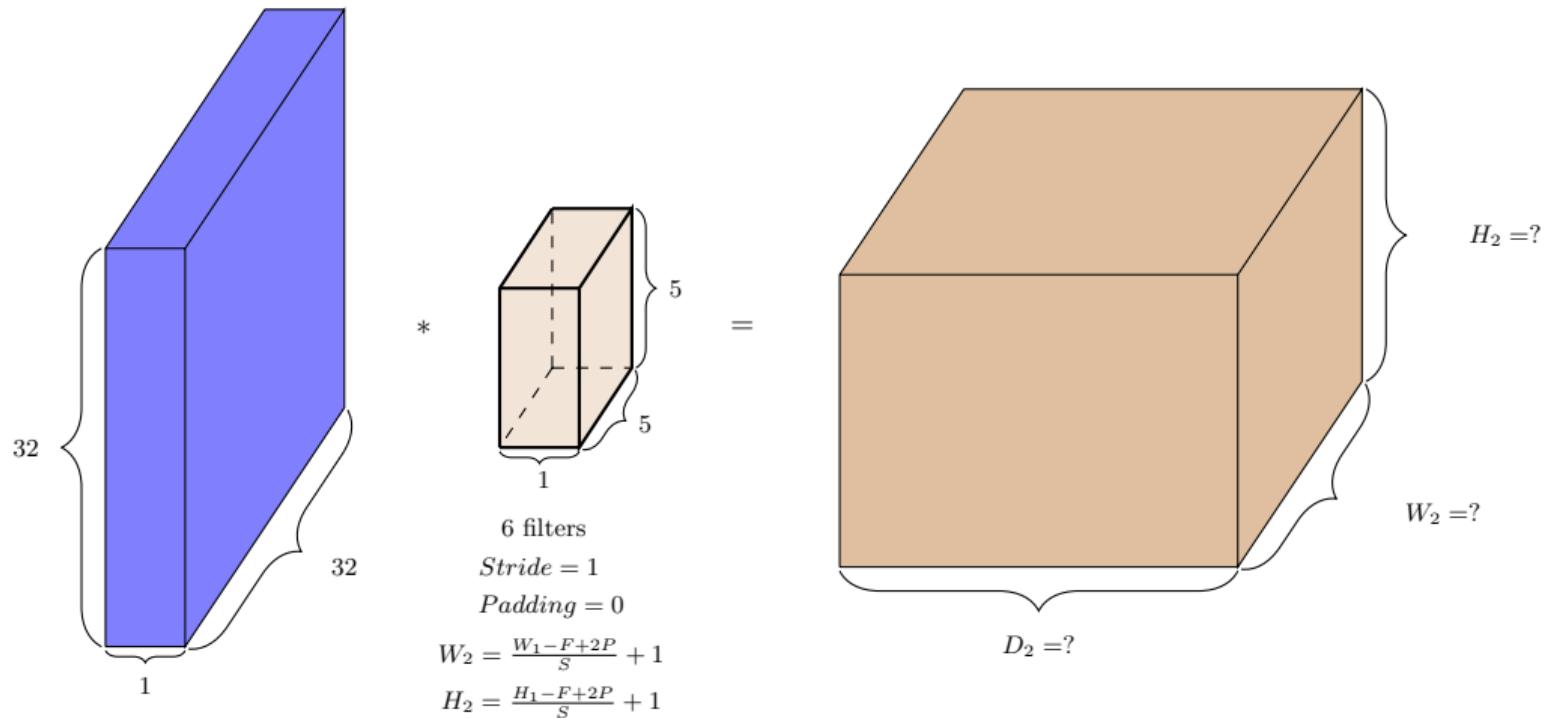
Let us do a few exercises



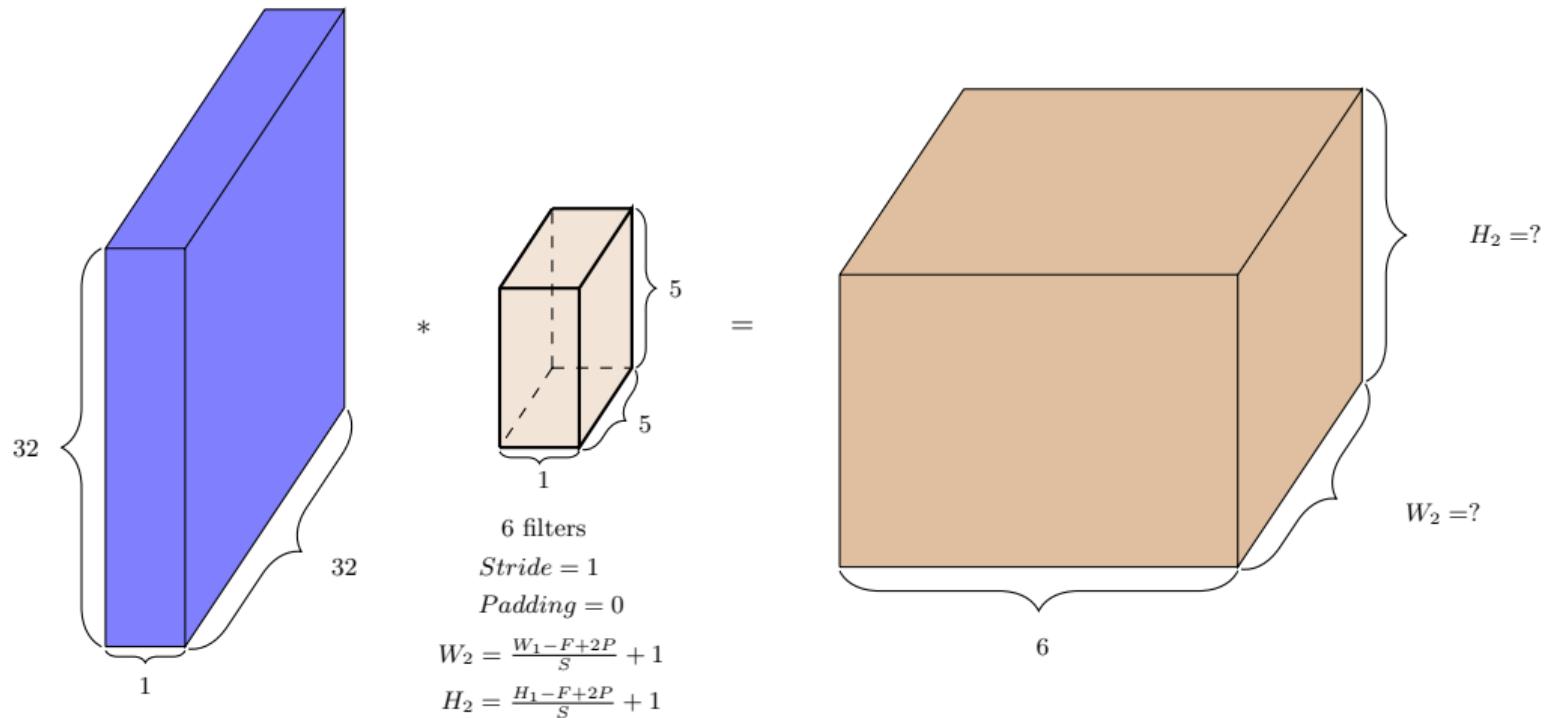
Let us do a few exercises



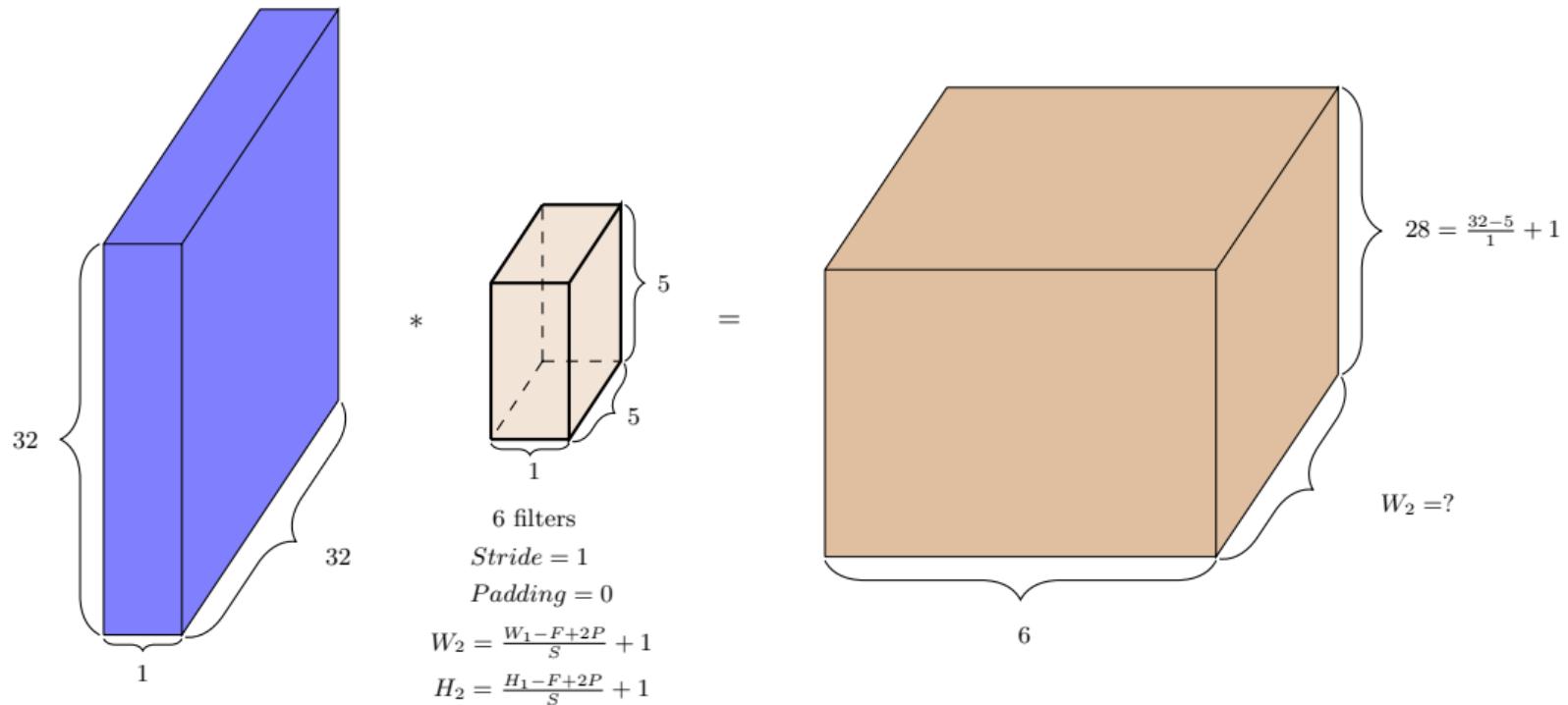
Let us do a few exercises



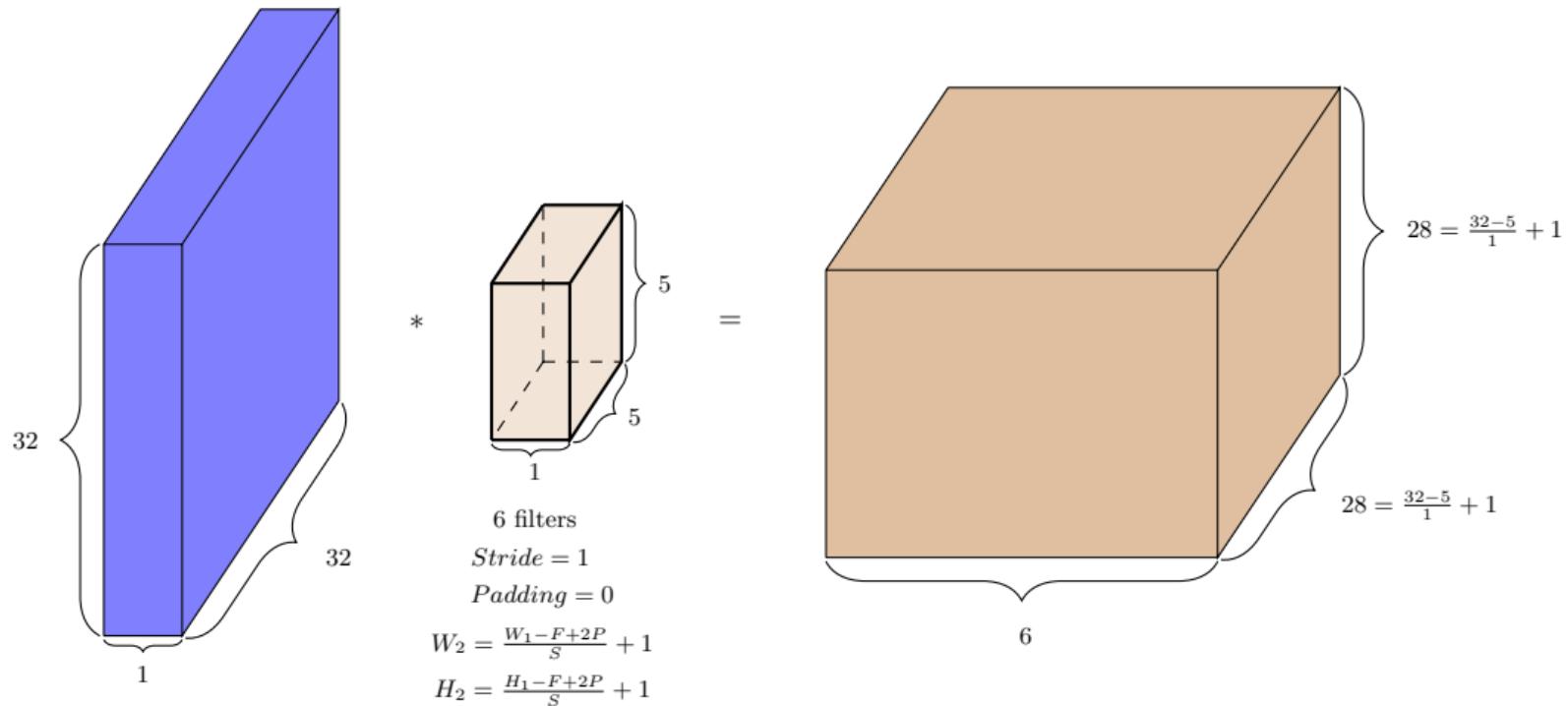
Let us do a few exercises



Let us do a few exercises



Let us do a few exercises



Module 11.3 : Convolutional Neural Networks

Putting things into perspective

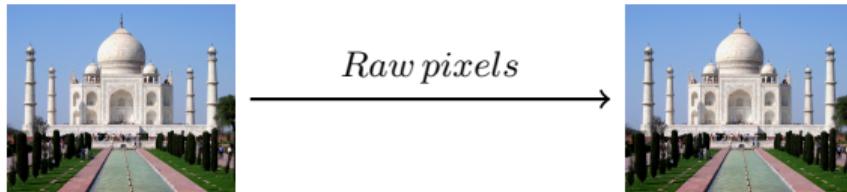
- What is the connection between this operation (convolution) and neural networks?

Putting things into perspective

- What is the connection between this operation (convolution) and neural networks?
- We will try to understand this by considering the task of “image classification”



Features





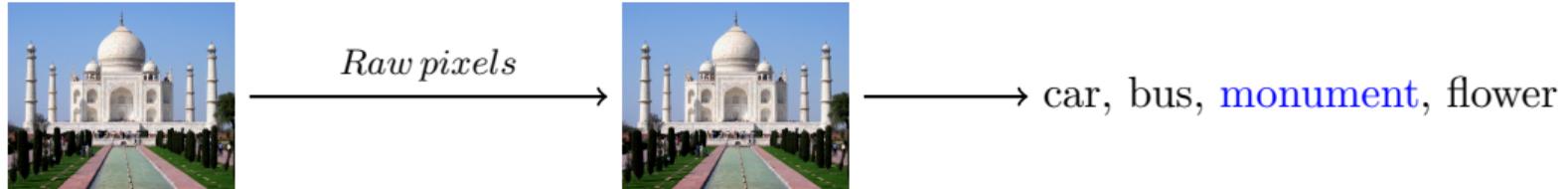
Raw pixels



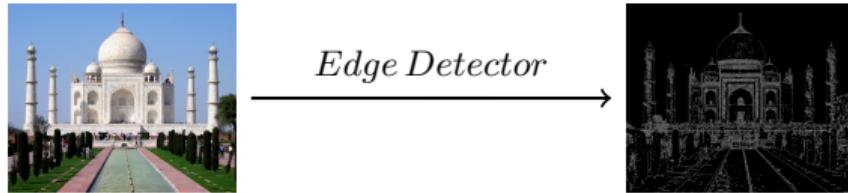
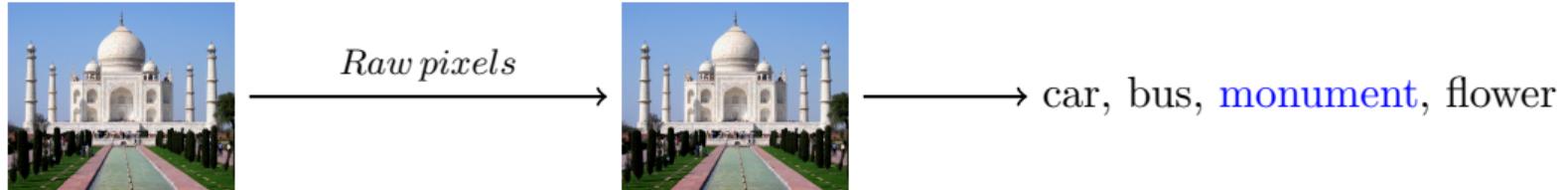
Features

car, bus, **monument**, flower

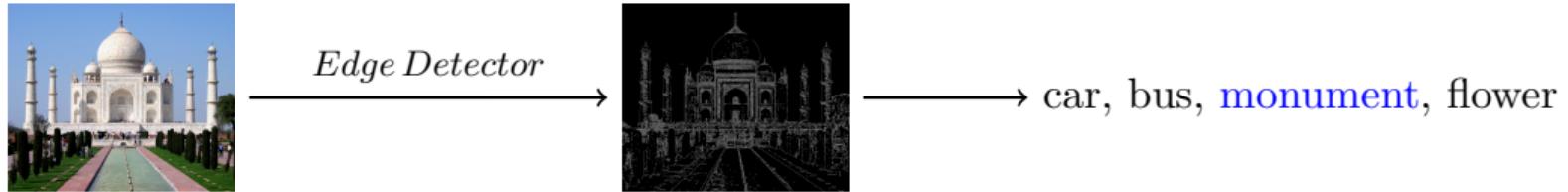
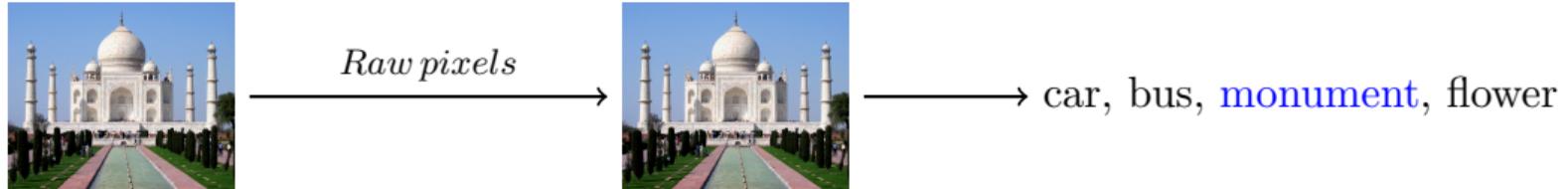
Features



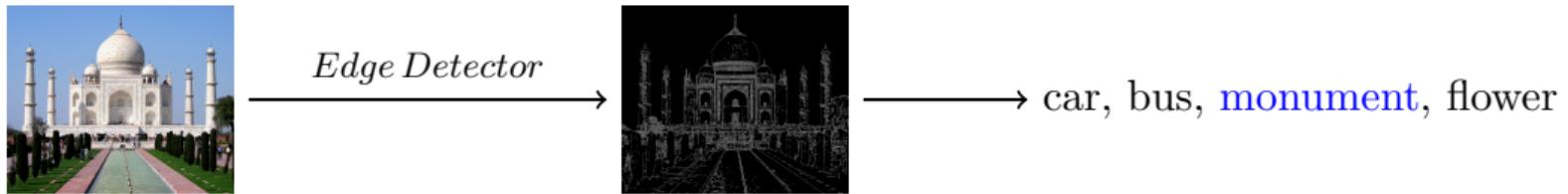
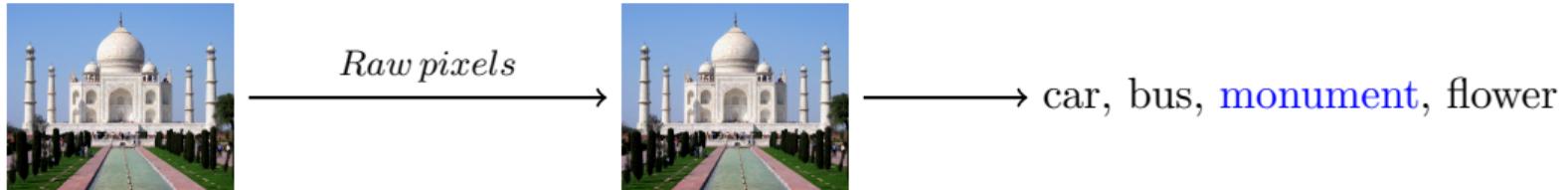
Features



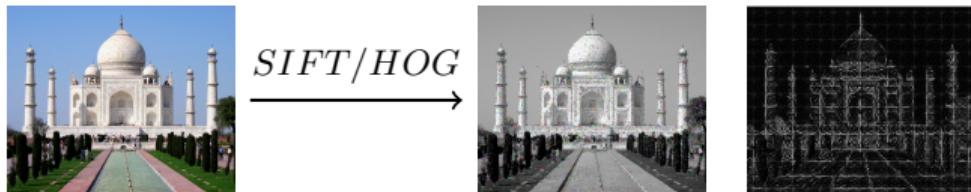
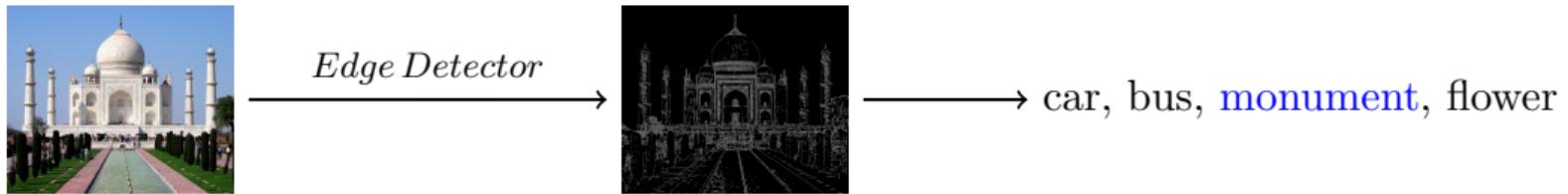
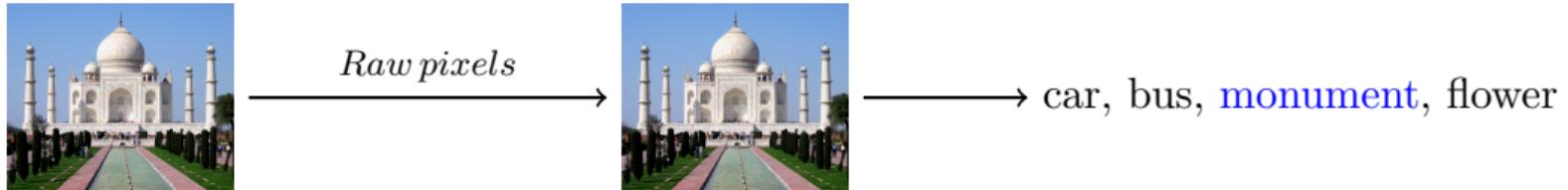
Features



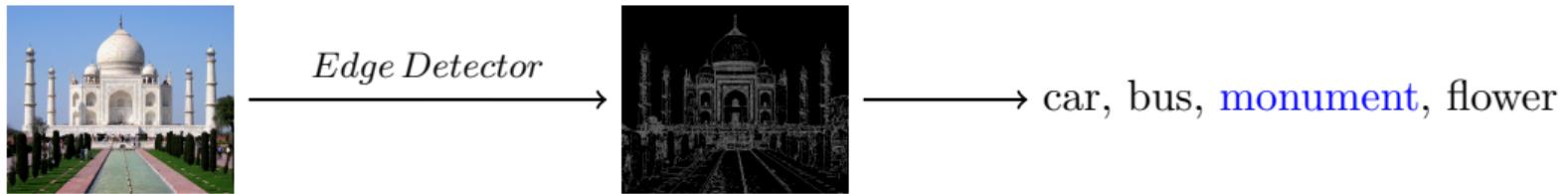
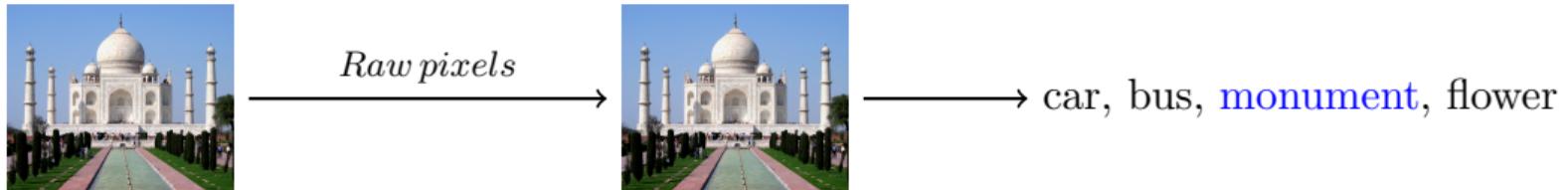
Features



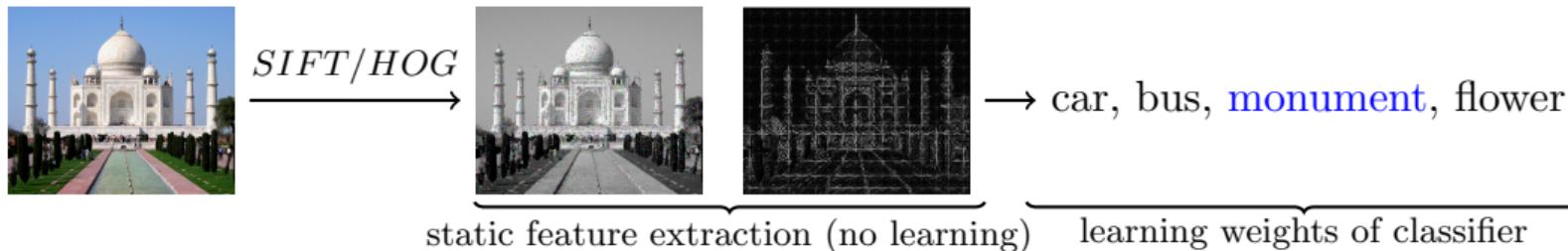
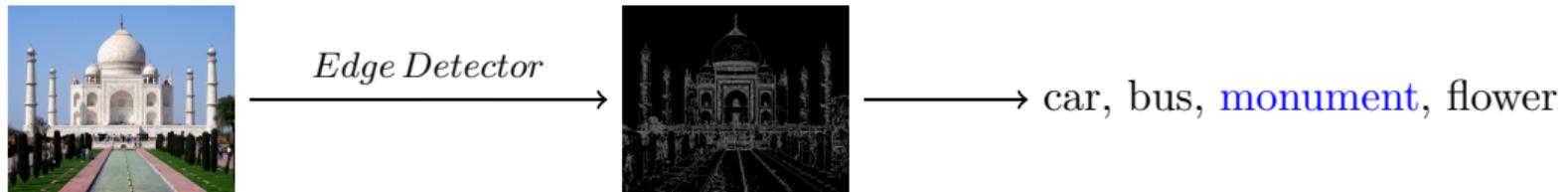
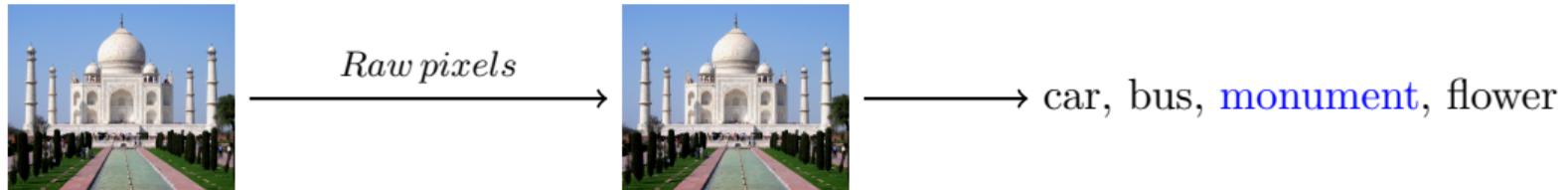
Features



Features



Features



Input



Features

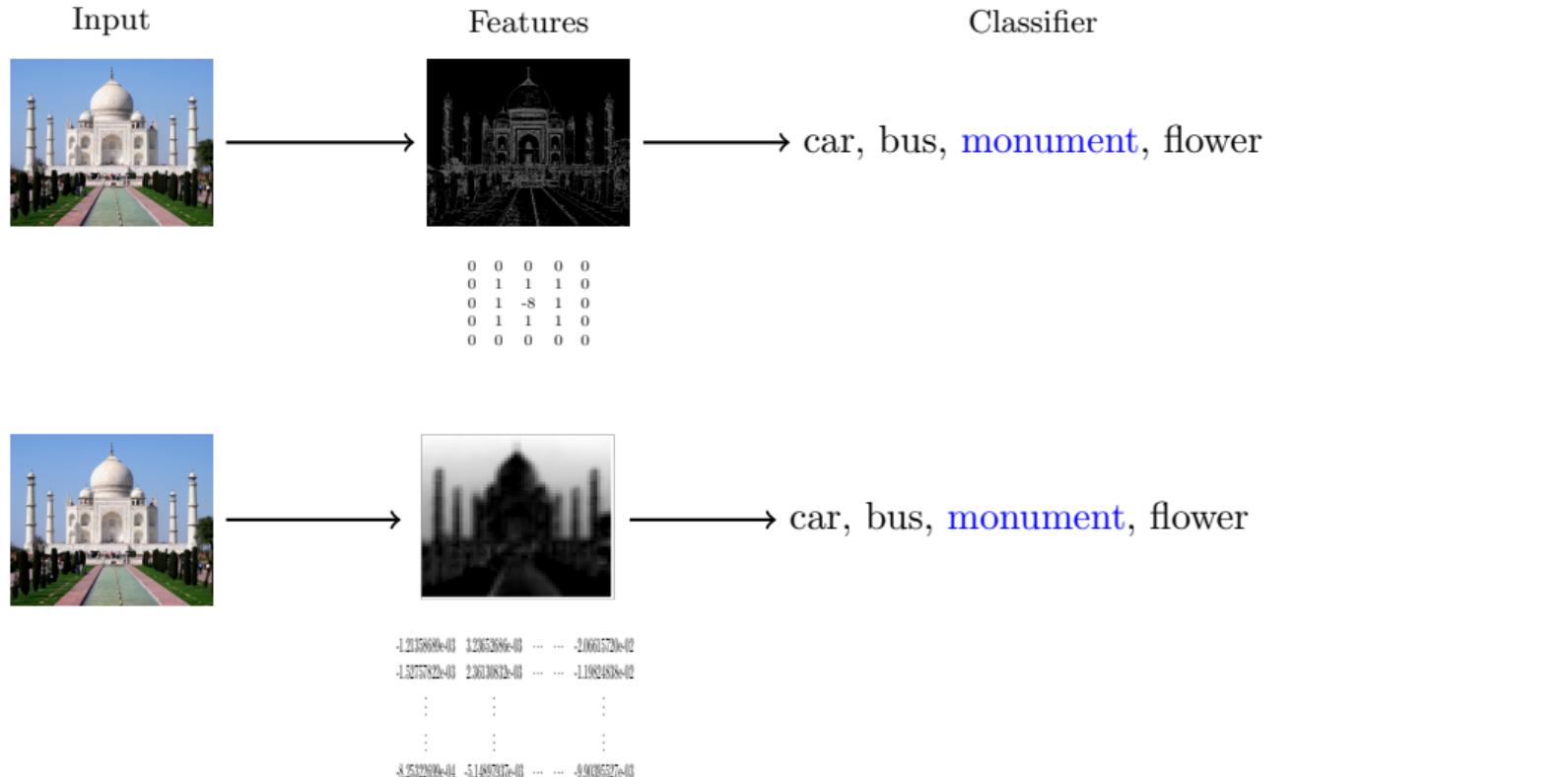


Classifier

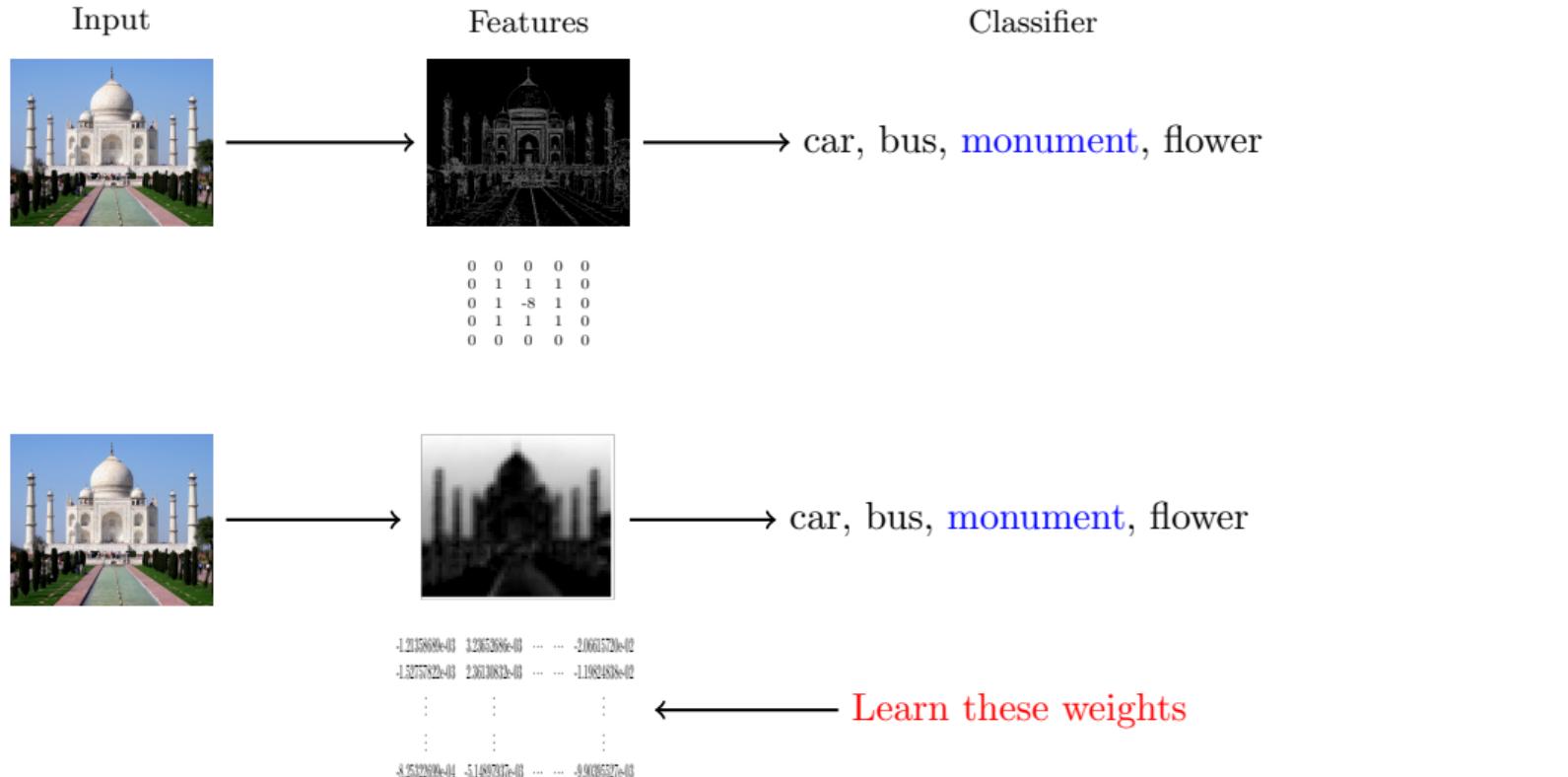
car, bus, **monument**, flower

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -8 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

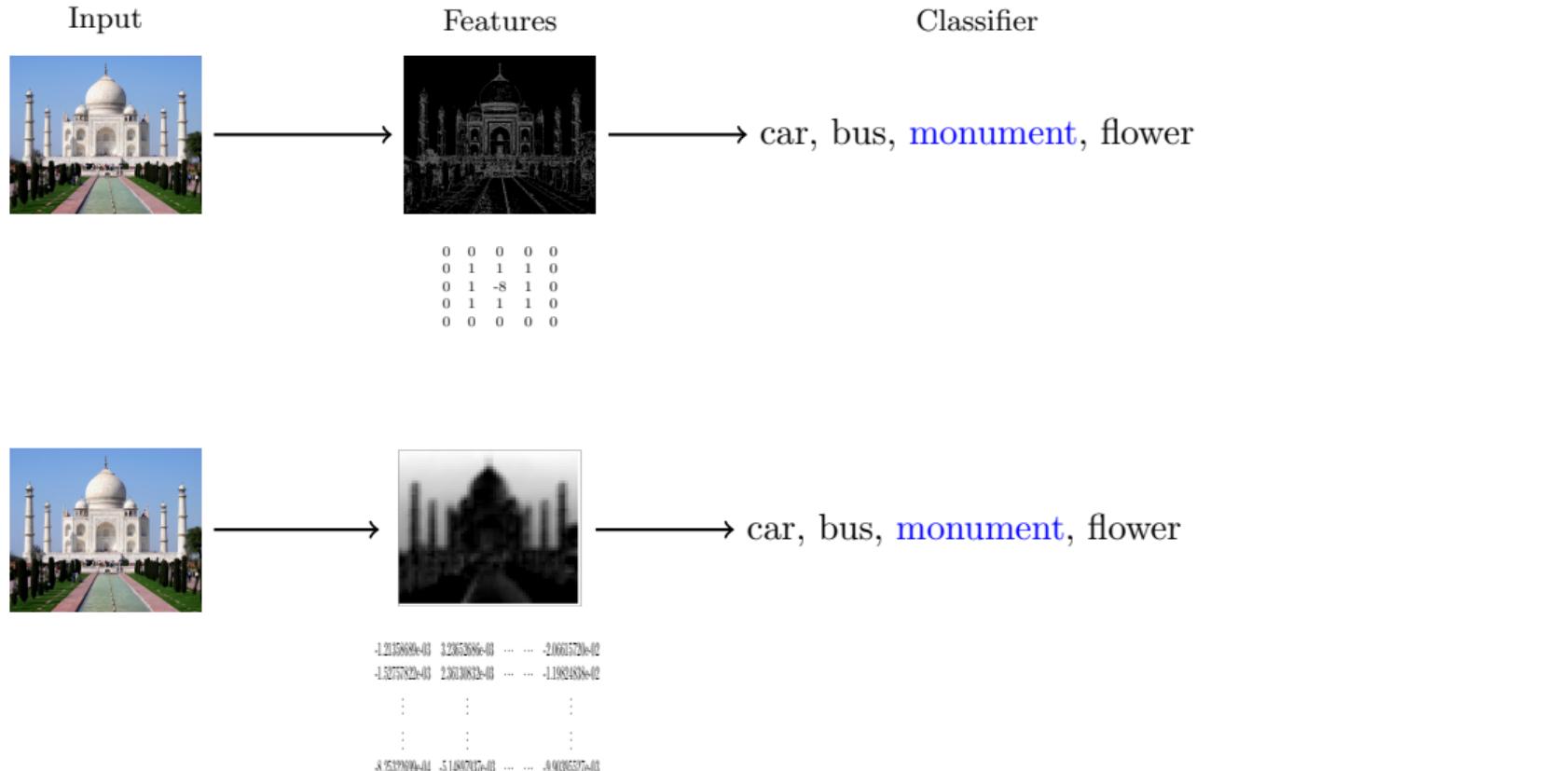
- Instead of using handcrafted kernels such as edge detectors **can we learn meaningful kernels/filters in addition to learning the weights of the classifier?**



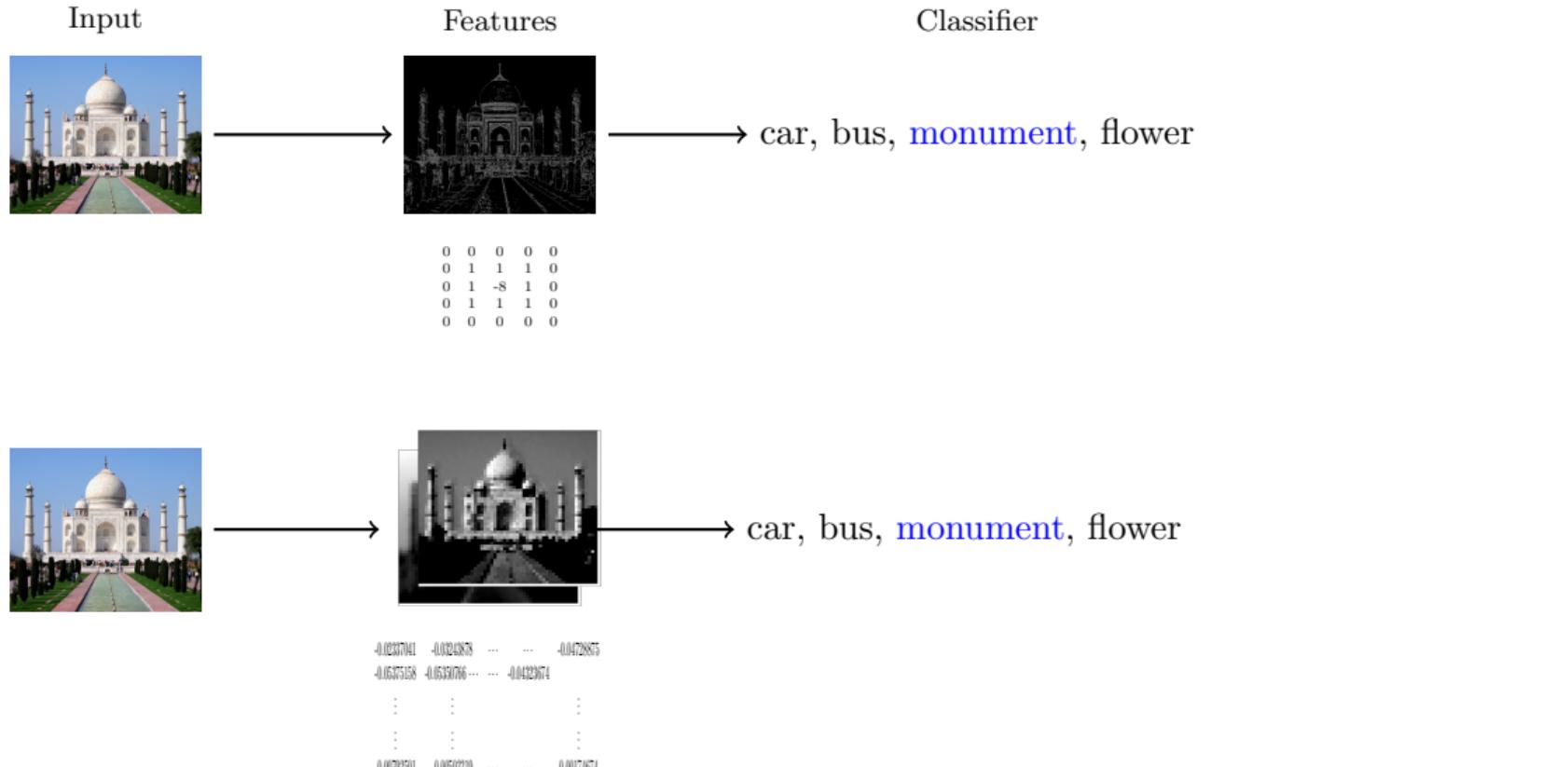
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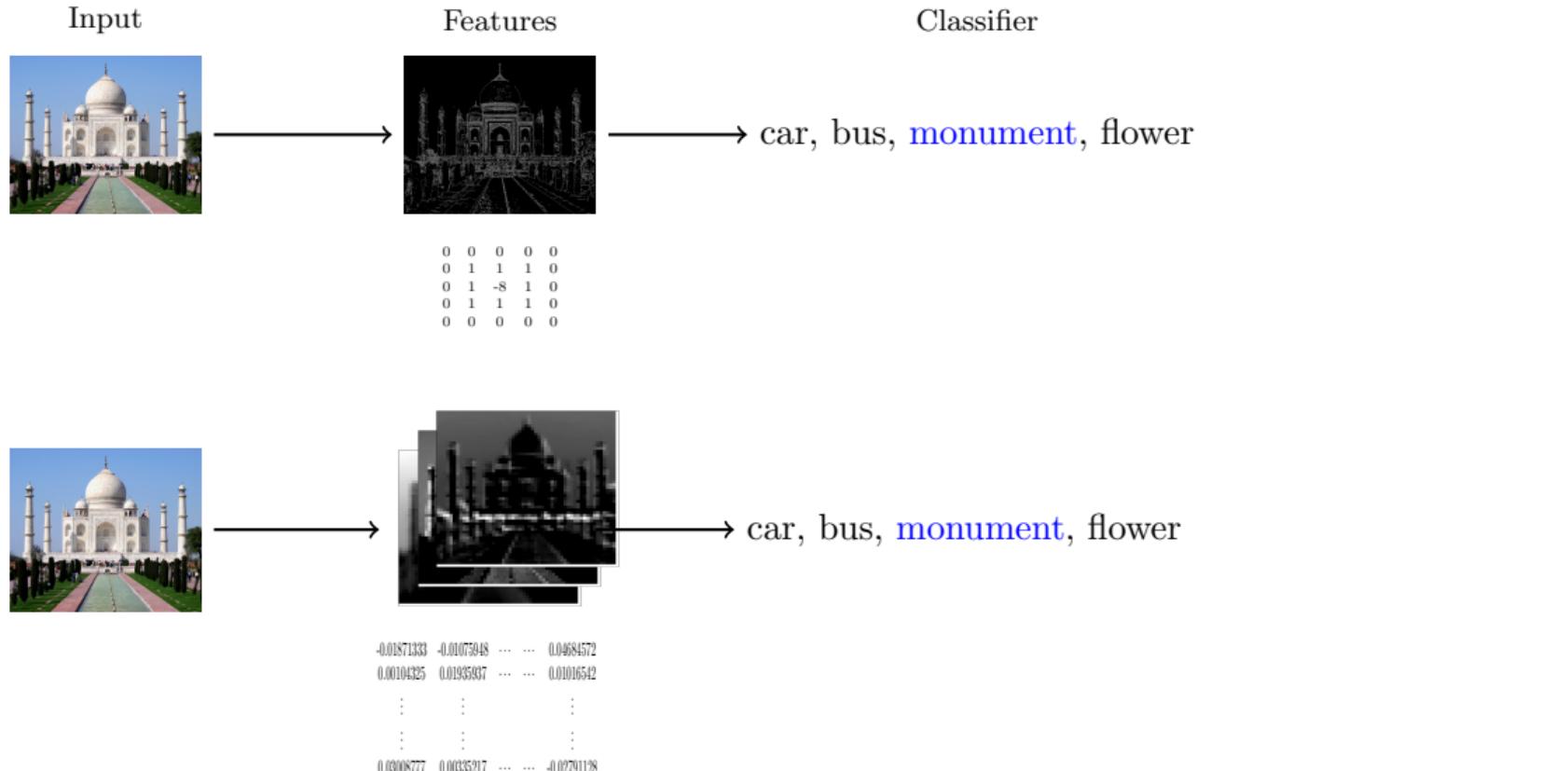
- Instead of using handcrafted kernels such as edge detectors **can we learn meaningful kernels/filters in addition to learning the weights of the classifier?**



- Even better: Instead of using handcrafted kernels (such as edge detectors) can we learn **multiple meaningful kernels/filters in addition to learning the weights of the classifier?**

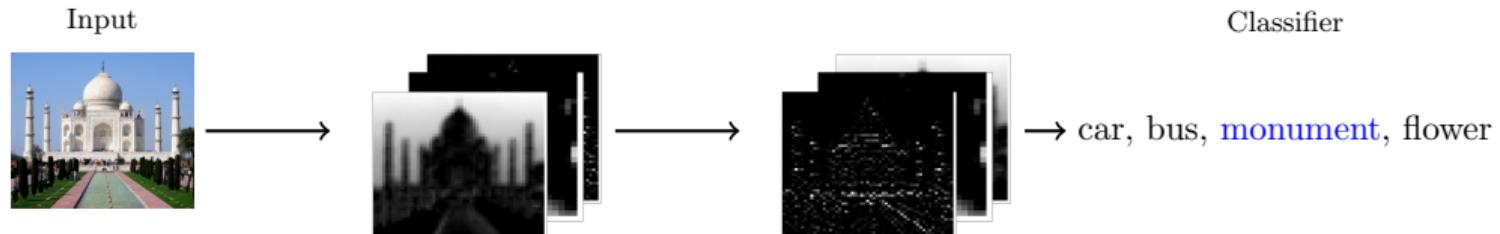


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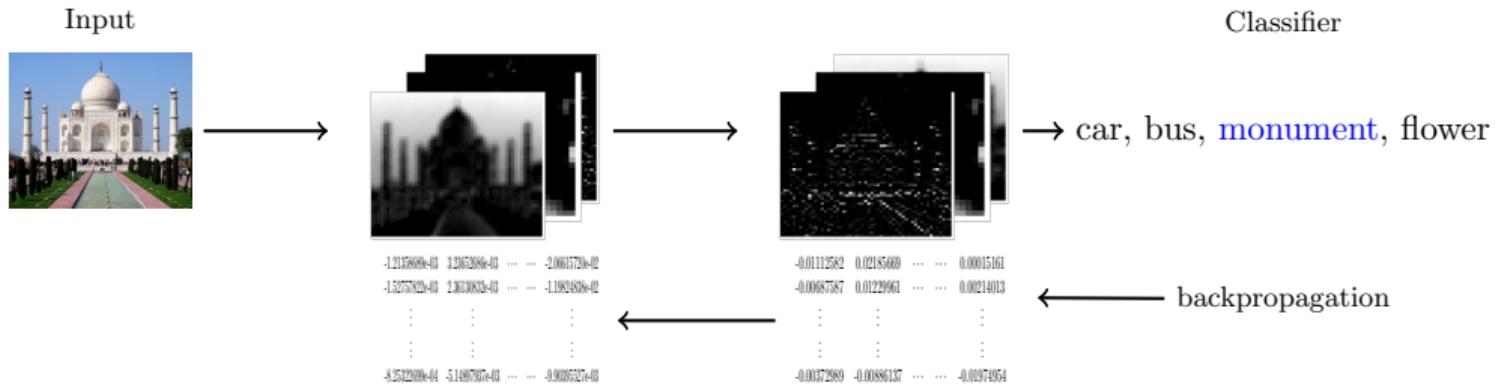


- Even better: Instead of using handcrafted kernels (such as edge detectors) can we learn **multiple meaningful kernels/filters** in addition to learning the weights of the classifier?

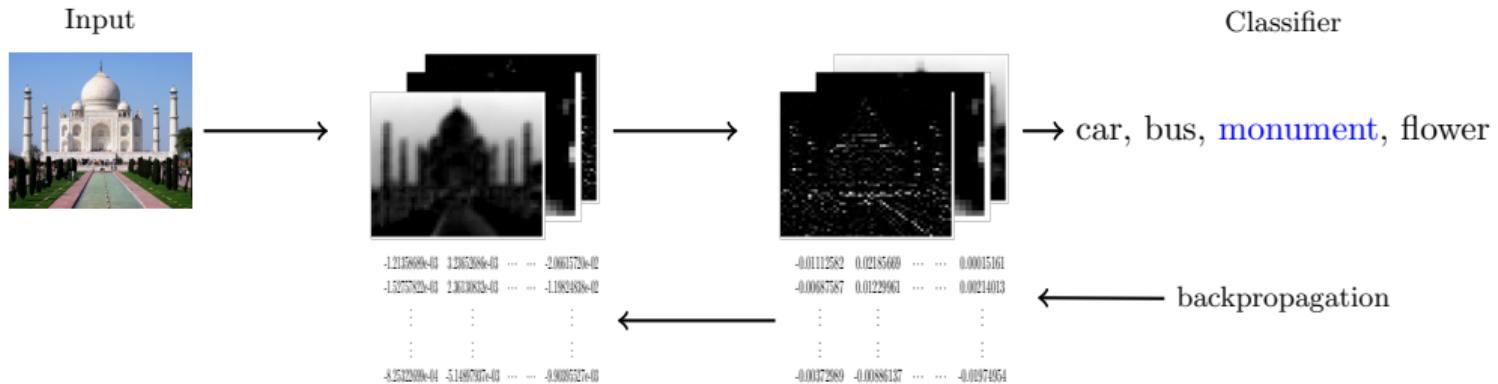
- Can we learn multiple **layers** of meaningful kernels/filters in addition to learning the weights of the classifier?



- Can we learn multiple **layers** of meaningful kernels/filters in addition to learning the weights of the classifier?
- Yes, we can !



- Can we learn multiple **layers** of meaningful kernels/filters in addition to learning the weights of the classifier?
- Yes, we can !
- Simply by treating these kernels as parameters and learning them in addition to the weights of the classifier (using back propagation)



- Can we learn multiple **layers** of meaningful kernels/filters in addition to learning the weights of the classifier?
- Yes, we can !
- Simply by treating these kernels as parameters and learning them in addition to the weights of the classifier (using back propagation)
- Such a network is called a Convolutional Neural Network.

- Okay, I get it that the idea is to learn the kernel/filters by just treating them as parameters of the classification model

- Okay, I get it that the idea is to learn the kernel/filters by just treating them as parameters of the classification model
- But how is this different from a regular feedforward neural network

- Okay, I get it that the idea is to learn the kernel/filters by just treating them as parameters of the classification model
- But how is this different from a regular feedforward neural network
- Let us see

2



16



10 classes(digits)



16



10 classes(digits)



16



10 classes(digits)



16



10 classes(digits)



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16



10 classes(digits)



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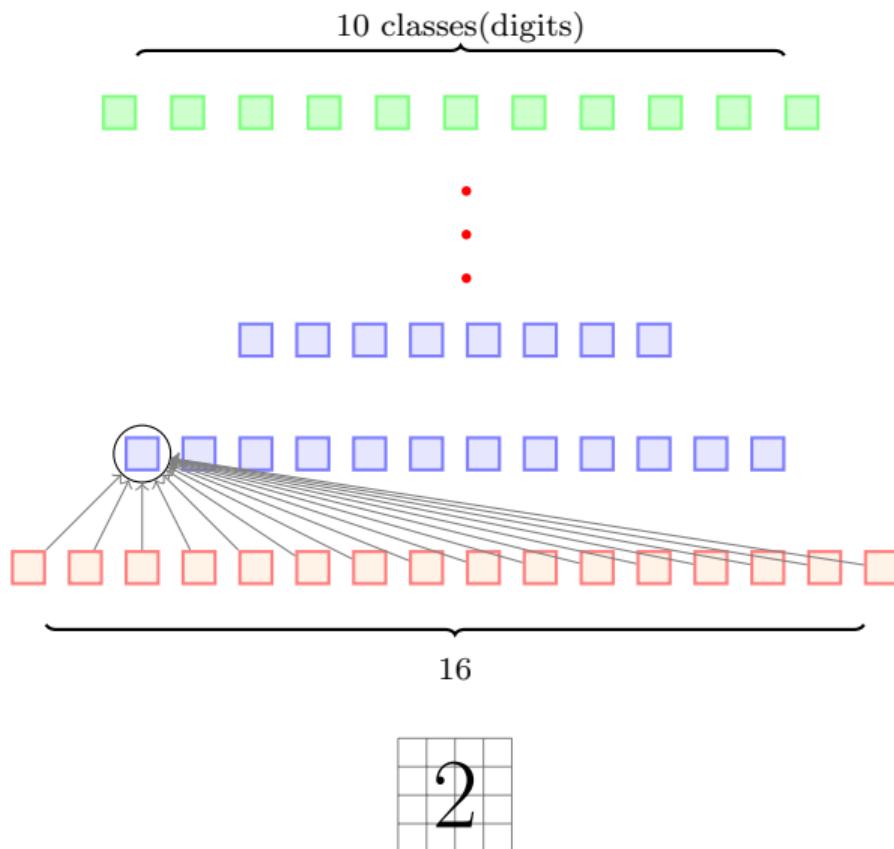
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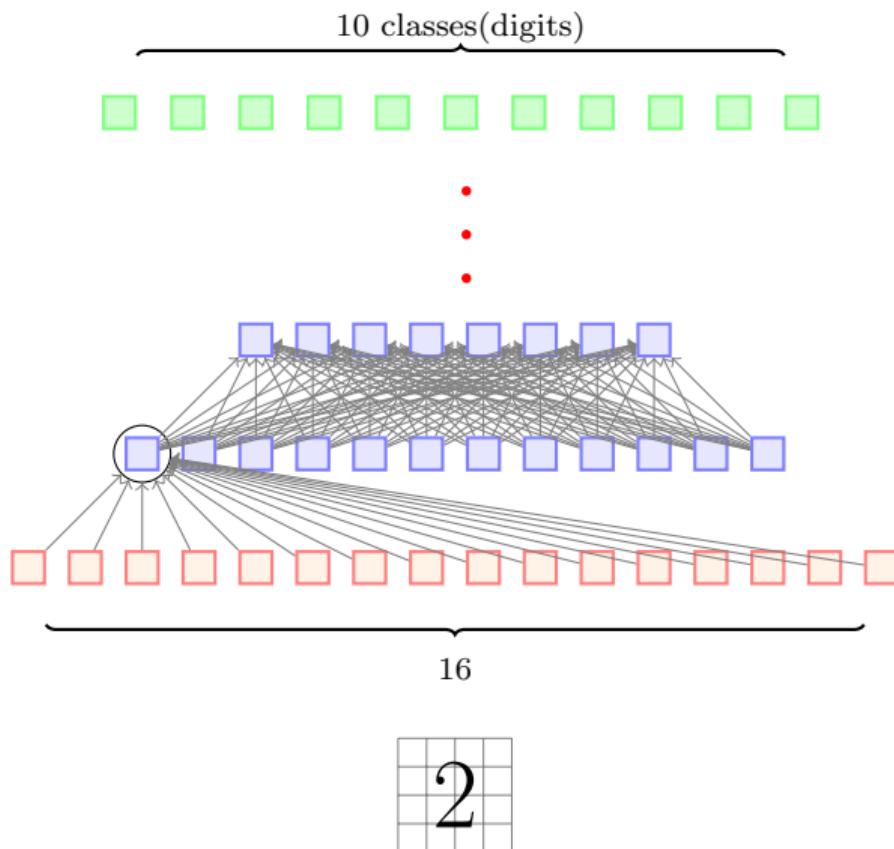
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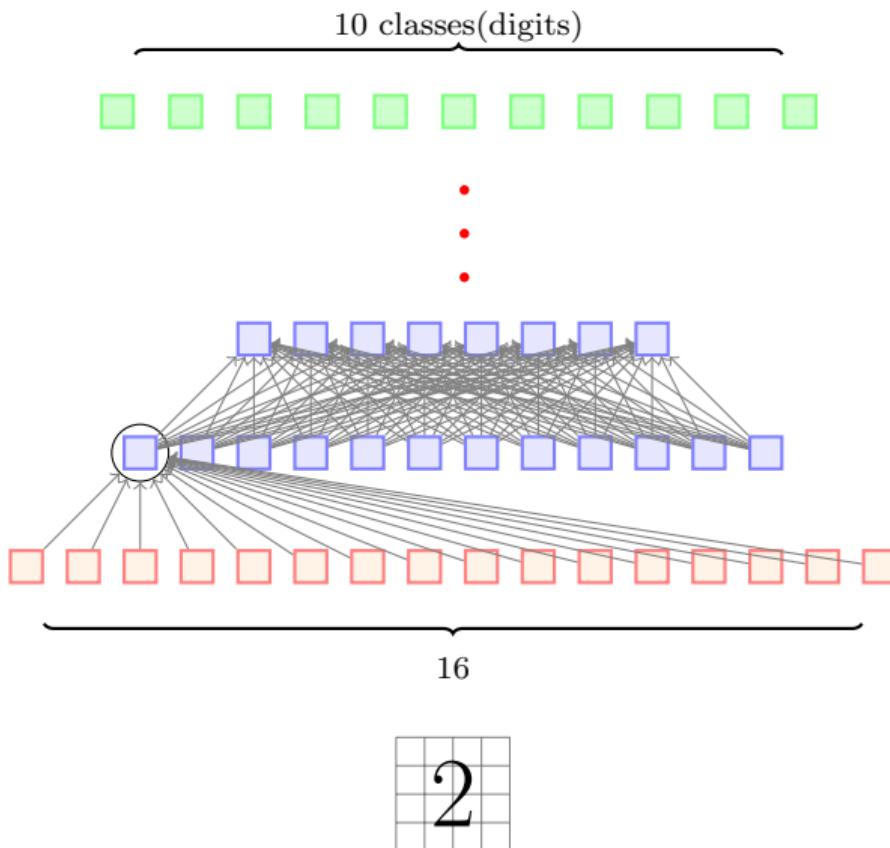


16

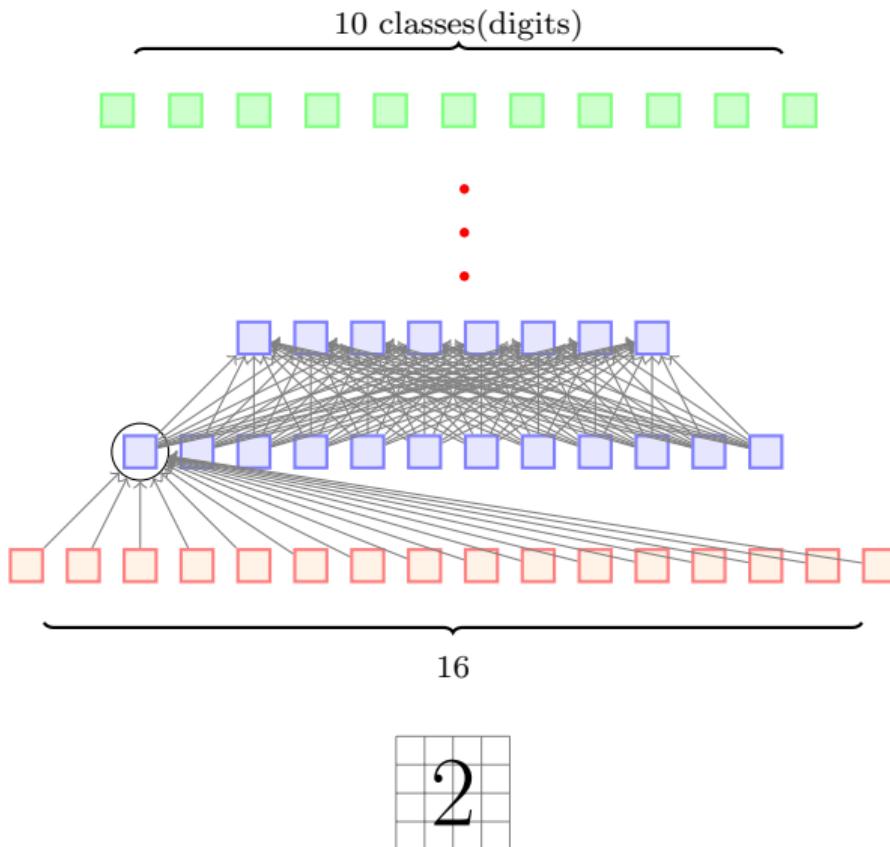




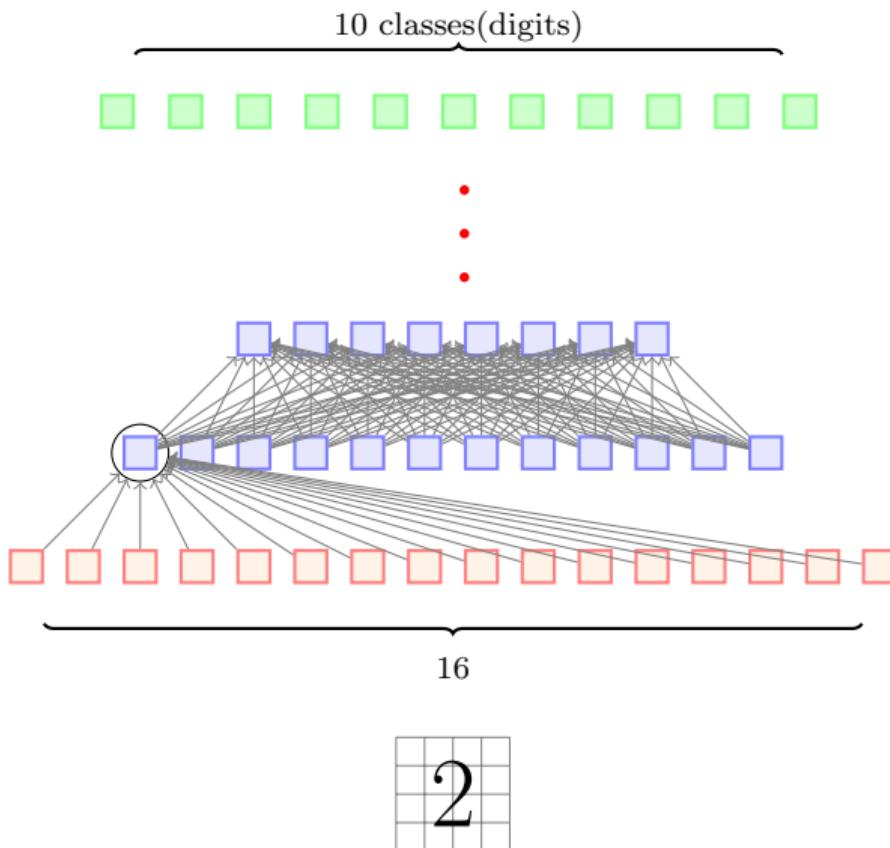




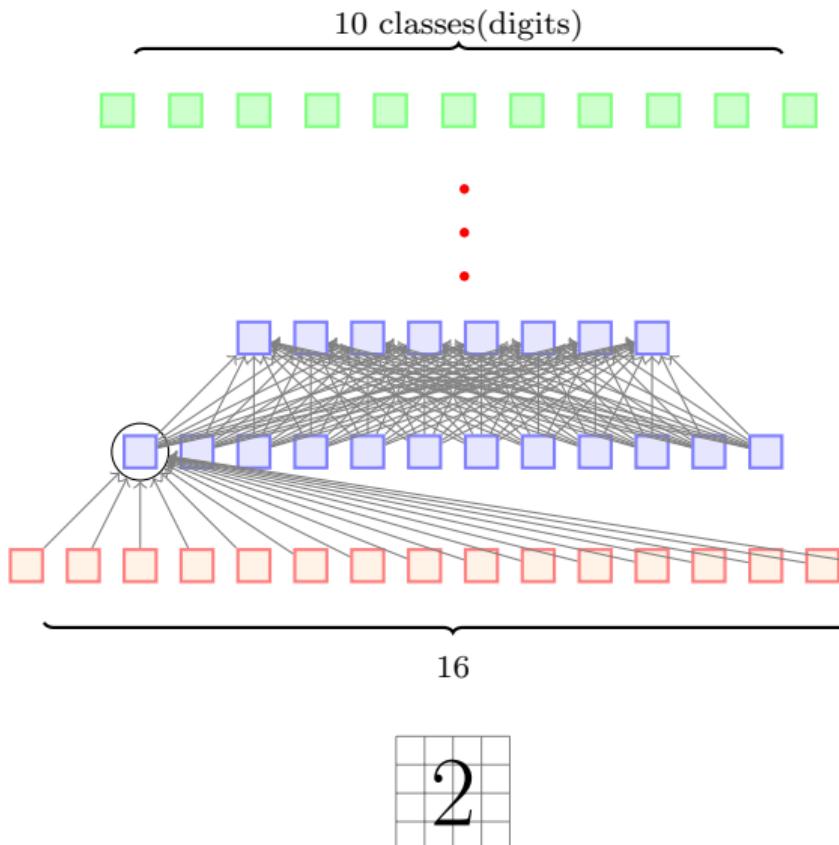
- This is what a regular feed-forward neural network will look like



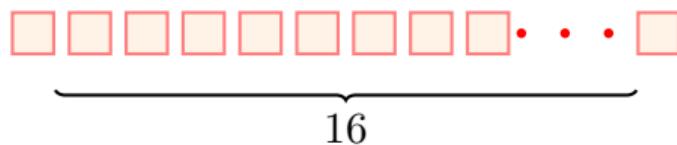
- This is what a regular feed-forward neural network will look like
- There are many dense connections here



- This is what a regular feed-forward neural network will look like
- There are many dense connections here
- For example all the 16 input neurons are contributing to the computation of h_{11}

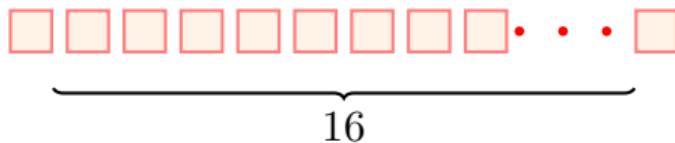


- This is what a regular feed-forward neural network will look like
 - There are many dense connections here
 - For example all the 16 input neurons are contributing to the computation of h_{11}
 - Contrast this to what happens in the case of convolution



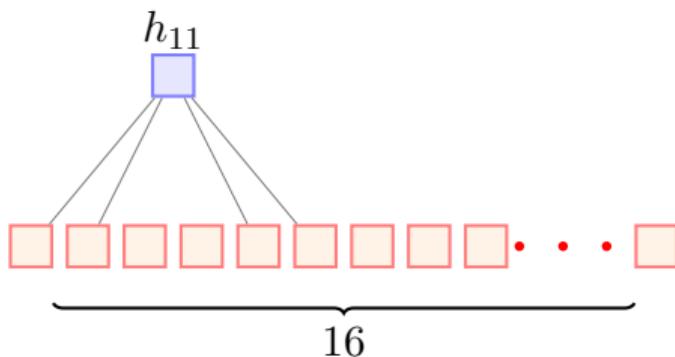
$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} * \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} = \square$$

- Only a few local neurons participate in the computation of h_{11}



A diagram illustrating a convolution operation. On the left is a 4x4 input grid with black dots. A 2x2 kernel with blue dots is applied to the input. The result is a single blue square. The diagram shows the receptive field of the central unit in the output layer, which covers four units in the input layer.

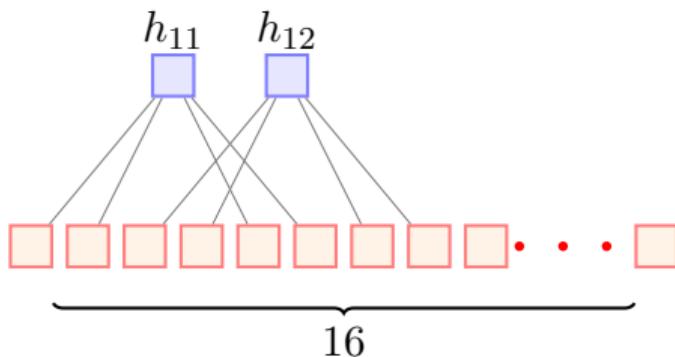
$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} \quad * \quad \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} \quad = \quad \square$$



- Only a few local neurons participate in the computation of h_{11}
- For example, only pixels 1, 2, 5, 6 contribute to h_{11}

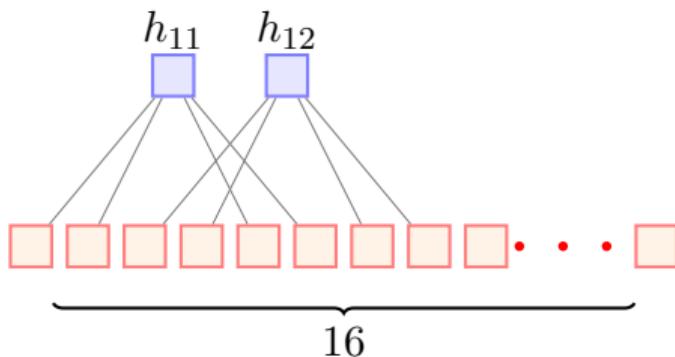
$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} * \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} = h_{11}$$

A diagram illustrating a convolution operation. On the left is a 4x4 input grid with black and red dots. A 2x2 kernel with blue dots is shown above it. An asterisk (*) indicates the convolution operation, and an equals sign (=) followed by a blue square represents the output neuron h_{11} . A large grey circle highlights the 2x2 receptive field of the central input pixel.



- Only a few local neurons participate in the computation of h_{11}
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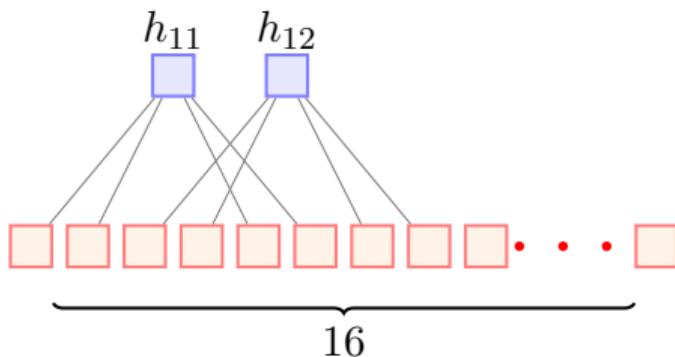
$$\begin{matrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix} * \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} = h_{12}$$



- Only a few local neurons participate in the computation of h_{11}
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A diagram illustrating a convolutional step. On the left is a 4x4 input grid with black and red dots. A 3x3 kernel with blue dots is applied to it. An arrow points from the input grid to the kernel. To the right of the kernel is the equation \ast . To the right of the equation is the output unit h_{13} , represented by a blue square.

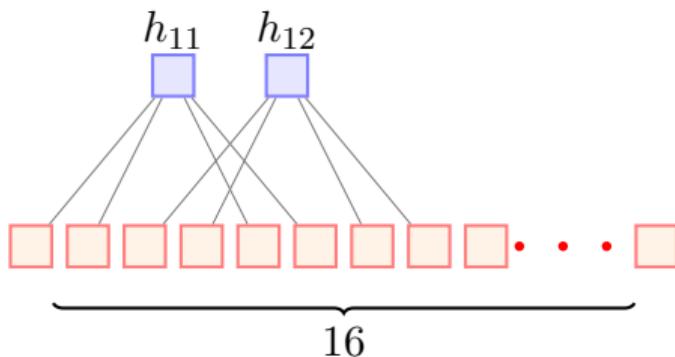
$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} \ast \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} = h_{13}$$



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$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} * \begin{matrix} \bullet & \bullet \\ \bullet & \bullet \end{matrix} = h_{14}$$

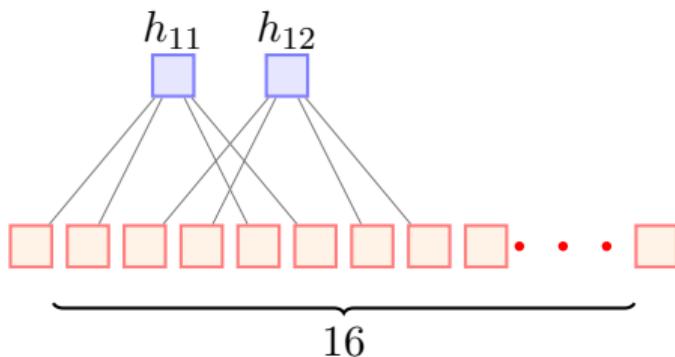
A diagram illustrating a convolutional step. A 4x4 input grid with black and red dots is multiplied by a 3x3 kernel with blue dots to produce a single output unit h_{14} .



- Only a few local neurons participate in the computation of h_{11}
- For example, only pixels 1, 2, 5, 6 contribute to h_{11}
- The connections are much sparser

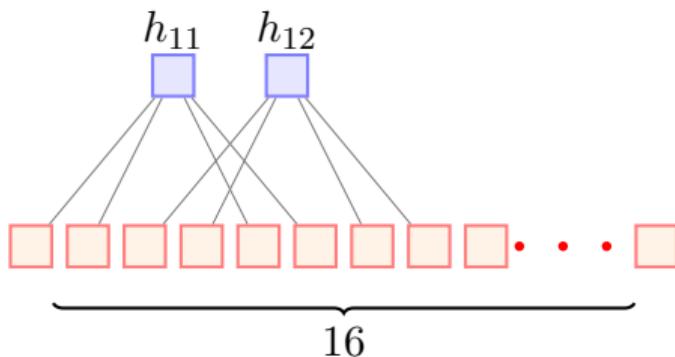
$$\begin{matrix}
 \bullet & \bullet & \bullet & \bullet \\
 \end{matrix} * \begin{matrix}
 \bullet & \bullet \\
 \bullet & \bullet \\
 \end{matrix} = h_{14}$$

A 4x4 input matrix with black dots at positions (1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,3), (4,4). A 2x2 kernel matrix with blue dots at positions (1,1), (1,2), (2,1), (2,2). A grey arrow highlights the receptive field of the bottom-right input pixel (4,4).



$$\begin{array}{|c|c|c|c|} \hline
 \bullet & \bullet & \bullet & \bullet \\ \hline
 \bullet & \bullet & \bullet & \bullet \\ \hline
 \bullet & \bullet & \color{red}{\bullet} & \color{red}{\bullet} \\ \hline
 \bullet & \color{red}{\bullet} & \color{red}{\bullet} & \color{red}{\bullet} \\ \hline
 \end{array}
 * \begin{array}{|c|c|} \hline
 \color{blue}{\bullet} & \color{blue}{\bullet} \\ \hline
 \color{blue}{\bullet} & \color{blue}{\bullet} \\ \hline
 \end{array} = h_{14}$$

- Only a few local neurons participate in the computation of h_{11}
- For example, only pixels 1, 2, 5, 6 contribute to h_{11}
- The connections are much sparser
- We are taking advantage of the structure of the image(interactions between neighboring pixels are more interesting)



$$\begin{matrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{matrix} * \begin{matrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{matrix} = h_{14}$$

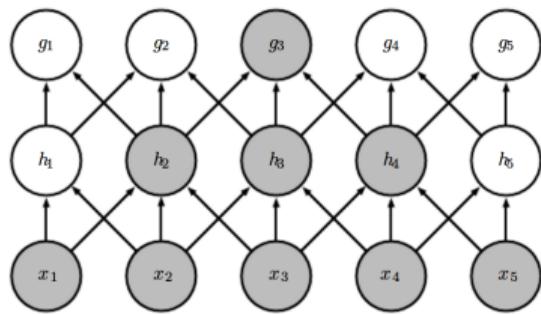
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- For example, only pixels 1, 2, 5, 6 contribute to h_{11}
- The connections are much sparser
- We are taking advantage of the structure of the image(interactions between neighboring pixels are more interesting)
- This **sparse connectivity** reduces the number of parameters in the model

- But is sparse connectivity really good thing ?

* Goodfellow-et-al-2016

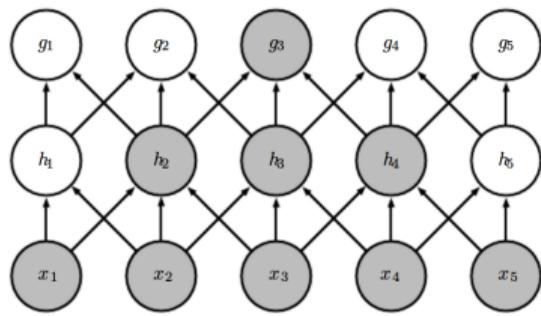
- But is sparse connectivity really good thing ?
- Aren't we losing information (by losing interactions between some input pixels)

* Goodfellow-et-al-2016



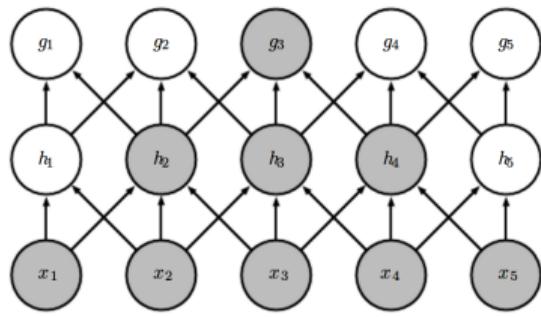
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- The two highlighted neurons (x_1 & x_5)* do not interact in *layer 1*

* Goodfellow-et-al-2016

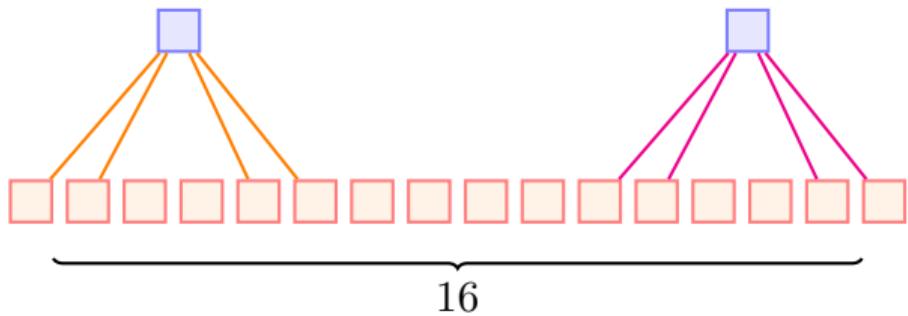


- But is sparse connectivity really good thing ?
- Aren't we losing information (by losing interactions between some input pixels)
- Well, not really
- The two highlighted neurons (x_1 & x_5)* do not interact in *layer 1*
- But they indirectly contribute to the computation of g_3 and hence interact indirectly

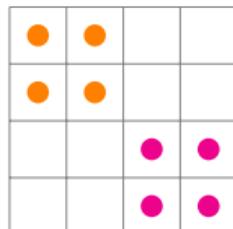
* Goodfellow-et-al-2016

- Another characteristic of CNNs is **weight sharing**

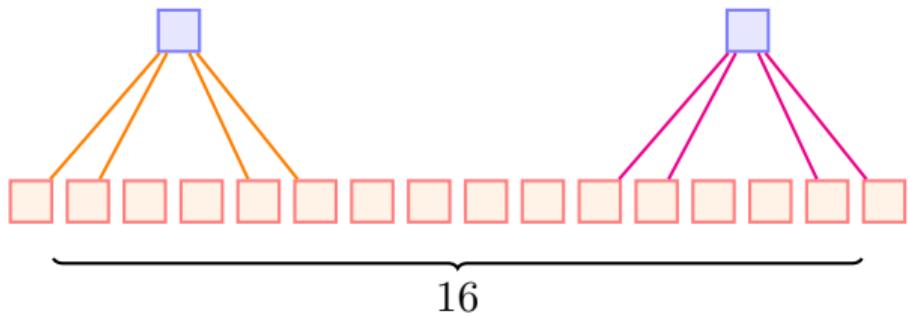
- Another characteristic of CNNs is **weight sharing**
- Consider the following network



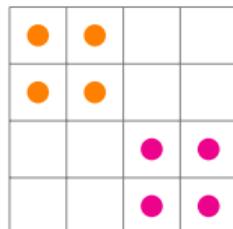
- Kernel 1
- Kernel 2



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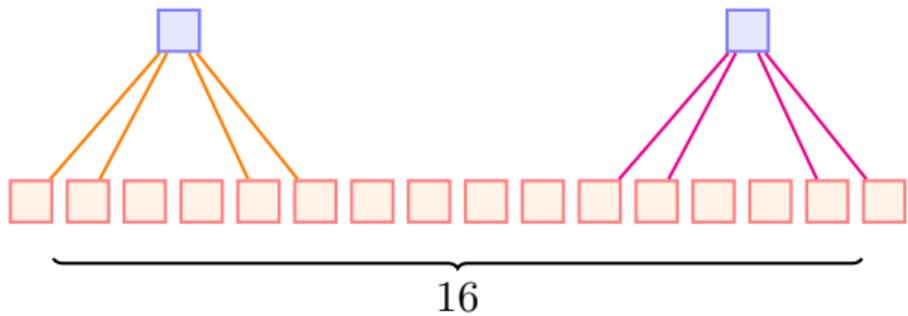


- Kernel 1
- Kernel 2

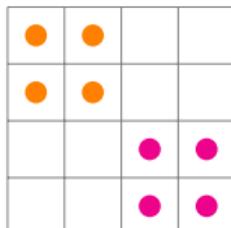


4x4 Image

- Another characteristic of CNNs is **weight sharing**
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- Do we want the kernel weights to be different for different portions of the image?

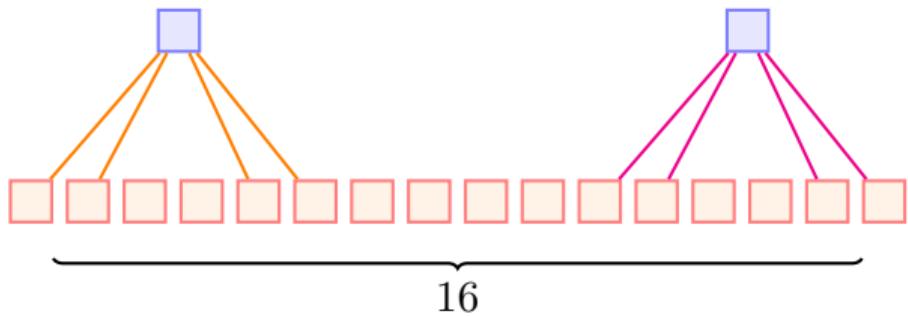


- Kernel 1
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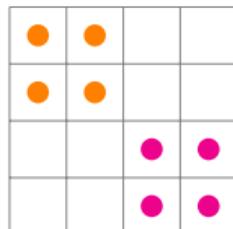


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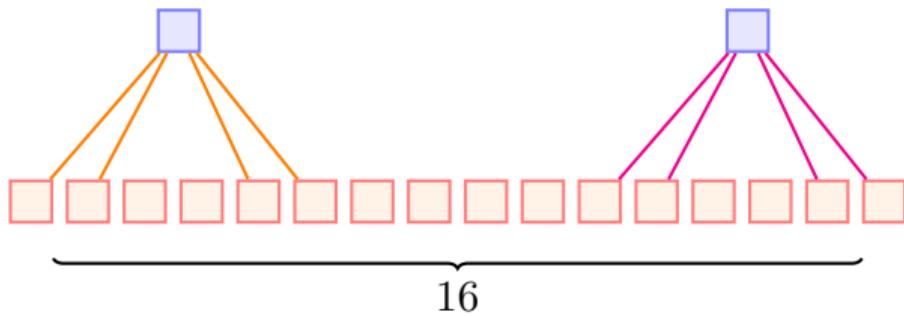
- Kernel 1
- Kernel 2



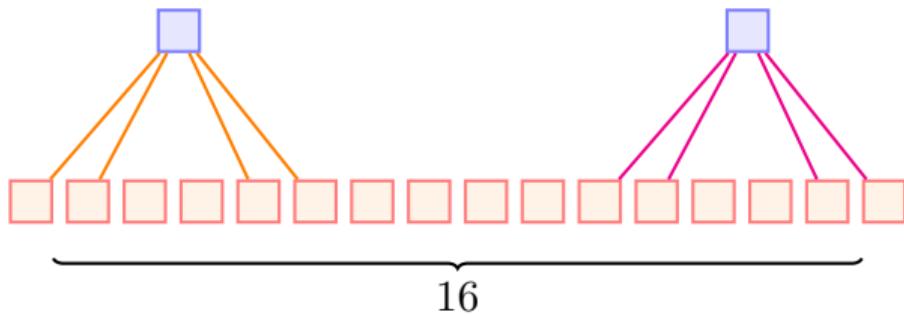
4x4 Image

- Another characteristic of CNNs is **weight sharing**
- Consider the following network
- Do we want the kernel weights to be different for different portions of the image?
- Imagine that we are trying to learn a kernel that detects edges
- Shouldn't we be applying the same kernel at all the portions of the image?

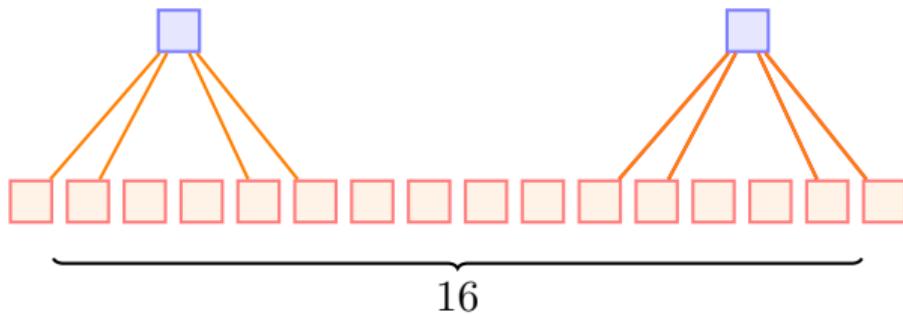
- In other words shouldn't the *orange* and *pink* kernels be the same



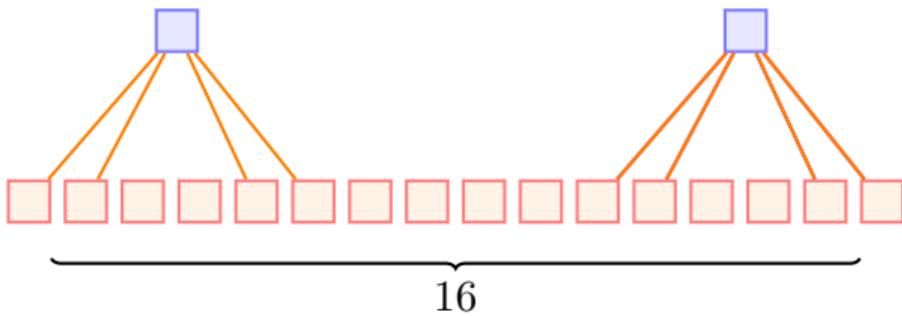
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- Yes, indeed

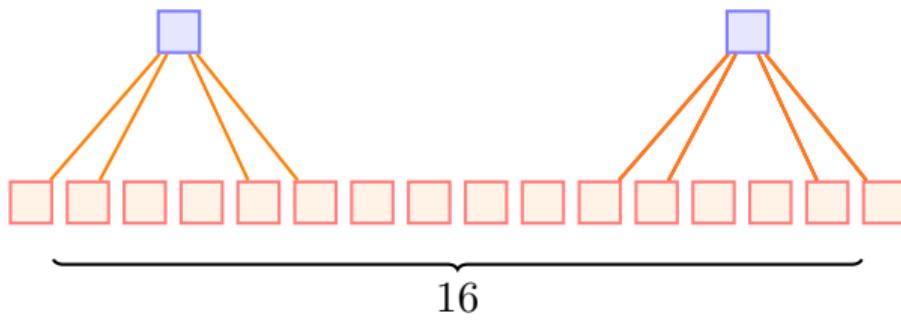


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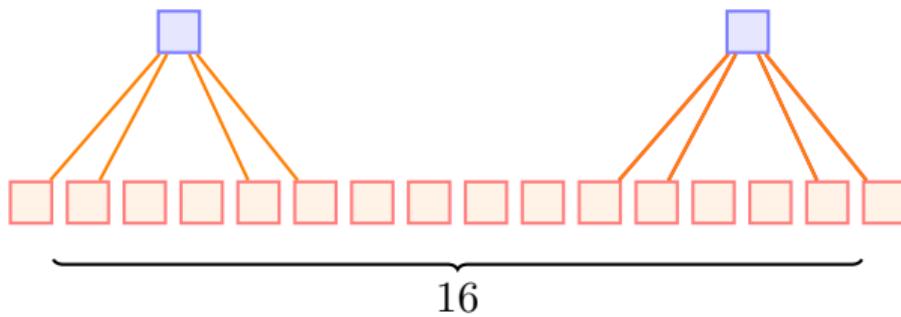


- In other words shouldn't the *orange* and *pink* kernels be the same
- Yes, indeed
- This would make the job of learning easier(instead of trying to learn the same weights/kernels at different locations again and again)

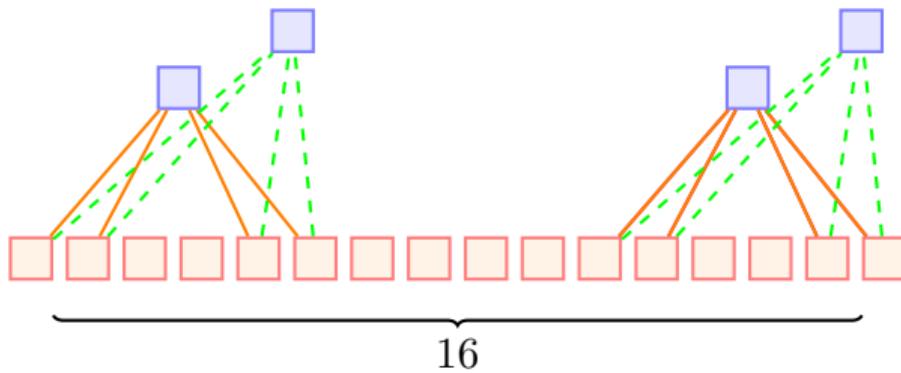




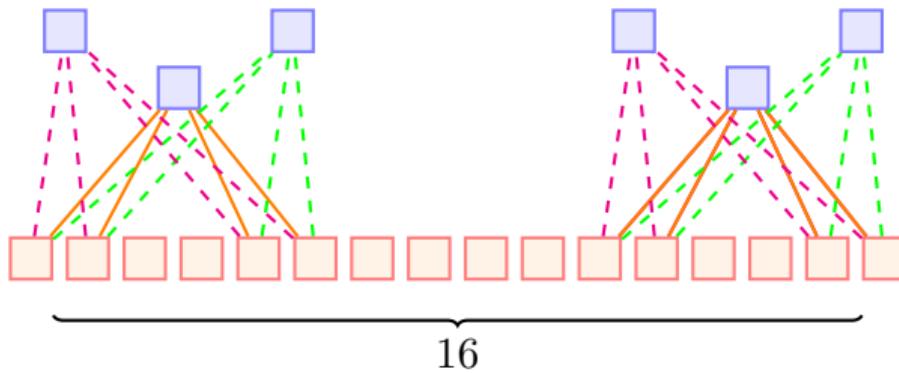
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- But does that mean we can have only one kernel?



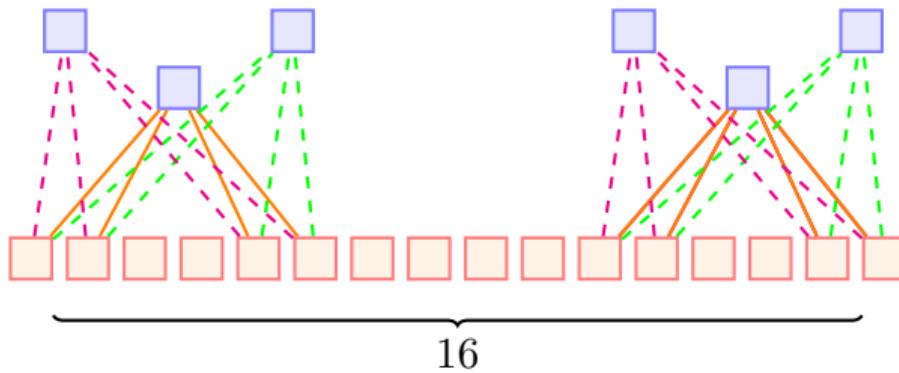
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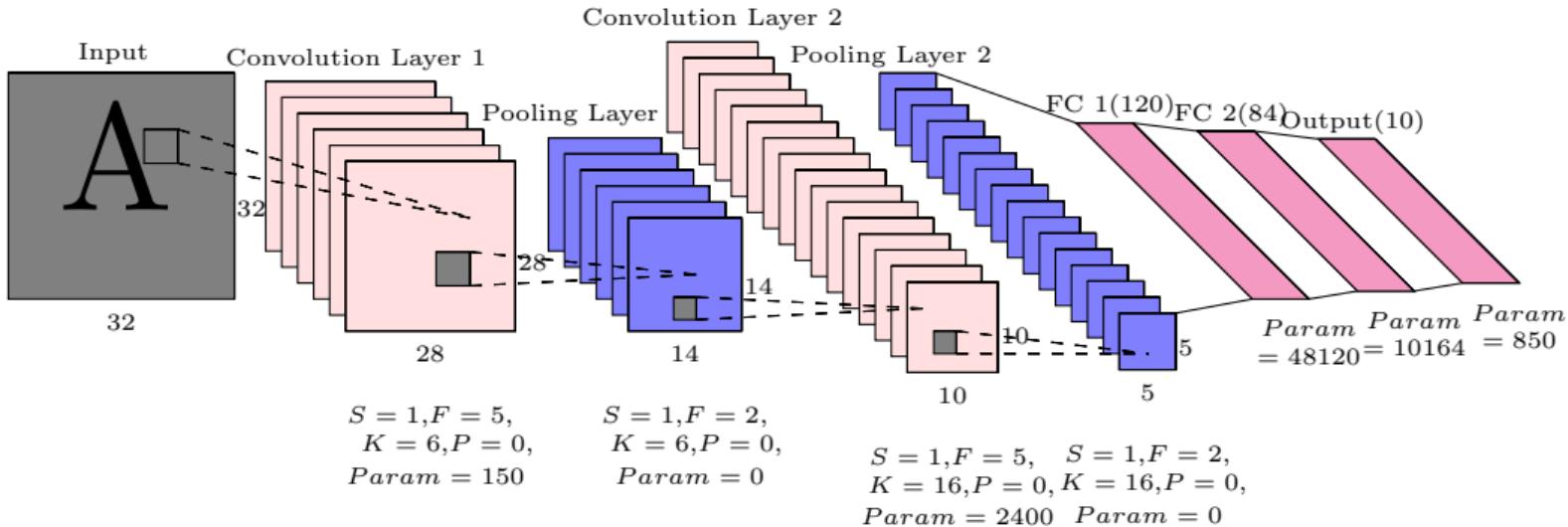
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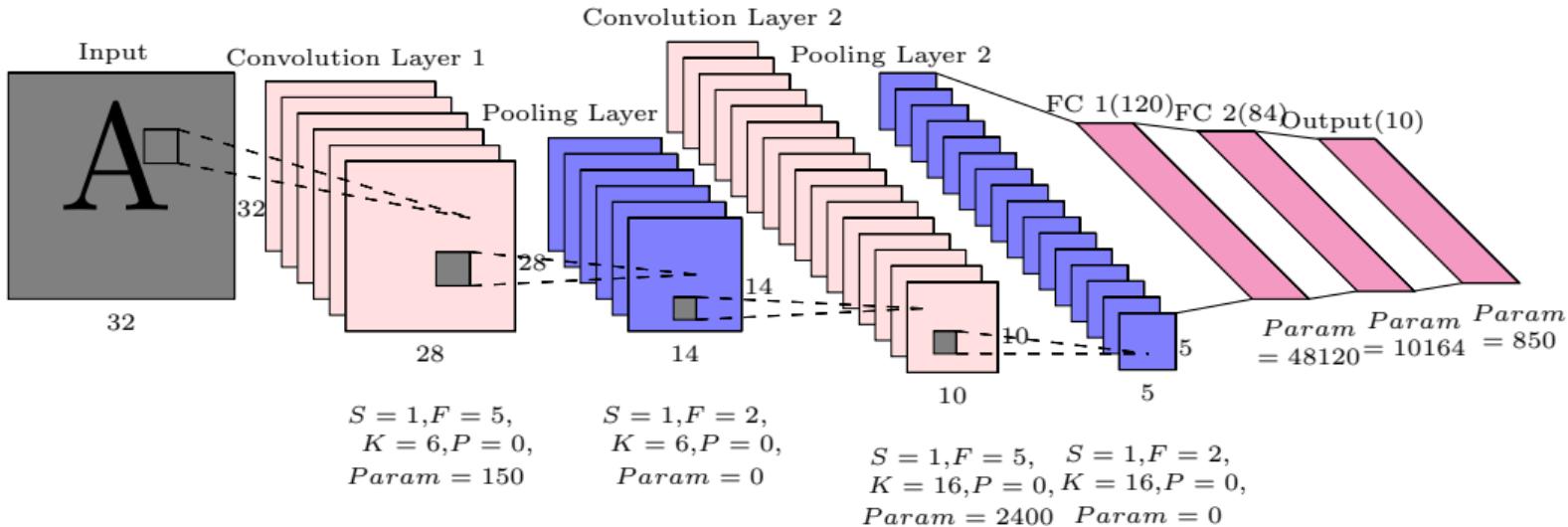


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- But does that mean we can have only one kernel?
- No, we can have many such kernels but the kernels will be shared by all locations in the image
- This is called “weight sharing”

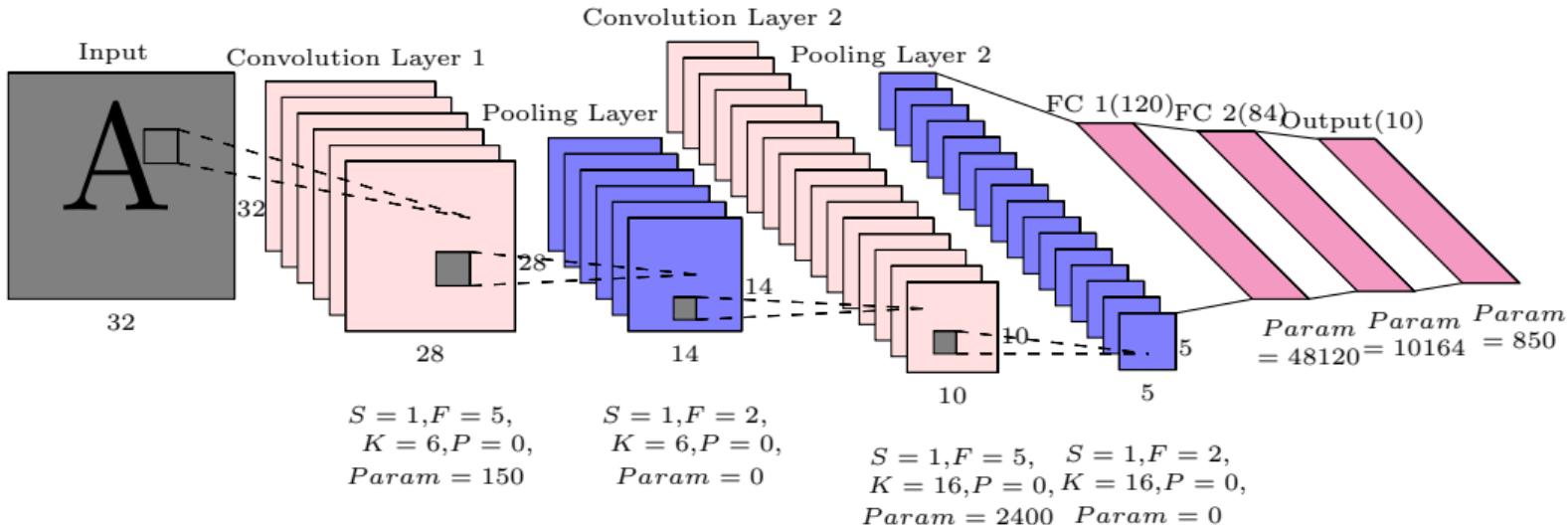
- So far, we have focused only on the convolution operation

- So far, we have focused only on the convolution operation
- Let us see what a full convolutional neural network looks like

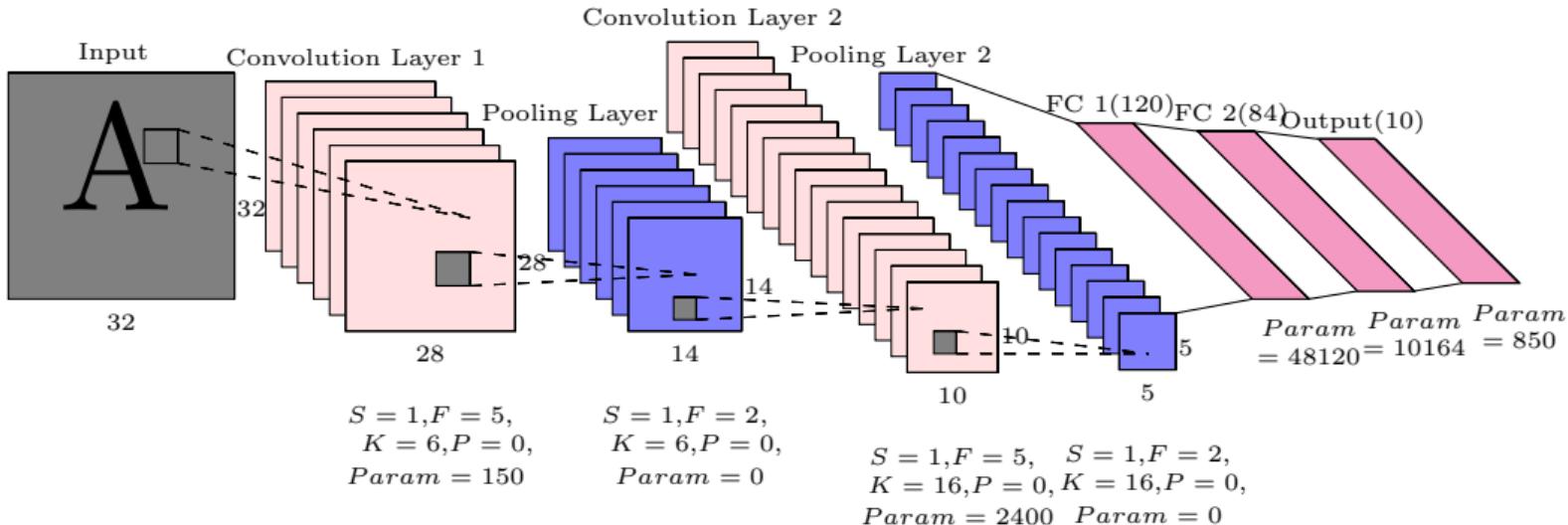




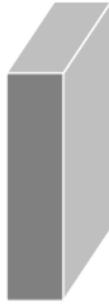
- It has alternate convolution and pooling layers



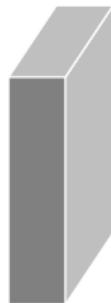
- It has alternate convolution and pooling layers
- What does a pooling layer do?



- It has alternate convolution and pooling layers
- What does a pooling layer do?
- Let us see



Input

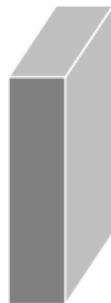


*



Input

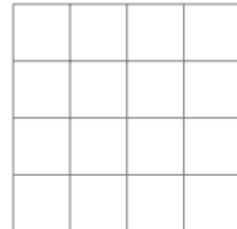
1 filter



*

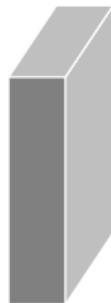


=



Input

1 filter



*

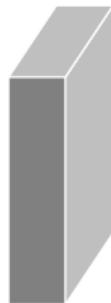


=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

Input

1 filter



*



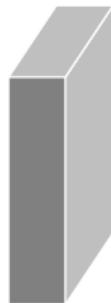
=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
→
2x2 filters (stride 2)

Input

1 filter



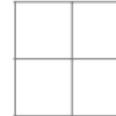
*



=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

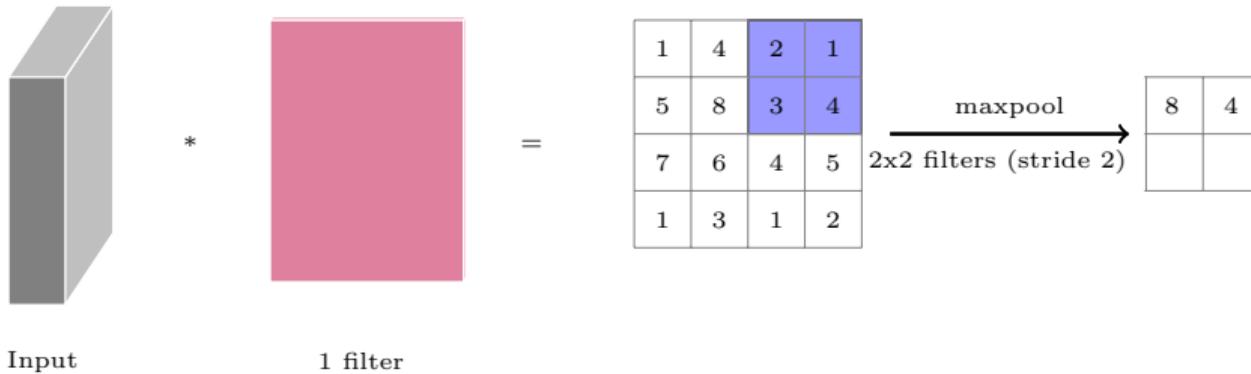
maxpool
2x2 filters (stride 2)



Input

1 filter

$$\text{Input} \quad * \quad \text{1 filter} = \begin{array}{|c|c|c|c|} \hline 1 & 4 & 2 & 1 \\ \hline 5 & 8 & 3 & 4 \\ \hline 7 & 6 & 4 & 5 \\ \hline 1 & 3 & 1 & 2 \\ \hline \end{array} \xrightarrow[\text{2x2 filters (stride 2)}]{\text{maxpool}} \begin{array}{|c|c|} \hline 8 & \\ \hline & \\ \hline \end{array}$$



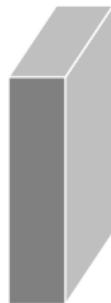
$$\text{Input} \quad * \quad \text{1 filter} =$$

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

8	4
7	

2x2 filters (stride 2)



*



=

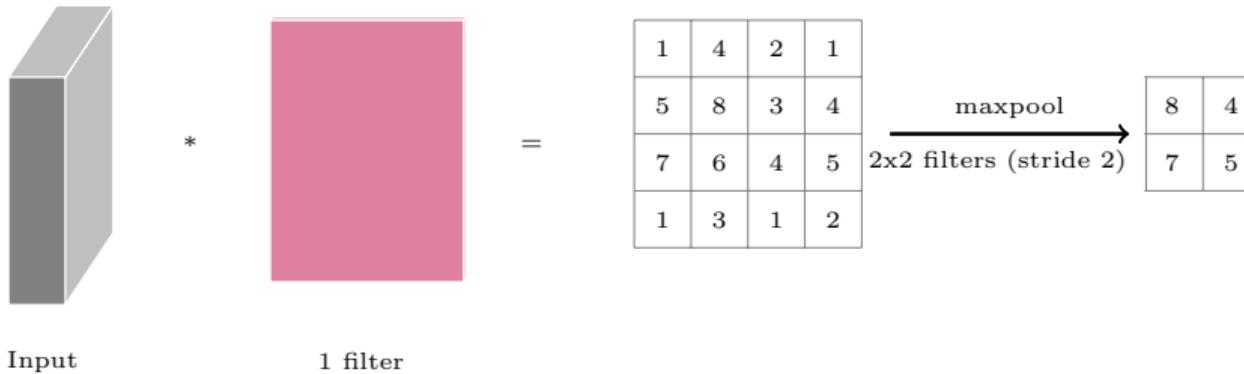
1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

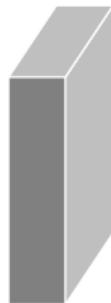
Input

1 filter



$$\text{Input} \quad * \quad \text{1 filter} = \begin{array}{|c|c|c|c|} \hline 1 & 4 & 2 & 1 \\ \hline 5 & 8 & 3 & 4 \\ \hline 7 & 6 & 4 & 5 \\ \hline 1 & 3 & 1 & 2 \\ \hline \end{array} \xrightarrow[\text{2x2 filters (stride 2)}]{\text{maxpool}} \begin{array}{|c|c|} \hline 8 & 4 \\ \hline 7 & 5 \\ \hline \end{array}$$

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2



*



=

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

$$\text{Input} \quad * \quad \text{1 filter} =$$

The diagram illustrates the convolution of a 4x4 input with a 2x2 filter. The input is represented by a gray 3D block and the filter by a red 2D square. An equals sign follows the multiplication symbol. To the right of the equals sign is a 4x4 grid of numbers representing the input. Below the input is a 2x2 grid of numbers representing the filter. A horizontal arrow labeled "maxpool" points from the input grid to a 2x2 output grid. Below the input grid is the text "2x2 filters (stride 2)".

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool

2x2 filters (stride 2)

8	4
7	5

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool

2x2 filters (stride 1)

$$\text{Input} \quad * \quad \text{1 filter} =$$

The diagram illustrates a convolution operation. On the left, a vertical grey bar labeled "Input" is multiplied by a red square labeled "1 filter". An equals sign follows. To the right of the equals sign is a 4x4 grid of numbers representing the input. To the right of the input is a 2x2 grid of numbers representing the filter. A horizontal arrow labeled "maxpool" points from the input grid to a 2x2 output grid on the right.

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

8	4
7	5

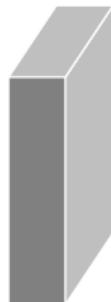
2x2 filters (stride 2)

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

8		

2x2 filters (stride 1)



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	

$$\text{Input} \quad * \quad \text{1 filter} =$$

The diagram illustrates a convolution operation. On the left, a vertical grey bar labeled "Input" is multiplied by a red square labeled "1 filter". An equals sign follows. To the right of the equals sign is a 4x4 grid representing the input features:

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

Below the input grid is the text "2x2 filters (stride 2)". An arrow labeled "maxpool" points to the resulting output grid:

8	4
7	5

maxpool
2x2 filters (stride 2)

8	4
7	5

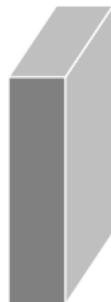
The diagram illustrates a convolution operation followed by maxpooling. On the left, a vertical grey bar labeled "Input" is multiplied by a red square labeled "1 filter". An equals sign follows. To the right of the equals sign is a 4x4 grid representing the input features:

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

Below the input grid is the text "2x2 filters (stride 1)". An arrow labeled "maxpool" points to the resulting output grid:

8	8	4

maxpool
2x2 filters (stride 1)



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8		

$$\text{Input} \quad * \quad \text{1 filter} =$$

The diagram illustrates a convolution operation. On the left, a vertical grey bar labeled "Input" is multiplied by a red square labeled "1 filter". An equals sign follows. To the right of the equals sign is a 4x4 grid of numbers representing the input. To the right of the input is a 2x2 grid of numbers representing the filter. An arrow labeled "maxpool" points from the input grid to a 2x2 output grid on the right.

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

8	4
7	5

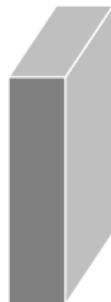
2x2 filters (stride 2)

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

8	8	4
8	8	

2x2 filters (stride 1)



*



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

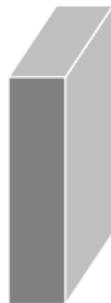
Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5



*



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

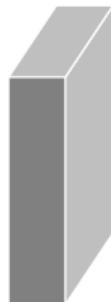
Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5
7		



*



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

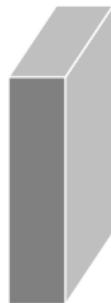
Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5
7	6	



*



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1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 2)

8	4
7	5

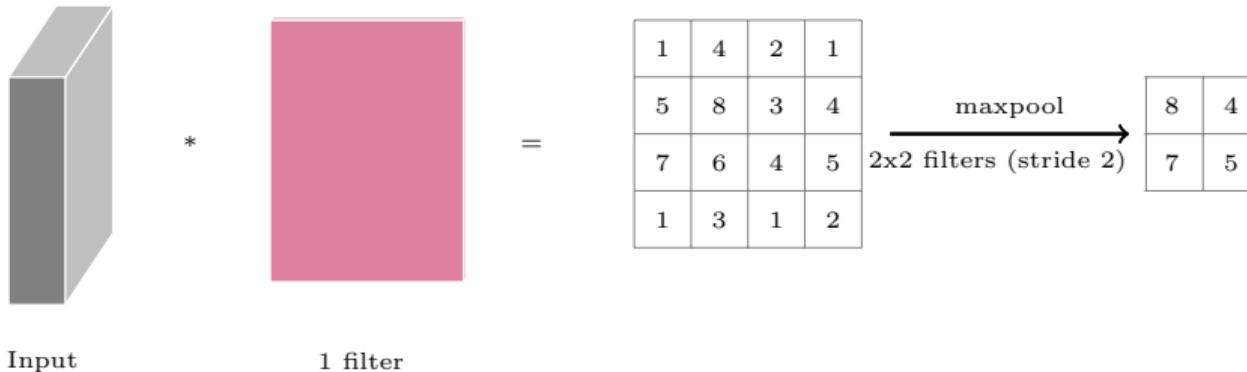
Input

1 filter

1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool
2x2 filters (stride 1)

8	8	4
8	8	5
7	6	5



1	4	2	1
5	8	3	4
7	6	4	5
1	3	1	2

maxpool →

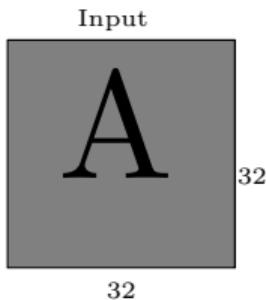
8	8	4
8	8	5
7	6	5

2x2 filters (stride 1)

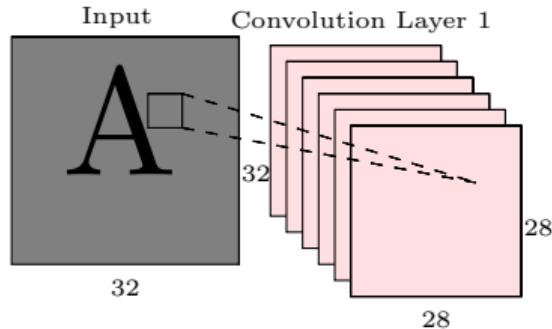
- Instead of max pooling we can also do average pooling

We will now see some case studies where convolution neural networks have been successful

LeNet-5 for handwritten character recognition

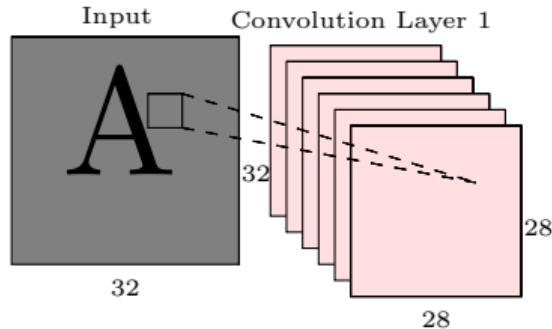


LeNet-5 for handwritten character recognition



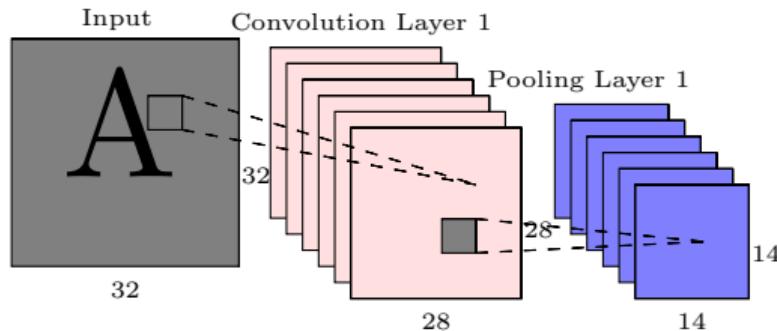
$$\begin{aligned} S &= 1, F = 5, \\ K &= 6, P = 0, \\ Param &=? \end{aligned}$$

LeNet-5 for handwritten character recognition



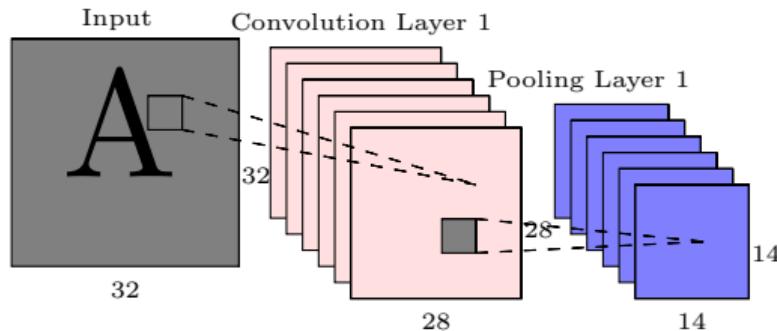
$$\begin{aligned} S &= 1, F = 5, \\ K &= 6, P = 0, \\ Param &= 150 \end{aligned}$$

LeNet-5 for handwritten character recognition



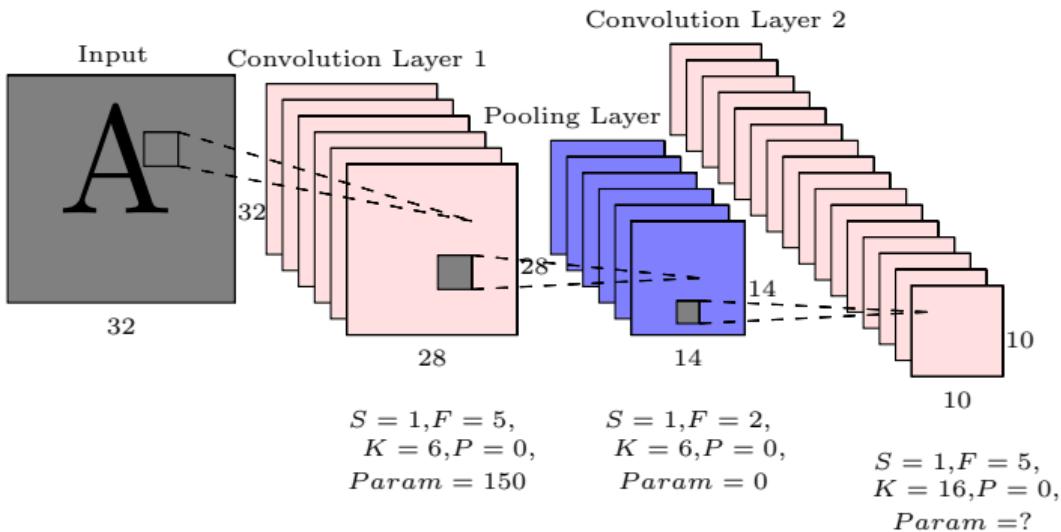
$$\begin{array}{ll} S = 1, F = 5, & S = 1, F = 2, \\ K = 6, P = 0, & K = 6, P = 0, \\ Param = 150 & Param = ? \end{array}$$

LeNet-5 for handwritten character recognition

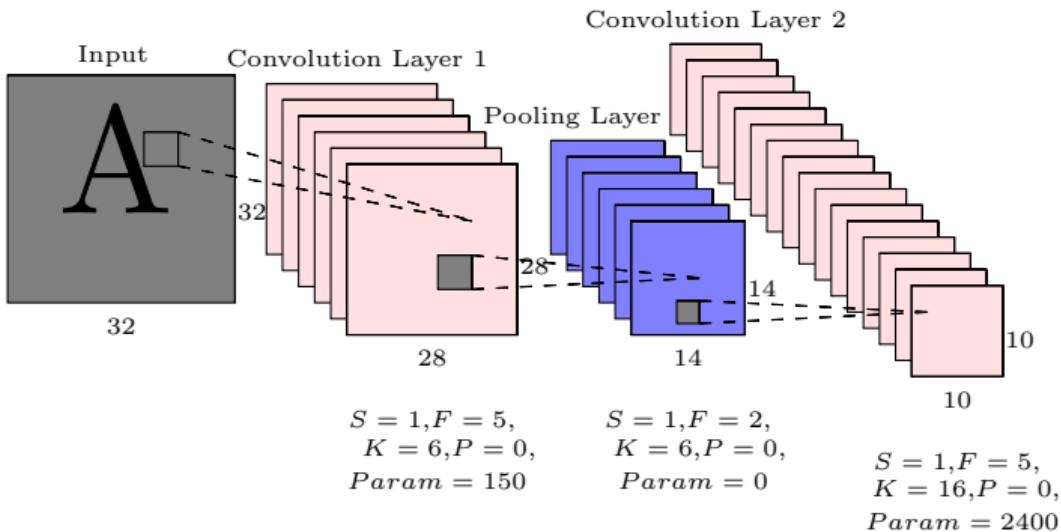


$$\begin{array}{ll} S = 1, F = 5, & S = 1, F = 2, \\ K = 6, P = 0, & K = 6, P = 0, \\ Param = 150 & Param = 0 \end{array}$$

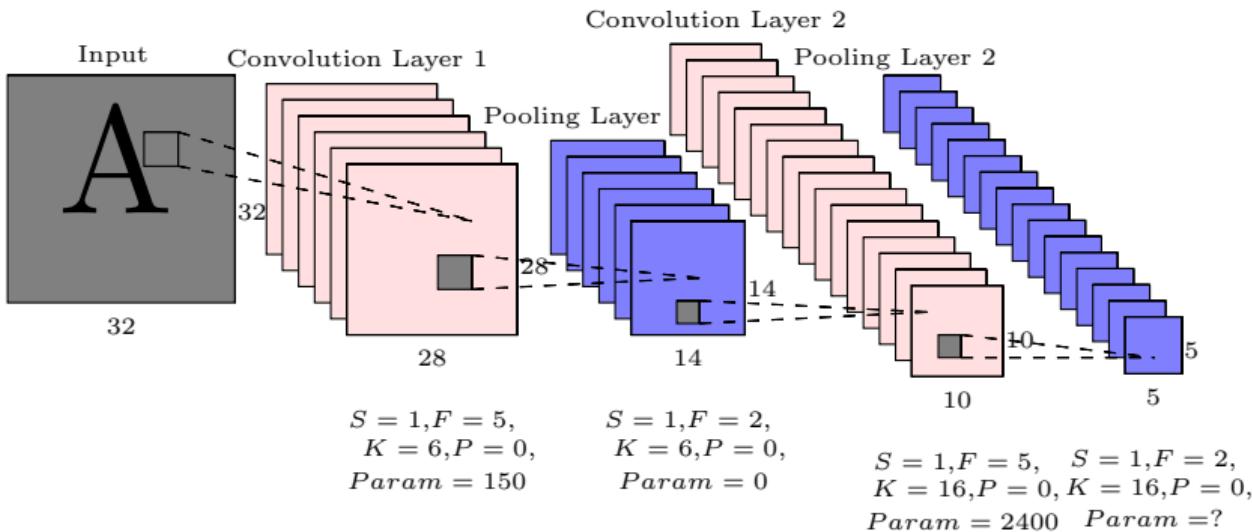
LeNet-5 for handwritten character recognition



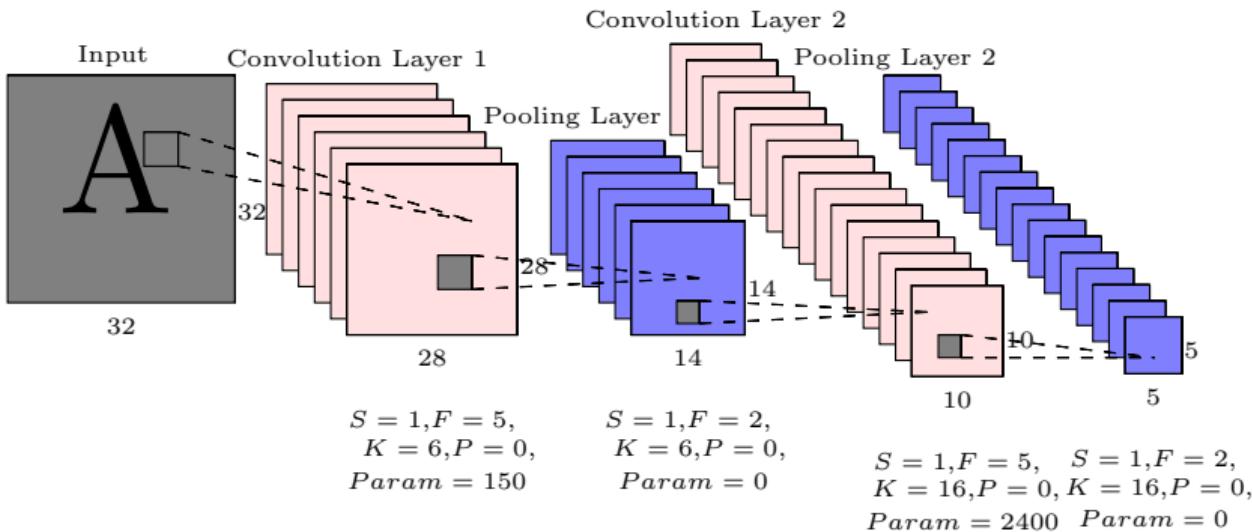
LeNet-5 for handwritten character recognition



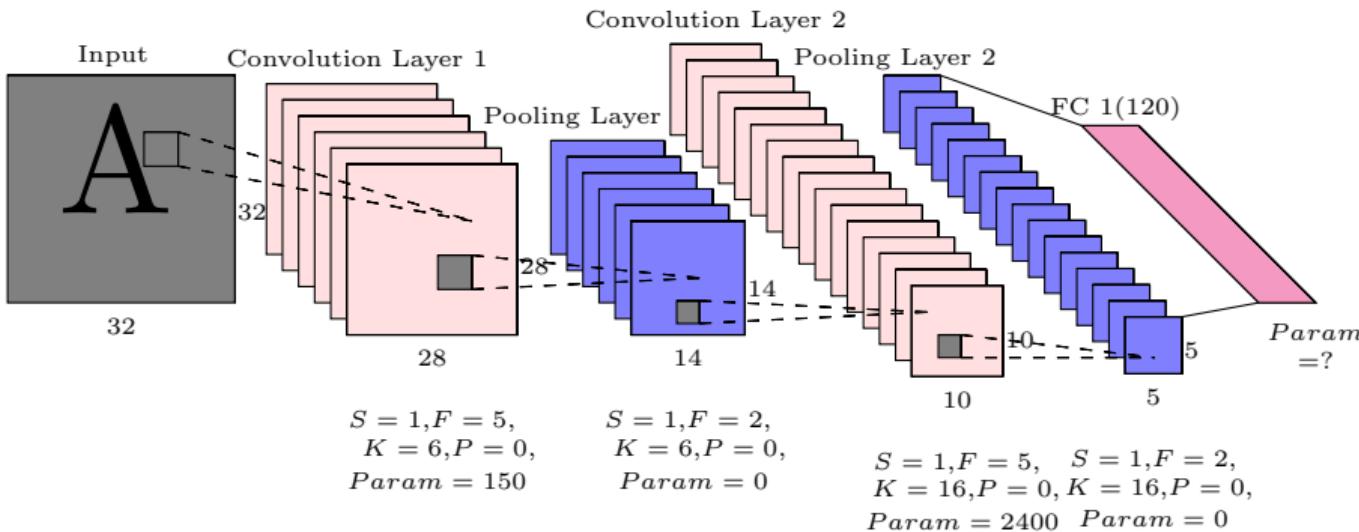
LeNet-5 for handwritten character recognition



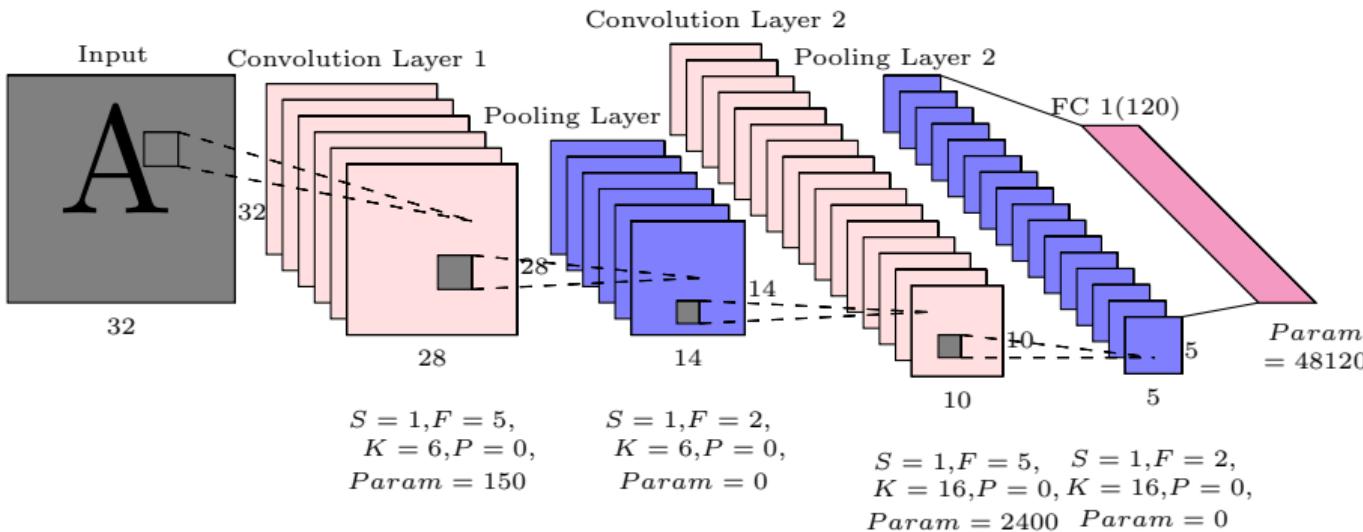
LeNet-5 for handwritten character recognition



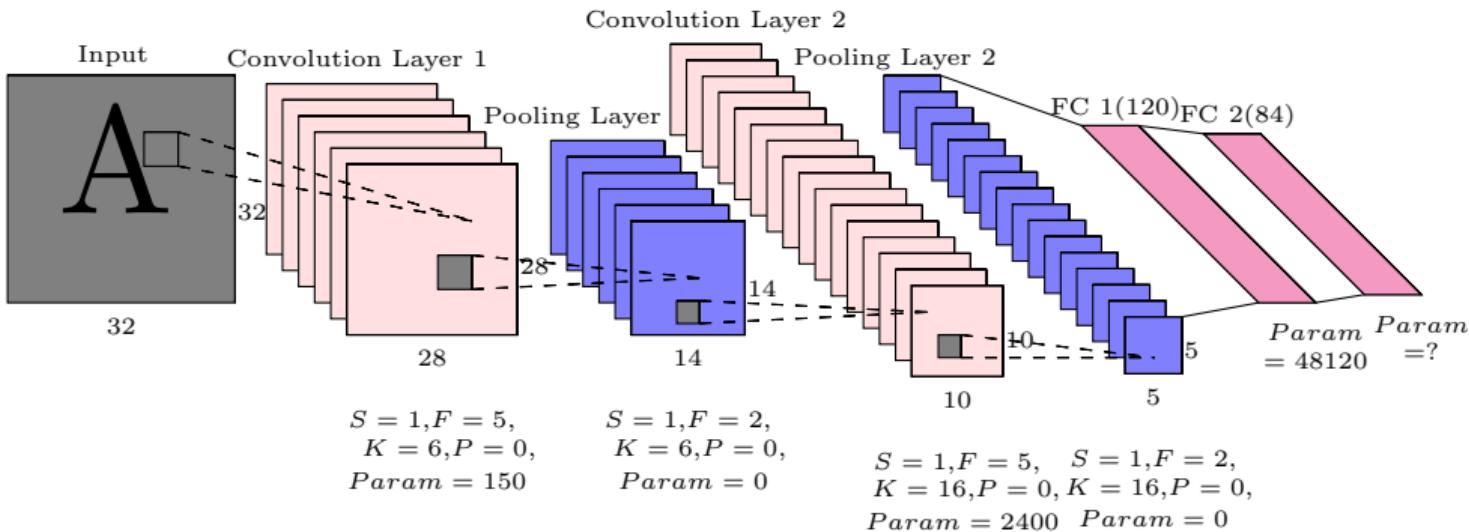
LeNet-5 for handwritten character recognition



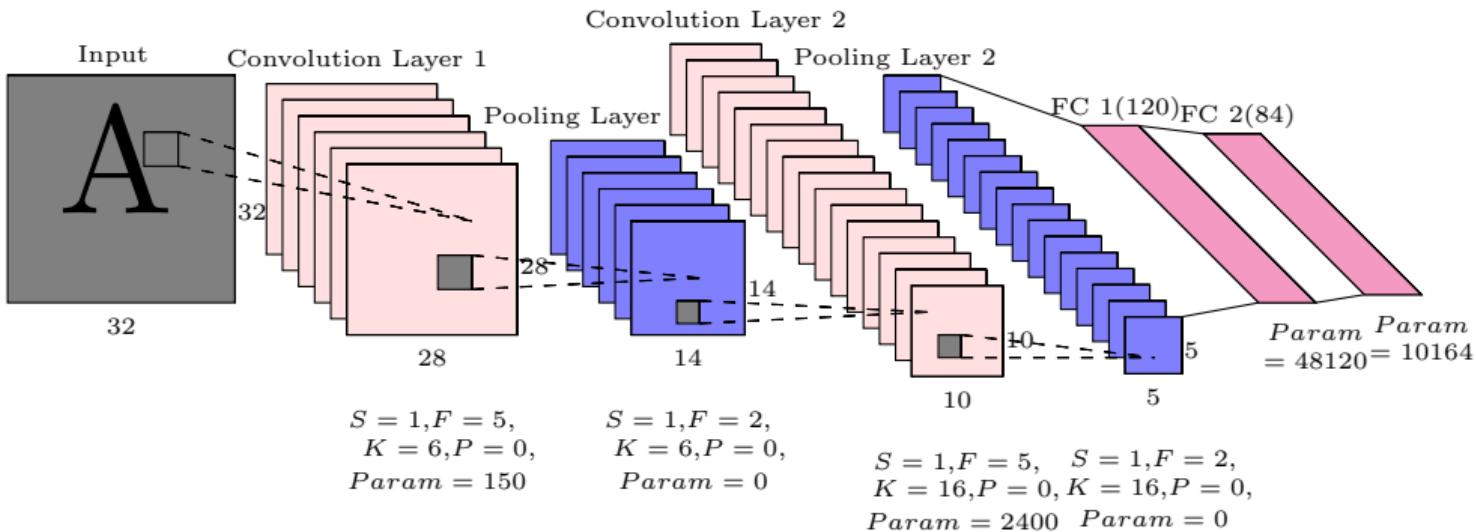
LeNet-5 for handwritten character recognition



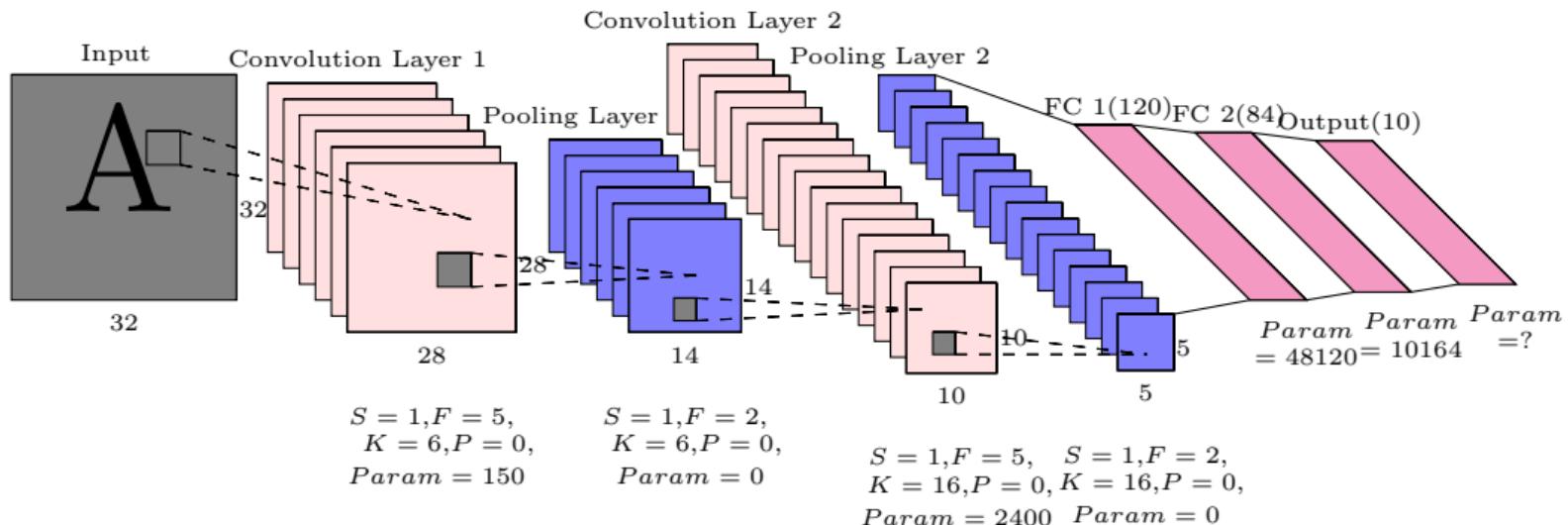
LeNet-5 for handwritten character recognition



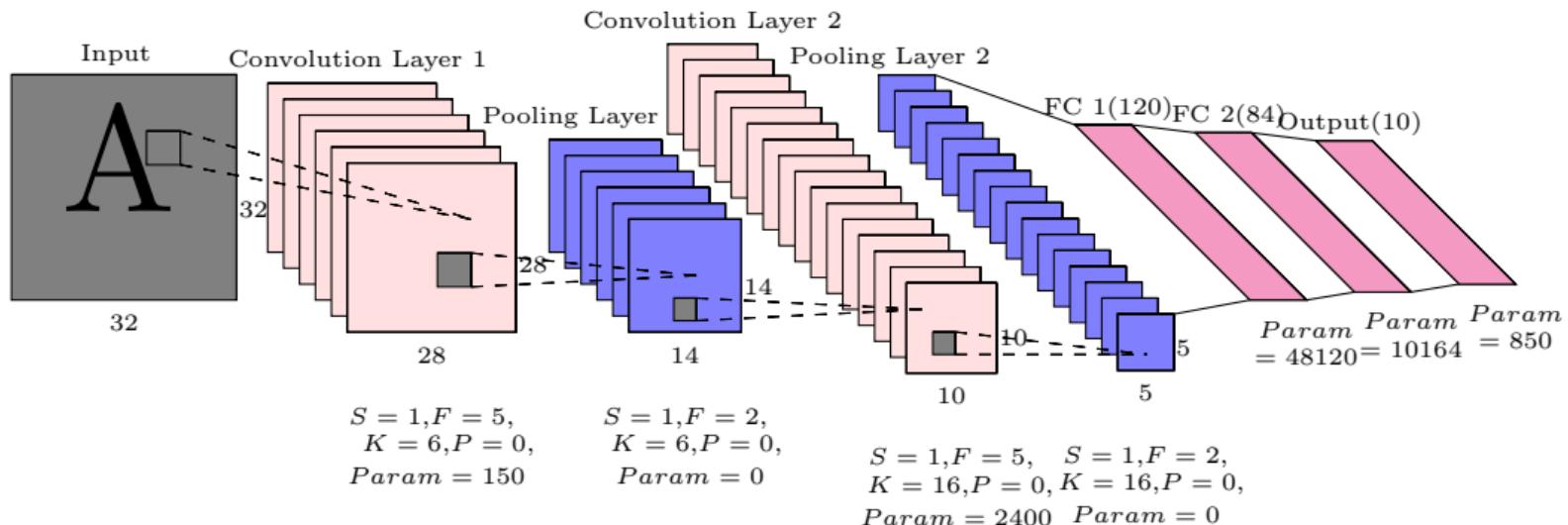
LeNet-5 for handwritten character recognition



LeNet-5 for handwritten character recognition



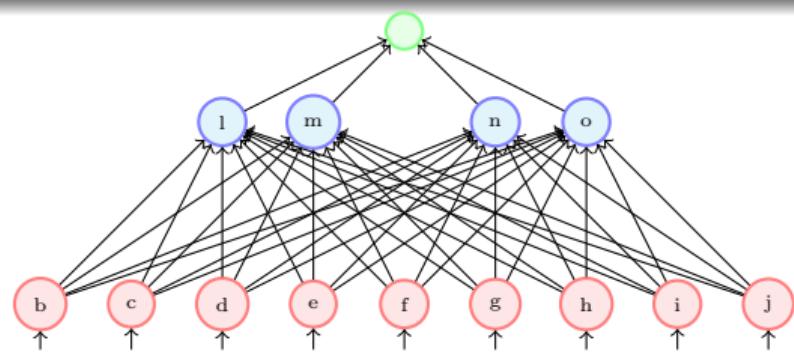
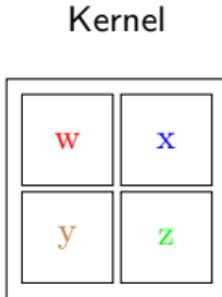
LeNet-5 for handwritten character recognition



- How do we train a convolutional neural network ?

Input

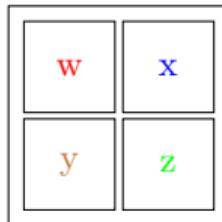
b	c	d
e	f	g
h	i	j



- A CNN can be implemented as a feedforward neural network

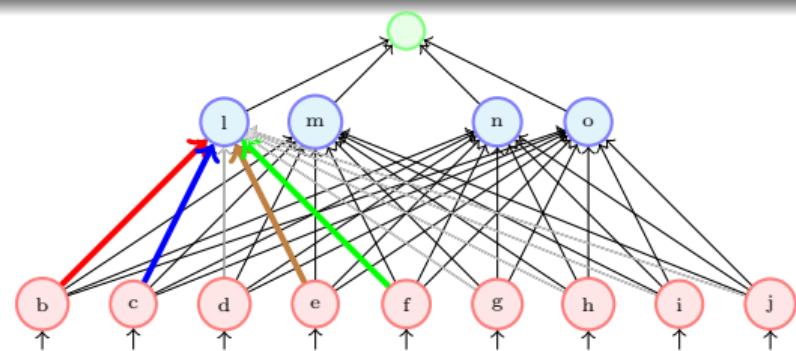
Input	
b	c
e	f
h	i

Kernel



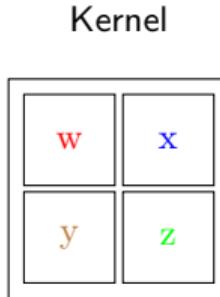
Output

ℓ	



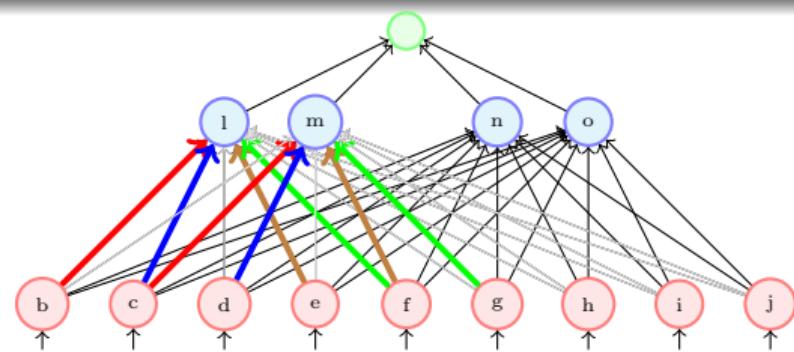
- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active

Input		
b	c	d
e	f	g
h	i	j



Output

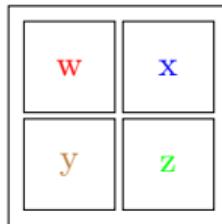
ℓ	m



- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active
- the rest of the weights (in gray) are zero

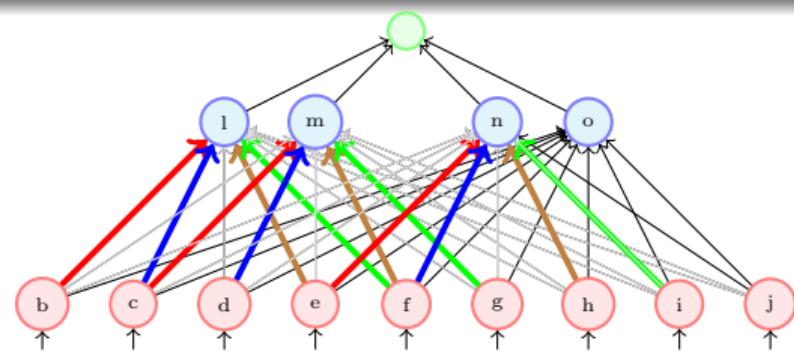
Input	
b	c
e	f
h	i

Kernel



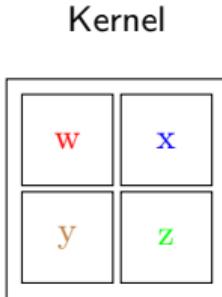
Output

ℓ	m
n	



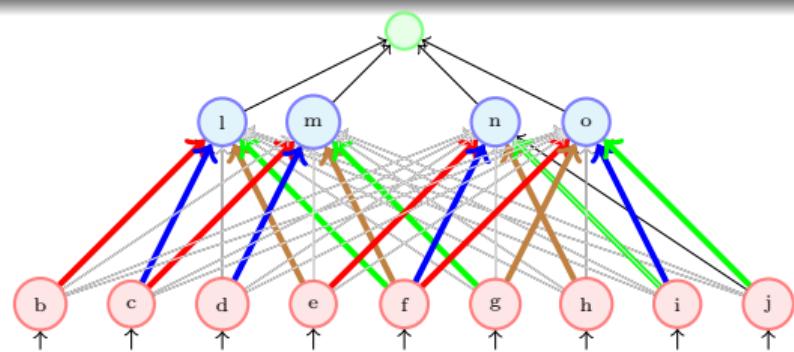
- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active
- the rest of the weights (in gray) are zero

Input		
b	c	d
e	f	g
h	i	j



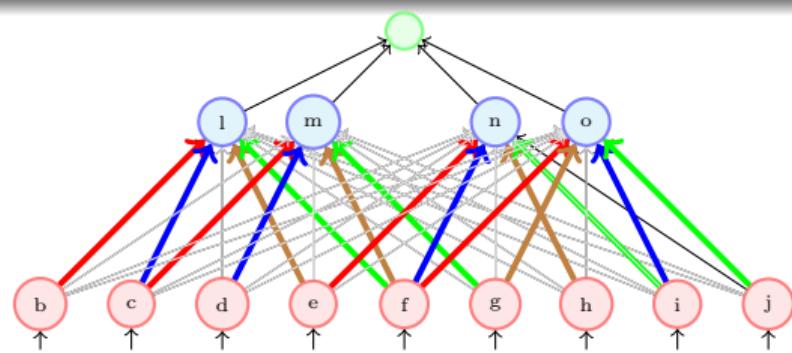
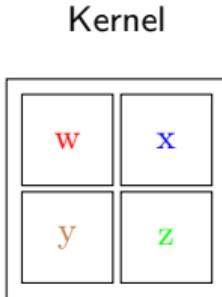
Output

ℓ	m
n	o



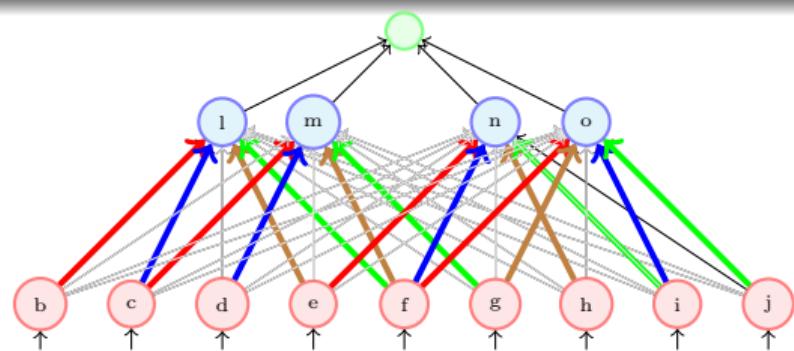
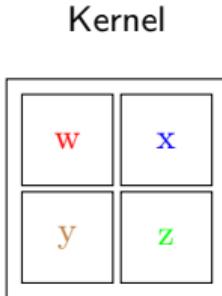
- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active
- the rest of the weights (in gray) are zero

Input		
b	c	d
e	f	g
h	i	j



- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active
- the rest of the weights (in gray) are zero

Input		
b	c	d
e	f	g
h	i	j



- We can thus train a convolution neural network using backpropagation by thinking of it as a feedforward neural network with sparse connections

- A CNN can be implemented as a feedforward neural network
- wherein only a few weights(in color) are active
- the rest of the weights (in gray) are zero

Module 11.4 : CNNs (success stories on ImageNet)

ImageNet Success Stories(roadmap for rest of the talk)

- AlexNet

ImageNet Success Stories(roadmap for rest of the talk)

- AlexNet
- ZFNet

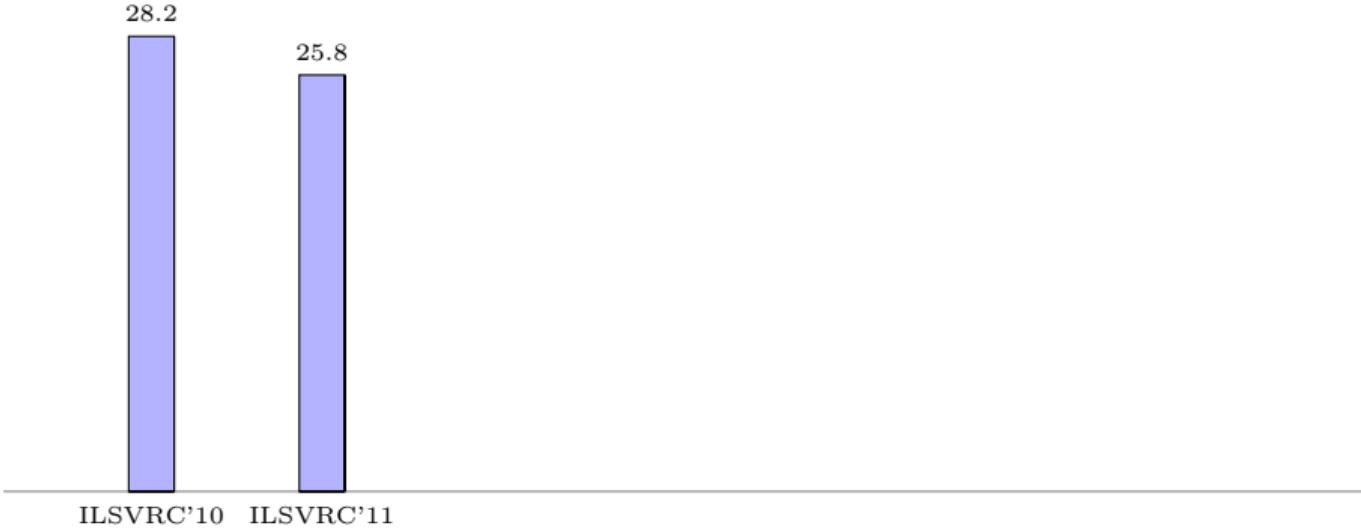
ImageNet Success Stories(roadmap for rest of the talk)

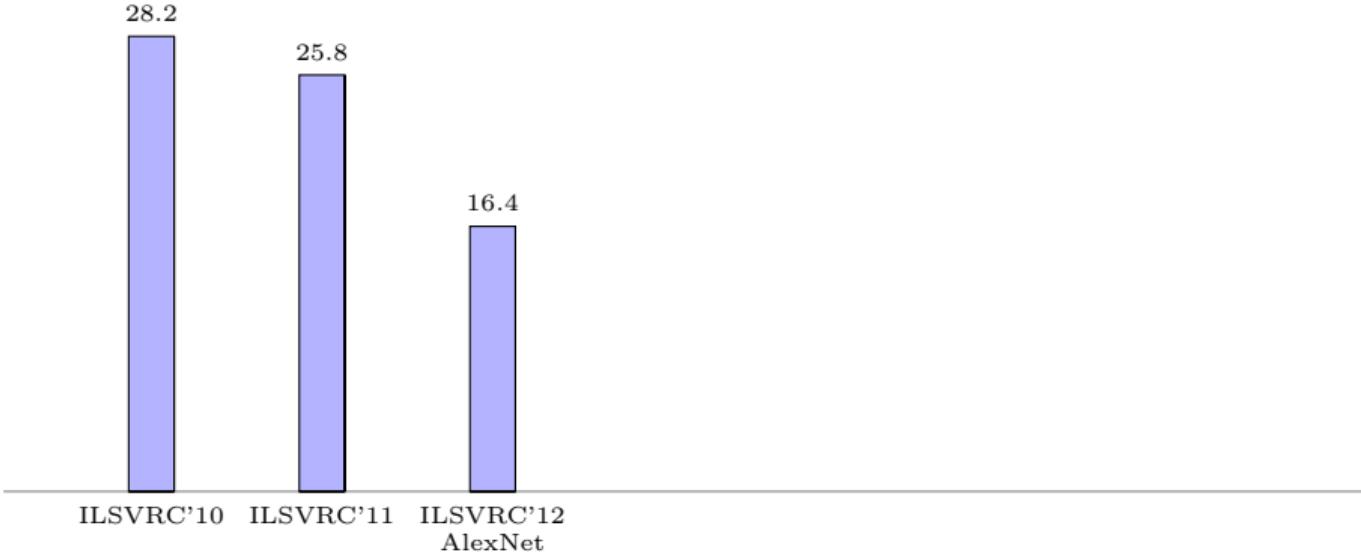
- AlexNet
- ZFNet
- VGGNet

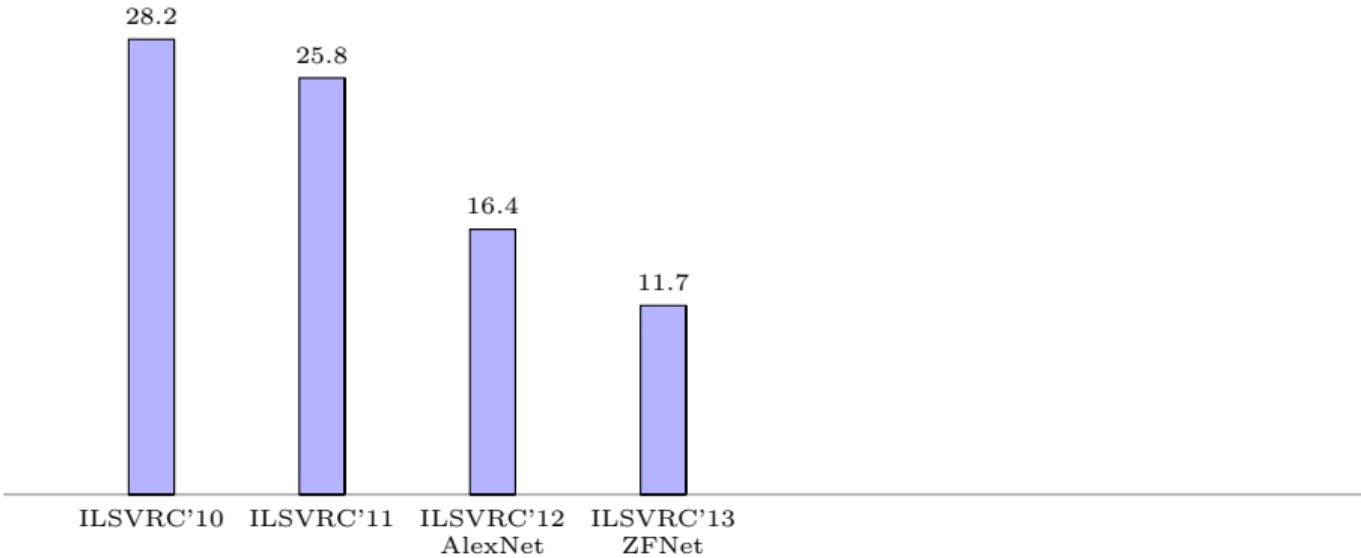
28.2

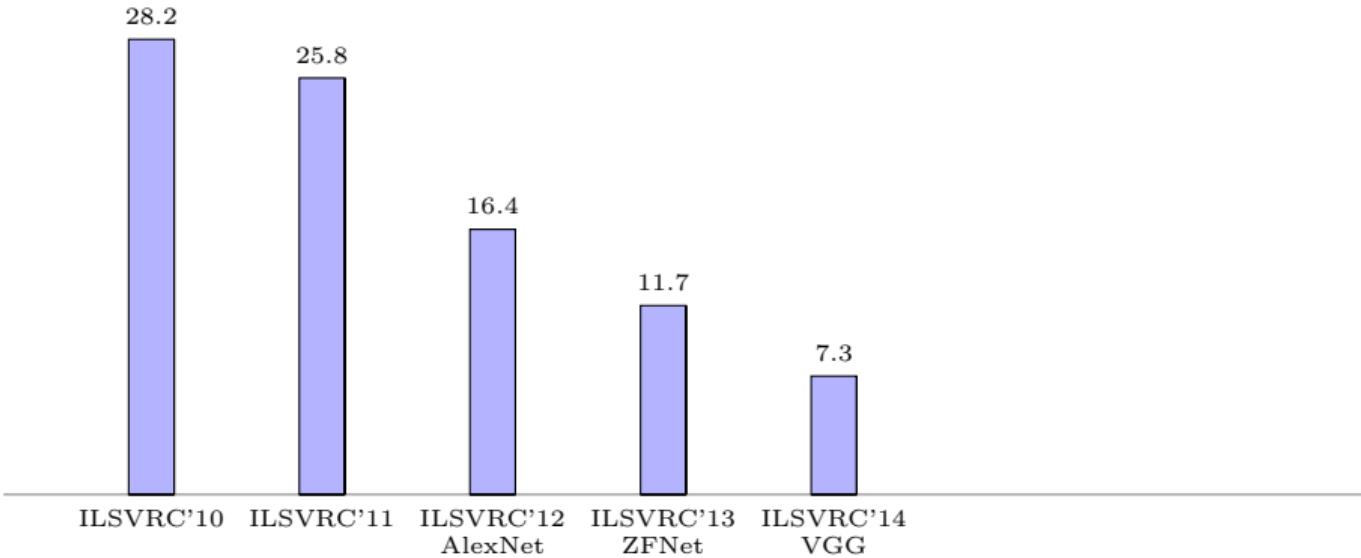


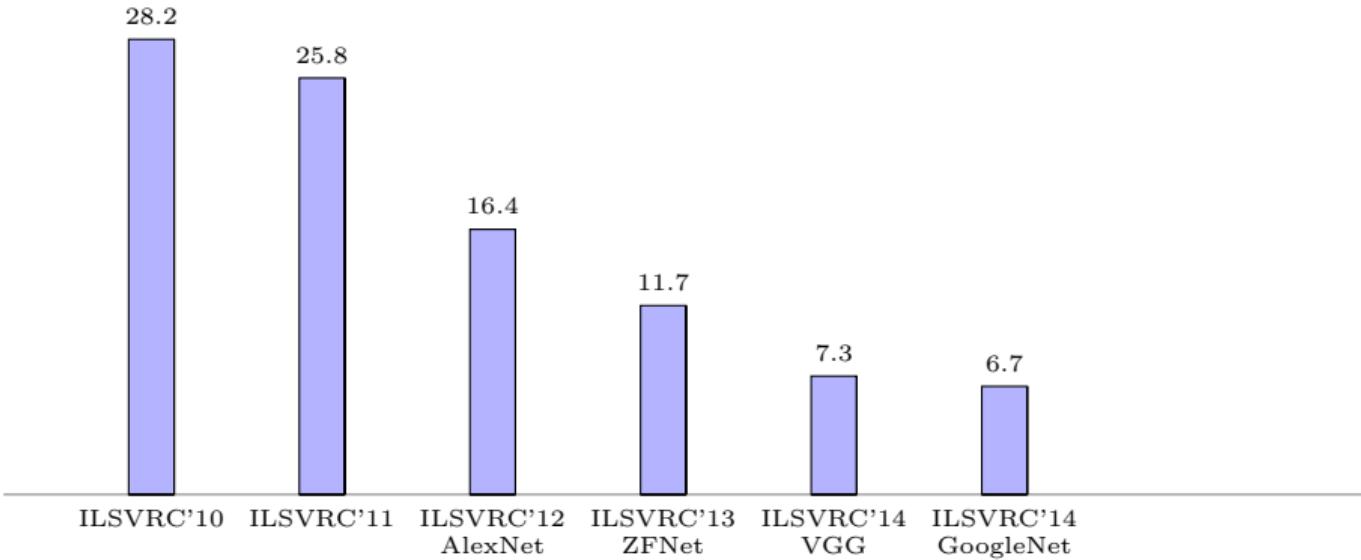
ILSVRC'10

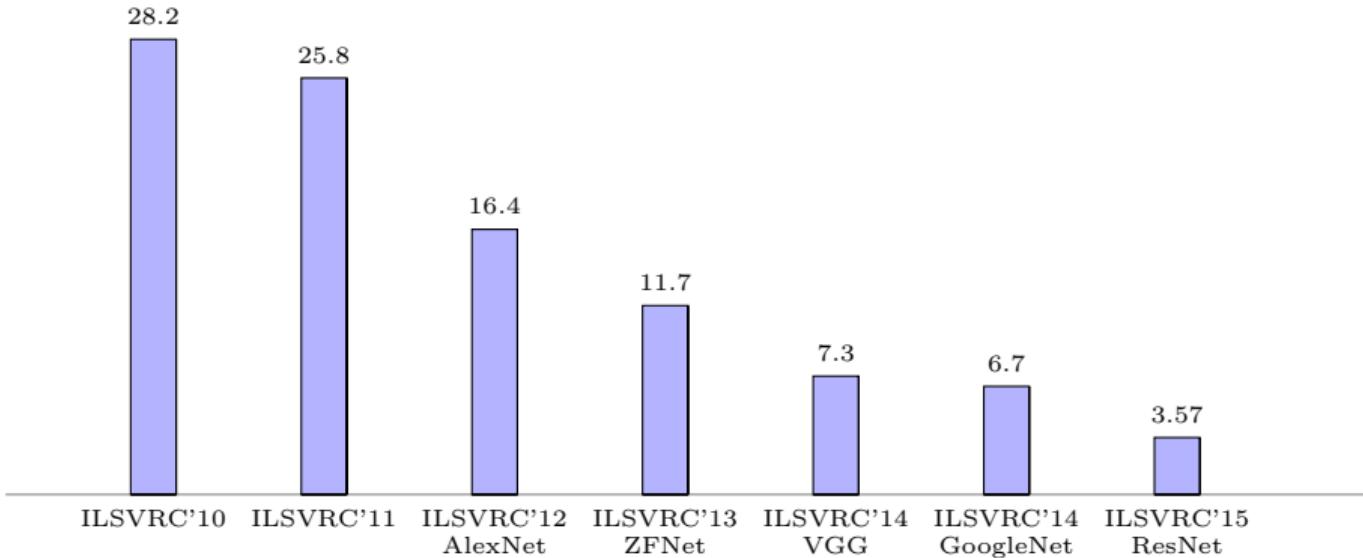


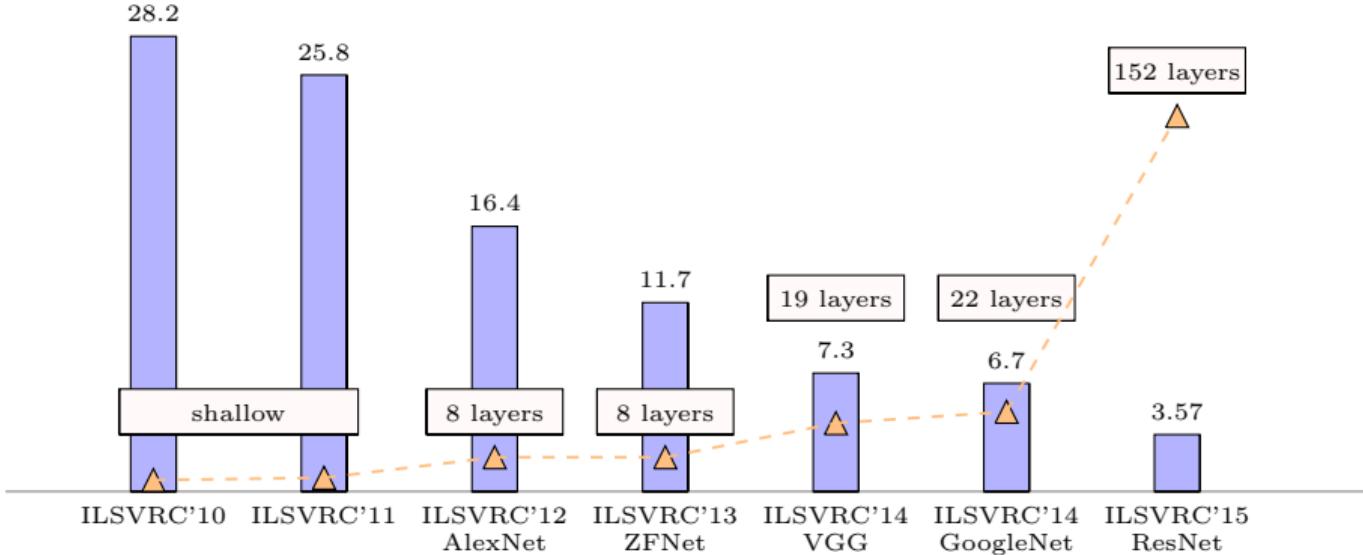


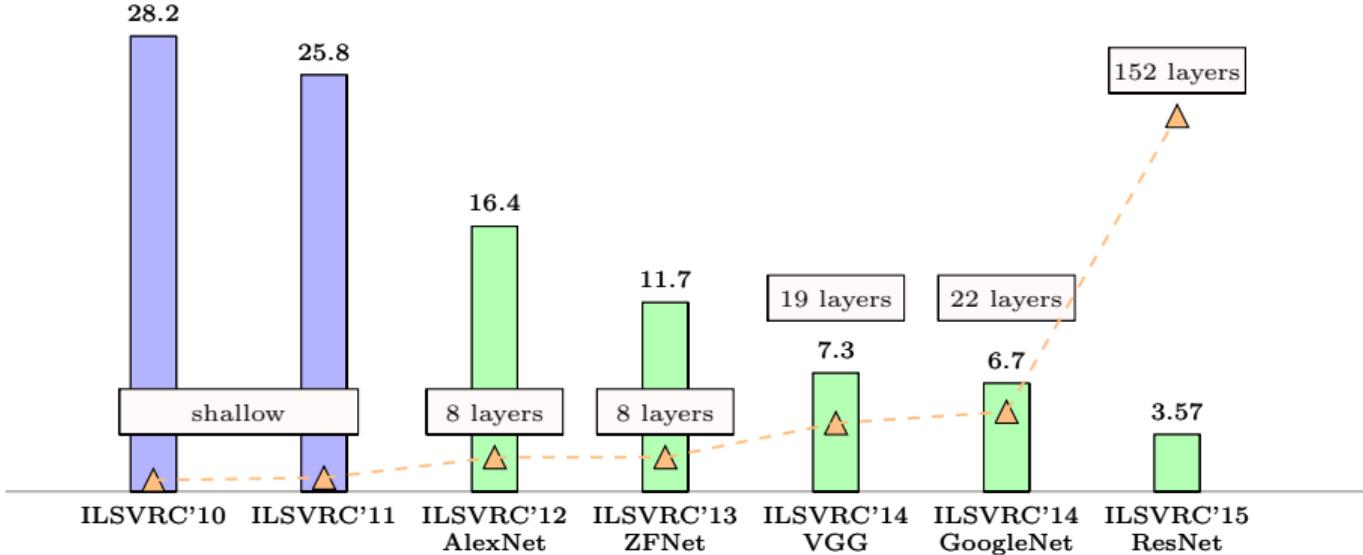








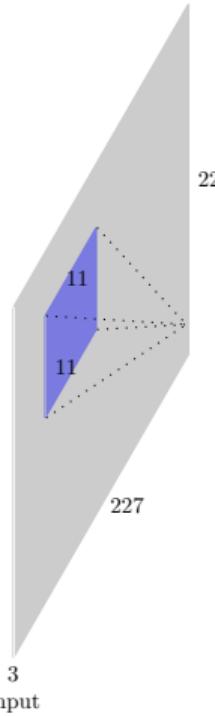




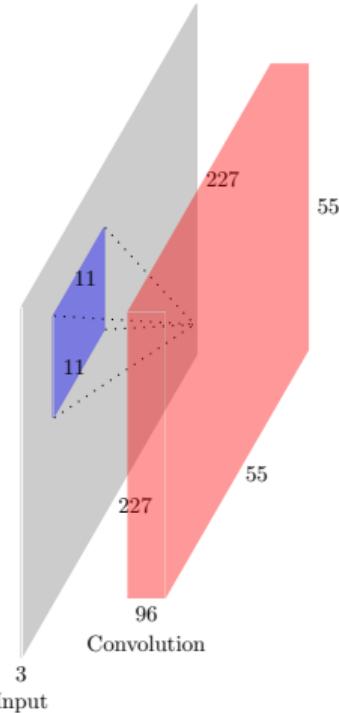
ImageNet Success Stories(roadmap for rest of the talk)

- AlexNet
- ZFNet
- VGGNet

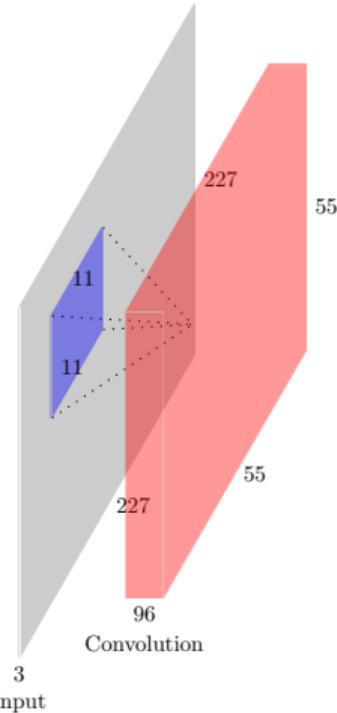




Input: $227 \times 227 \times 3$
Conv1: $K = 96, F = 11$
 $S = 4, P = 0$
Output: $W_2 = ?, H_2 = ?$
Parameters: ?



Input: $227 \times 227 \times 3$
Conv1: $K = 96, F = 11$
 $S = 4, P = 0$
Output: $W_2 = 55, H_2 = 55$
Parameters: ?



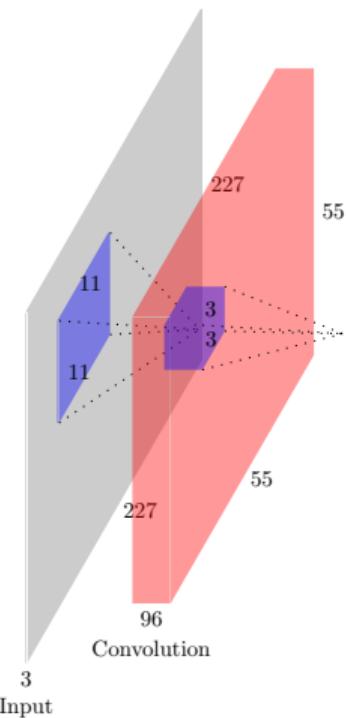
Input: $227 \times 227 \times 3$
Conv1: $K = 96, F = 11$
 $S = 4, P = 0$
Output: $W_2 = 55, H_2 = 55$
Parameters: $(11 \times 11 \times 3) \times 96 = 34K$

Max Pool Input: $55 \times 55 \times 96$

$$F = 3, S = 2$$

Output: $W_2 = ?, H_2 = ?$

Parameters: ?

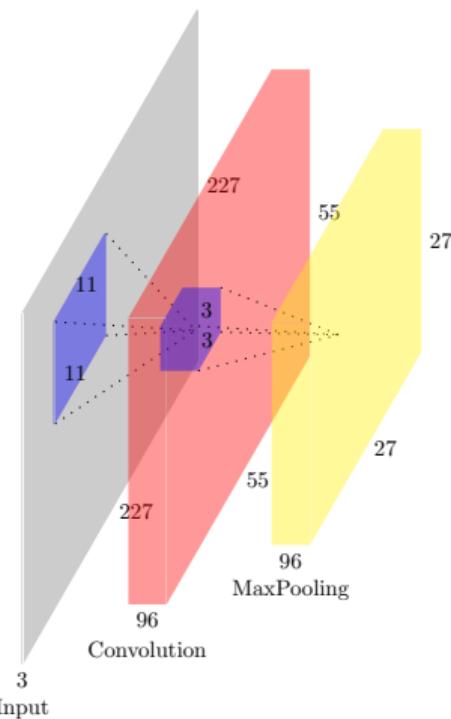


Max Pool Input: $55 \times 55 \times 96$

$$F = 3, S = 2$$

Output: $W_2 = 27, H_2 = 27$

Parameters: ?

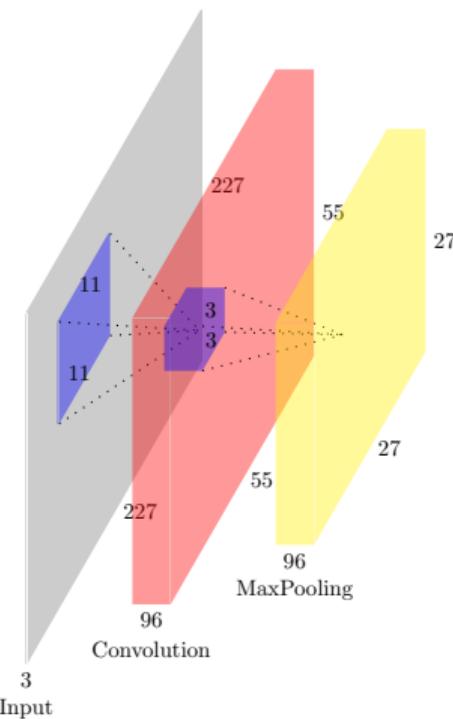


Max Pool Input: $55 \times 55 \times 96$

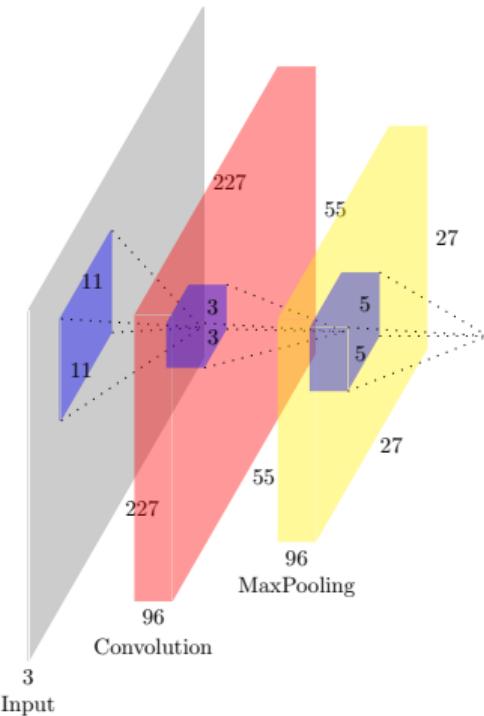
$$F = 3, S = 2$$

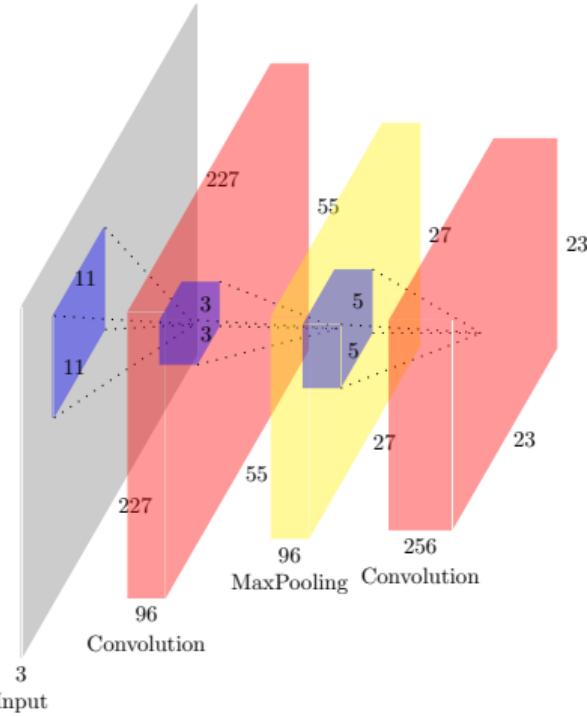
Output: $W_2 = 27, H_2 = 27$

Parameters: 0

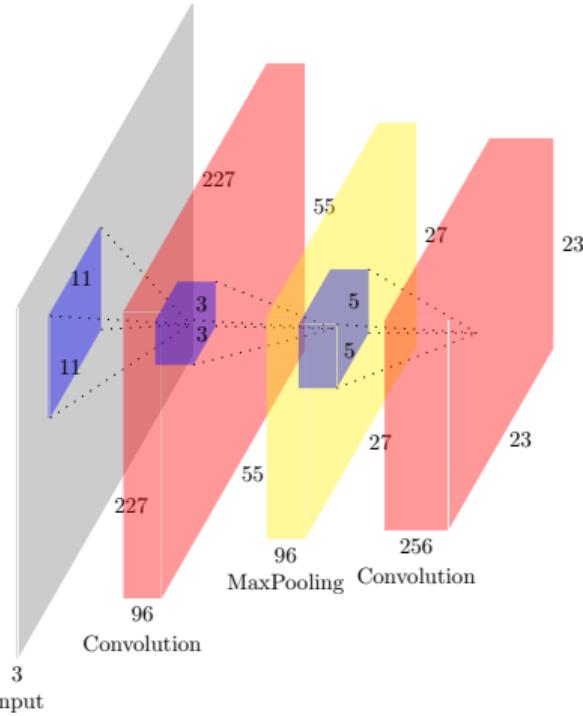


Input: $27 \times 27 \times 96$
Conv1: $K = 256, F = 5$
 $S = 1, P = 0$
Output: $W_2 = ?, H_2 = ?$
Parameters: ?



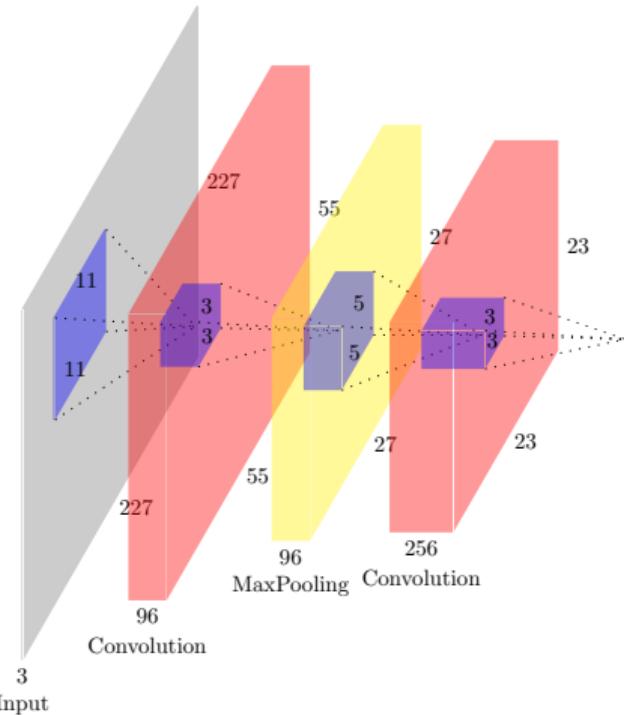


Input: $27 \times 27 \times 96$
 Conv1: $K = 256, F = 5$
 $S = 1, P = 0$
 Output: $W_2 = 23, H_2 = 23$
 Parameters: ?



Input: $27 \times 27 \times 96$
Conv1: $K = 256, F = 5$
$S = 1, P = 0$
Output: $W_2 = 23, H_2 = 23$
Parameters: $(5 \times 5 \times 96) \times 256 = 0.6M$

Max Pool Input: $23 \times 23 \times 256$
 $F = 3, S = 2$
Output: $W_2 = ?, H_2 = ?$
Parameters: ?

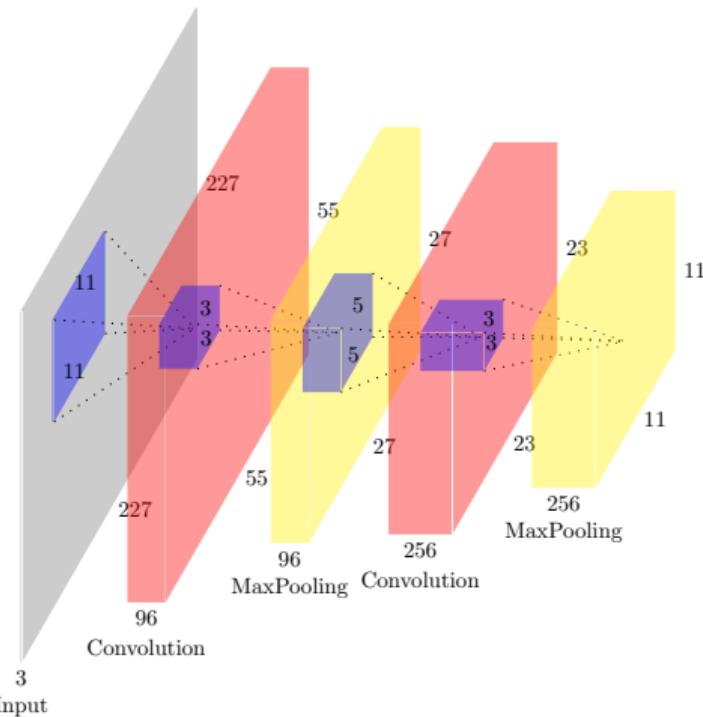


Max Pool Input: $23 \times 23 \times 256$

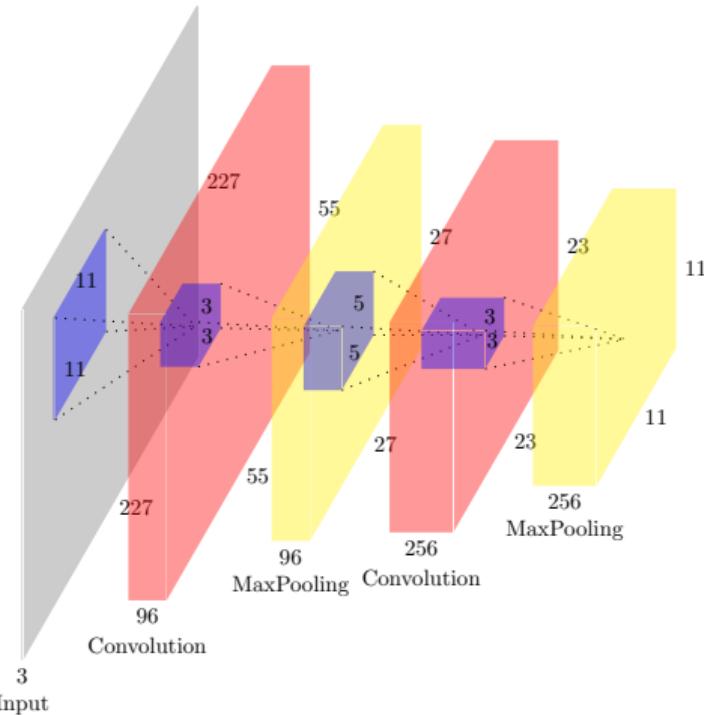
$$F = 3, S = 2$$

Output: $W_2 = 11, H_2 = 11$

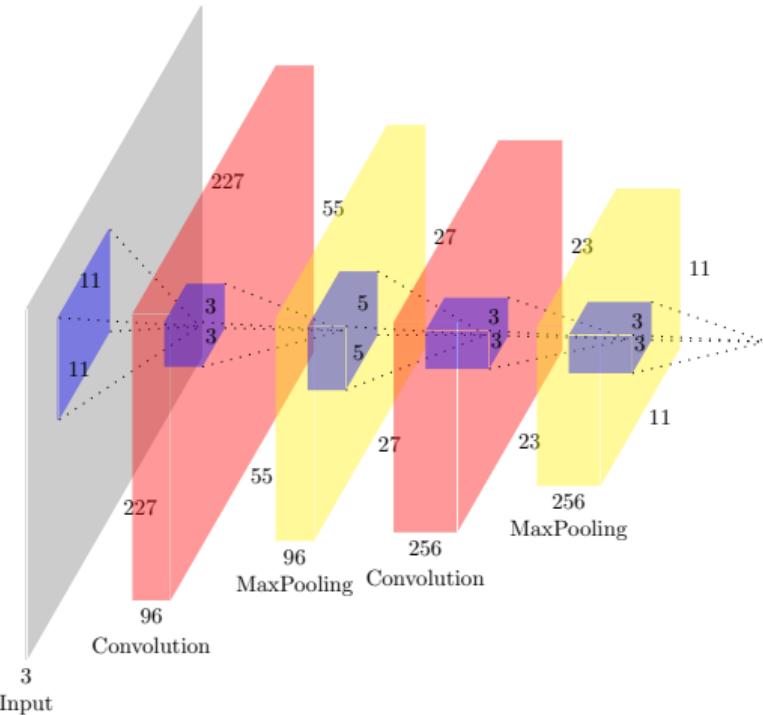
Parameters: ?



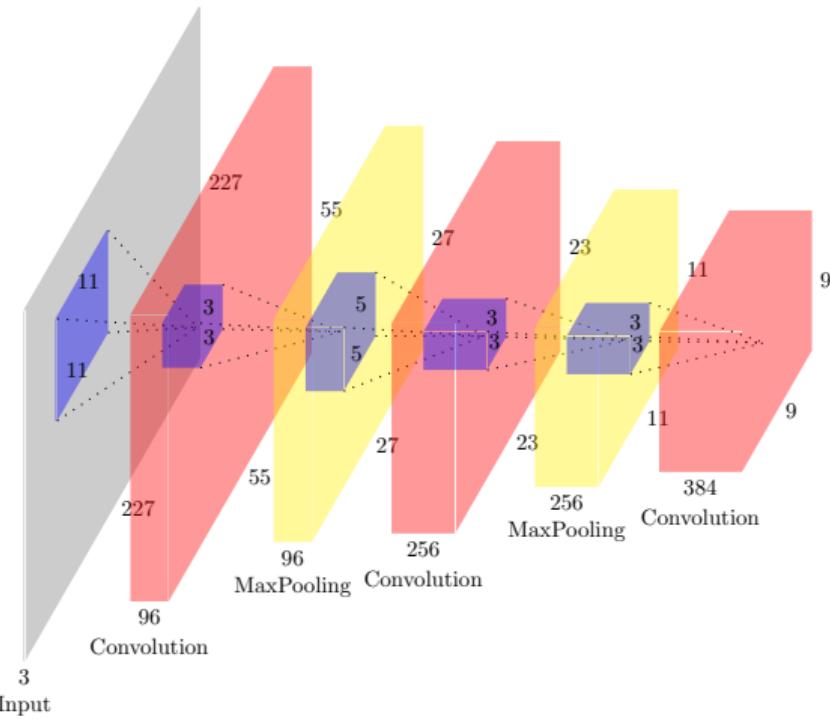
Max Pool Input: $23 \times 23 \times 256$
 $F = 3, S = 2$
Output: $W_2 = 11, H_2 = 11$
Parameters: 0

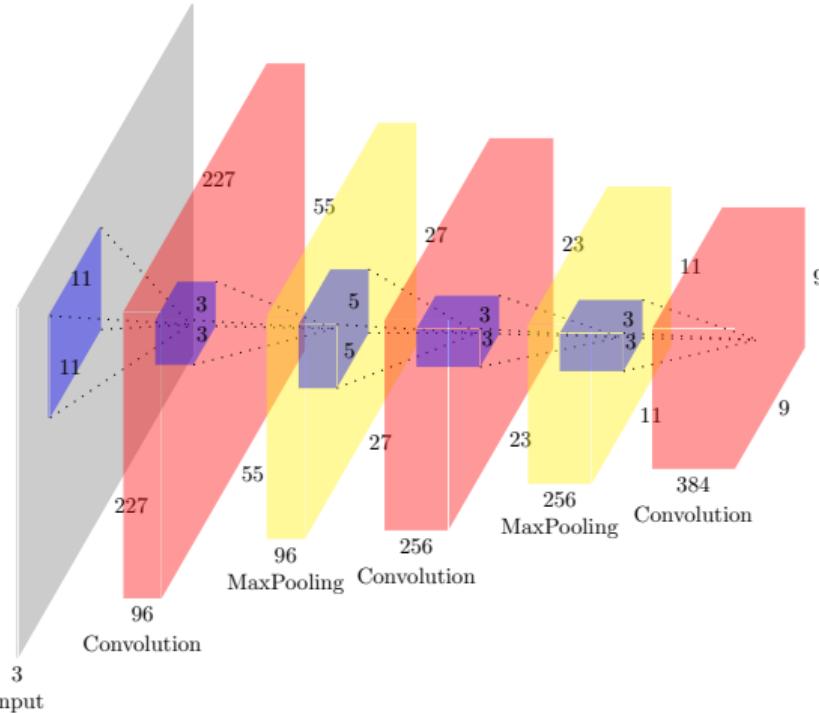


Input: $11 \times 11 \times 256$
Conv1: $K = 384, F = 3$
 $S = 1, P = 0$
Output: $W_2 = ?, H_2 = ?$
Parameters: ?



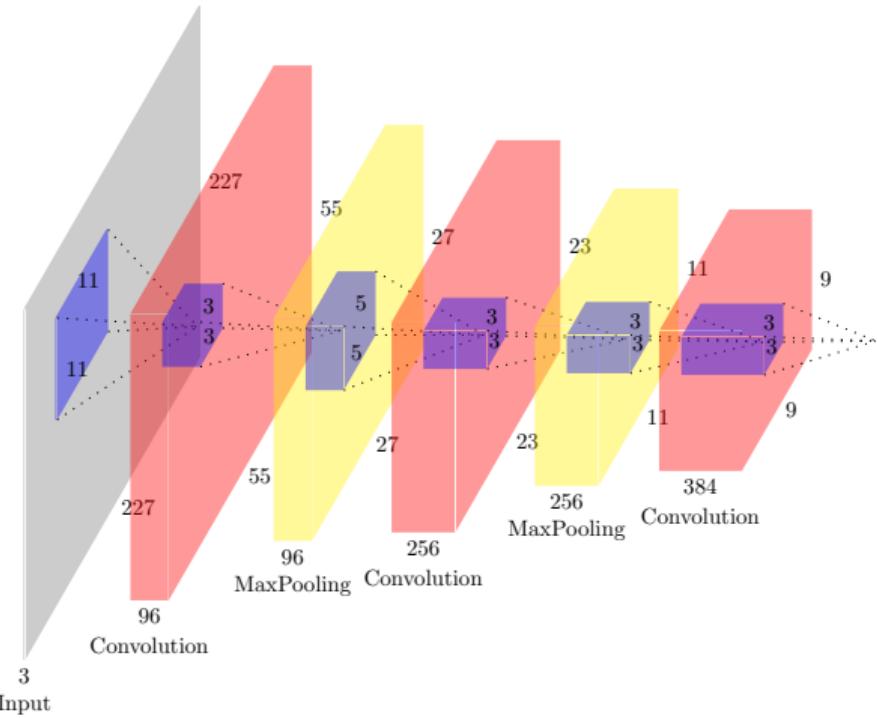
Input: $11 \times 11 \times 256$
 Conv1: $K = 384, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = 9, H_2 = 9$
 Parameters: ?



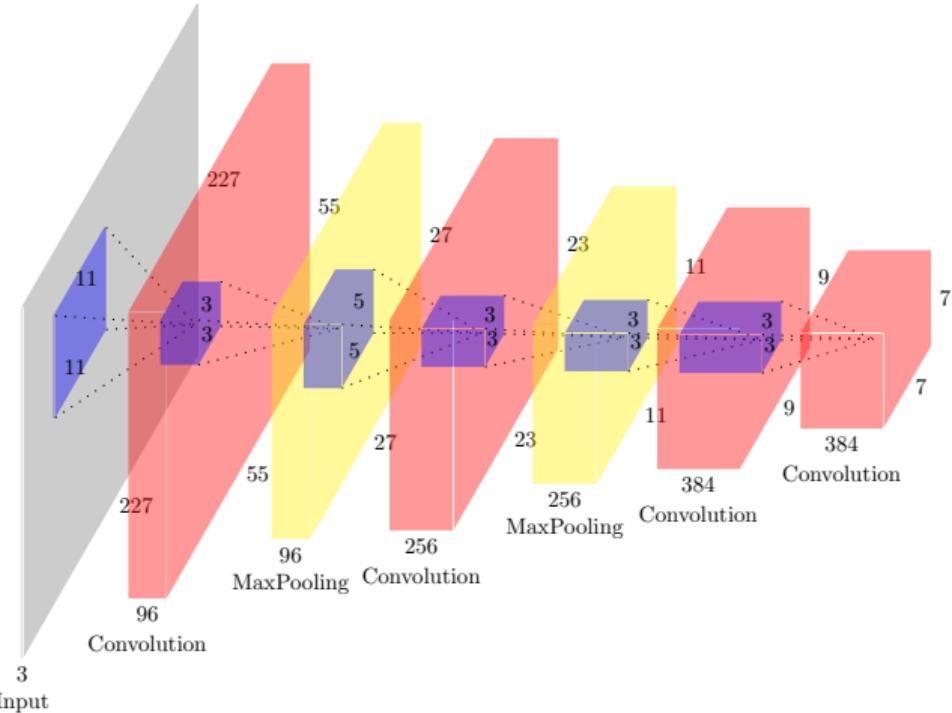


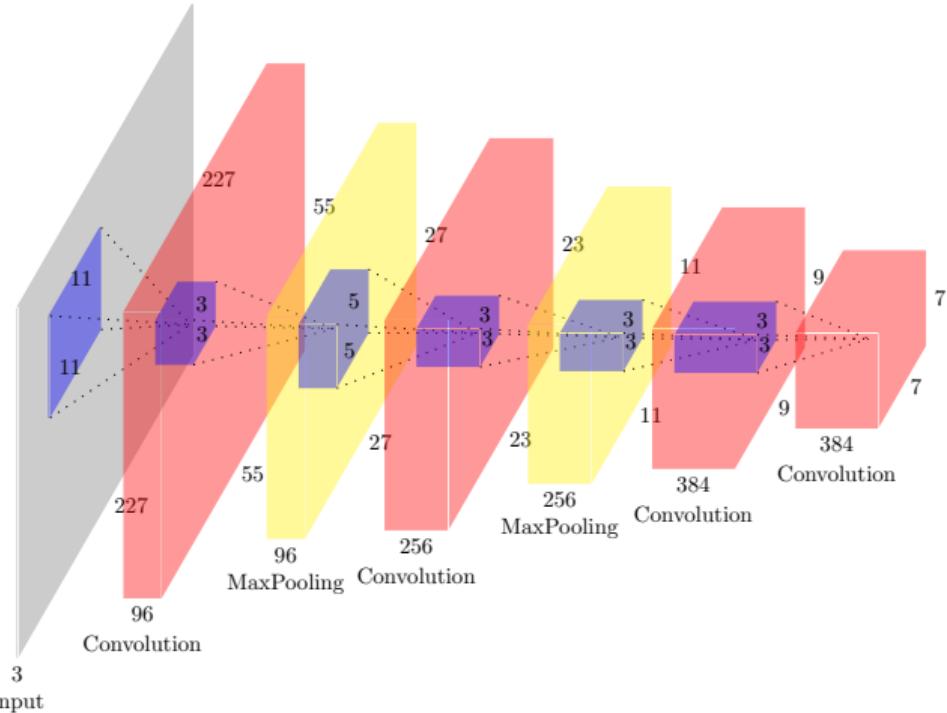
Input: $11 \times 11 \times 256$
Conv1: $K = 384, F = 3$
$S = 1, P = 0$
Output: $W_2 = 9, H_2 = 9$
Parameters: $(3 \times 3 \times 256) \times 384 = 0.8M$

Input: $9 \times 9 \times 384$
 Conv1: $K = 384, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = ?, H_2 = ?$
 Parameters: ?



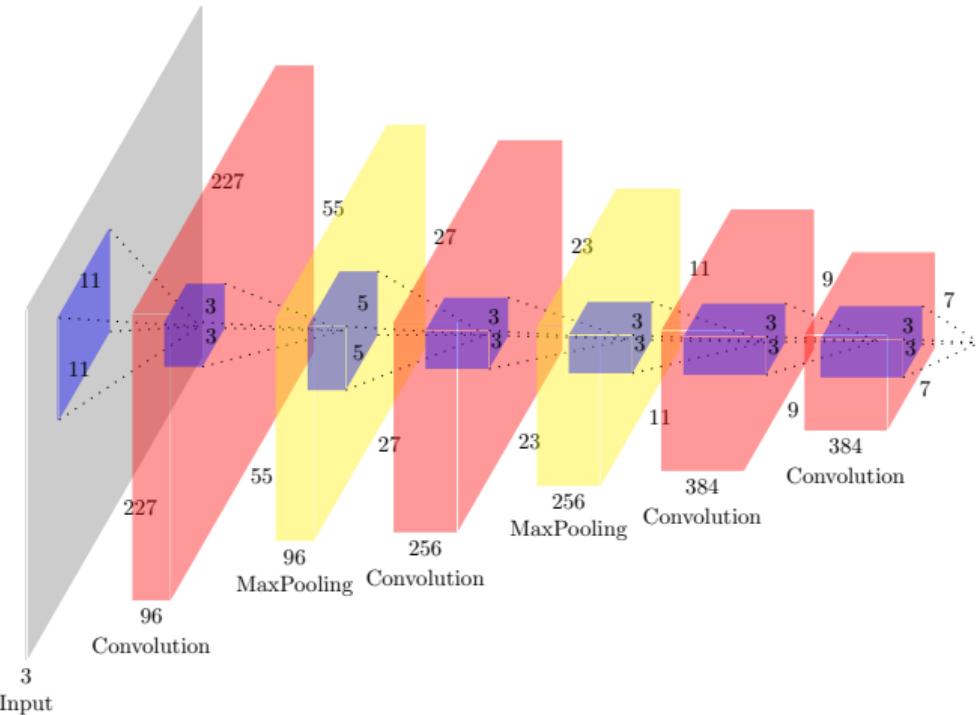
Input: $9 \times 9 \times 384$
 Conv1: $K = 384, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = 7, H_2 = 7$
 Parameters: ?



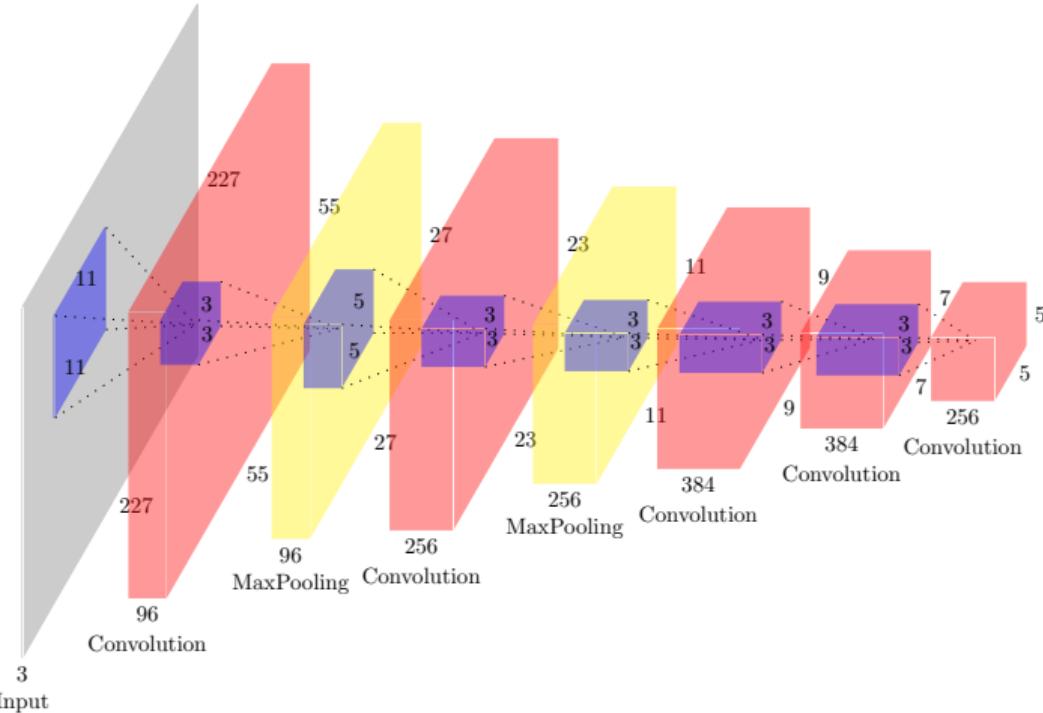


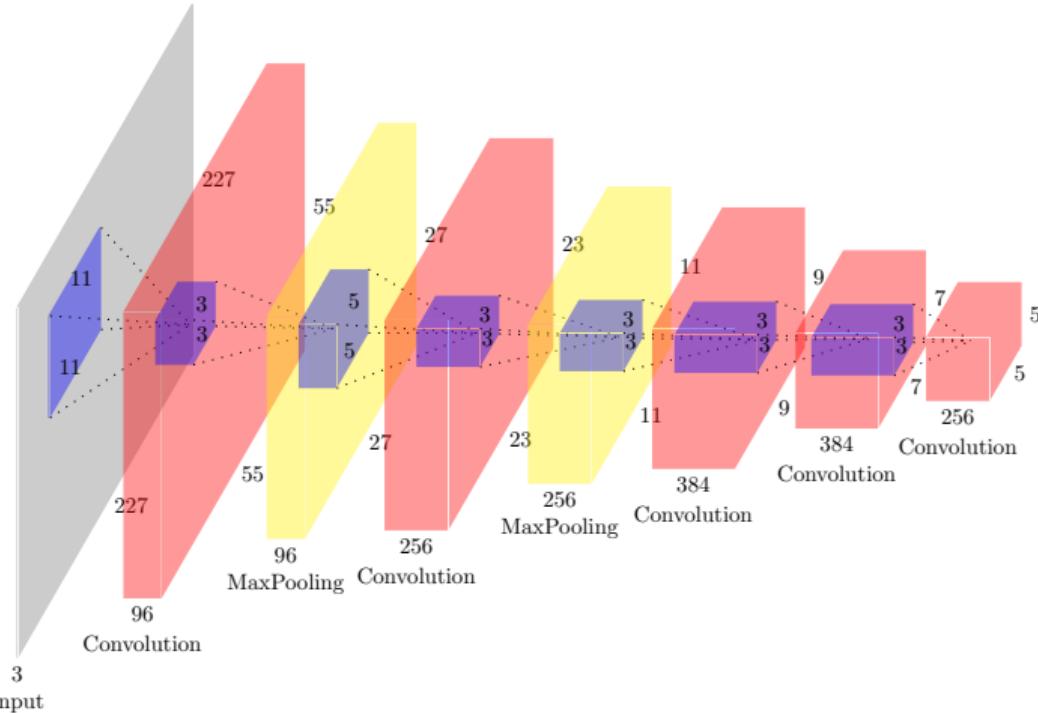
Input: $9 \times 9 \times 384$
Conv1: $K = 384, F = 3$
$S = 1, P = 0$
Output: $W_2 = 7, H_2 = 7$
Parameters: $(3 \times 3 \times 384) \times 384 = 1.327M$

Input: $7 \times 7 \times 384$
 Conv1: $K = 256, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = ?, H_2 = ?$
 Parameters: ?



Input: $7 \times 7 \times 384$
 Conv1: $K = 256, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = 5, H_2 = 5$
 Parameters: ?





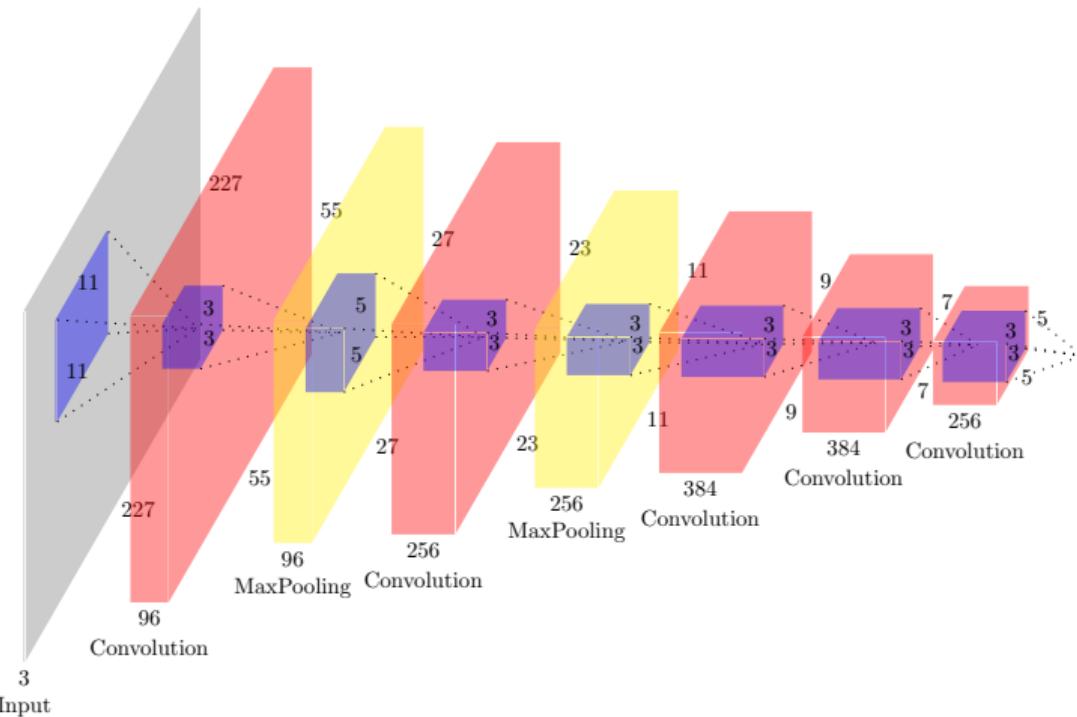
Input: $7 \times 7 \times 384$
 Conv1: $K = 256, F = 3$
 $S = 1, P = 0$
 Output: $W_2 = 5, H_2 = 5$
 Parameters: $(3 \times 3 \times 384) \times 256 = 0.8M$

Max Pool Input: $5 \times 5 \times 256$

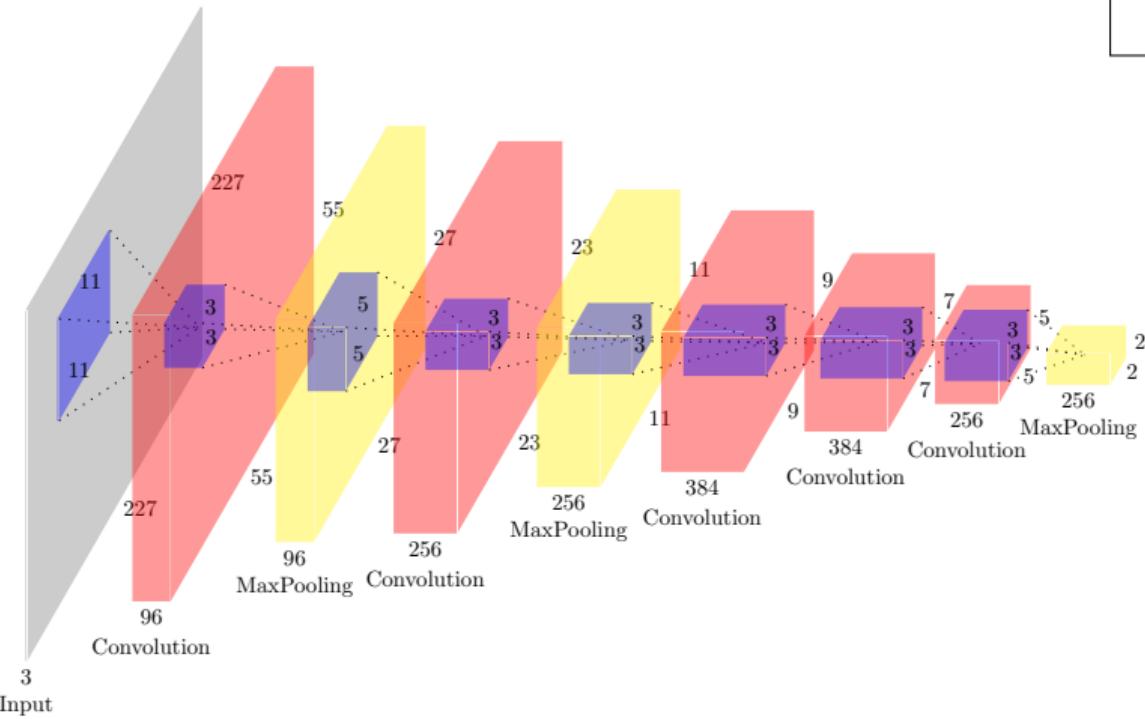
$$F = 3, S = 2$$

Output: $W_2 = ?, H_2 = ?$

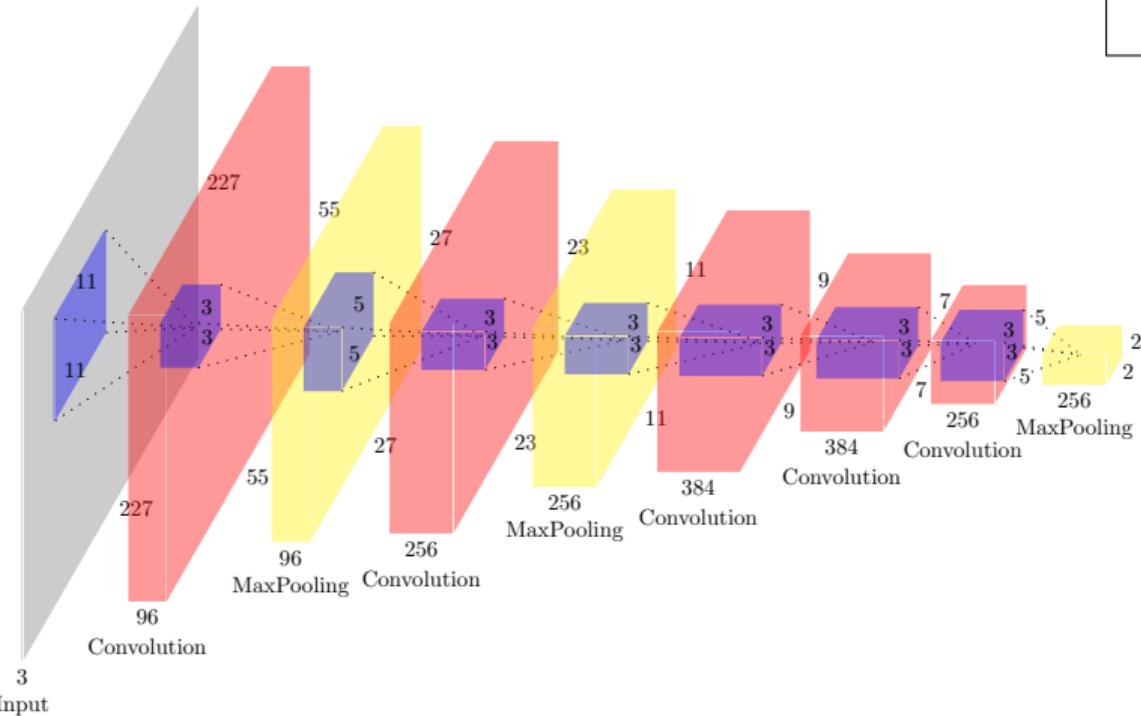
Parameters: ?

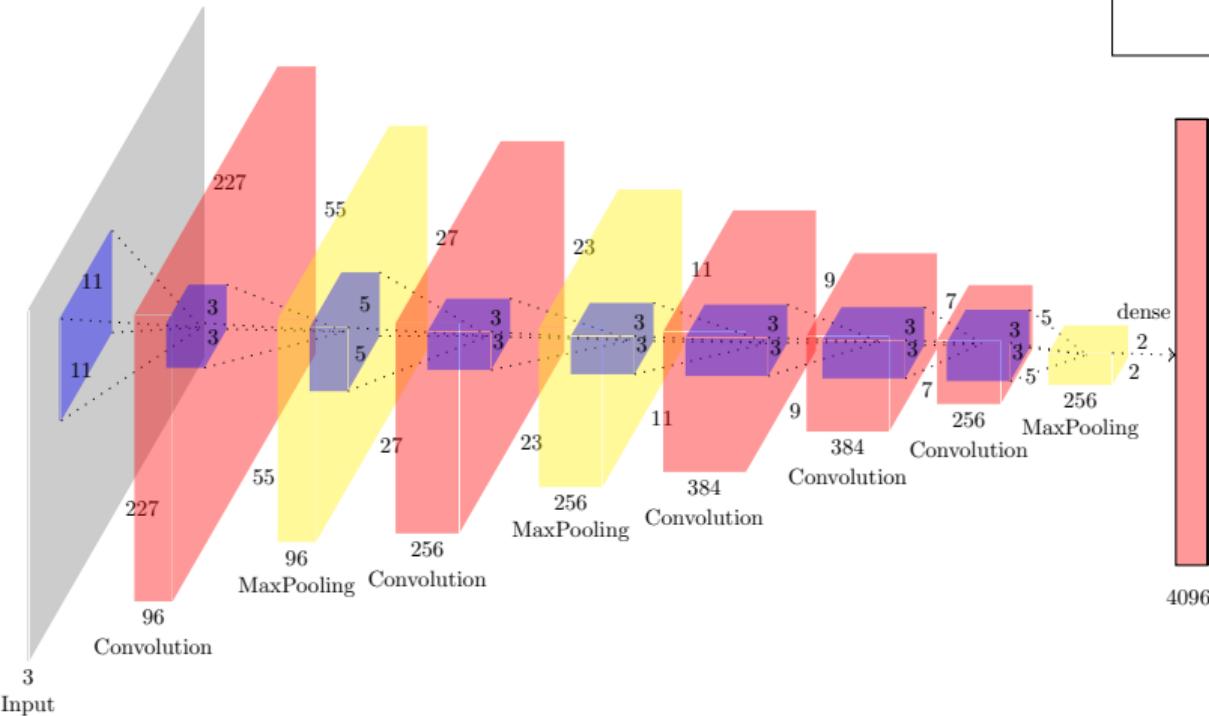


Max Pool Input: $5 \times 5 \times 256$
 $F = 3, S = 2$
Output: $W_2 = 2, H_2 = 2$
Parameters: ?

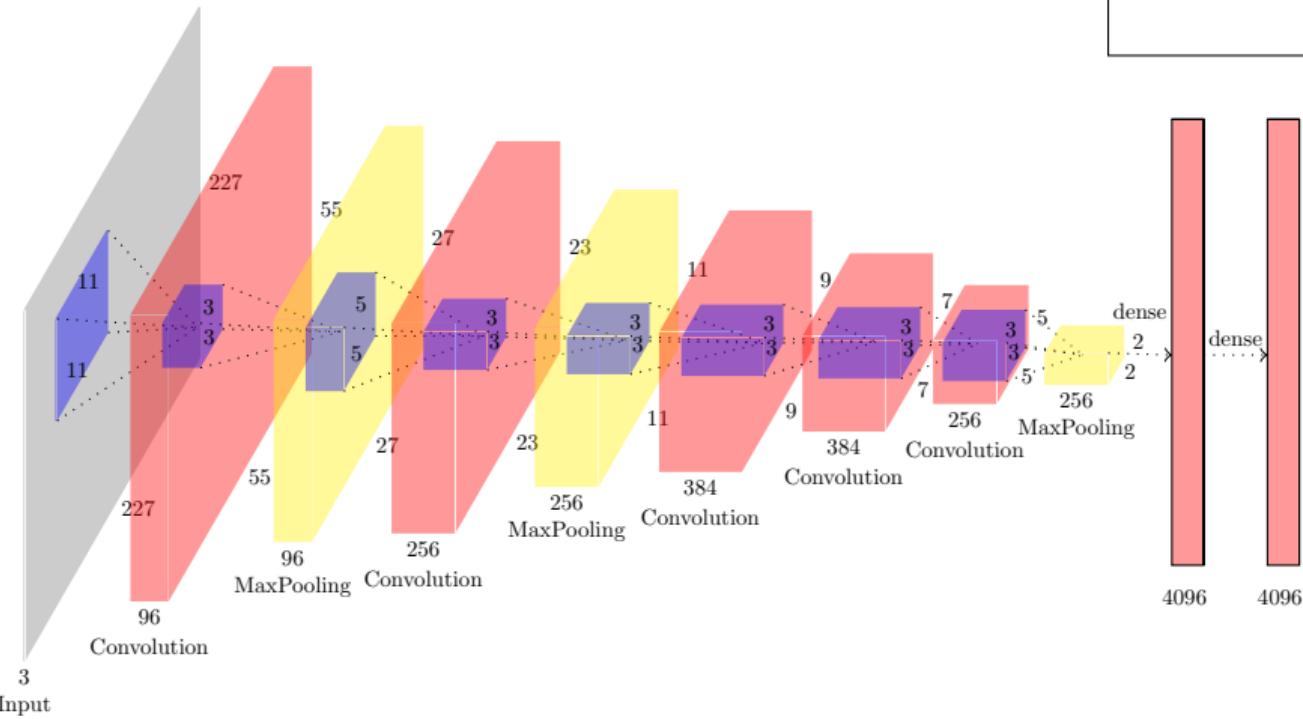


Max Pool Input: $5 \times 5 \times 256$
 $F = 3, S = 2$
Output: $W_2 = 2, H_2 = 2$
Parameters: 0





FC1

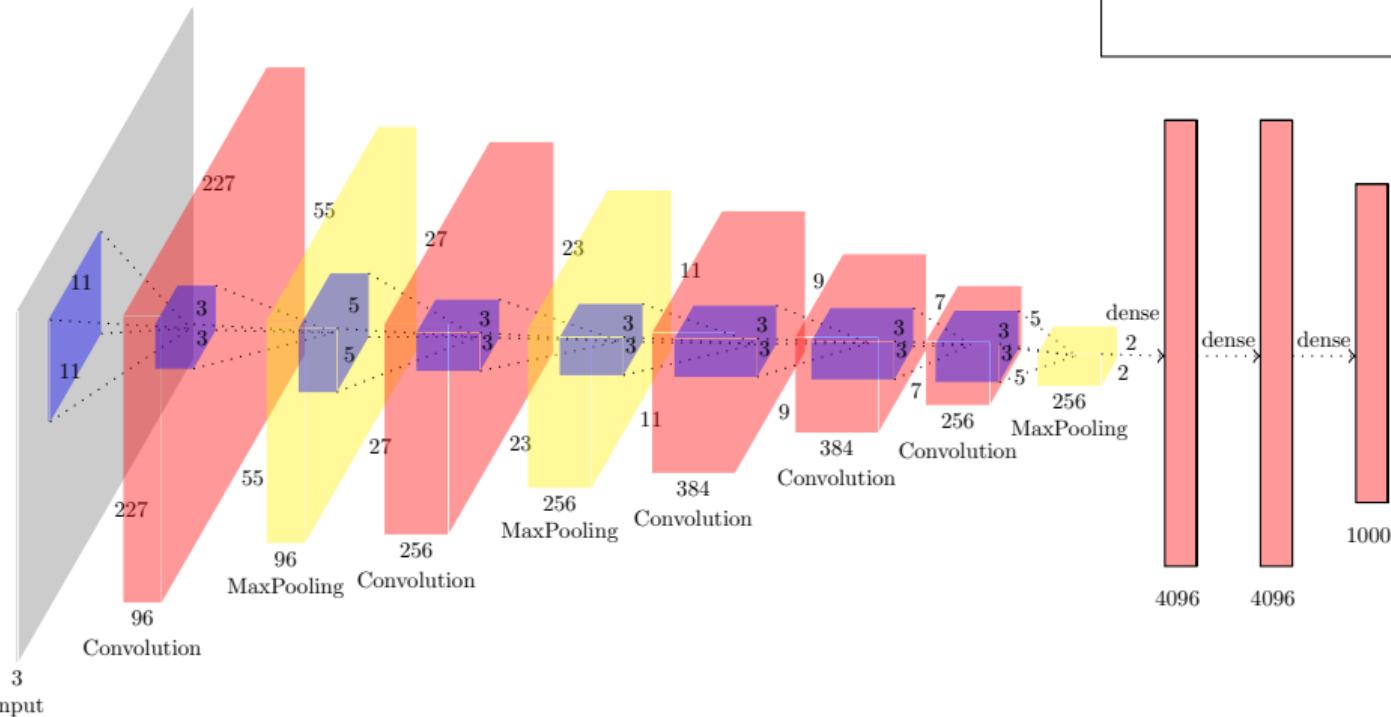


FC1

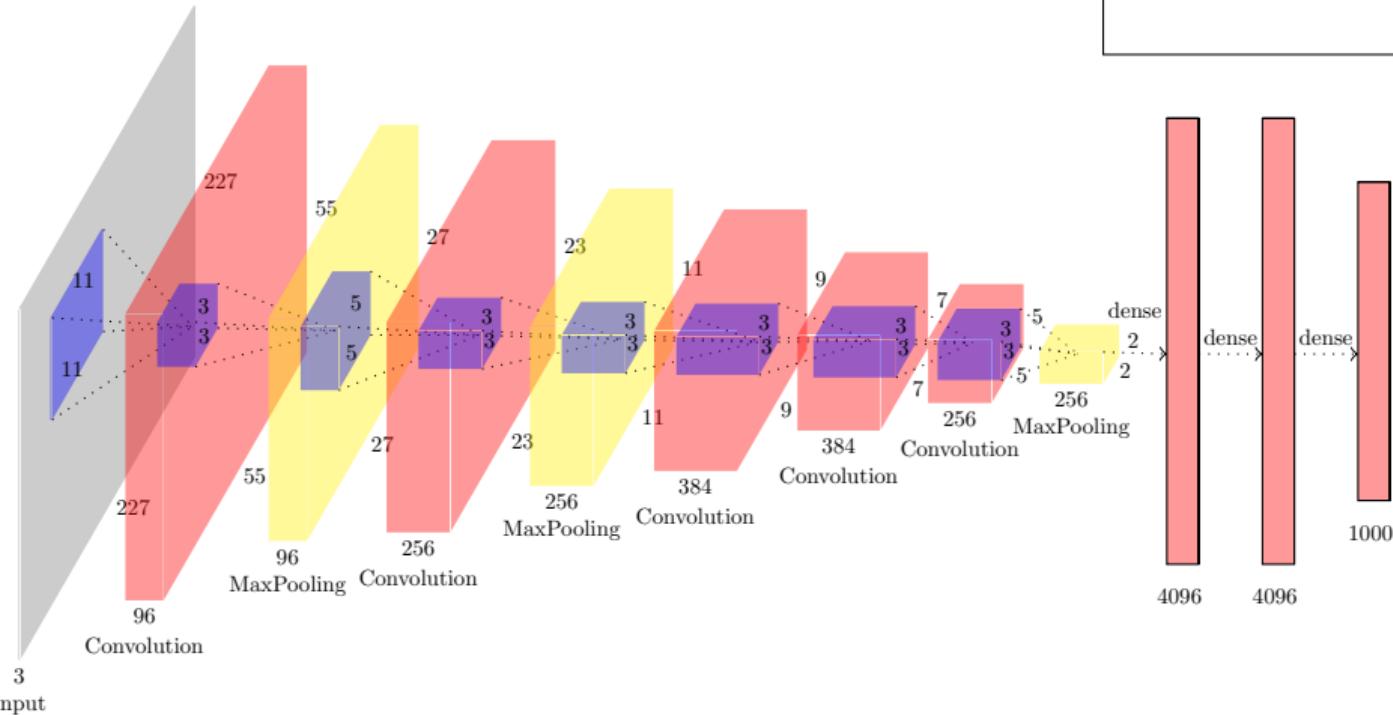
Parameters: $4096 \times 4096 = 16M$

FC1

Parameters: $4096 \times 1000 = 4M$



Total Parameters: 27.55M

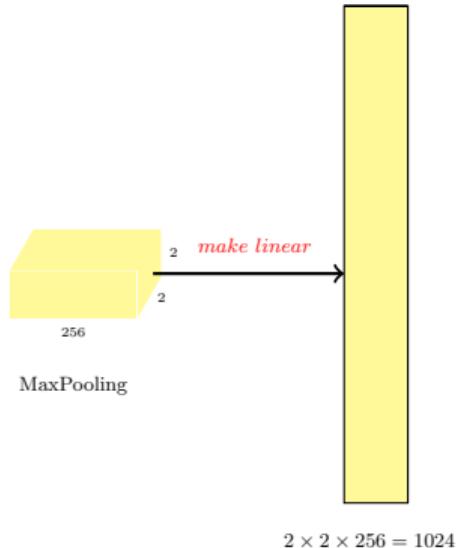


- Let us look at the connections in the fully connected layers in more detail

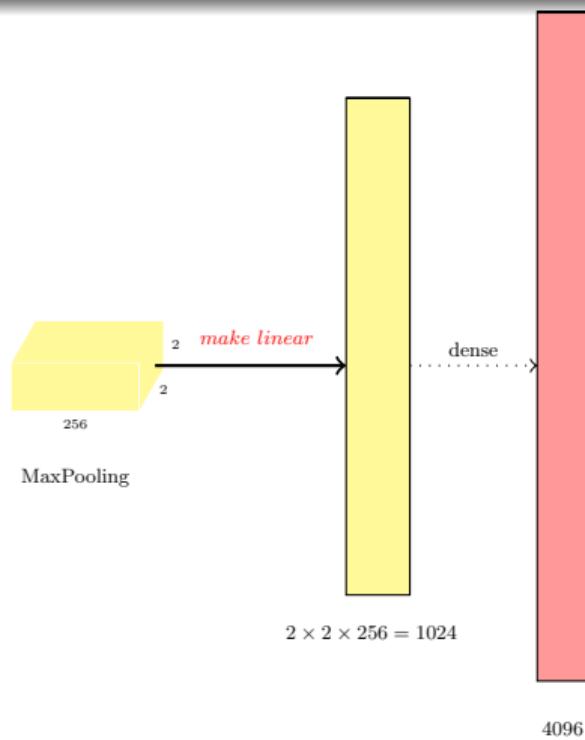


MaxPooling

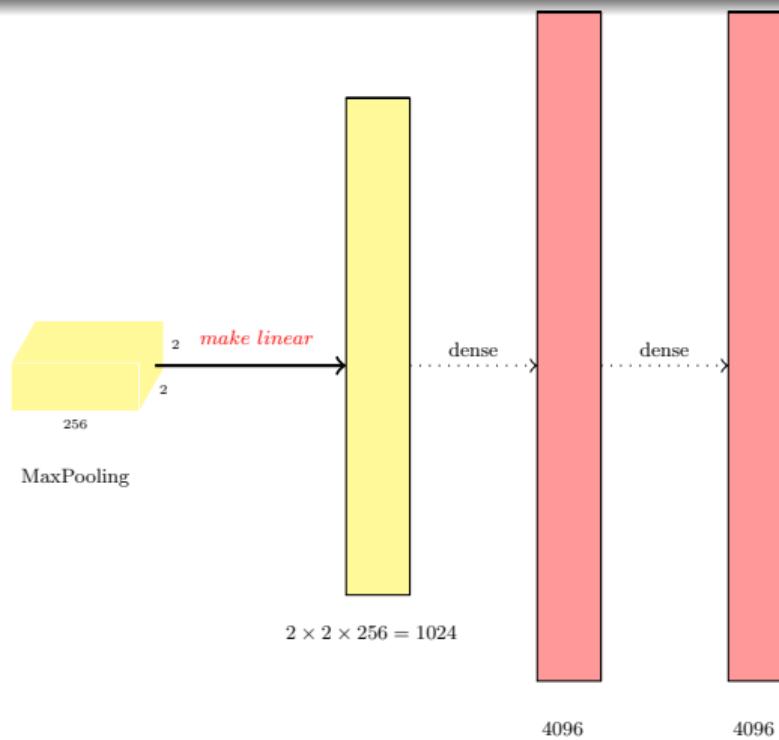
- Let us look at the connections in the fully connected layers in more detail
- We will first stretch out the last conv or maxpool layer to make it a 1d vector



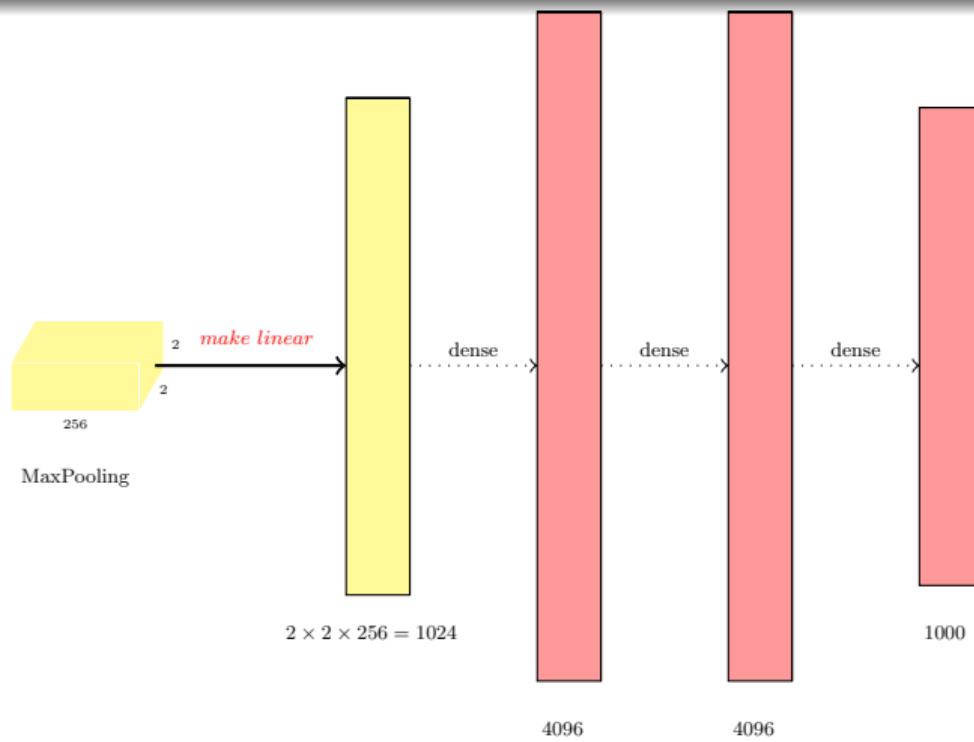
- Let us look at the connections in the fully connected layers in more detail
- We will first stretch out the last conv or maxpool layer to make it a 1d vector
- This 1d vector is then densely connected to other layers just as in a regular feedforward neural network



- Let us look at the connections in the fully connected layers in more detail
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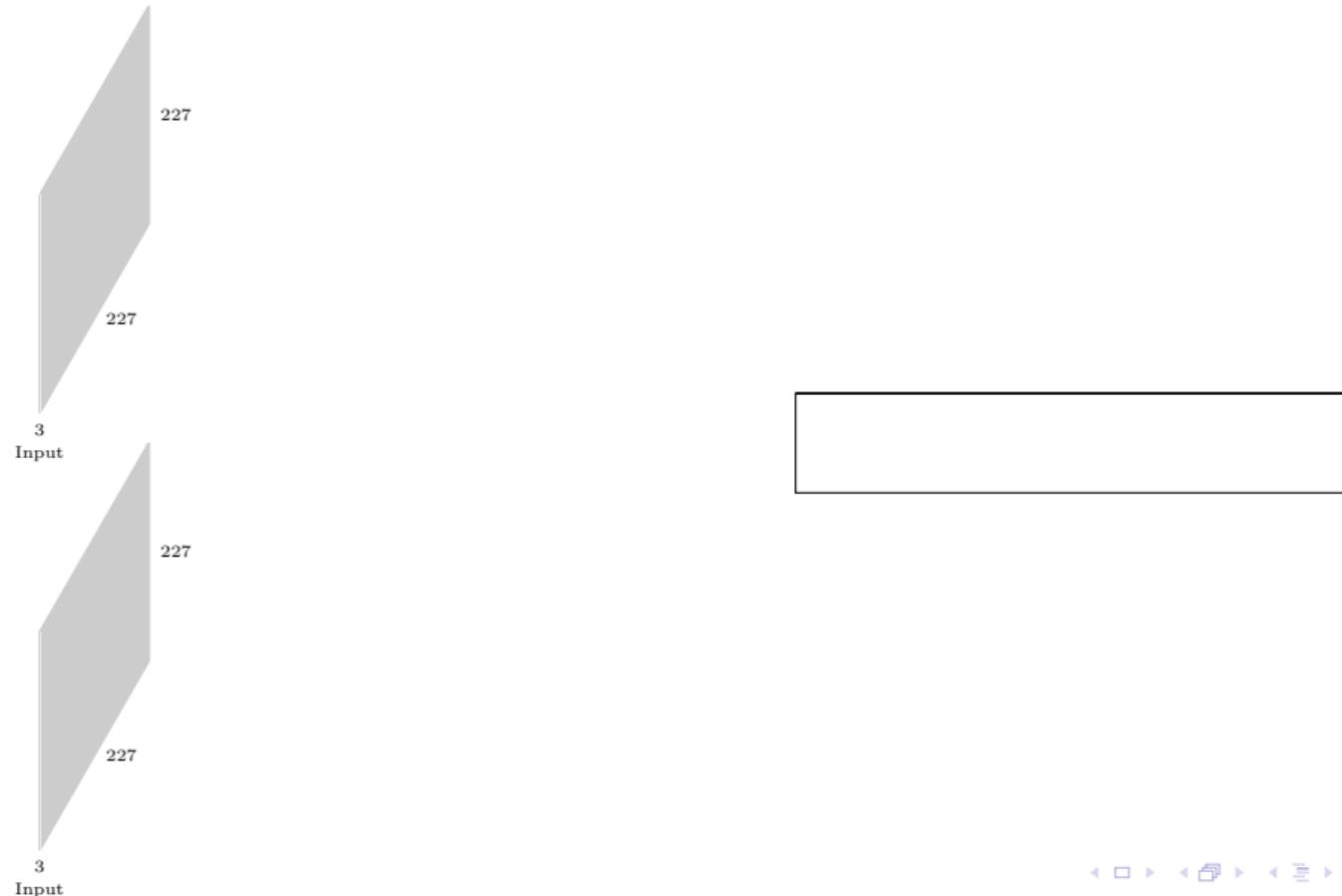


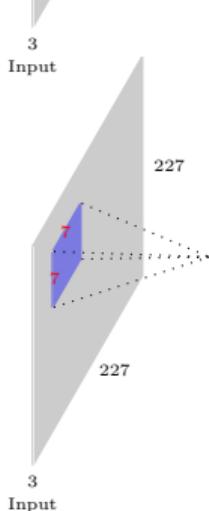
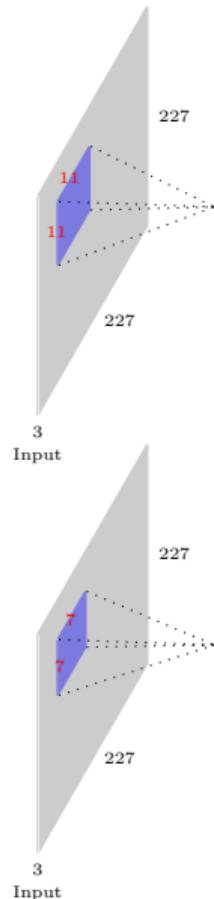
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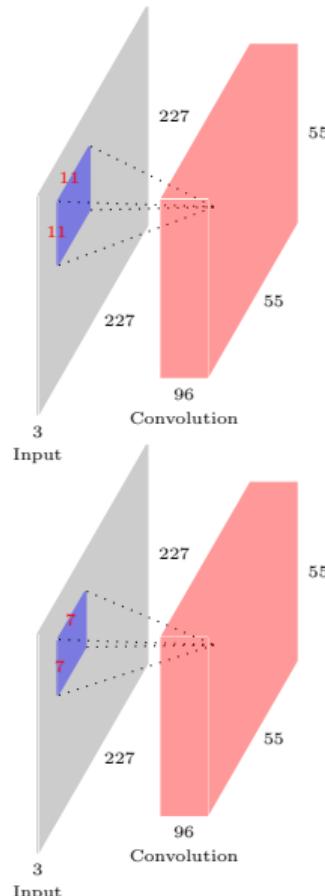
ImageNet Success Stories(roadmap for rest of the talk)

- AlexNet
- ZFNet
- VGGNet

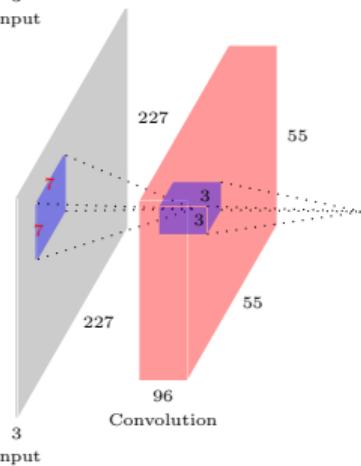
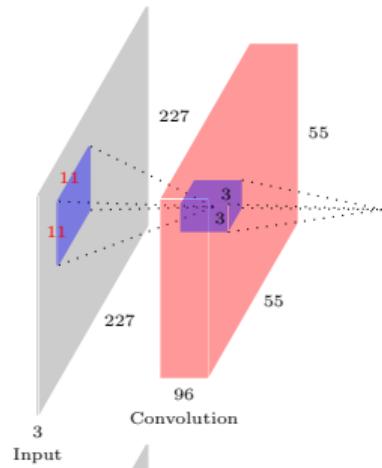




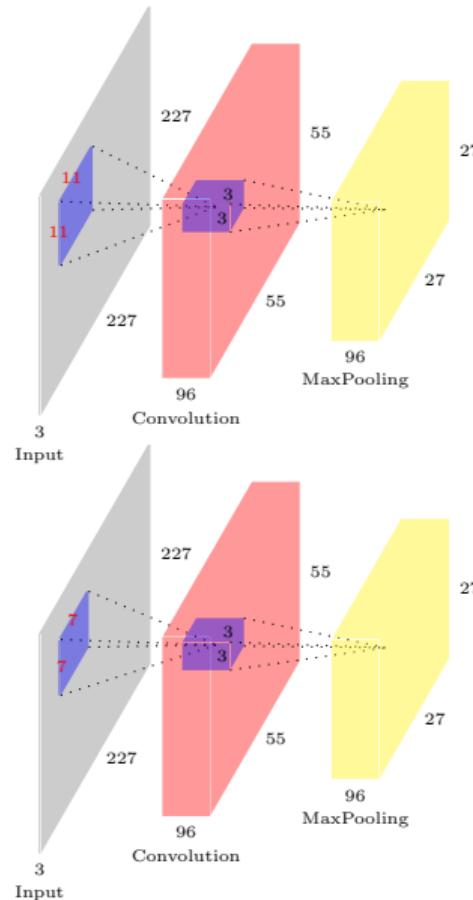
Layer1: $F = 11 \rightarrow 7$
Difference in Parameters
 $((11^2 - 7^2) \times 3) \times 96 = 20.7K$



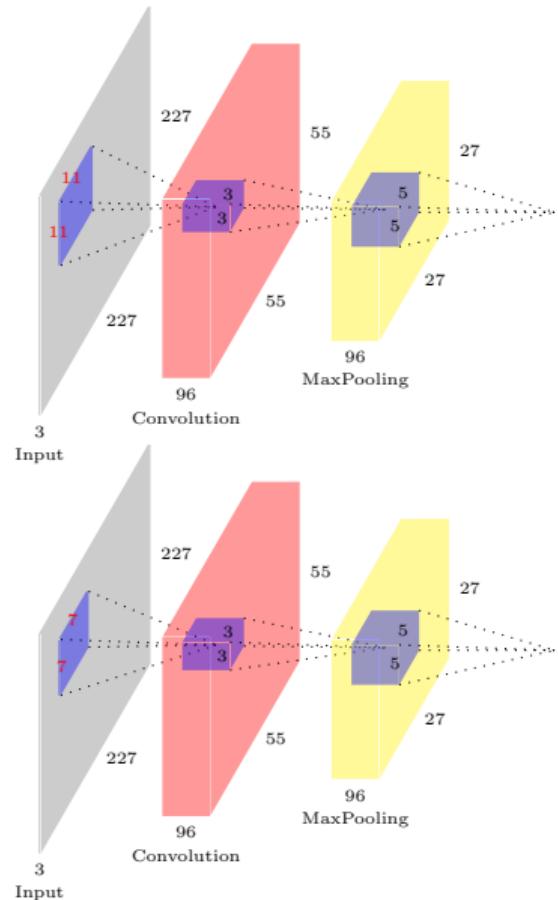
Layer1: $F = 11 \rightarrow 7$
 Difference in Parameters
 $((11^2 - 7^2) \times 3) \times 96 = 20.7K$



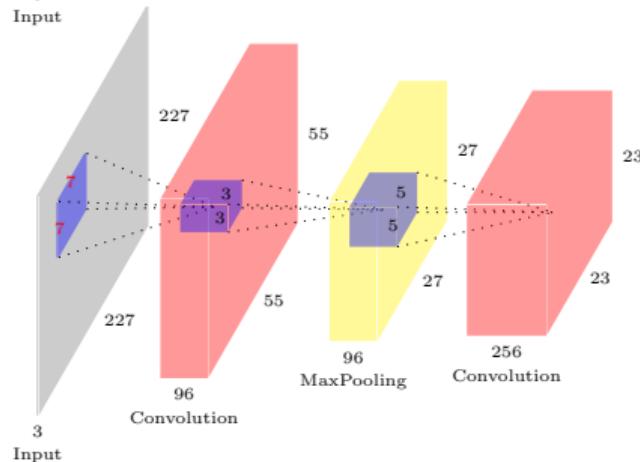
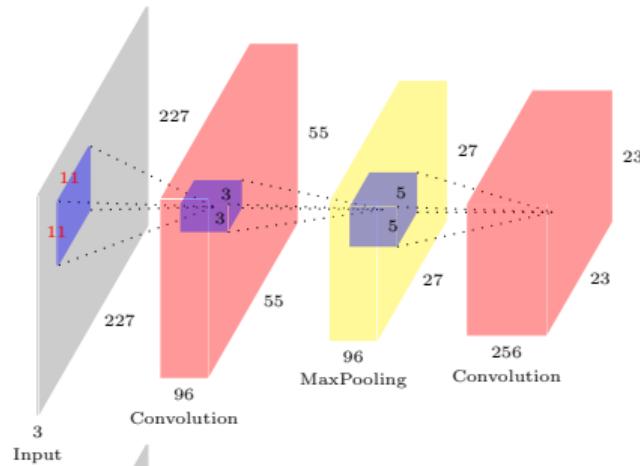
Layer2: No difference



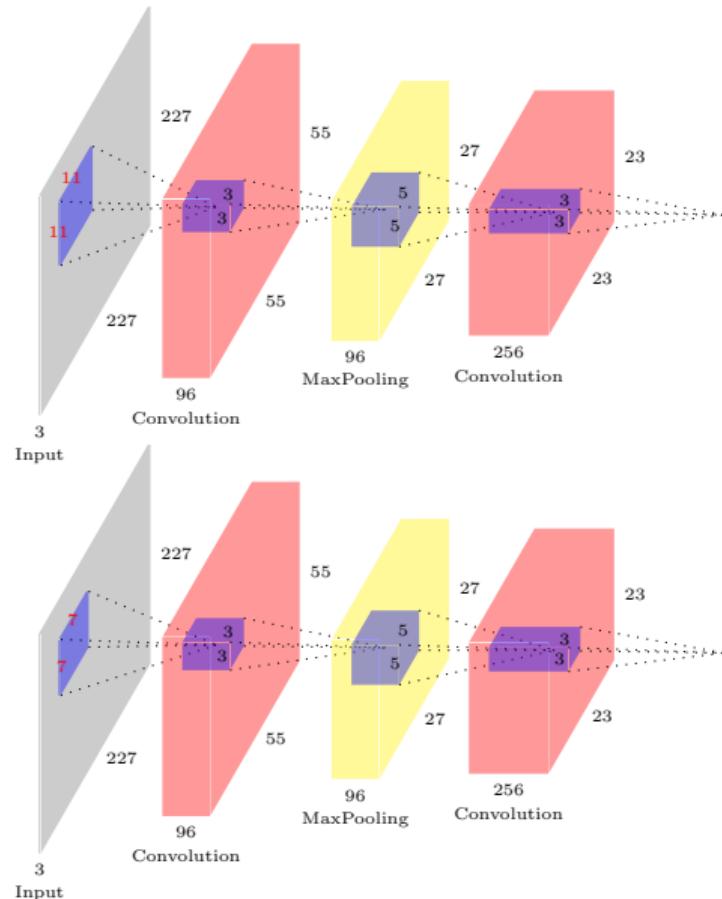
Layer2: No difference



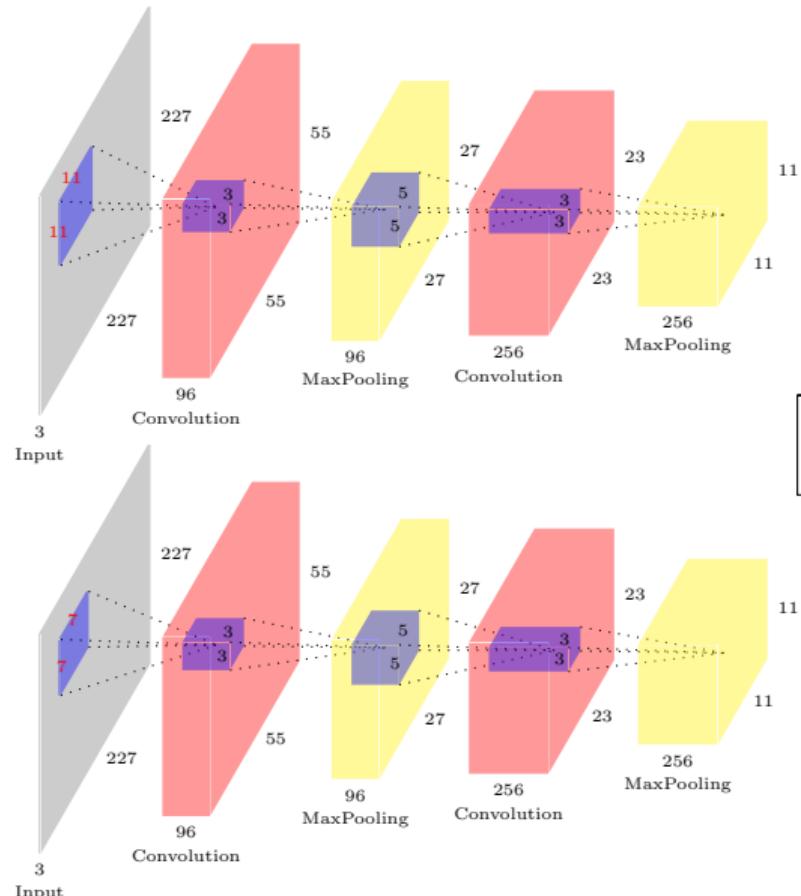
Layer3: No difference



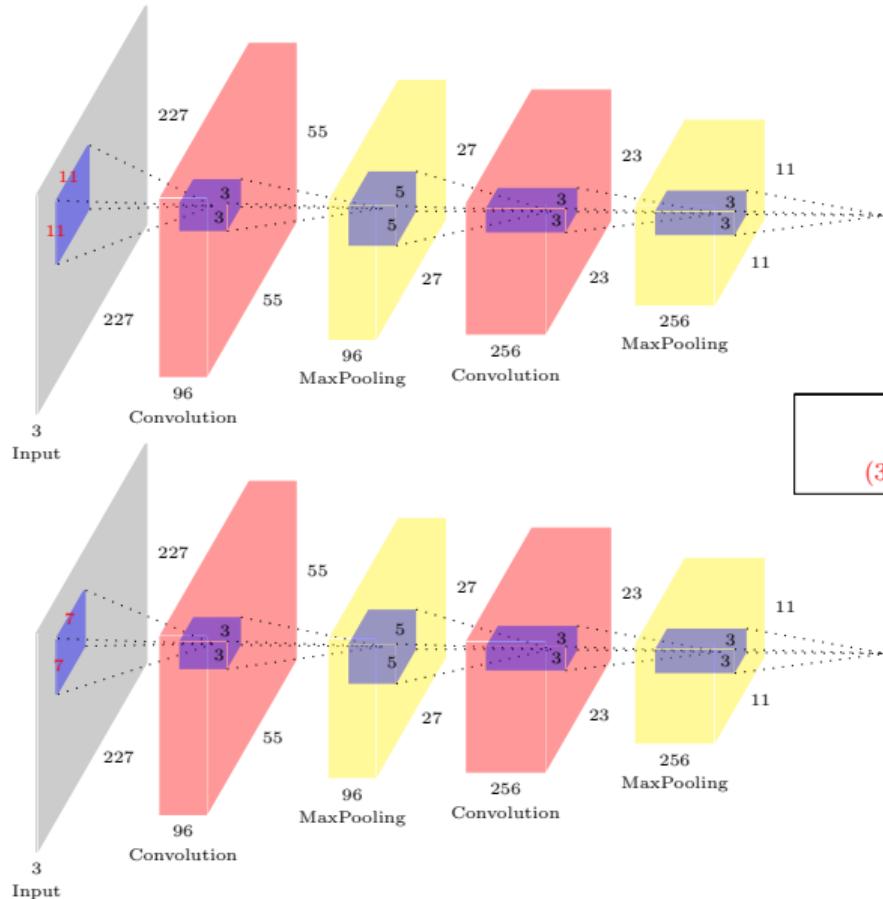
Layer3: No difference



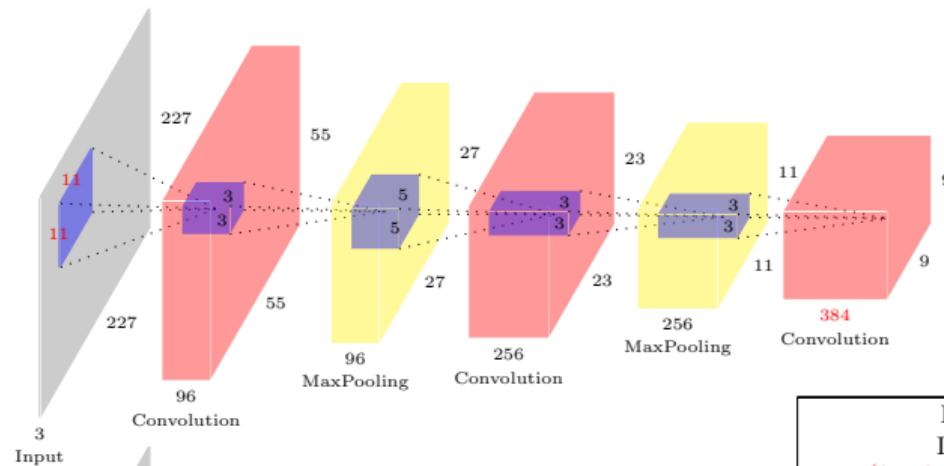
Layer4: No difference



Layer4: No difference



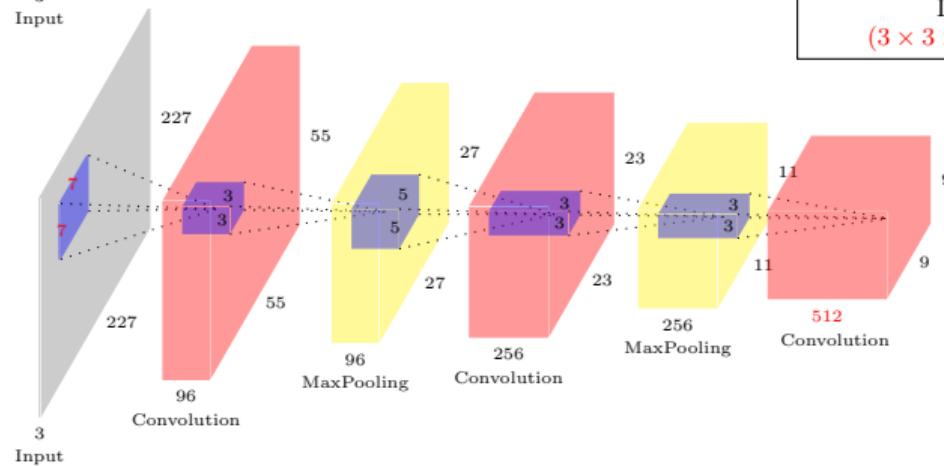
$$\begin{aligned} \text{Layer5: } K &= 384 \rightarrow 512 \\ \text{Difference in Parameters} \\ (3 \times 3 \times 256) \times (512 - 384) &= 0.29M \end{aligned}$$

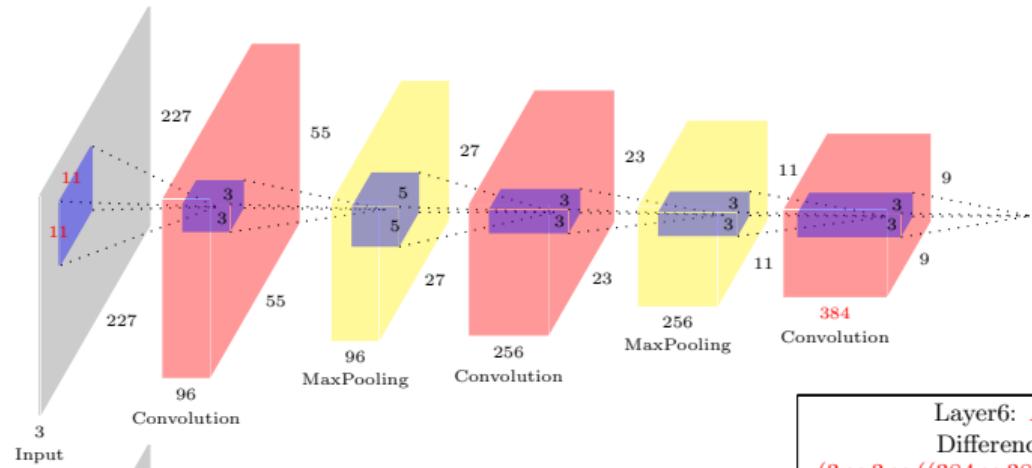


Layer5: $K = 384 \rightarrow 512$

Difference in Parameters

$$(3 \times 3 \times 256) \times (512 - 384) = 0.29M$$

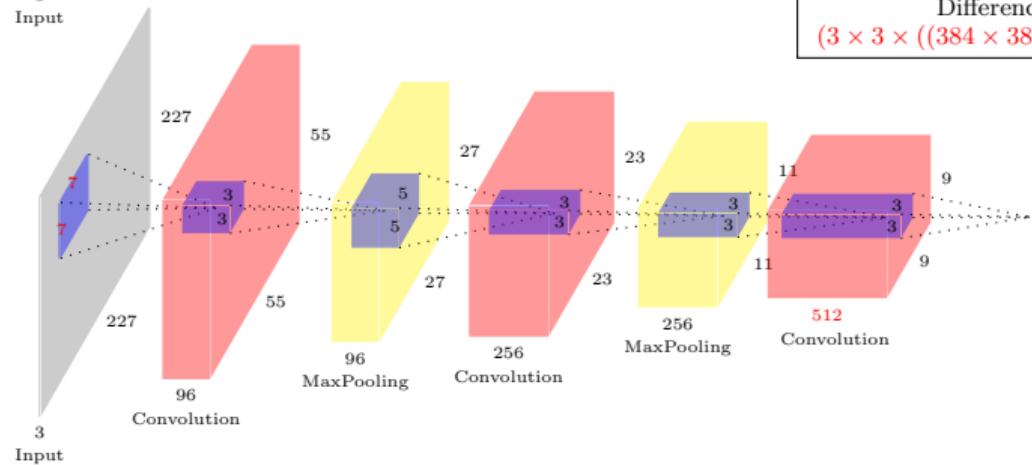


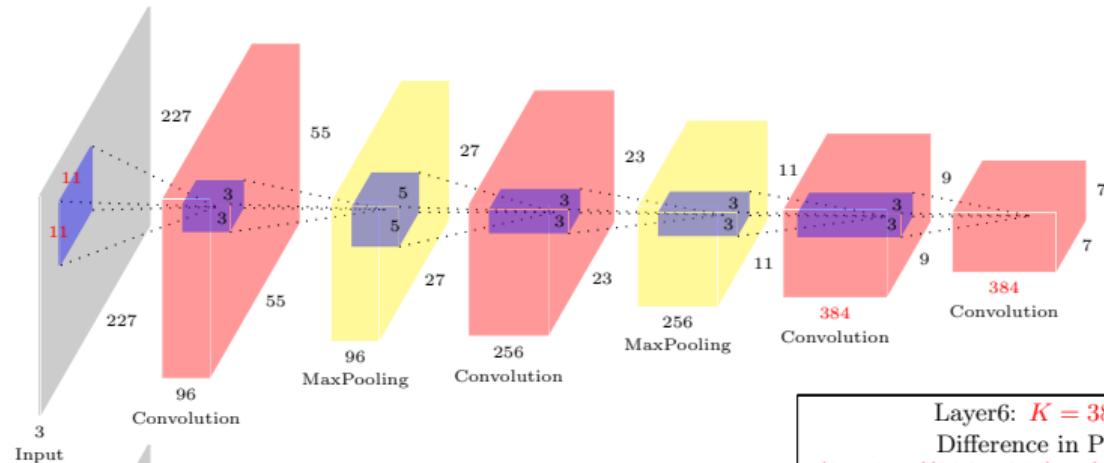


Layer6: $K = 384 \rightarrow 1024$

Difference in Parameters

$$(3 \times 3 \times ((384 \times 384) - (512 \times 1024))) = 0.8M$$

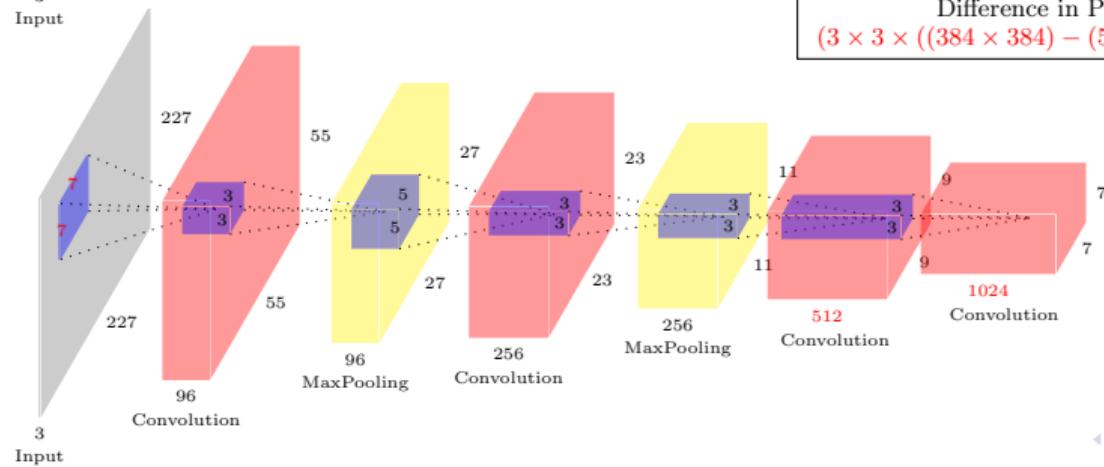


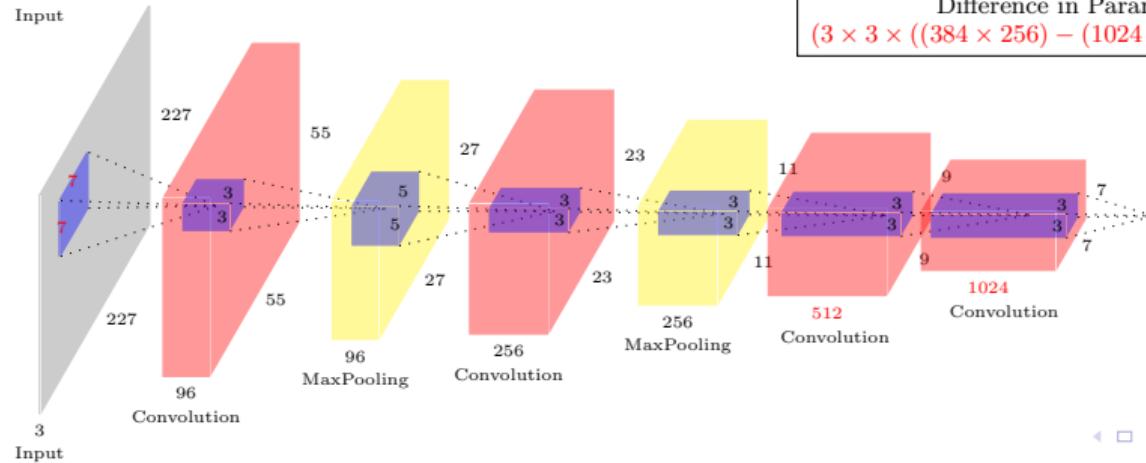
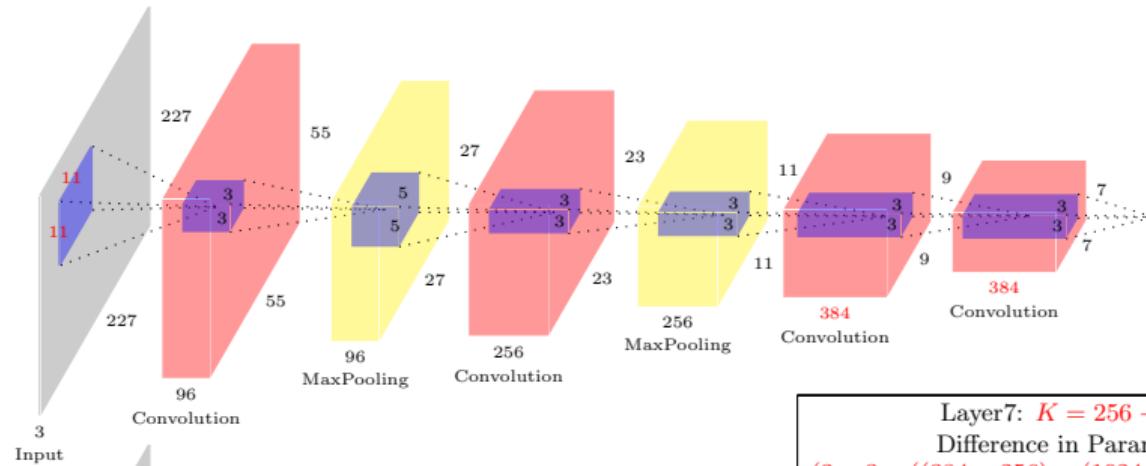


Layer6: $K = 384 \rightarrow 1024$

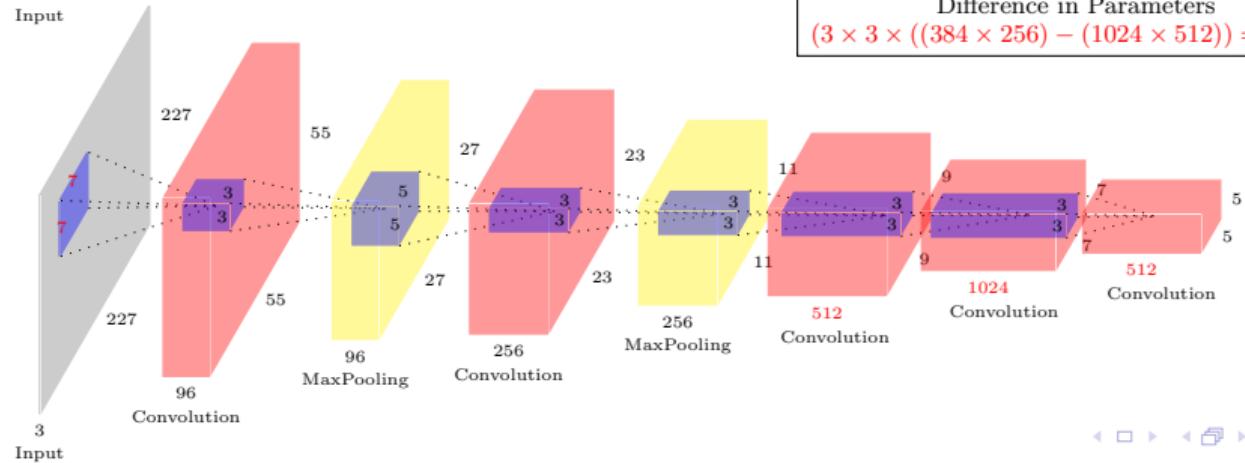
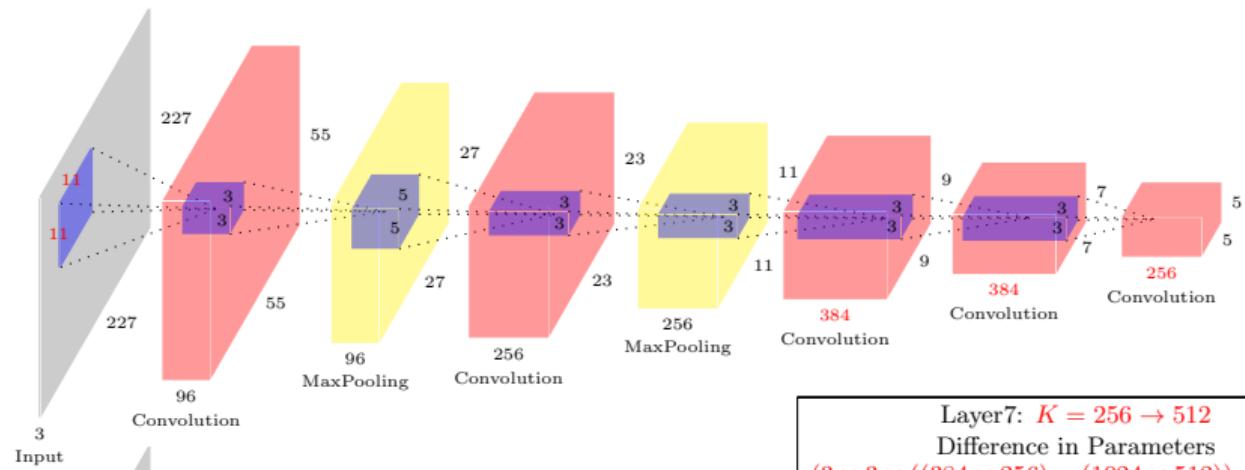
Difference in Parameters

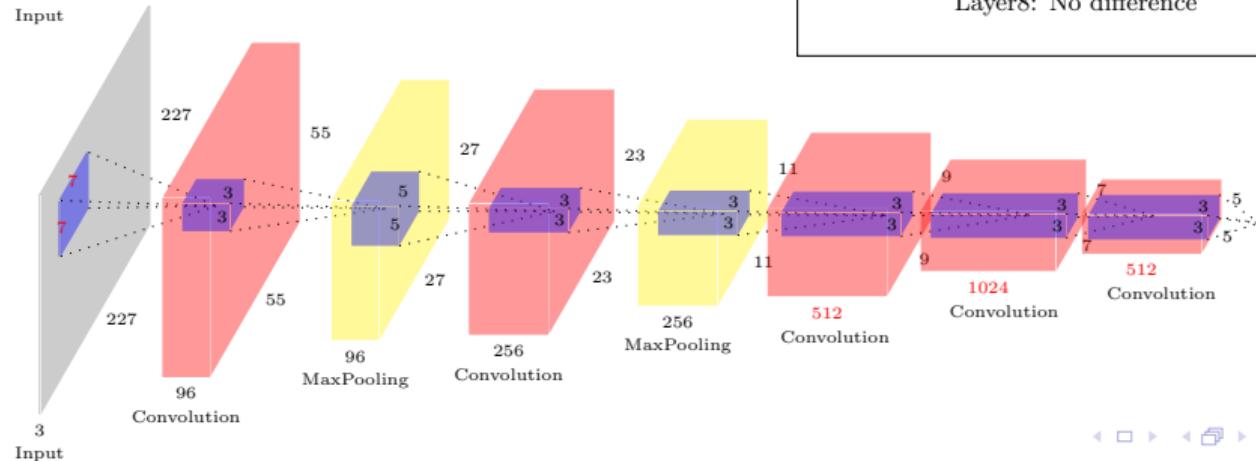
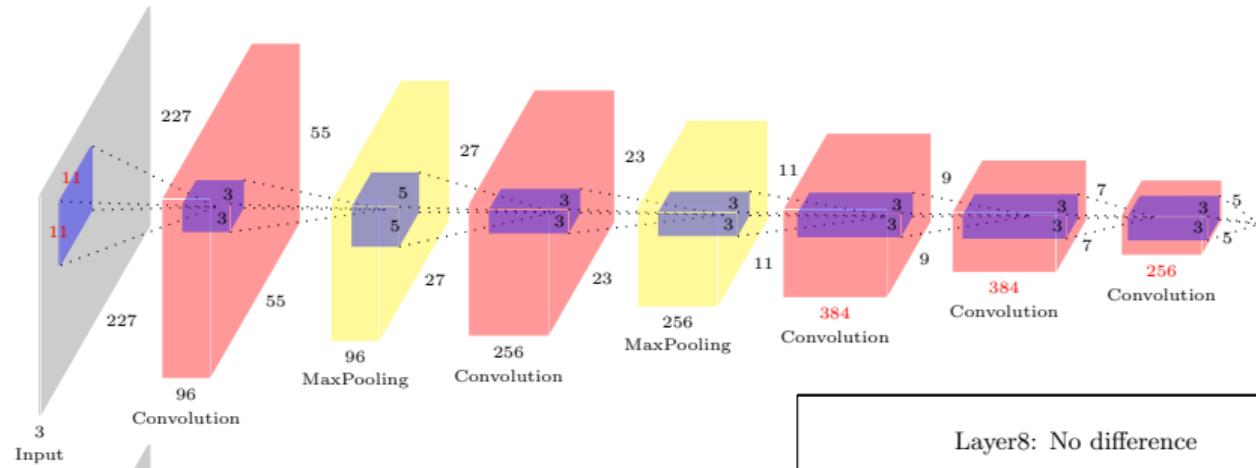
$$(3 \times 3 \times ((384 \times 384) - (512 \times 1024))) = 0.8M$$

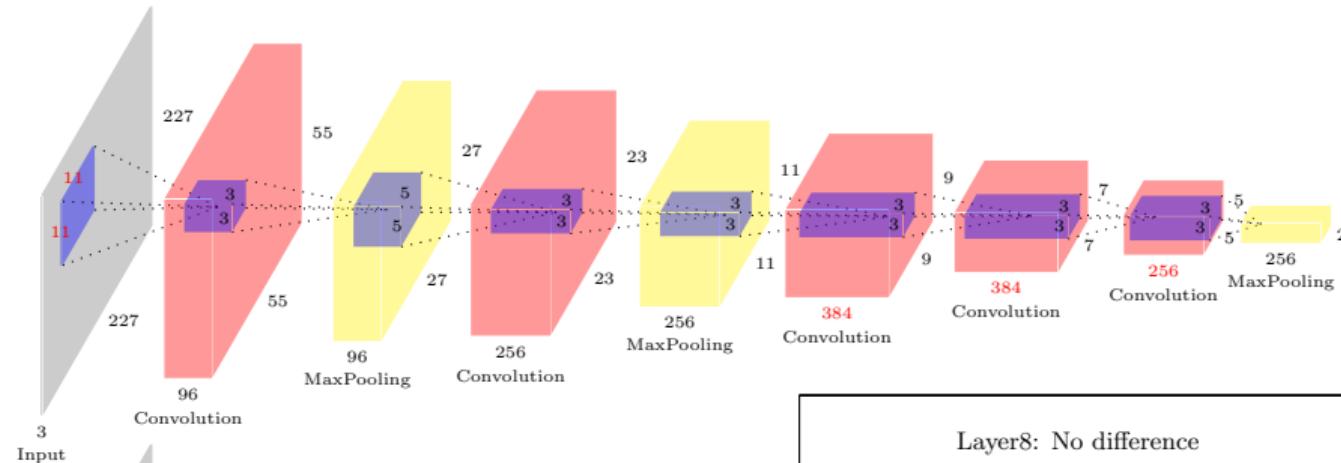




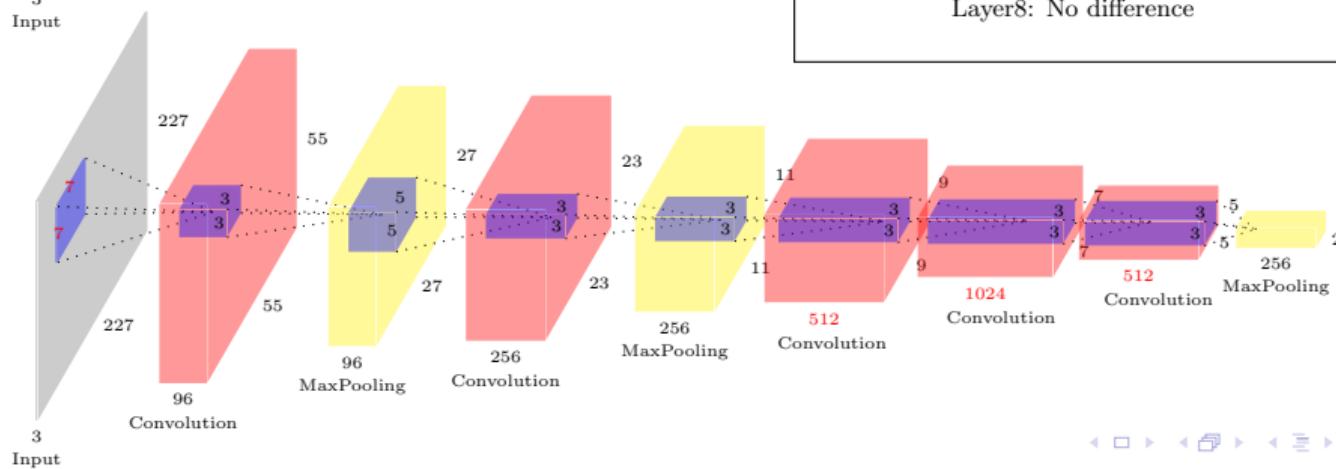
Layer7: $K = 256 \rightarrow 512$
Difference in Parameters
 $(3 \times 3 \times ((384 \times 256) - (1024 \times 512))) = 0.36M$

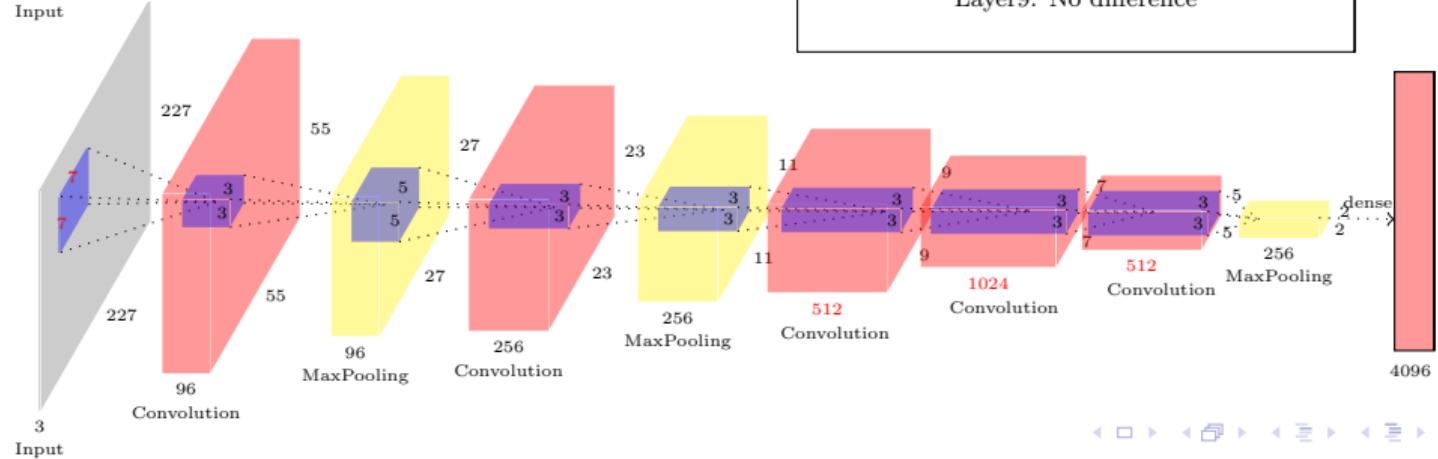
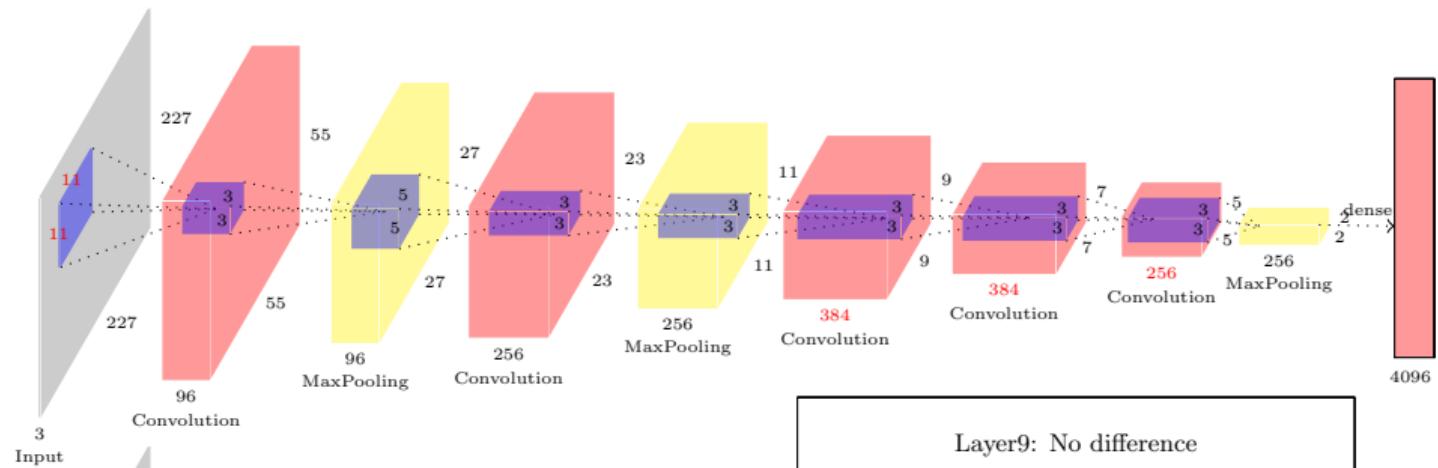


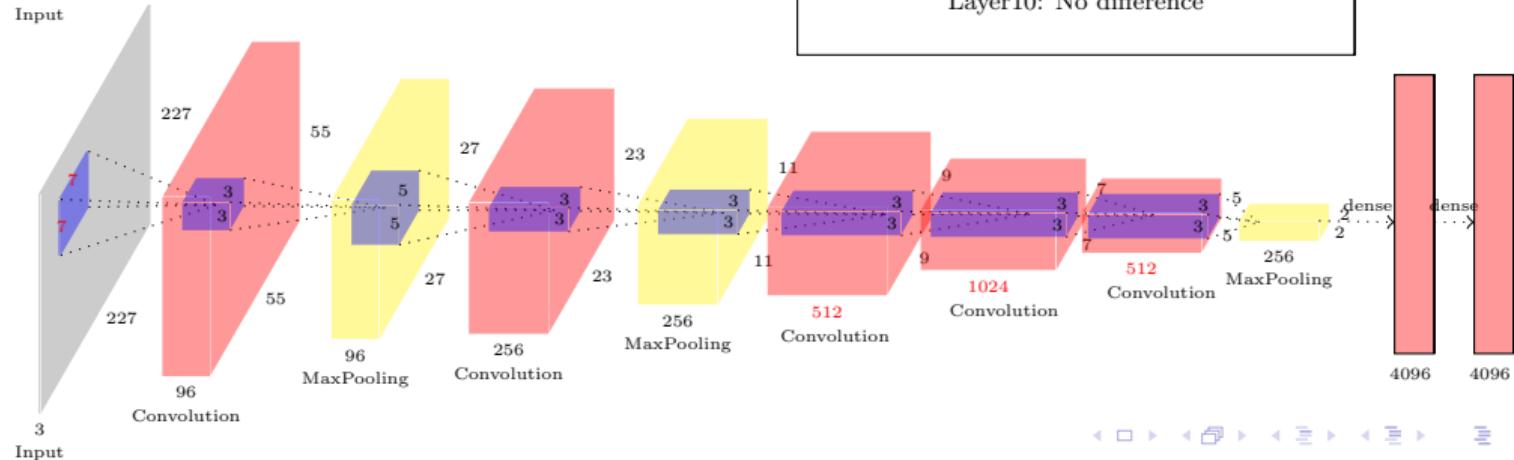
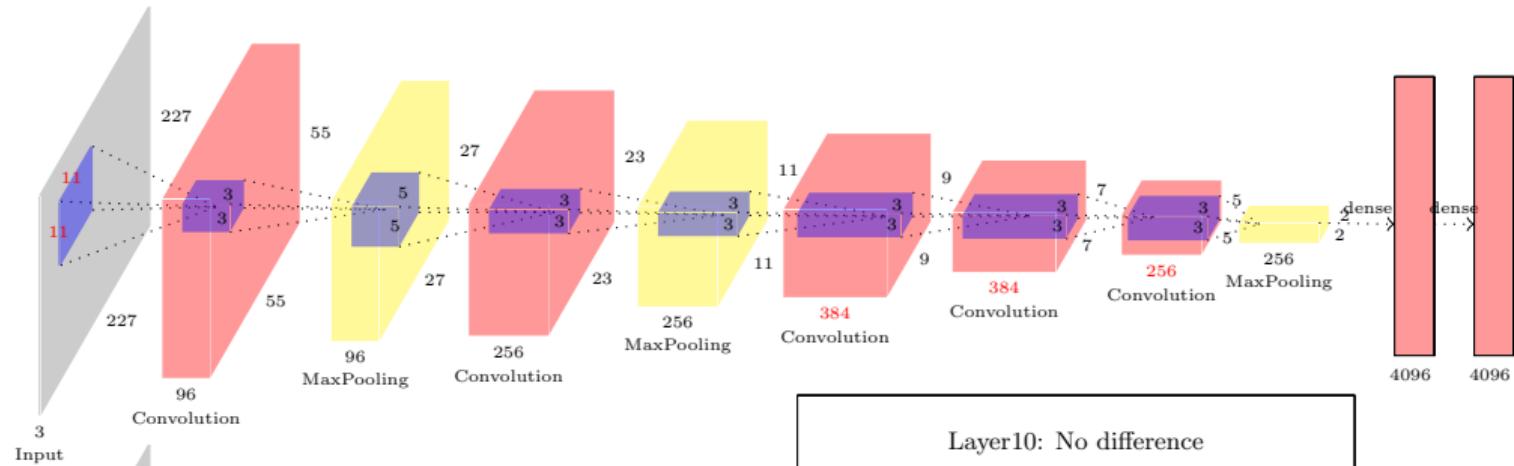


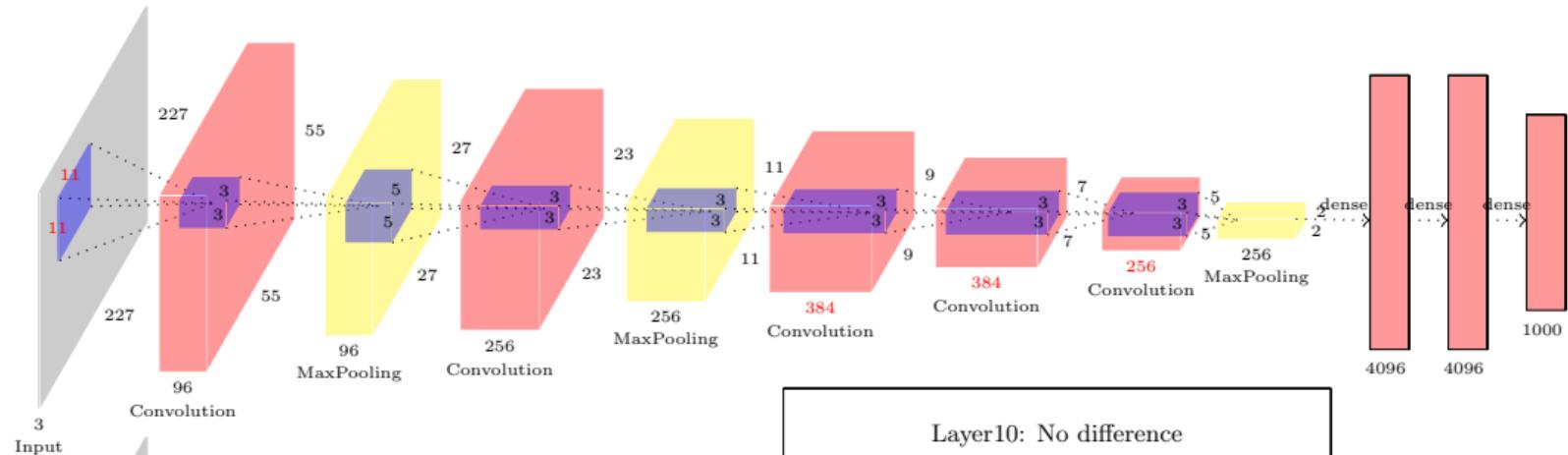


Layer8: No difference

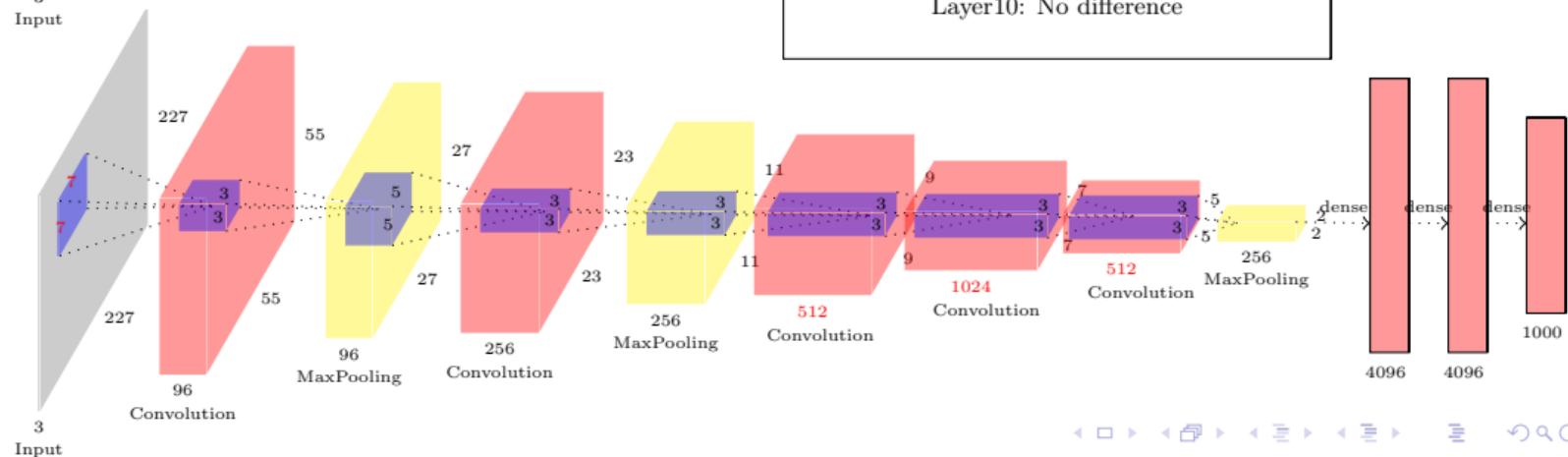


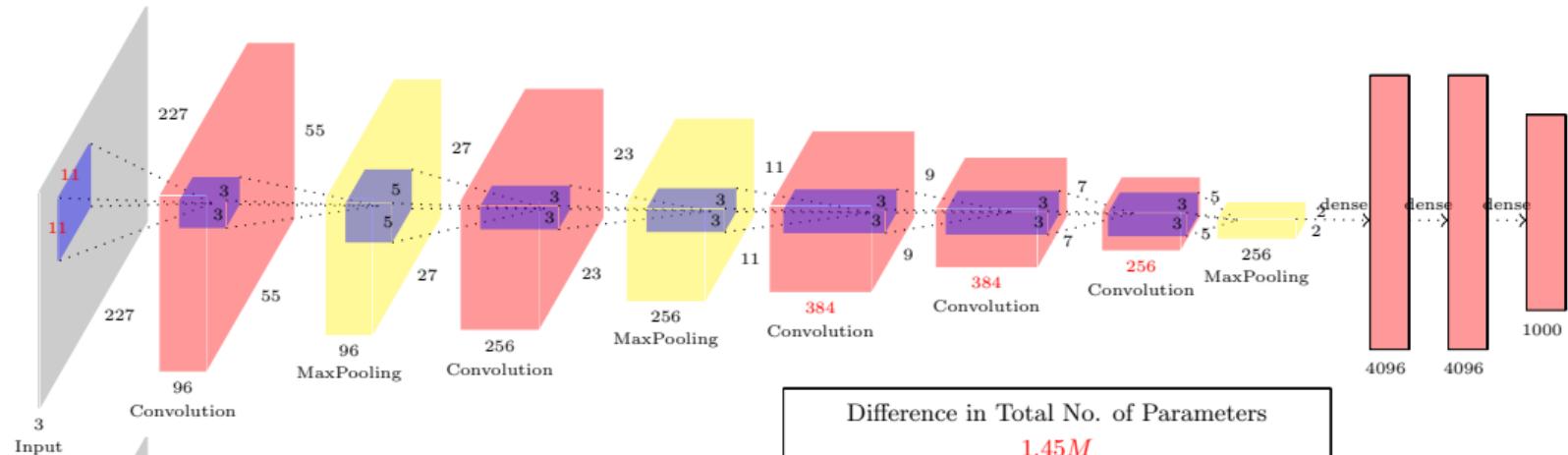






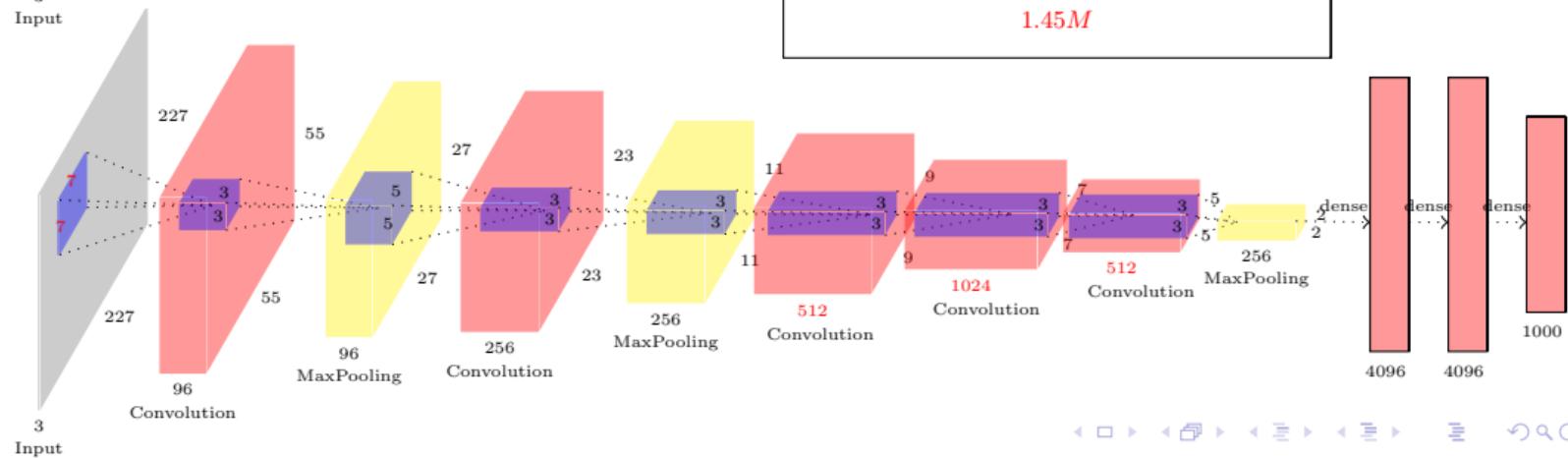
Layer10: No difference





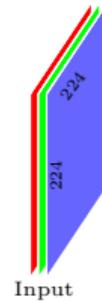
Difference in Total No. of Parameters

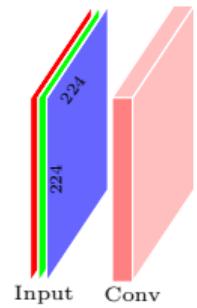
1.45*M*

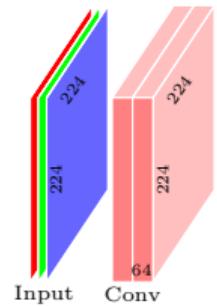


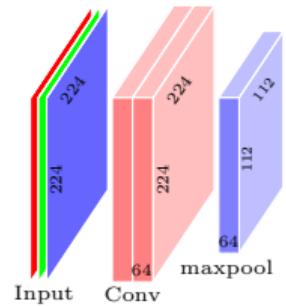
ImageNet Success Stories(roadmap for rest of the talk)

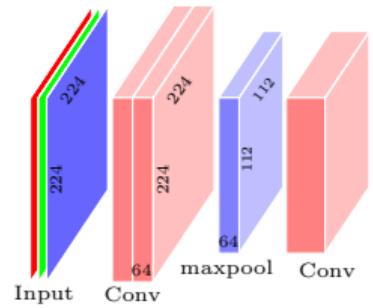
- AlexNet
- ZFNet
- VGGNet

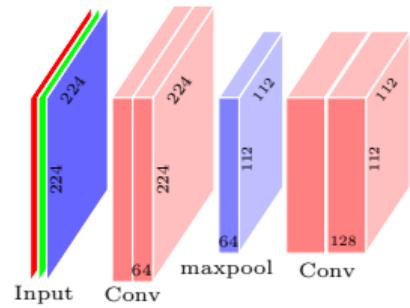


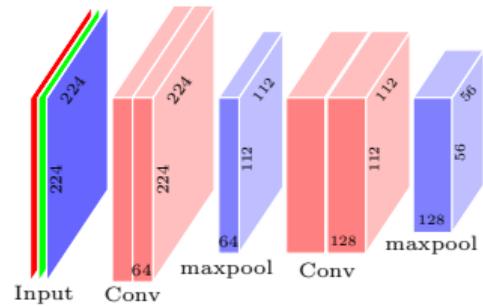


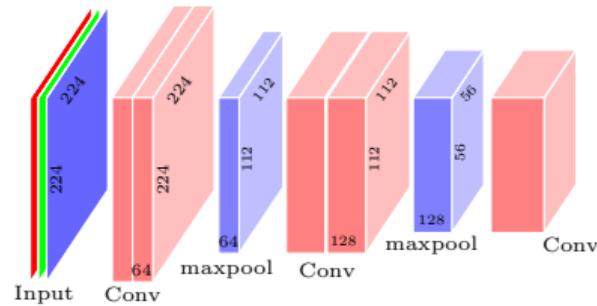


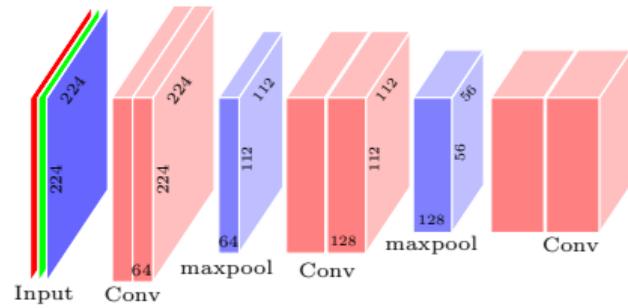


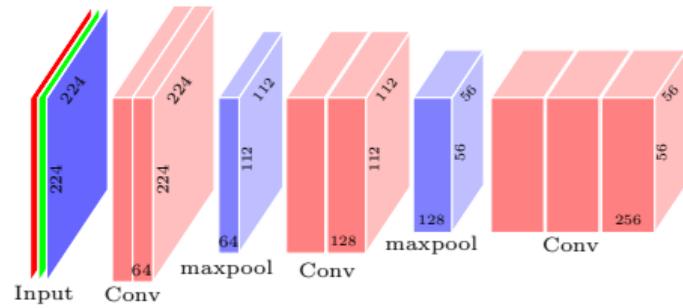


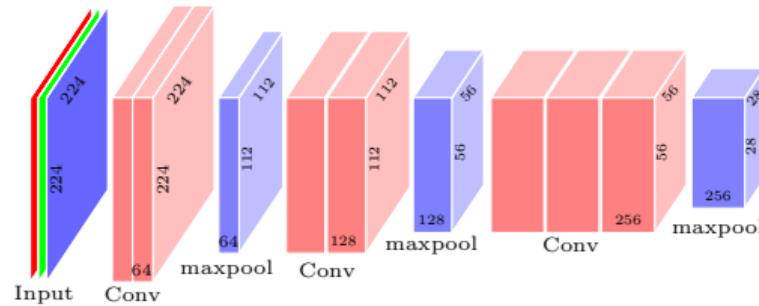


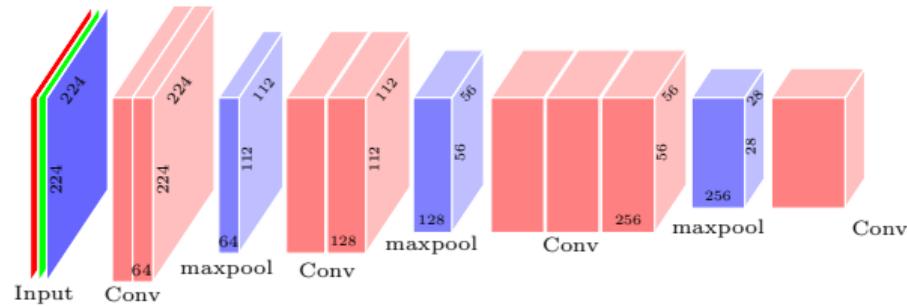


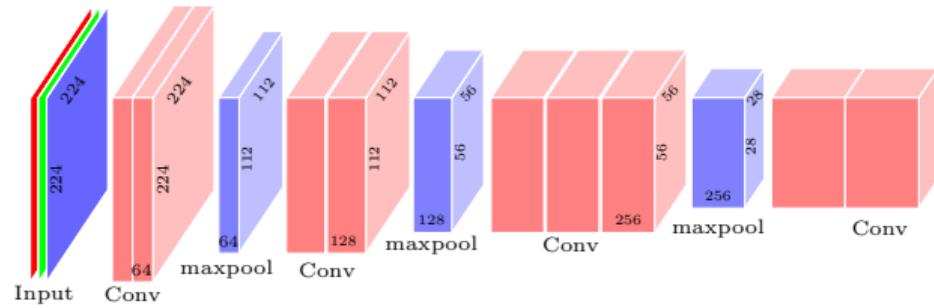


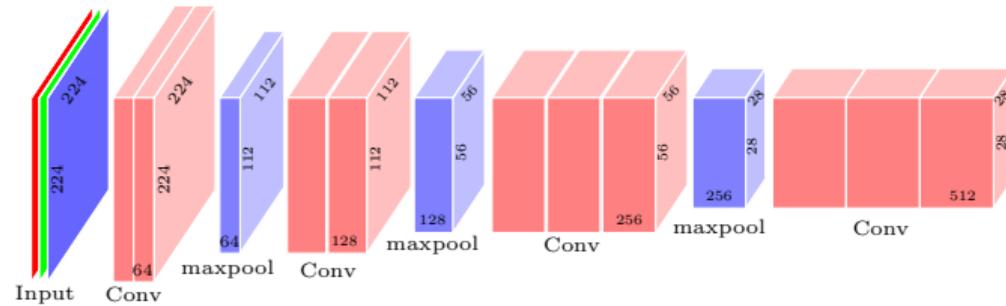


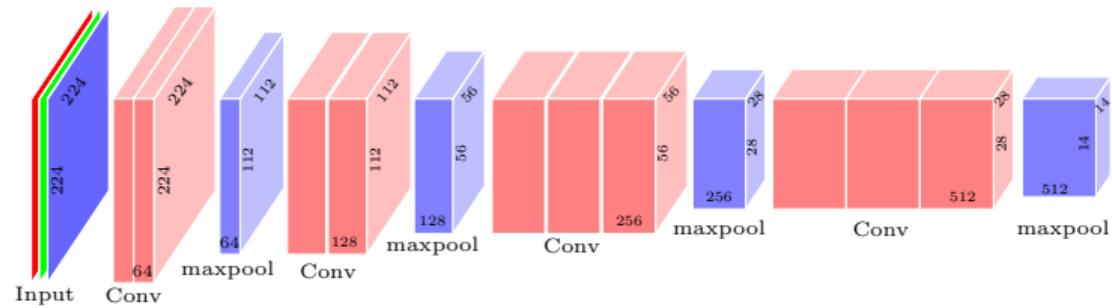


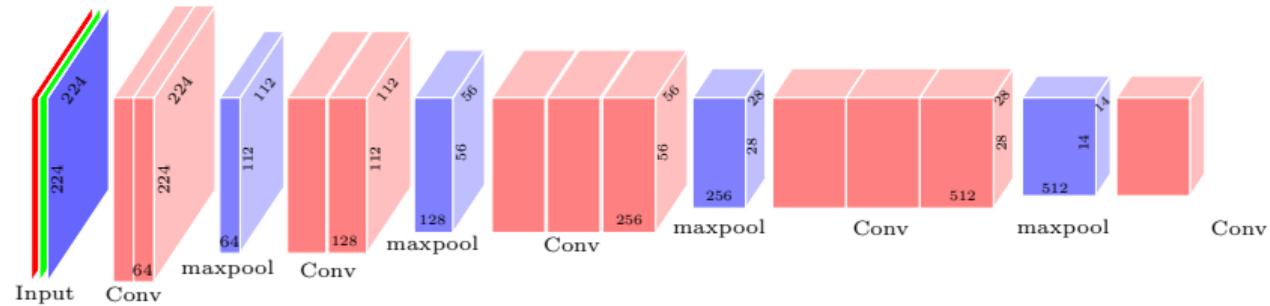


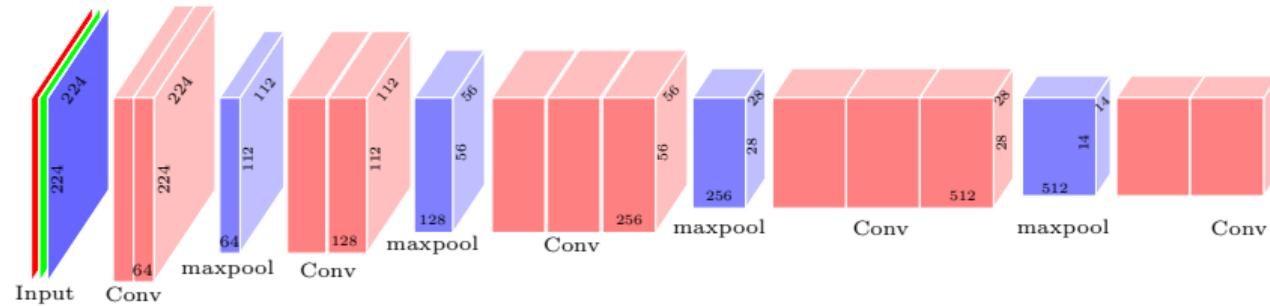


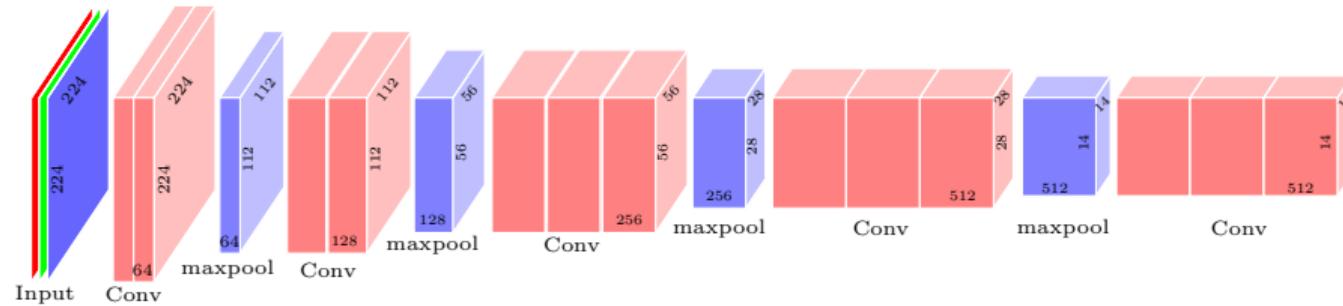


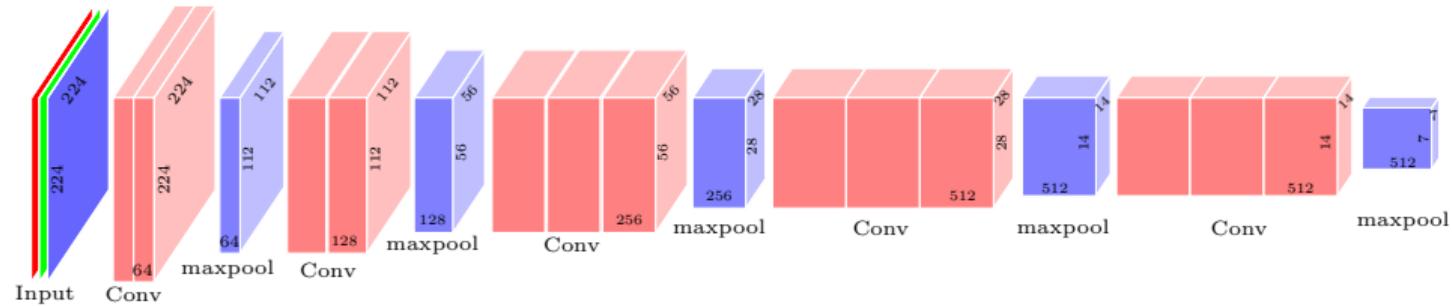


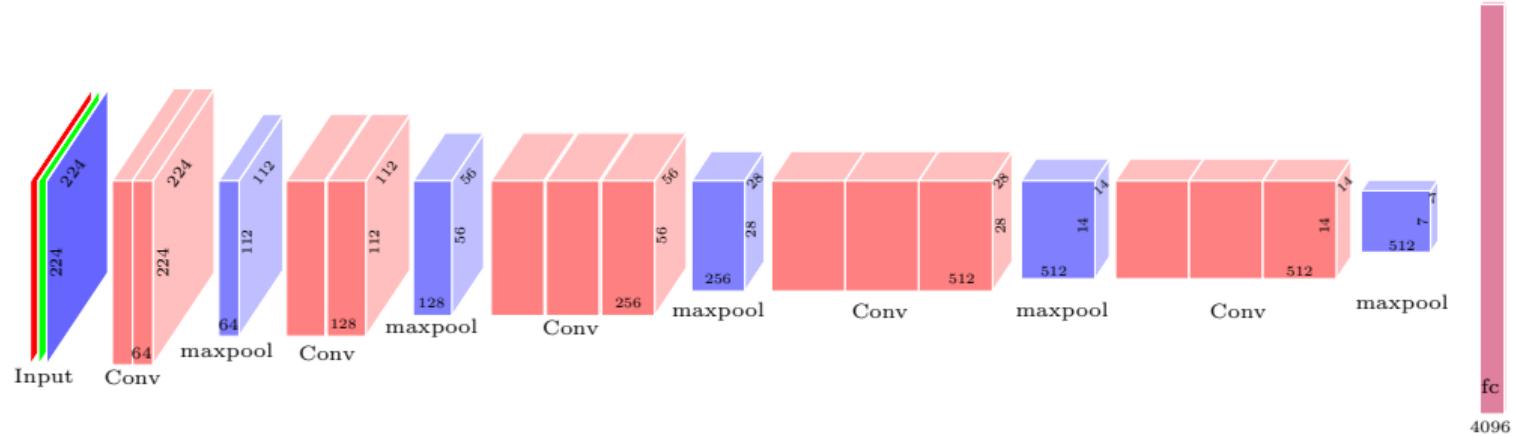


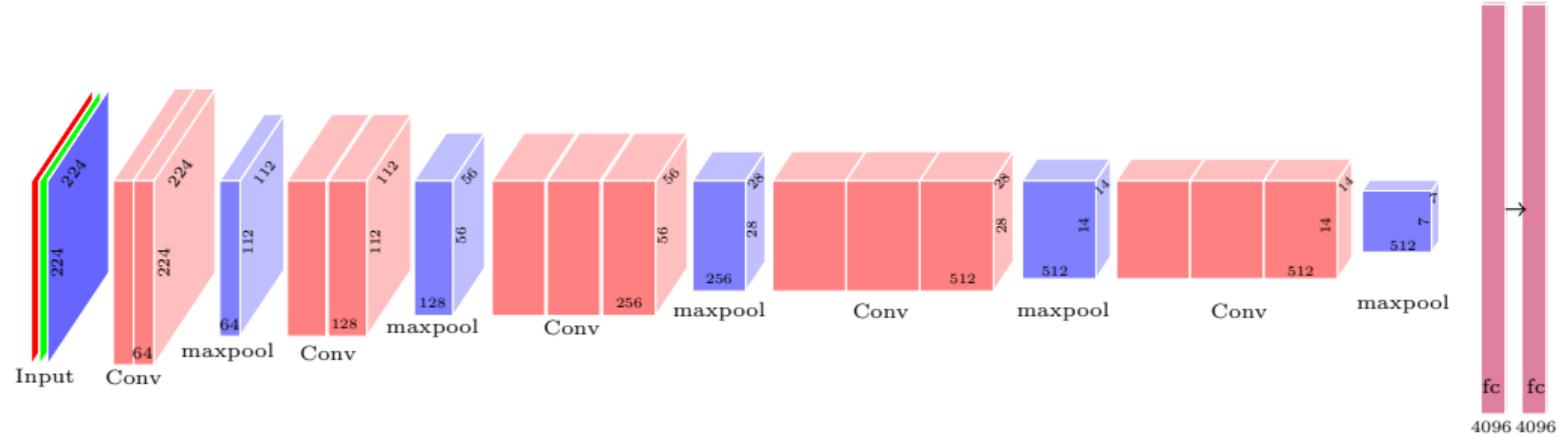


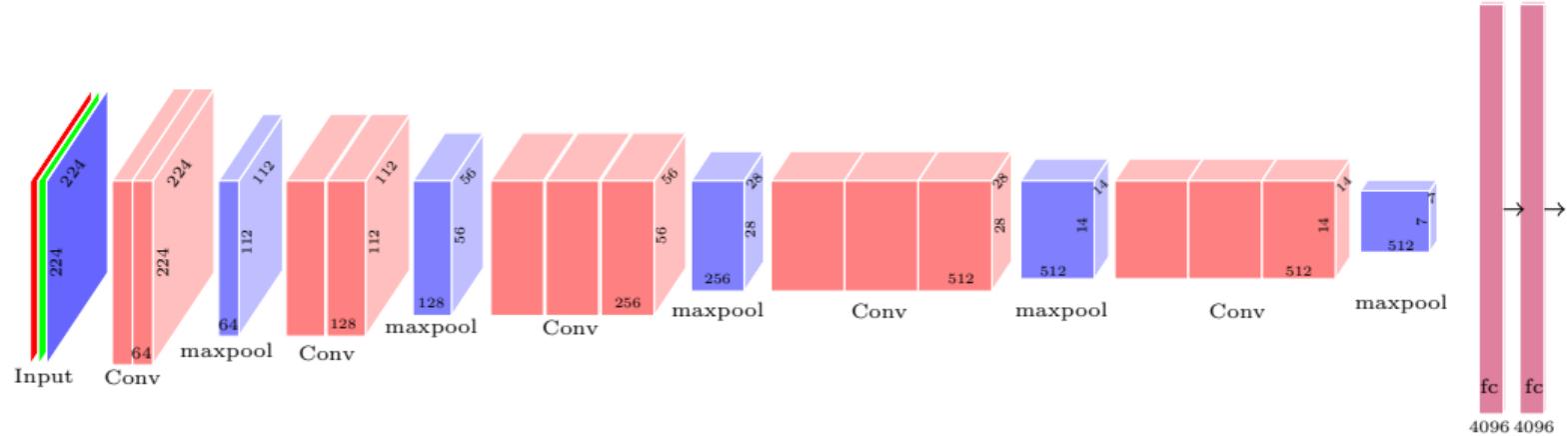


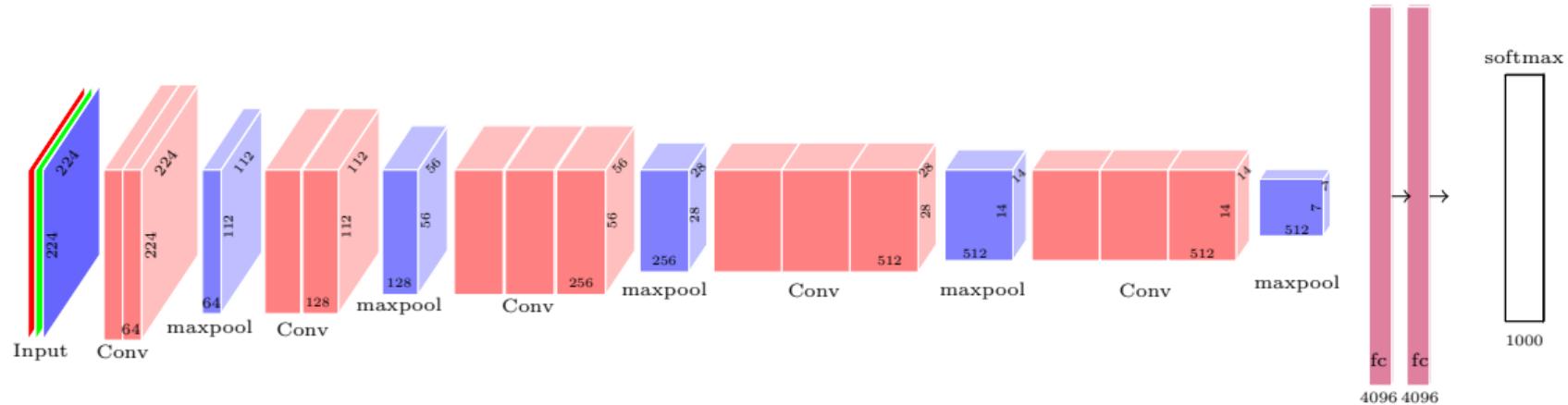


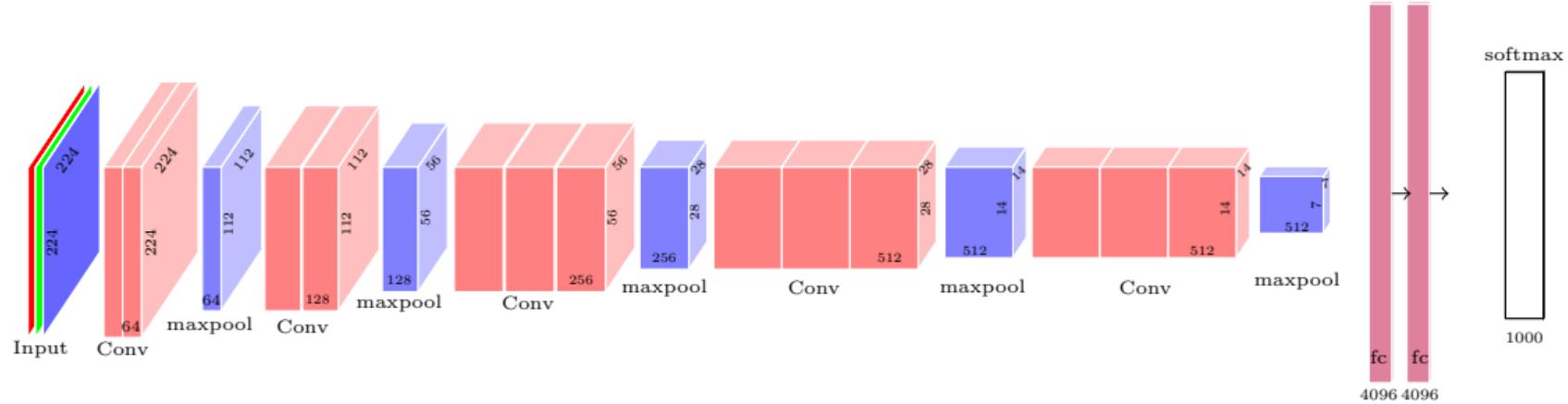




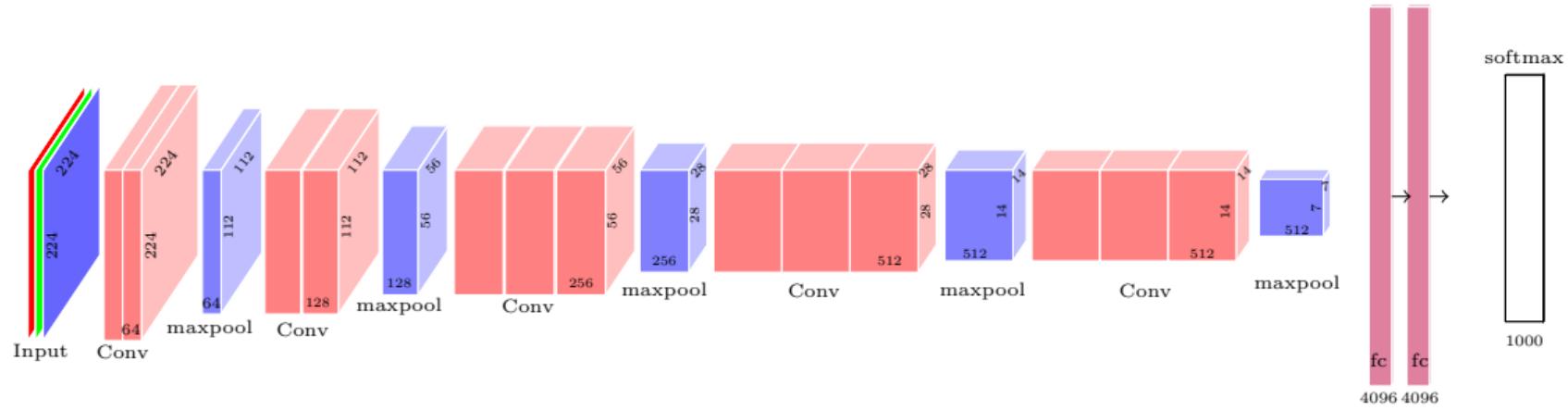




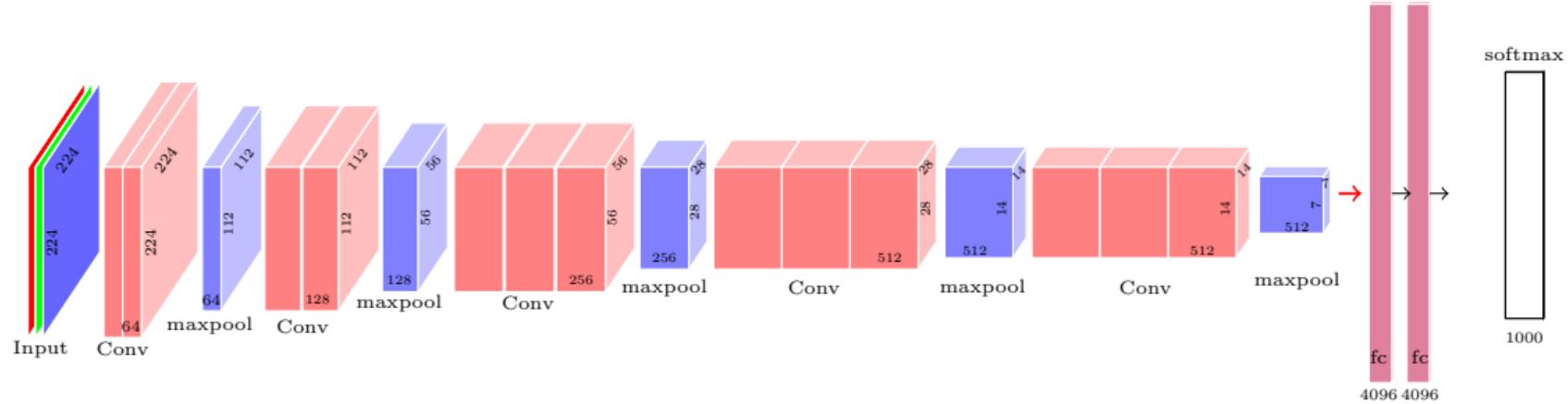




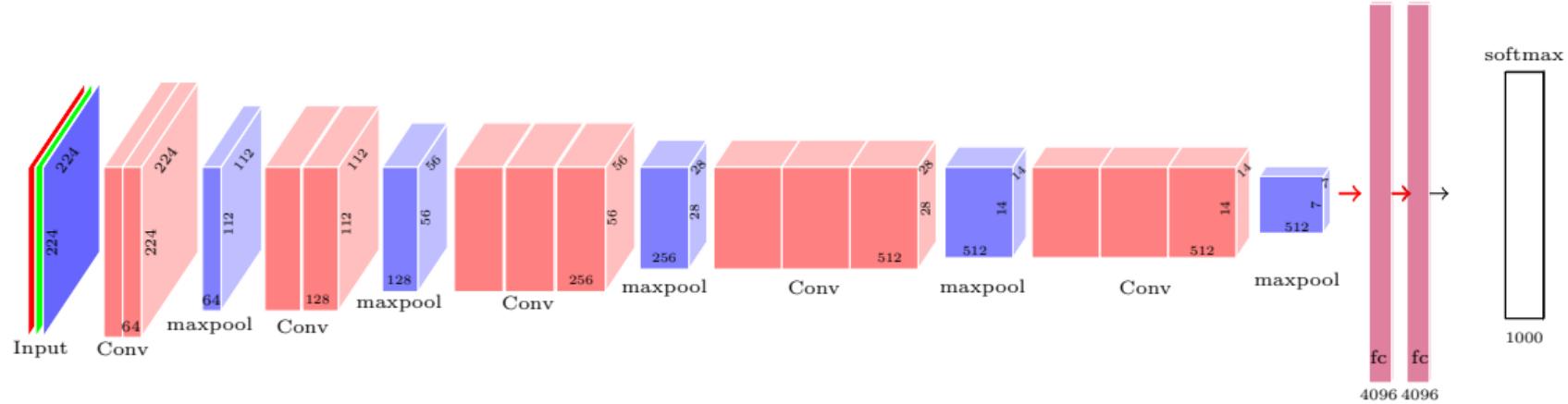
- Kernel size is 3×3 throughout



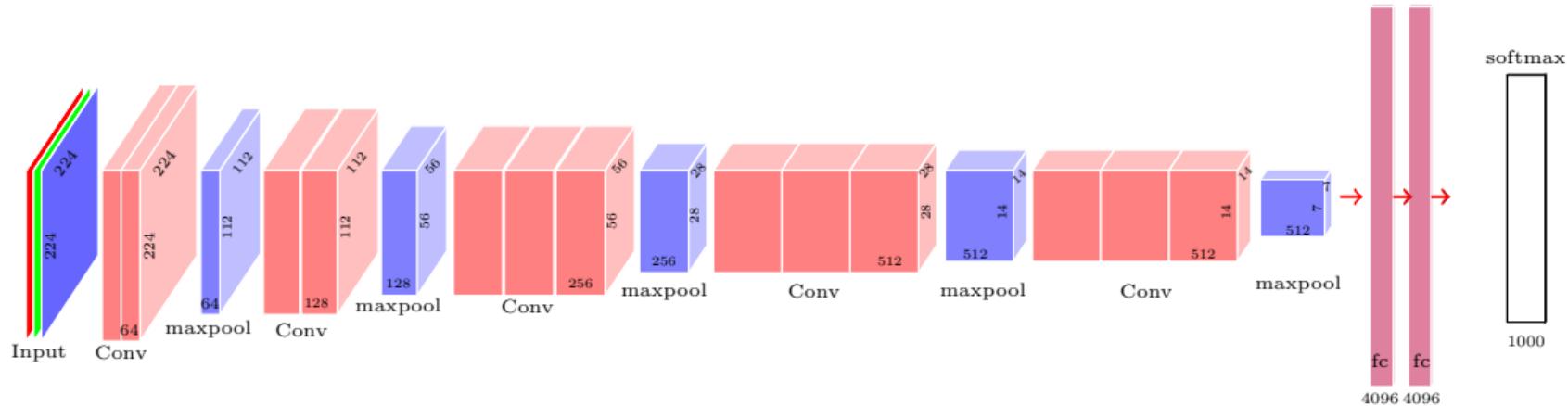
- Kernel size is 3×3 throughout
 - Total parameters in non FC layers = $\sim 16M$



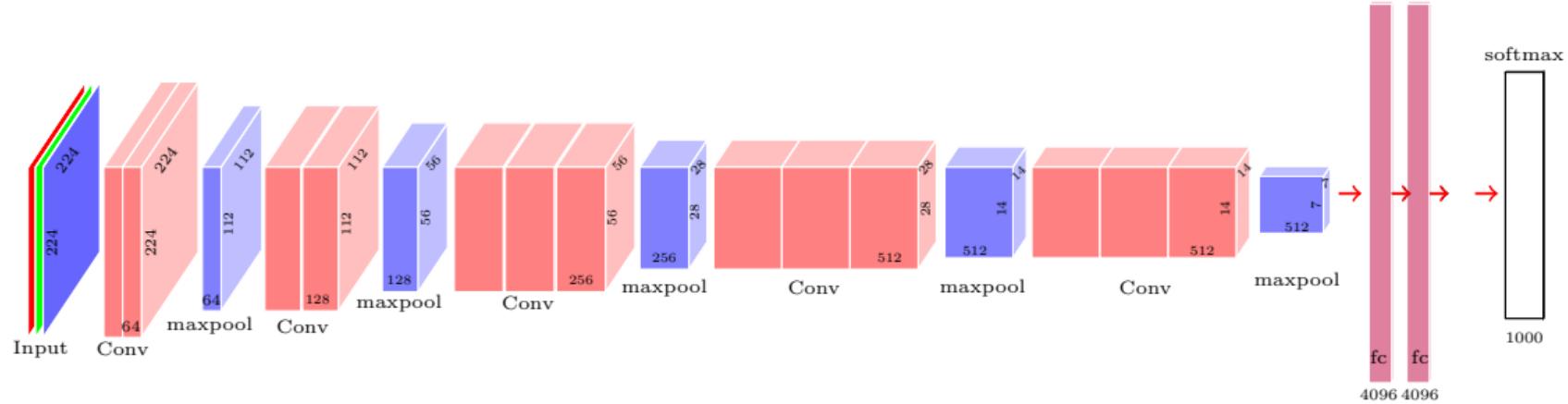
- Kernel size is 3×3 throughout
- Total parameters in non FC layers = $\sim 16M$
- Total Parameters in FC layers =



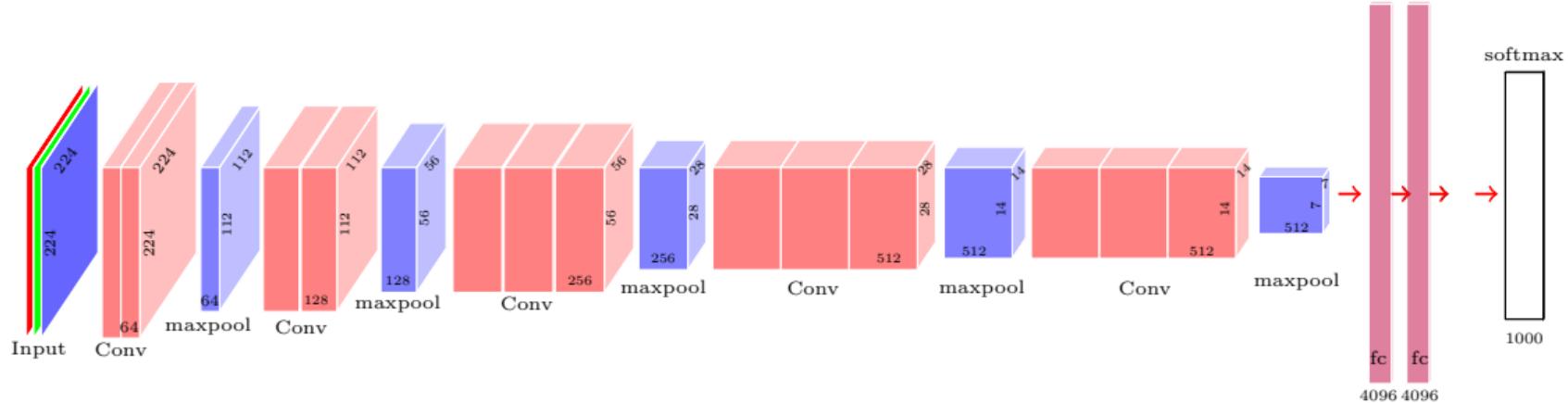
- Kernel size is 3×3 throughout
- Total parameters in non FC layers = $\sim 16M$
- Total Parameters in FC layers = $(512 \times 7 \times 7 \times 4096)$



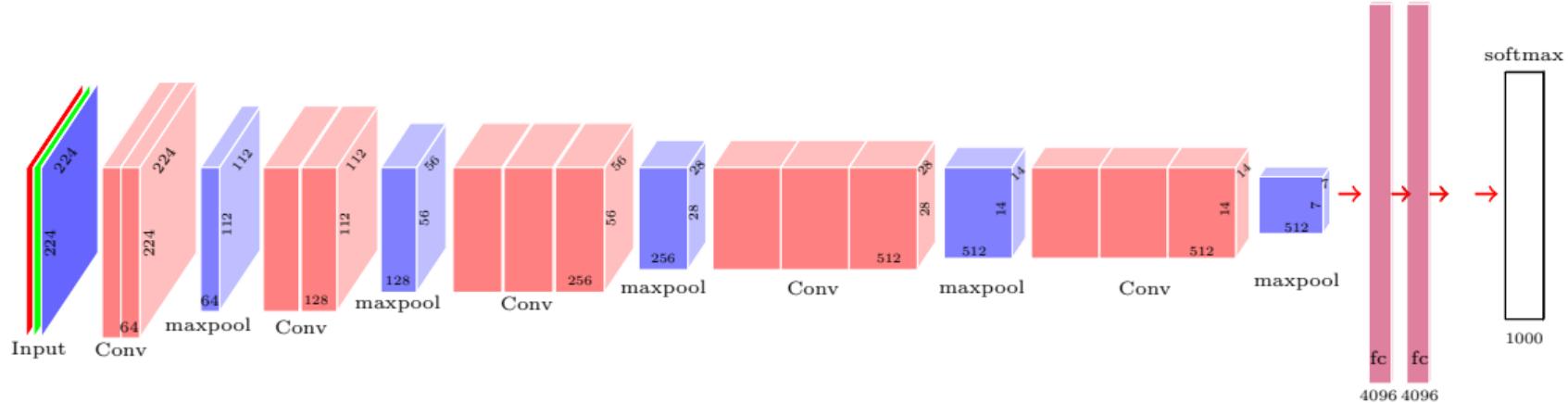
- Kernel size is 3×3 throughout
 - Total parameters in non FC layers = $\sim 16M$
 - Total Parameters in FC layers = $(512 \times 7 \times 7 \times 4096) + (4096 \times 4096)$



- Kernel size is 3×3 throughout
- Total parameters in non FC layers = $\sim 16M$
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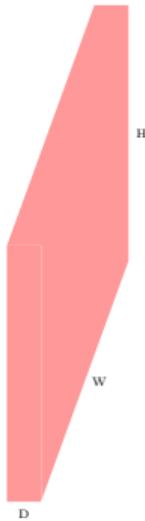
- Kernel size is 3×3 throughout
- Total parameters in non FC layers = $\sim 16M$
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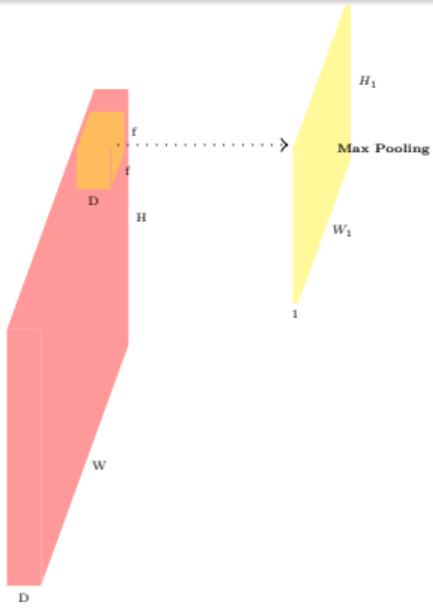


- Kernel size is 3×3 throughout
- Total parameters in non FC layers = $\sim 16M$
- Total Parameters in FC layers = $(512 \times 7 \times 7 \times 4096) + (4096 \times 4096) + (4096 \times 1024) = \sim 122M$
- Most parameters are in the first FC layer ($\sim 102M$)

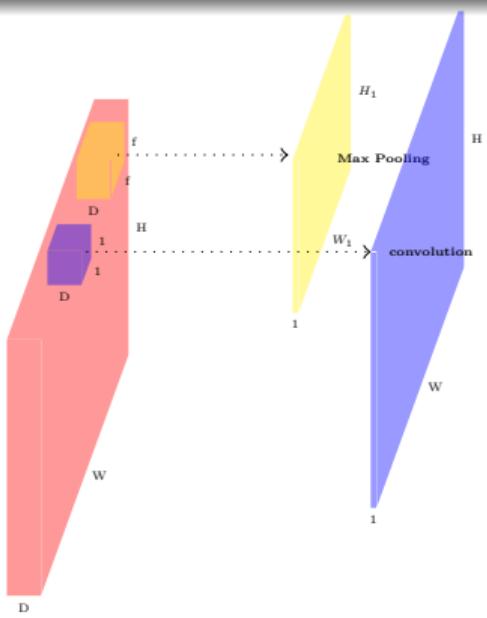
Module 11.5 : Image Classification continued (GoogLeNet and ResNet)

- Consider the output at a certain layer of a convolutional neural network

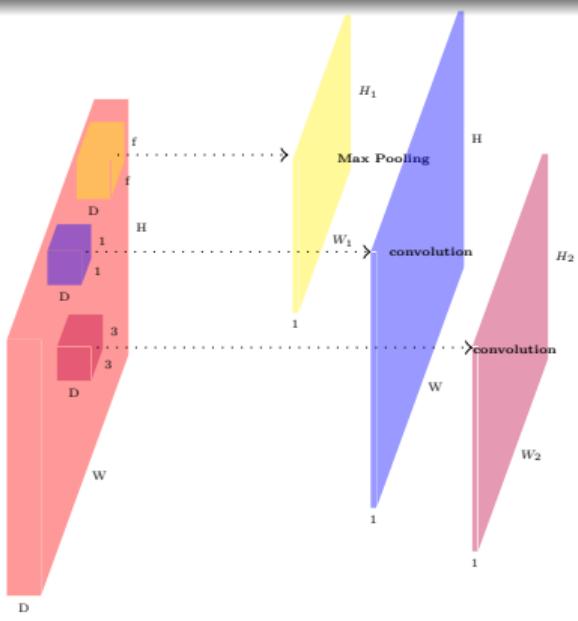




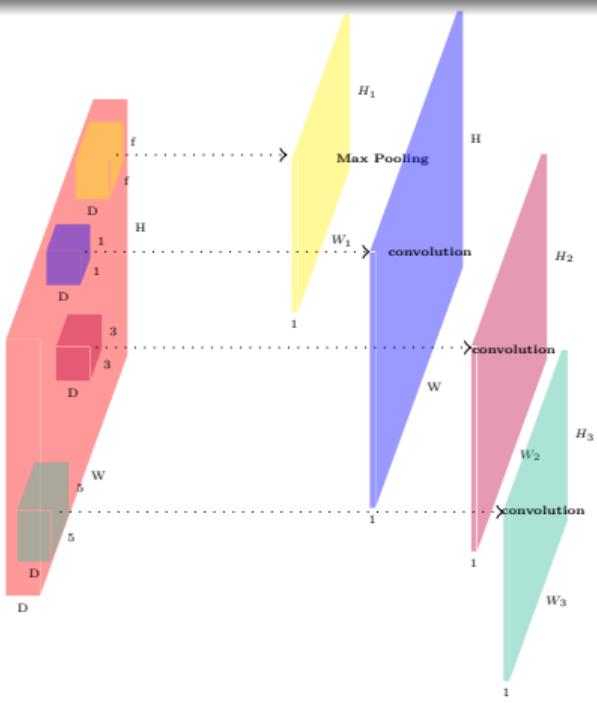
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer



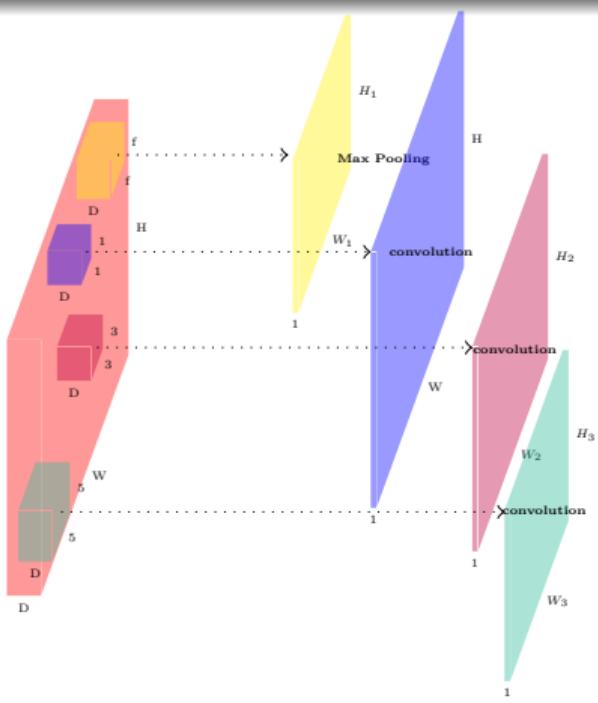
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer
- Or a 1×1 convolution



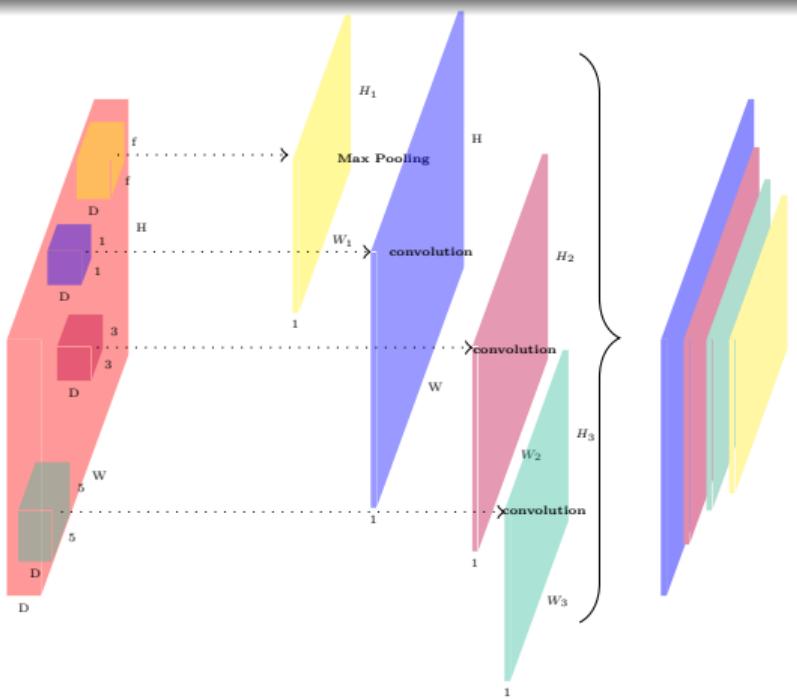
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer
- Or a 1×1 convolution
- Or a 3×3 convolution



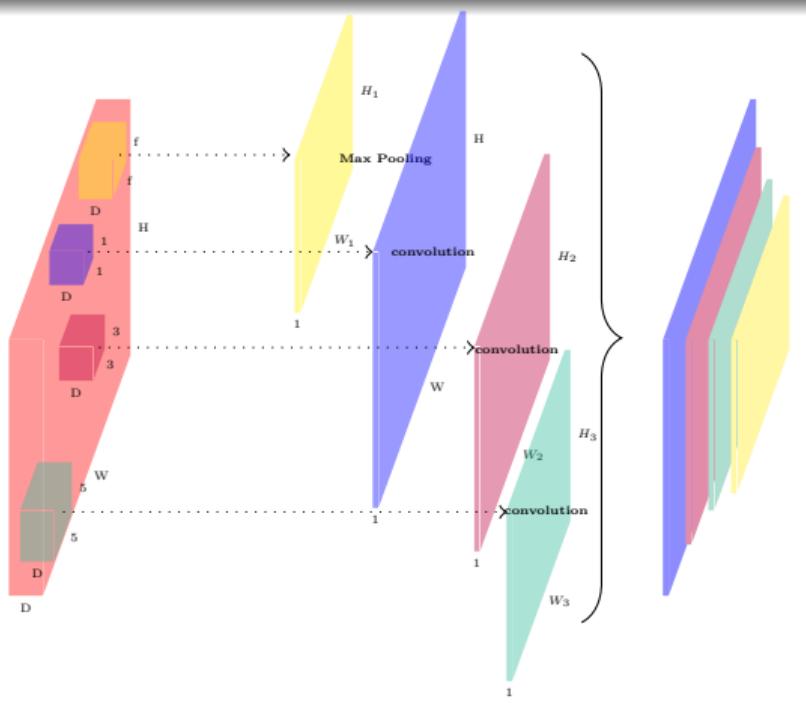
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer
- Or a 1×1 convolution
- Or a 3×3 convolution
- Or a 5×5 convolution



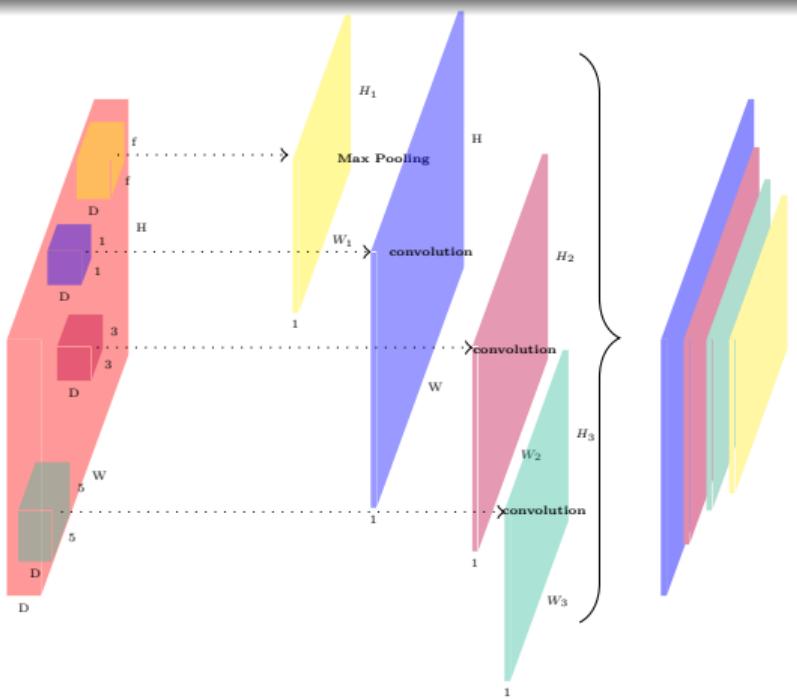
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer
- Or a 1×1 convolution
- Or a 3×3 convolution
- Or a 5×5 convolution
- Question:** Why choose between these options (convolution, maxpooling, filter sizes)?



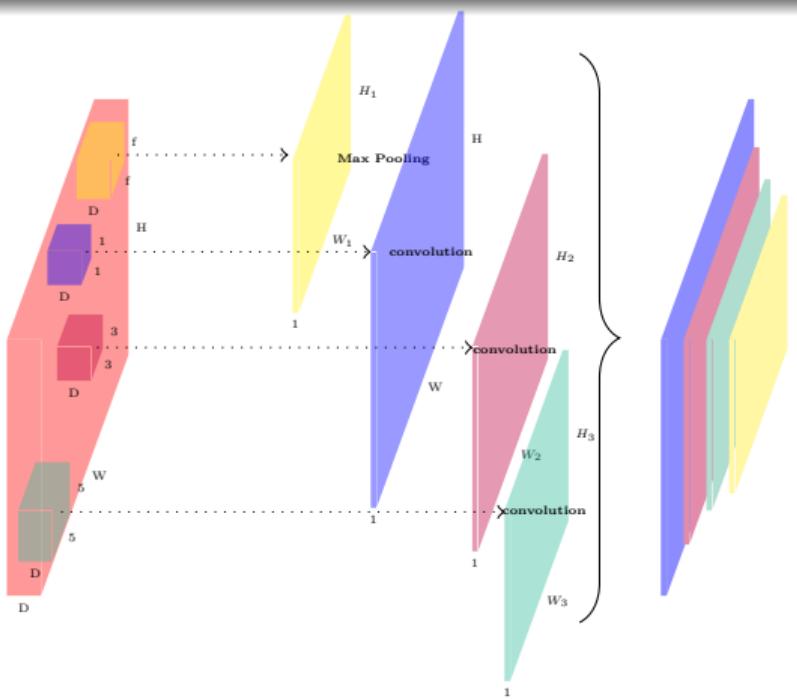
- Consider the output at a certain layer of a convolutional neural network
- After this layer we could apply a max-pooling layer
- Or a 1×1 convolution
- Or a 3×3 convolution
- Or a 5×5 convolution
- Question:** Why choose between these options (convolution, maxpooling, filter sizes)?
- Idea:** Why not apply all of them at the same time and then concatenate the feature maps?



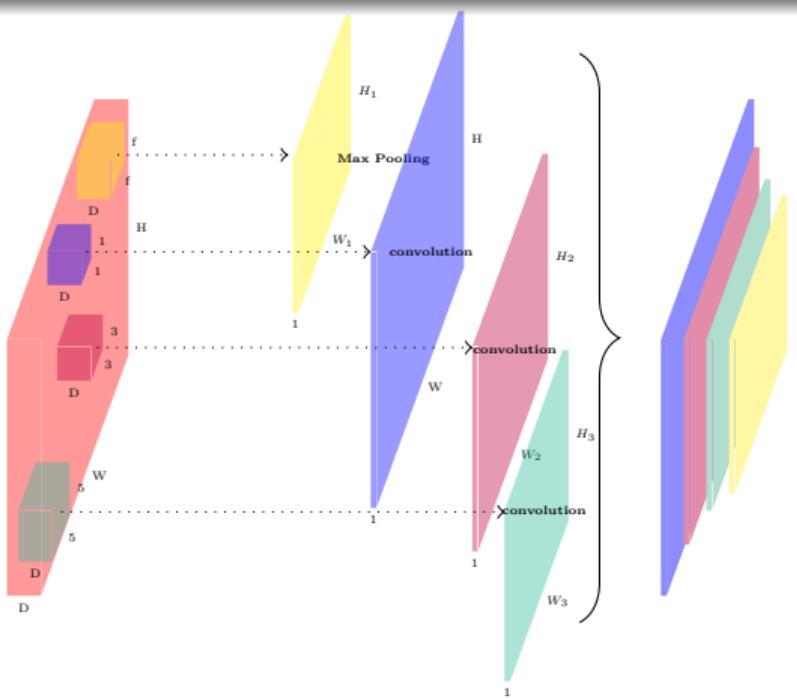
- Well this naive idea could result in a large number of computations



- Well this naive idea could result in a large number of computations
- If $P = 0$ & $S = 1$ then convolving a $W \times H \times D$ input with a $F \times F \times D$ filter results in a $(W - F + 1)(H - F + 1)$ sized output

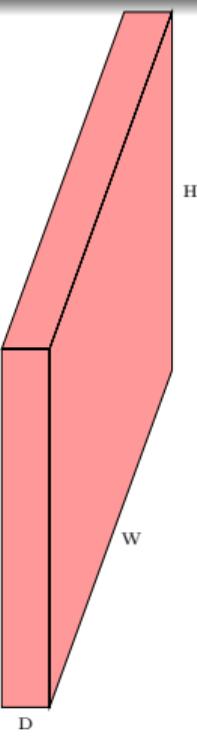


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- Each element of the output requires $O(F \times F \times D)$ computations

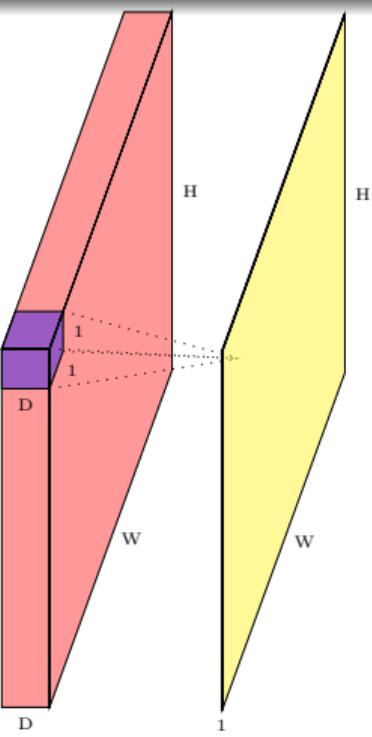


- Well this naive idea could result in a large number of computations
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- Each element of the output requires $O(F \times F \times D)$ computations
- Can we reduce the number of computations?

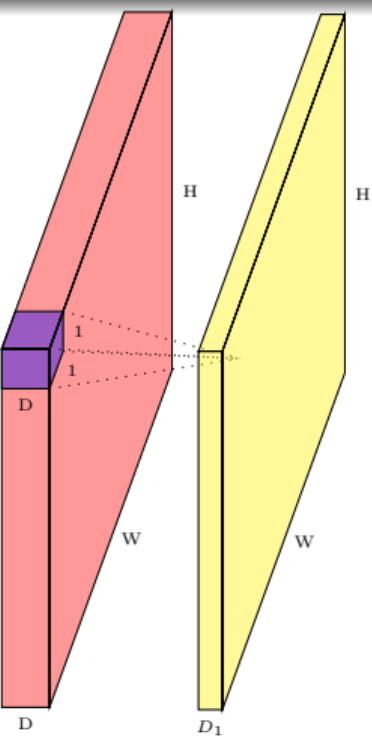
- Yes, by using 1×1 convolutions



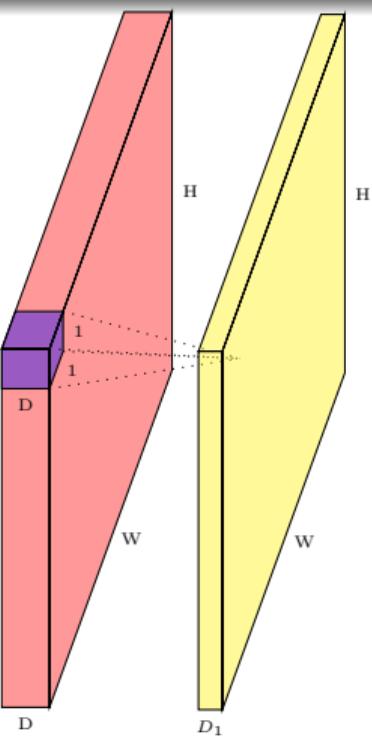
- Yes, by using 1×1 convolutions
- Huh?? What does a 1×1 convolution do ?



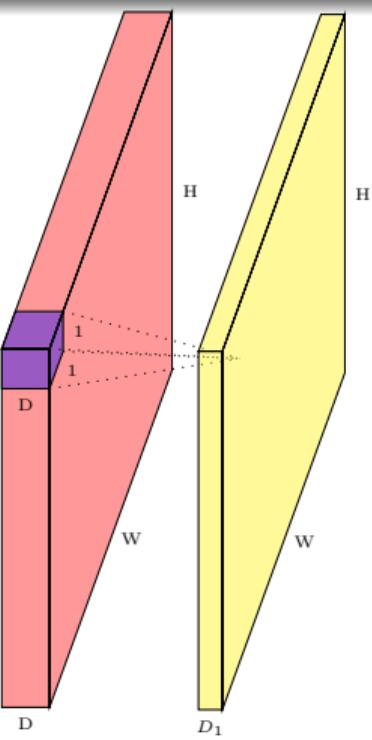
- Yes, by using 1×1 convolutions
- Huh?? What does a 1×1 convolution do ?
- It aggregates along the depth



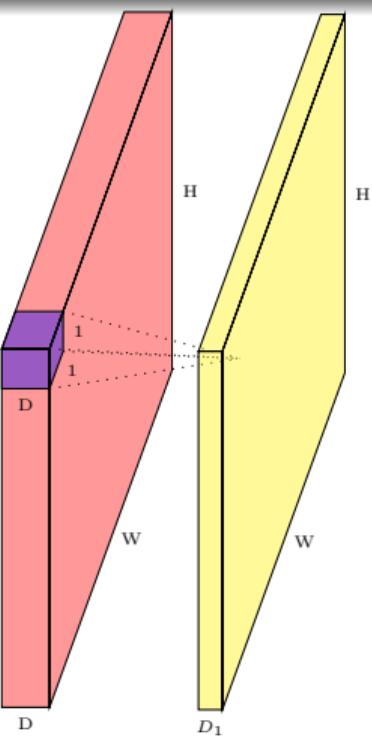
- Yes, by using 1×1 convolutions
- Huh?? What does a 1×1 convolution do ?
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- So convolving a $D \times W \times H$ input with $D_1 1 \times 1$ ($D_1 < D$) filters will result in a $D_1 \times W \times H$ output ($S = 1, P = 0$)



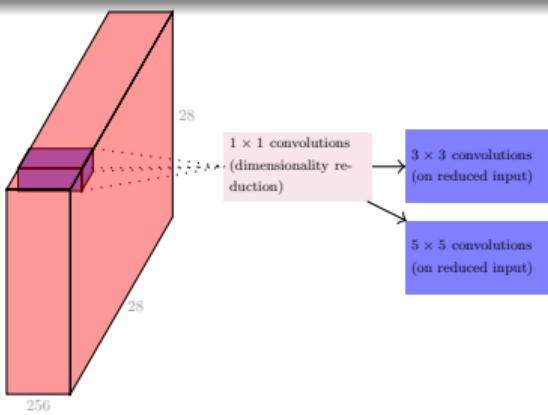
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- If $D_1 < D$ then this effectively reduces the dimension of the input and hence the computations



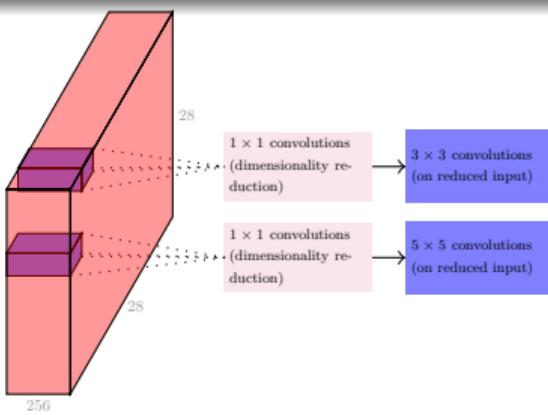
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- Specifically instead of $O(F \times F \times D)$ we will need $O(F \times F \times D_1)$ computations



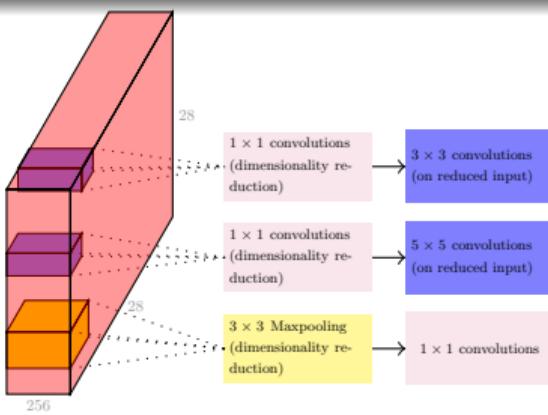
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- If $D_1 < D$ then this effectively reduces the dimension of the input and hence the computations
- Specifically instead of $O(F \times F \times D)$ we will need $O(F \times F \times D_1)$ computations
- We could then apply subsequent 3×3 , 5×5 filter on this reduced output



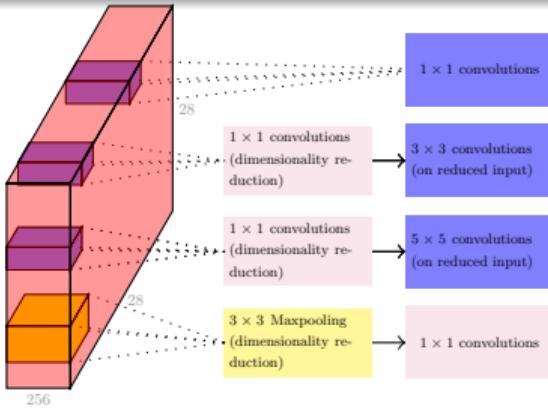
- But we might want to use different dimensionality reductions before the 3×3 and 5×5 filters



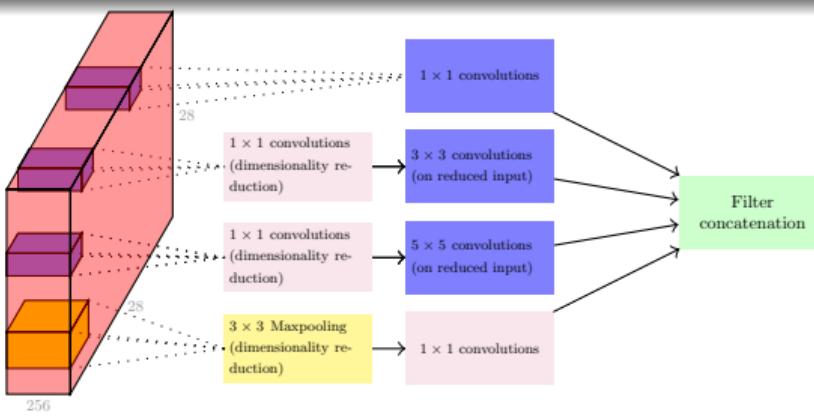
- But we might want to use different dimensionality reductions before the 3×3 and 5×5 filters
- So we can use D_1 and D_2 1×1 filters before the 3×3 and 5×5 filters respectively



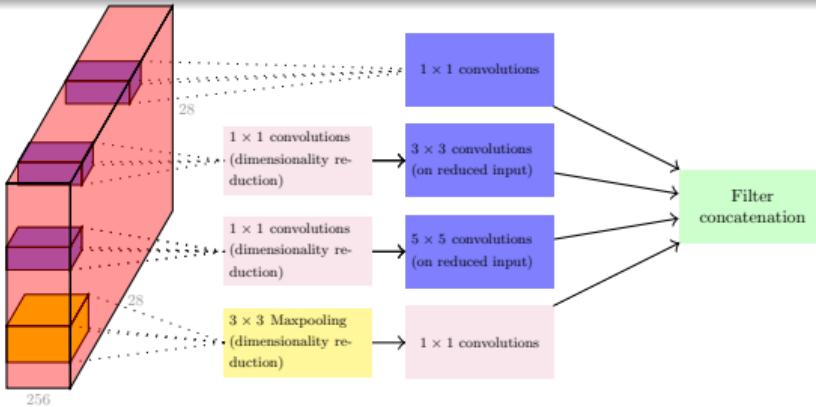
- But we might want to use different dimensionality reductions before the 3×3 and 5×5 filters
- So we can use D_1 and D_2 1×1 filters before the 3×3 and 5×5 filters respectively
- We can then add the maxpooling layer followed by dimensionality reduction



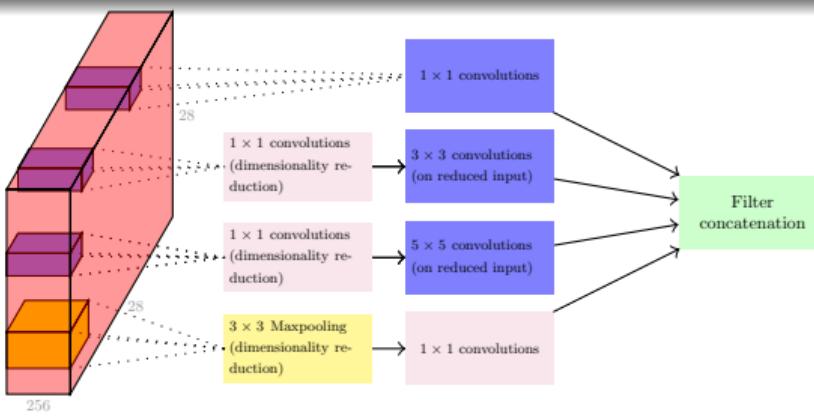
- But we might want to use different dimensionality reductions before the 3×3 and 5×5 filters
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- We can then add the maxpooling layer followed by dimensionality reduction
- And a new set of 1×1 convolutions



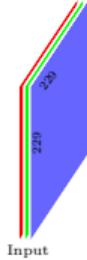
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- And finally we concatenate all these layers

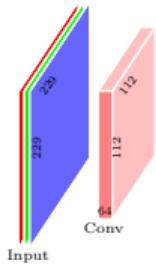


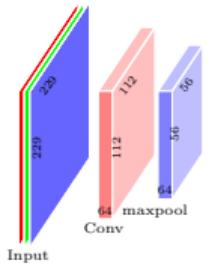
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- This is called the **Inception module**

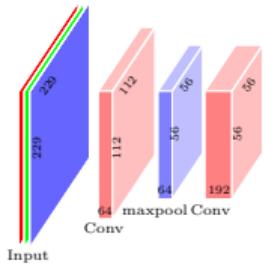


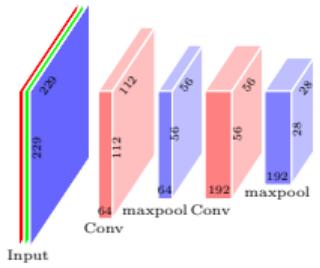
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- This is called the **Inception module**
- We will now see **GoogLeNet** which contains many such inception modules

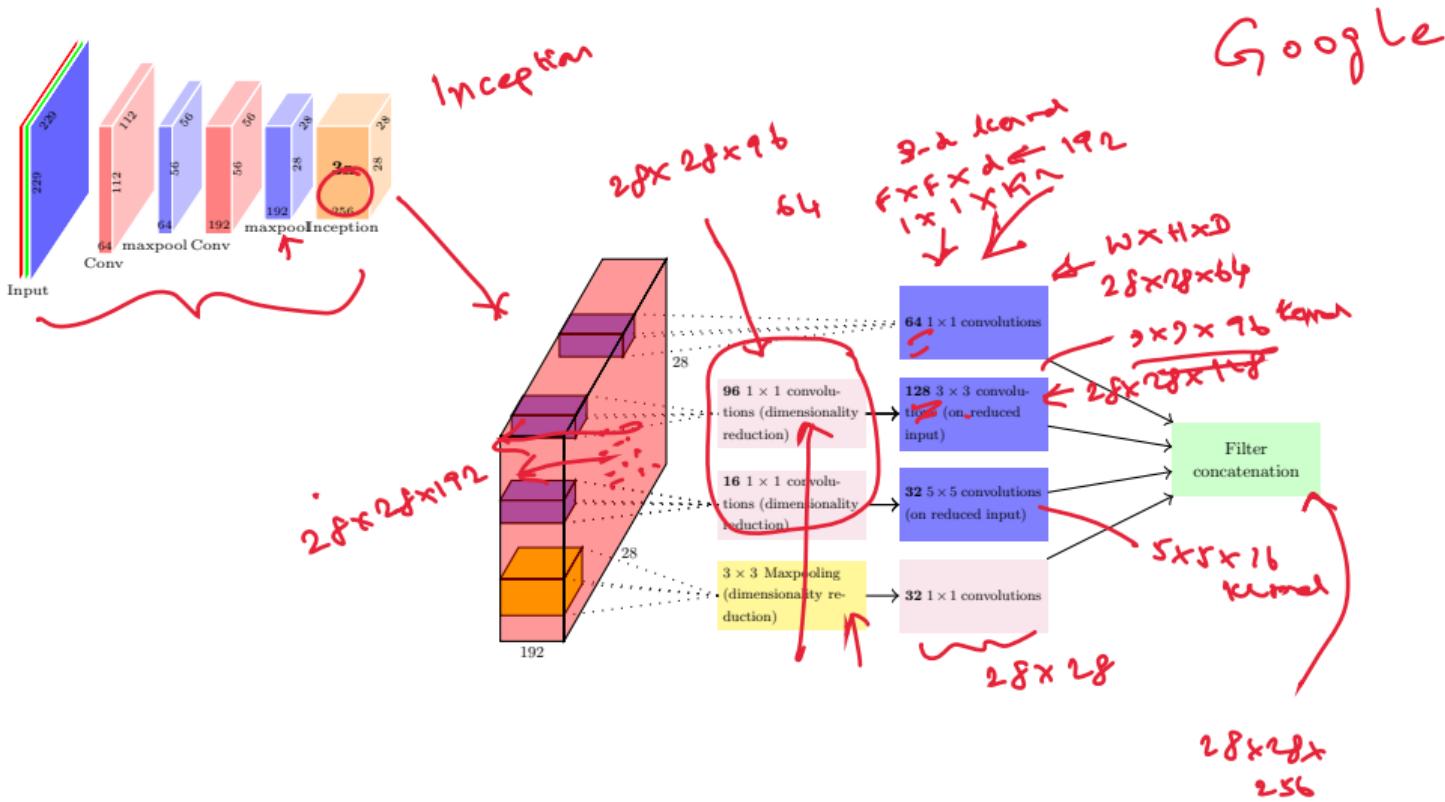


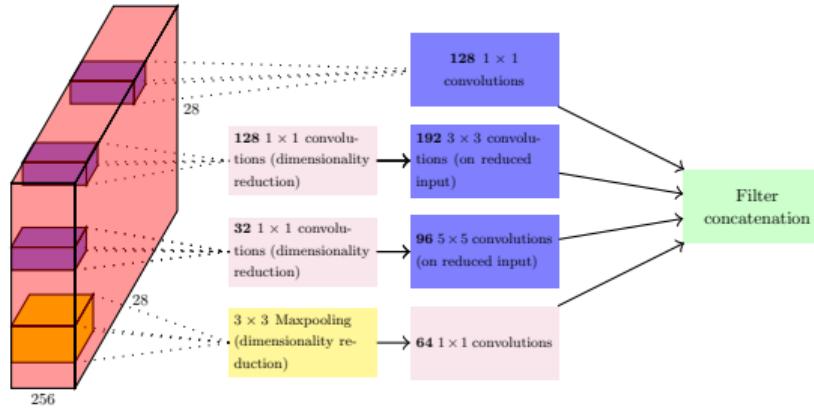
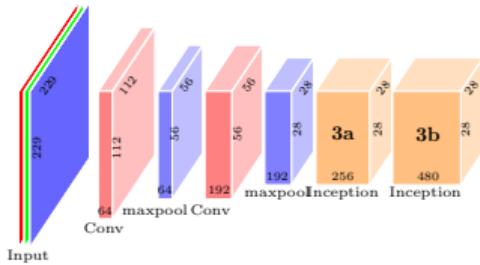


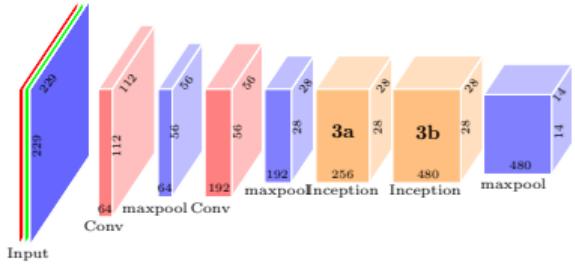


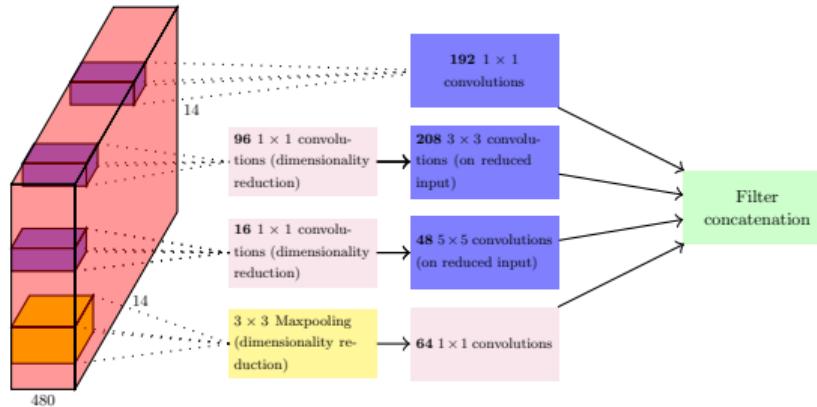
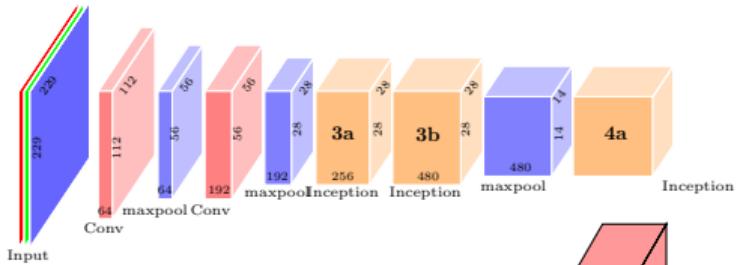


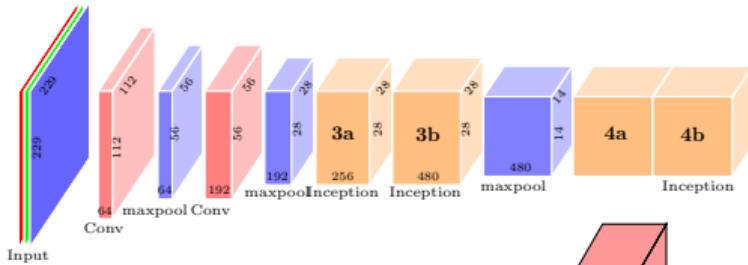




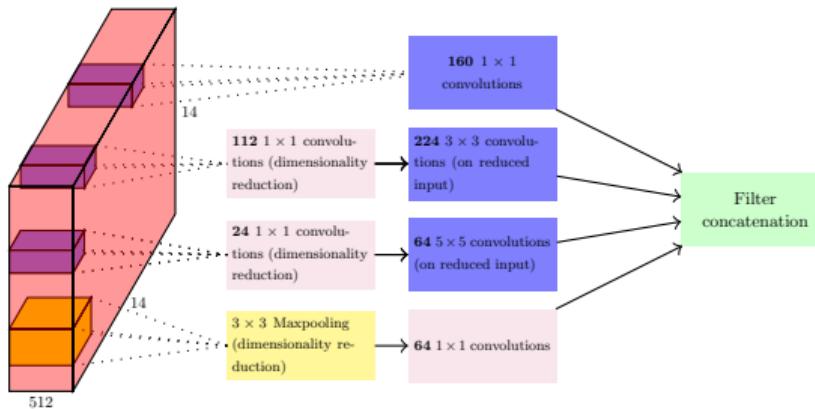


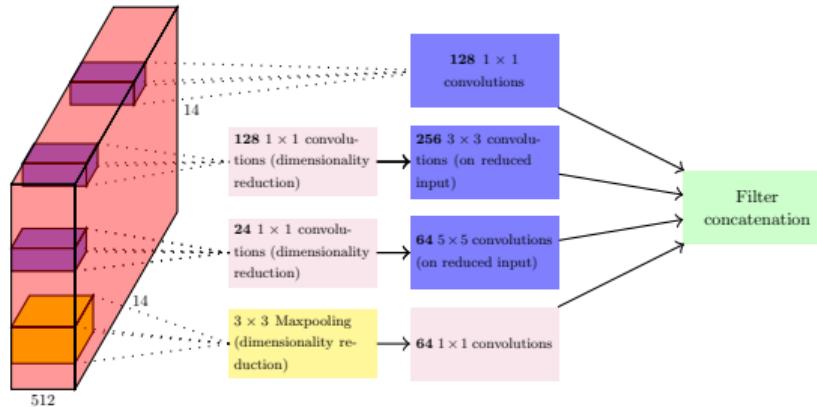
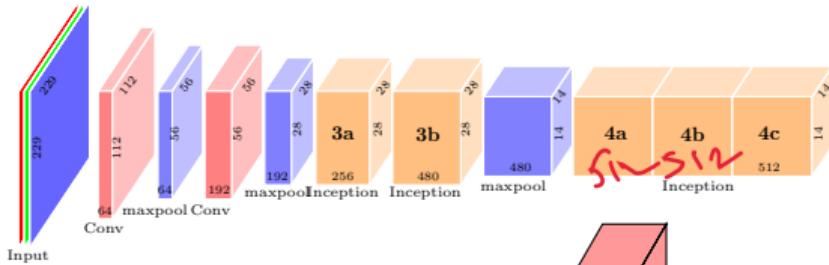


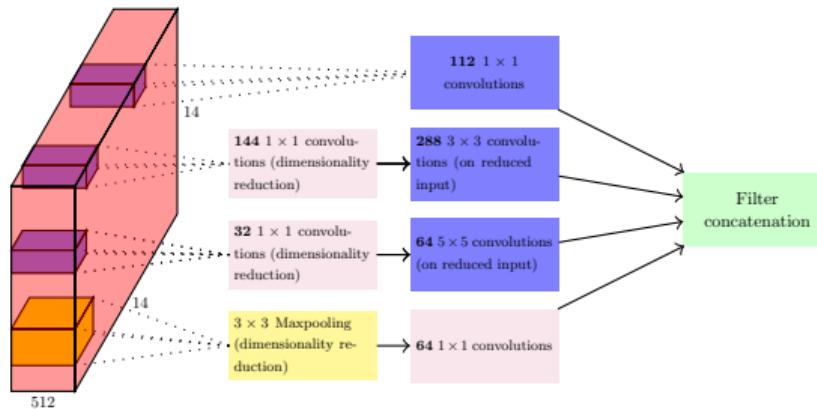
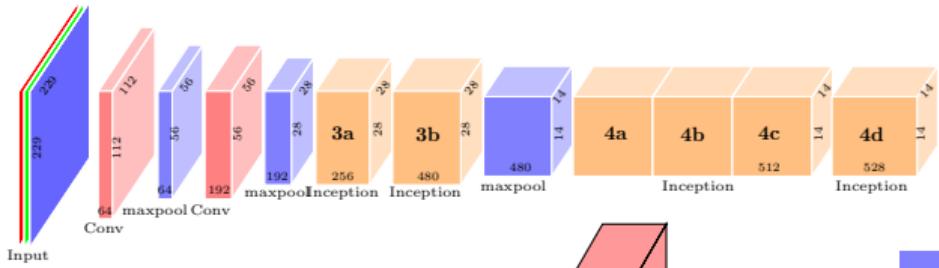


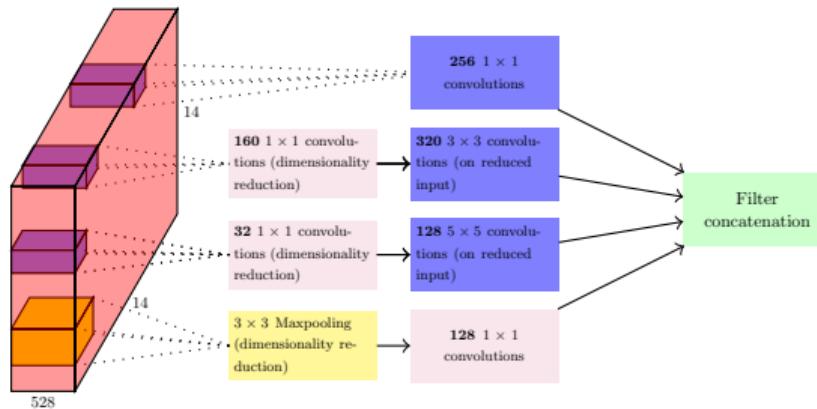
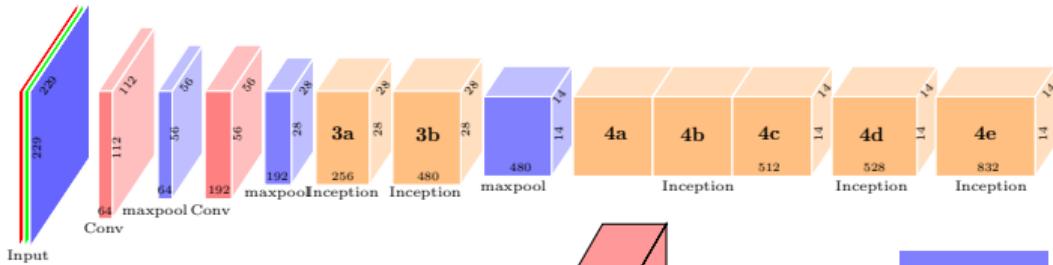


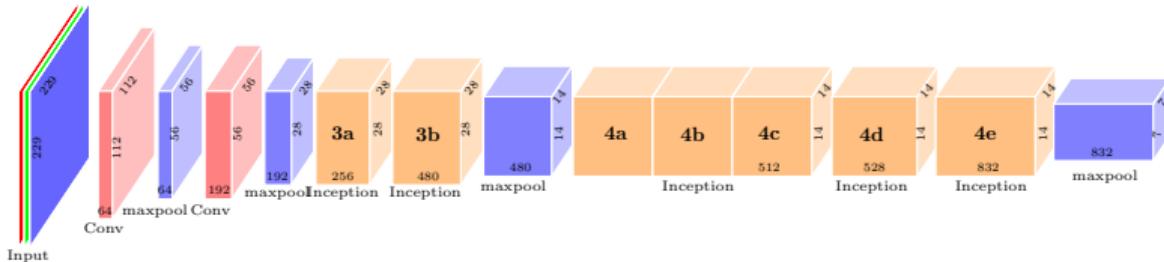
$$160 + 224 \\ + 64 \times 64$$

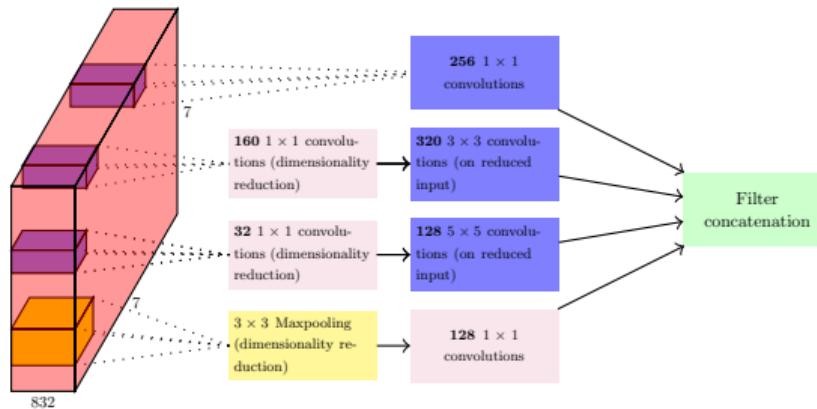
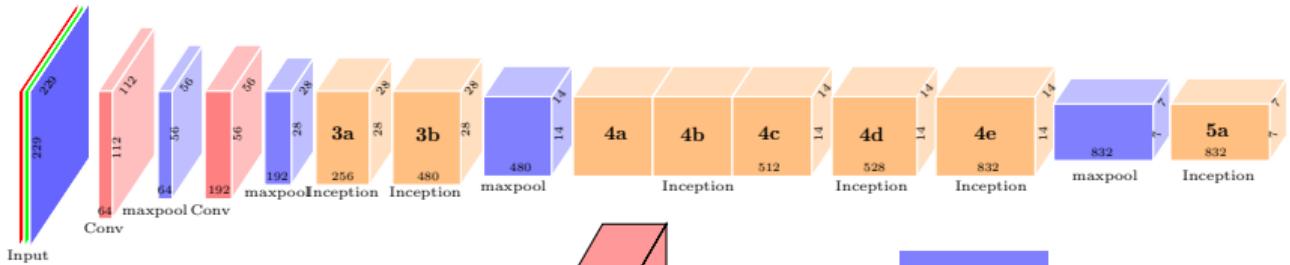


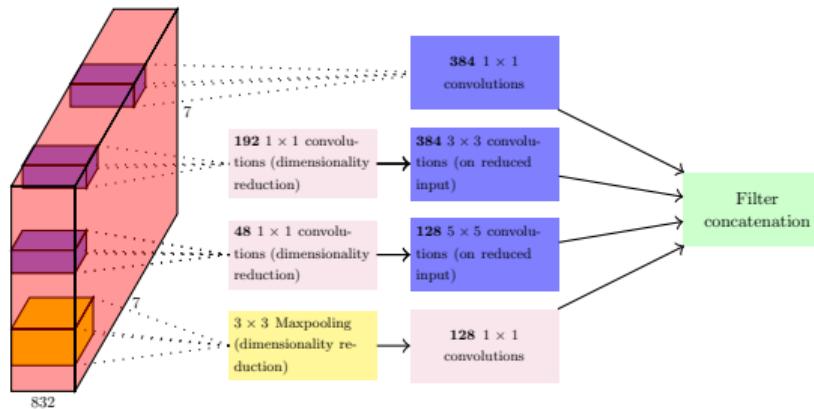
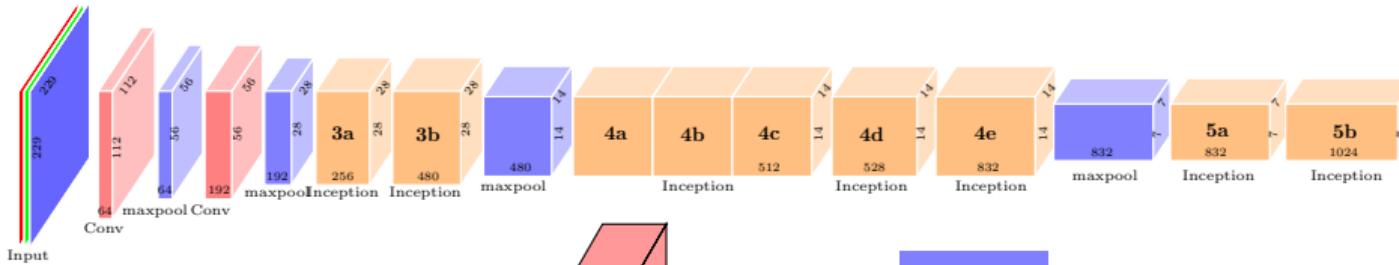


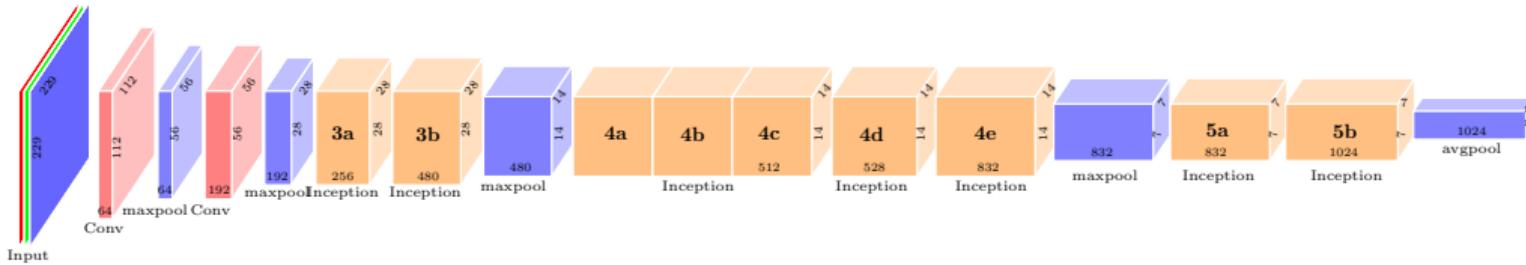


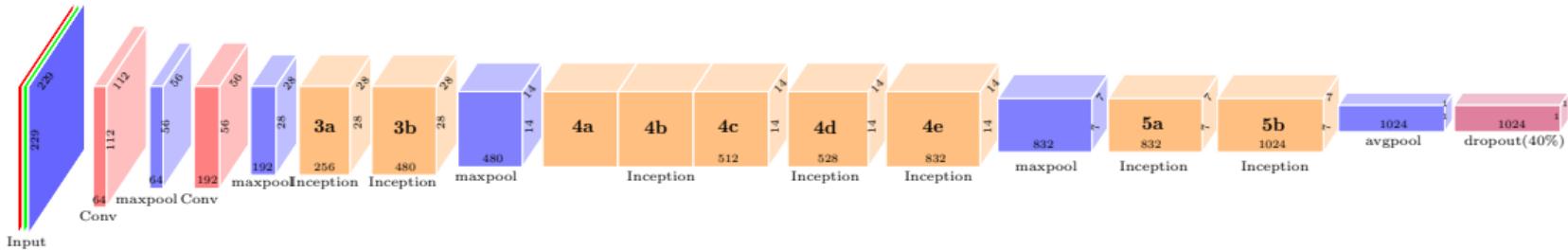


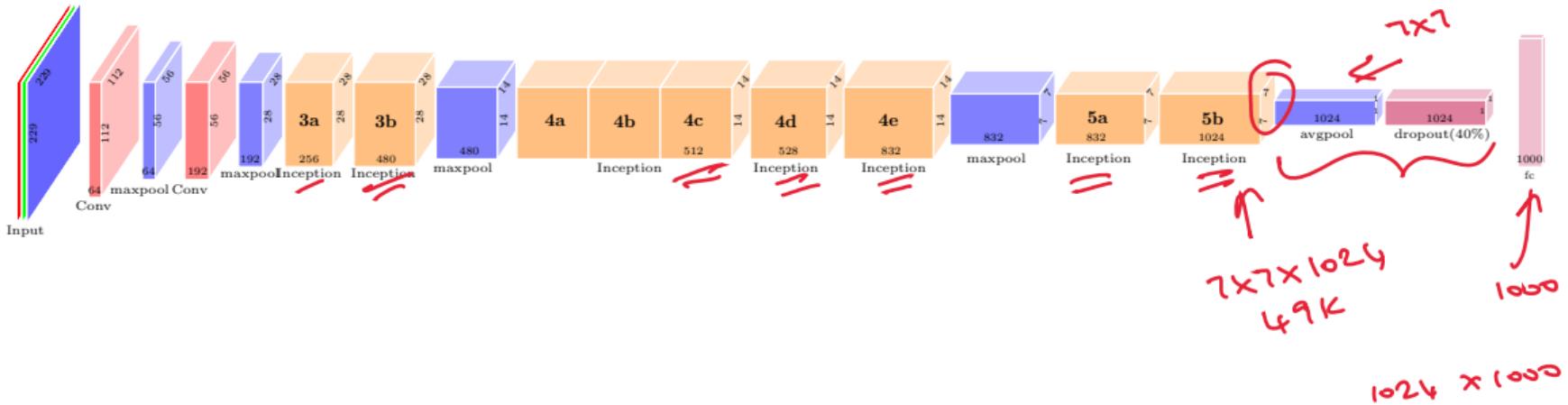


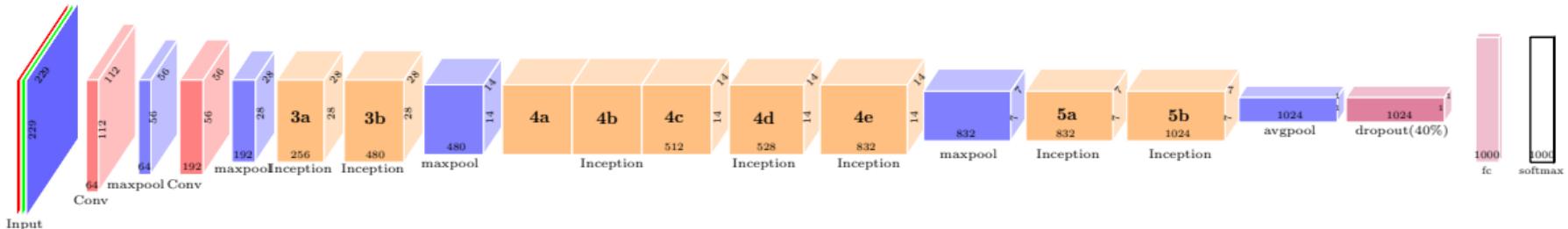






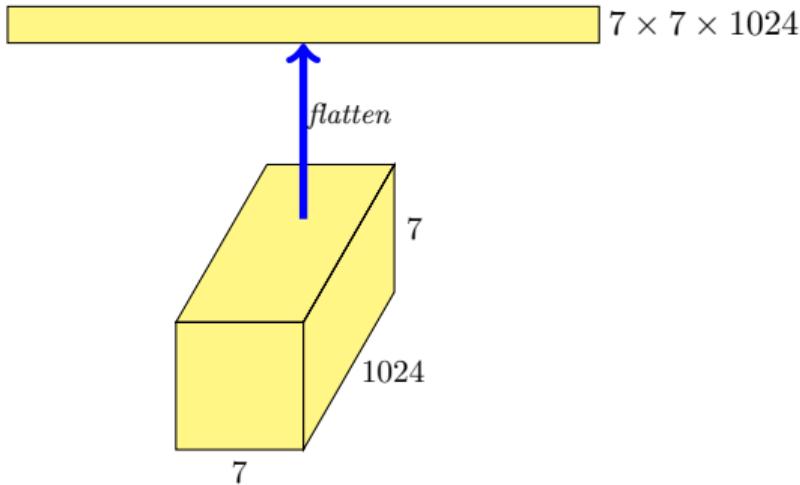


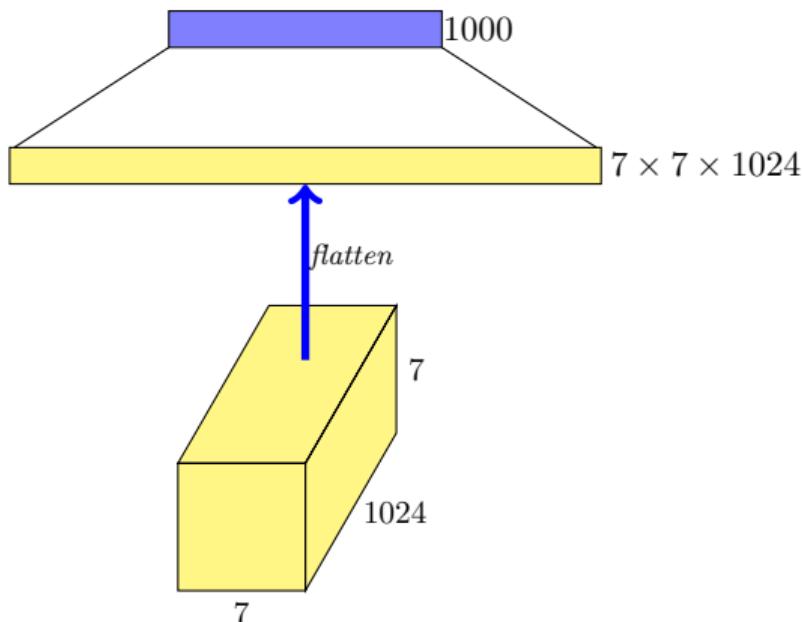




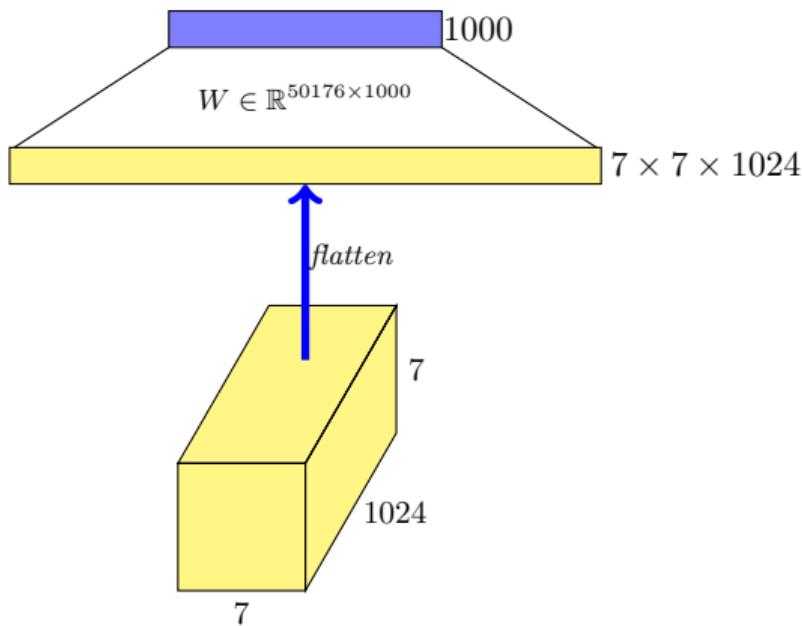
- **Important Trick:** Got rid of the fully connected layer

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- Notice that output of the last layer is $7 \times 7 \times 1024$ dimensional

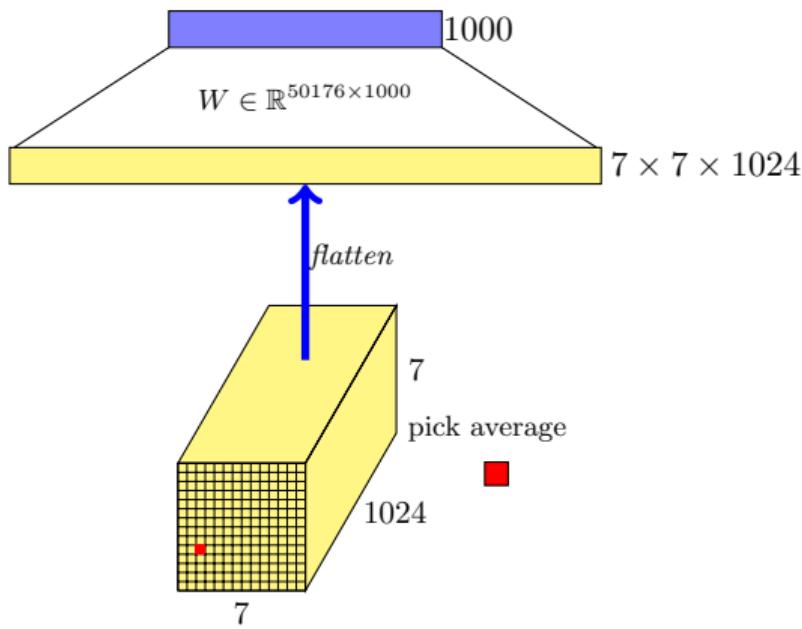




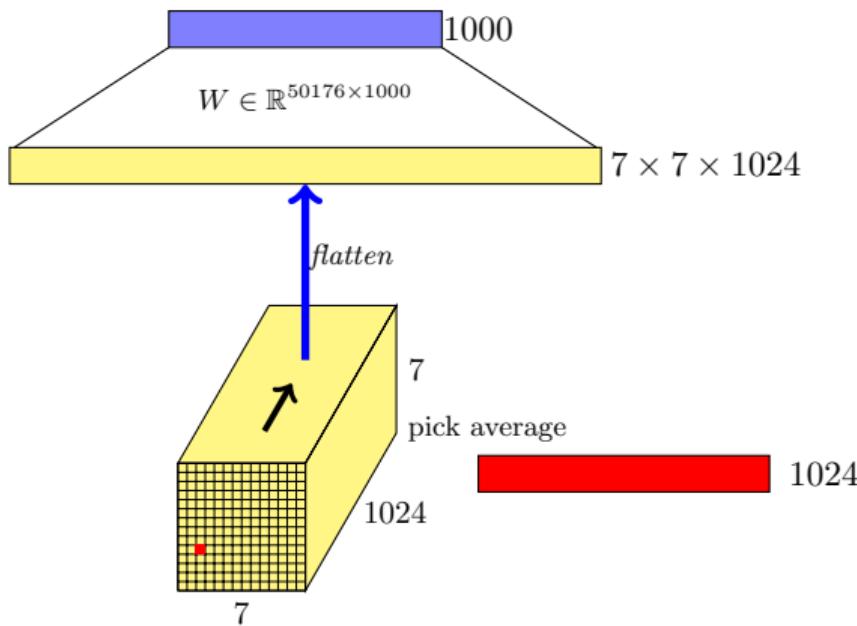
- **Important Trick:** Got rid of the fully connected layer
- Notice that output of the last layer is $7 \times 7 \times 1024$ dimensional
- What if we were to add a fully connected layer with 1000 nodes (for 1000 classes) on top of this



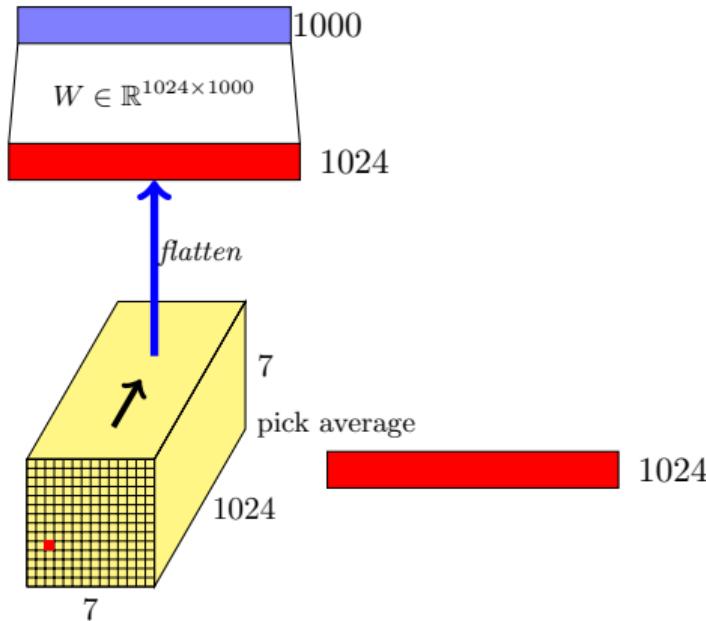
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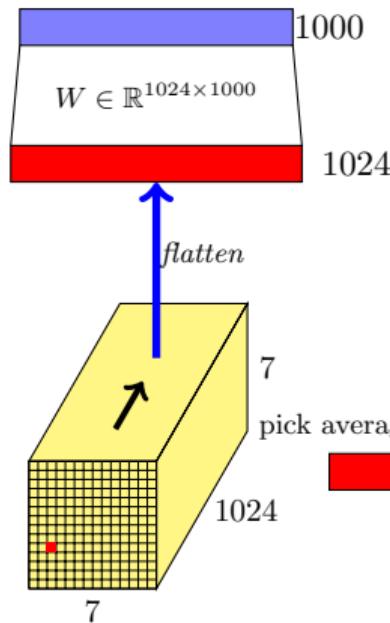
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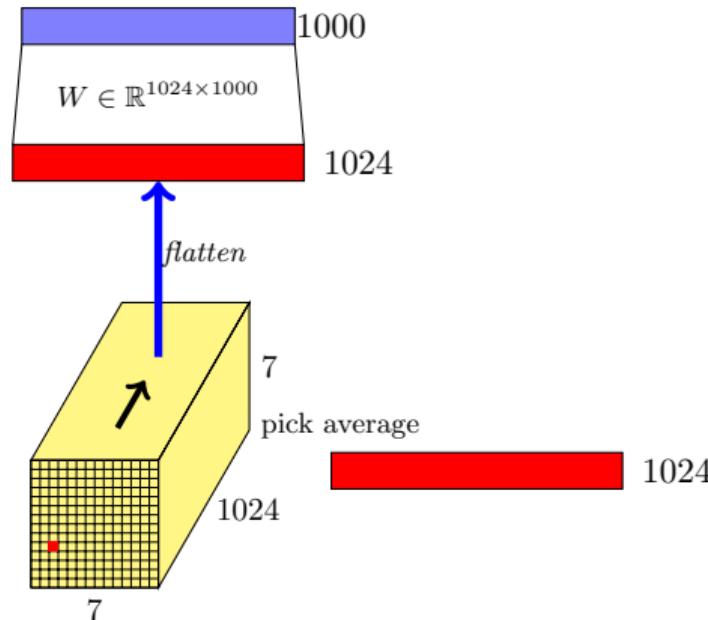
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 - Instead they use an average pooling of size 7×7 on each of the 1024 feature maps
 - This results in a 1024 dimensional output
 - Significantly reduces the number of parameters

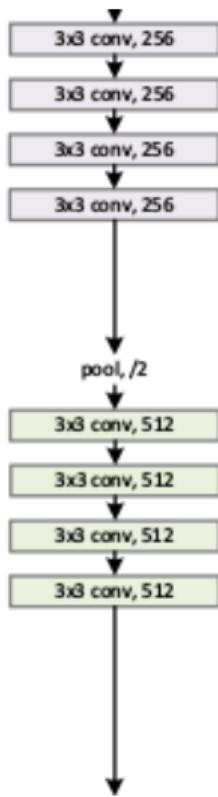


- 12× less parameters than AlexNet

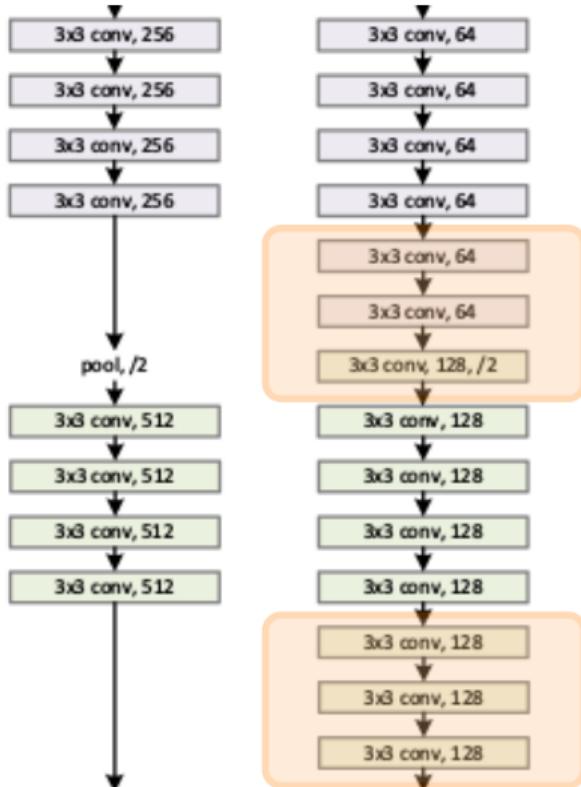


- 12× less parameters than AlexNet
- 2× more computations

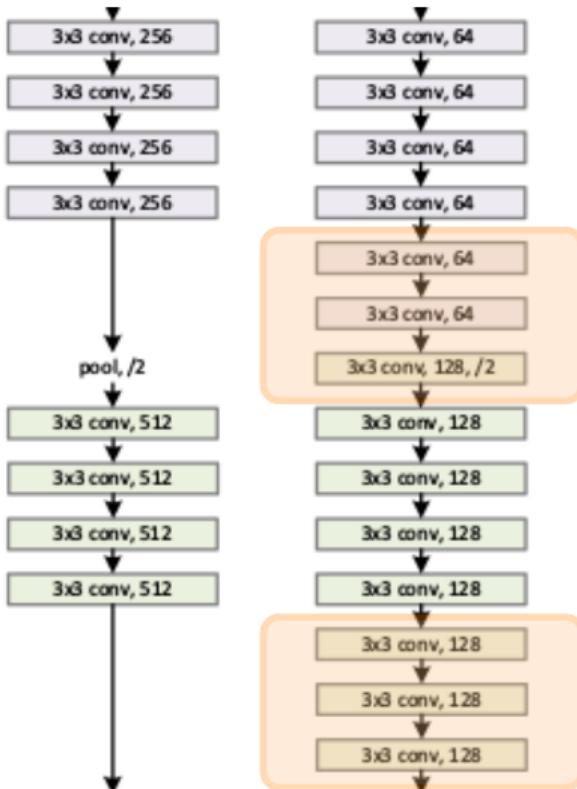
- GoogLeNet
- ResNet



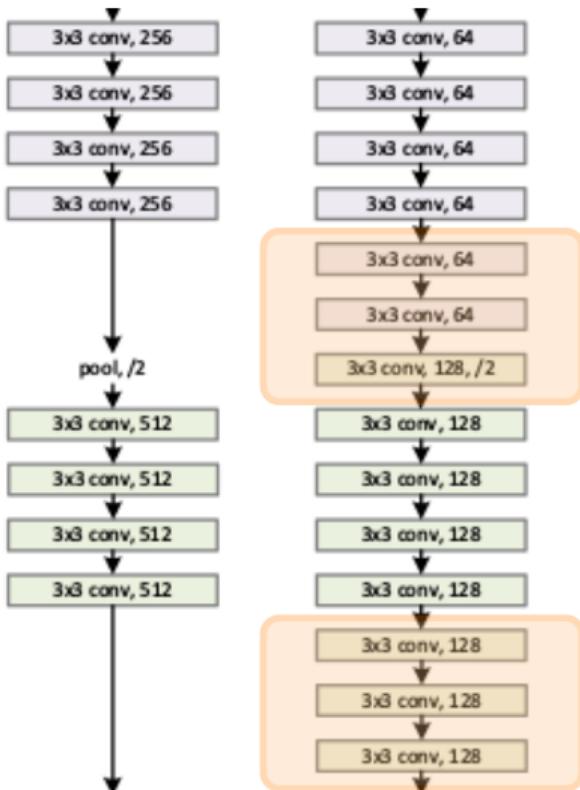
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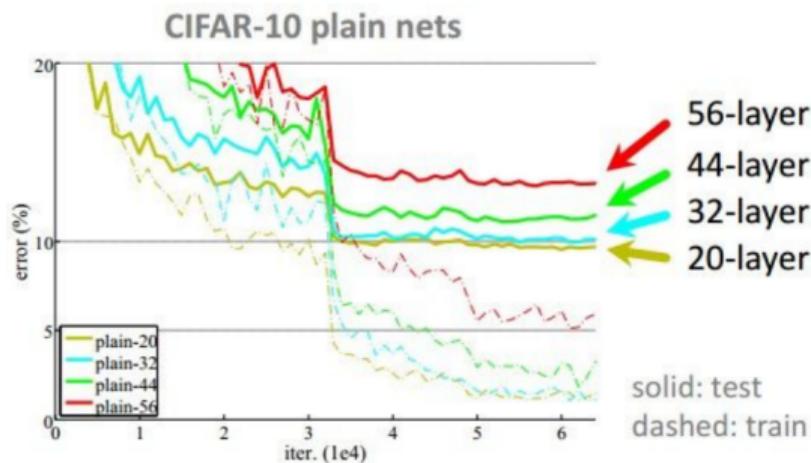


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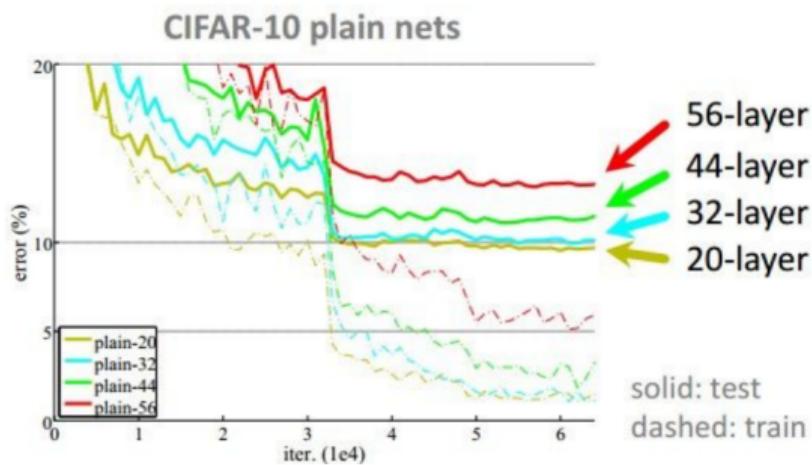


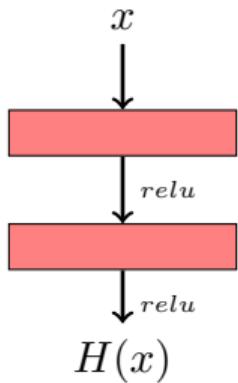
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- Now suppose we construct a deeper network which has few more layers (in orange)
- Intuitively, if the shallow network works well then the deep network should also work well by simply learning to compute identity functions in the new layers
- Essentially, the solution space of a shallow neural network is a subset of the solution space of a deep neural network

- But in practice it is observed that this doesn't happen

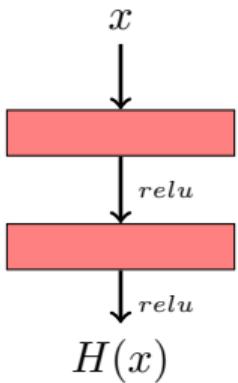


- But in practice it is observed that this doesn't happen
- Notice that the deep layers have a higher error rate on the test set

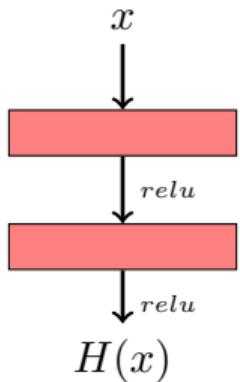




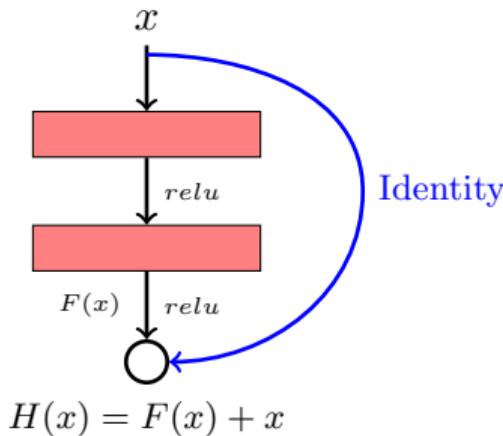
- Consider any two stacked layers in a CNN



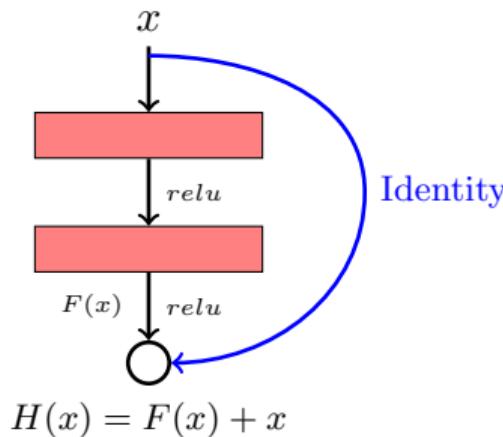
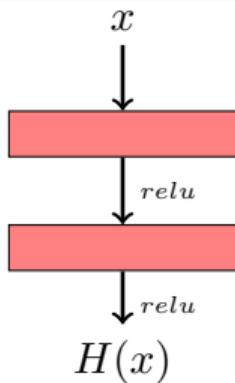
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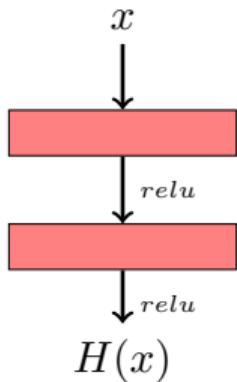


- Consider any two stacked layers in a CNN
- The two layers are essentially learning some function of the input
- What if we enable it to learn only a residual function of the input?

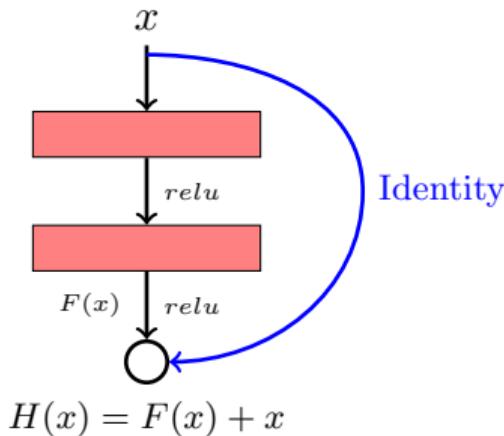


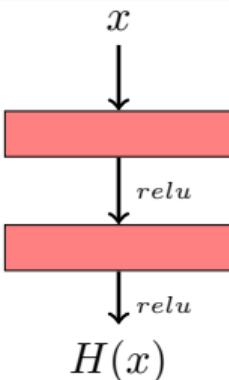
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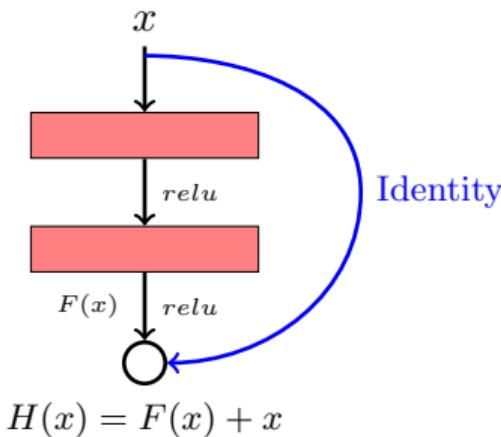


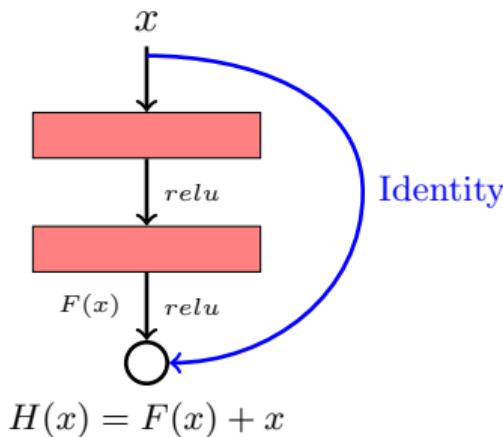
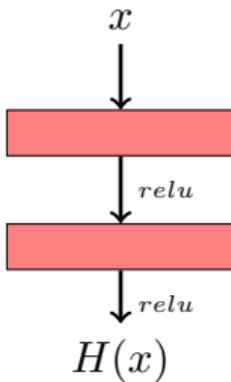
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- Why would this help?
- Remember our argument that a deeper version of a shallow network would do just fine by learning identity transformations in the new layers
- This identity connection from the input allows a ResNet to retain a copy of the input
- Using this idea they were able to train really deep networks



ResNet, 152 layers

1st place in all five main tracks

- **ImageNet Classification:** “Ultra-deep” 152-layer nets



ResNet, 152 layers

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ResNet, 152 layers

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- **ImageNet Classification:** “Ultra-deep” 152-layer nets
- **ImageNet Detection:** 16% better than the 2nd best system
- **ImageNet Localization:** 27% better than the 2nd best system



ResNet, 152 layers

1st place in all five main tracks

- **ImageNet Classification:** “Ultra-deep” 152-layer nets
- **ImageNet Detection:** 16% better than the 2nd best system
- **ImageNet Localization:** 27% better than the 2nd best system
- **COCO Detection:** 11% better than the 2nd best system



ResNet, 152 layers

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- **ImageNet Classification:** “Ultra-deep” 152-layer nets
- **ImageNet Detection:** 16% better than the 2nd best system
- **ImageNet Localization:** 27% better than the 2nd best system
- **COCO Detection:** 11% better than the 2nd best system
- **COCO Segmentation:** 12% better than the 2nd best system



ResNet, 152 layers

Bag of tricks

- Batch Normalizaton after every CONV layer



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ResNet, 152 layers

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- Batch Normalizaton after every CONV layer
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ResNet, 152 layers

Bag of tricks

- Batch Normalizaton after every CONV layer
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- Mini-batch size 256



Bag of tricks

- Batch Normalizaton after every CONV layer
- Xavier/2 initialization from [He et al]
- SGD + Momentum(0.9)
- Learning rate:0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5

ResNet, 152 layers



ResNet, 152 layers

Bag of tricks

- Batch Normalizaton after every CONV layer
- Xavier/2 initialization from [He et al]
- SGD + Momentum(0.9)
- Learning rate:0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used