On Decompositions, Generation Methods, and related concepts in the theory of Matching Covered Graphs

Student: Janmenjaya Panda

Guide: Nishad Kothari

Indian Institute of Technology Madras

May 24, 2024

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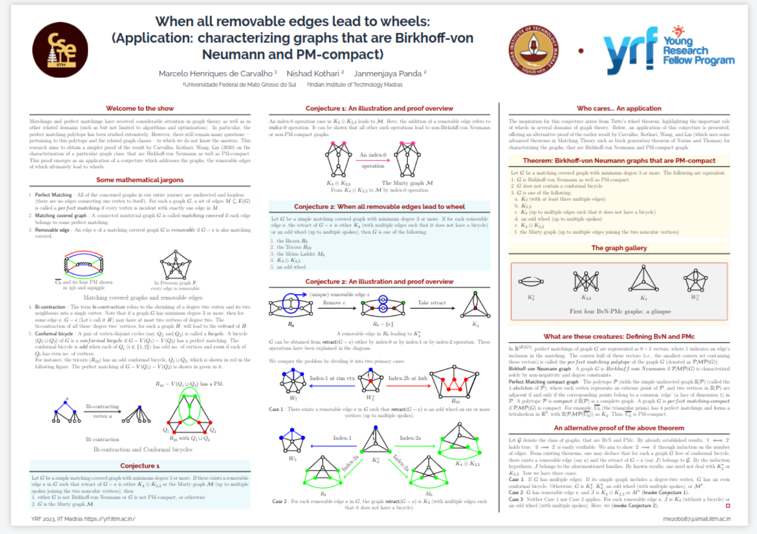

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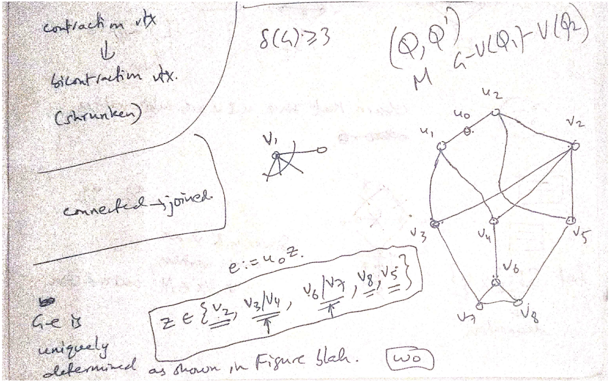

Figure: Check computationally if v_6 and v_7 lie in the same vertex orbit.

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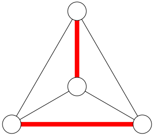

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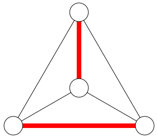

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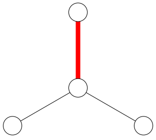

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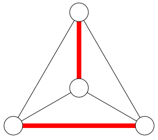

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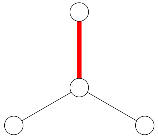

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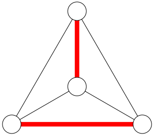

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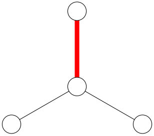

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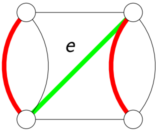

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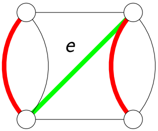

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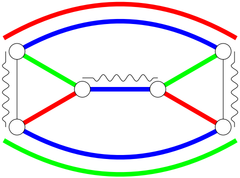

The triangular prism $\overline{C_6}$ and

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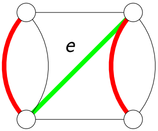

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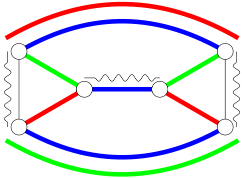

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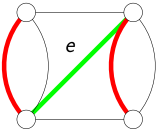

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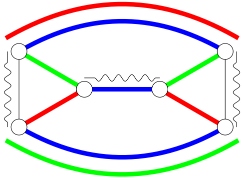

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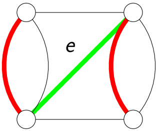

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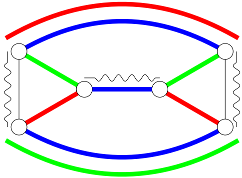

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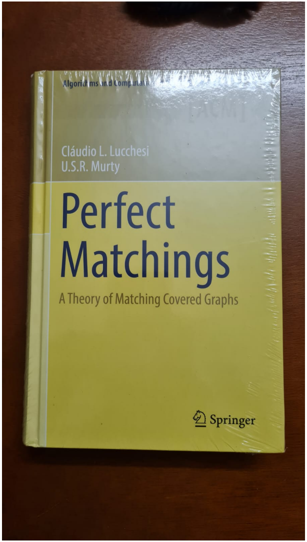

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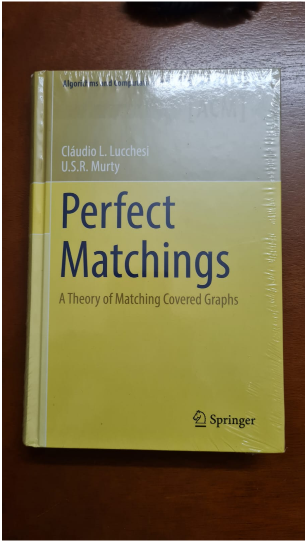

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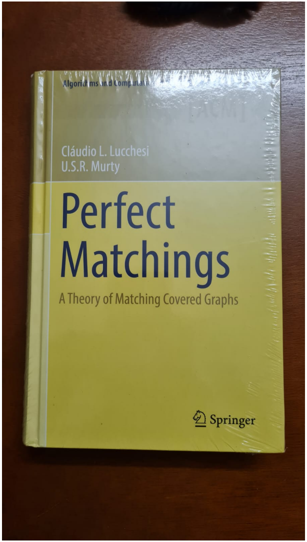

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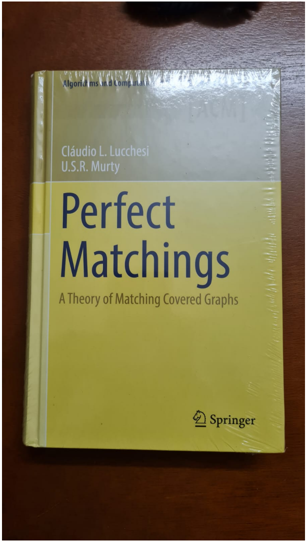

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Lucchesi and Murty, 2024

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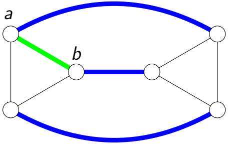

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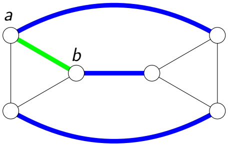

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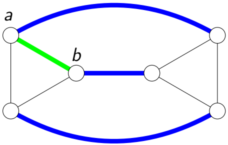

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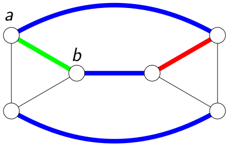

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where $o(H)$ denotes the # of odd components in graph H.

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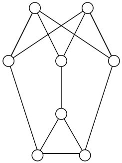

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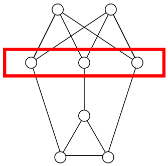

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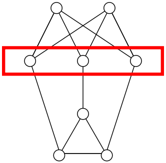

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Theorem: Lucchesi Murty, 2024

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Theorem: Lucchesi Murty, 2024

Let u and v be any two vertices in a matchable graph G. Then the graph $G - u - v$ is not matchable if and only if there is a barrier of G which contains both u and v.

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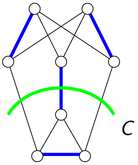

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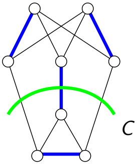

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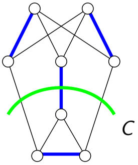

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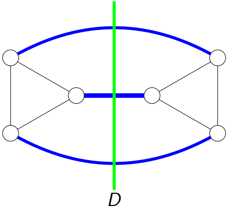

D: A cut that is not tight

Cuts and cut-contractions

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Given any cut $C := \partial(X)$ of a connected graph G, where X is a nonempty proper subset of V , we refer to the two graphs $G/(X \to x)$ and $G/(\overline{X} \to x)$ as the C-contractions of G, where $\overline{X} := V - X$.

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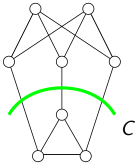

The graph G with the cut C leads to two C-contractions, that are — K_4 and $K_{3,3}$.

Towards the tight cut decomposition

A small DIY for all of you:

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Getting smaller MCGs thru tight cuts

Let G be a graph and C be a tight cut of it. Suppose that G_1 and G_2 are the C-contractions of G. The graph G is matching covered iff each of G_1 and G_2 are matching covered.

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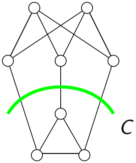

The graph G shown has a non-trivial tight cut C, that leads to two C-contractions, that are — K_4 and $K_{3,3}$, both of which are free of tight cuts.

Bricks and braces

For a matching covered graph that is free of non-trivial tight cuts, it is called a *brick* if it is non-bipartite and a *brace* if it is bipartite.

For example: K_4 is a brick and $K_{3,3}$ is a brace.

A natural question.

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Are the results of two TCD procedure unique?

A matching covered graph might have several tight cuts. Will they lead to different list of bricks and braces if we follow two different sequence of tight cut decomposition procedure?

Lovász Tight cut decomposition theorem

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The unique decomposition theorem: Lovász 1987

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Any two applications of the tight cut decomposition procedure to a matching covered graph G produce the same list of bricks and braces, up to multiple edges.

Lovász Tight cut decomposition theorem

The unique decomposition theorem: Lovász 1987

Any two applications of the tight cut decomposition procedure to a matching covered graph G produce the same list of bricks and braces, up to multiple edges.

- The list underlying simple graphs of the bricks and the braces of a matching covered graph are its invariant.

Why is it interesting?

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- Several properties concerning problems in matching theory are preserved under tight cut decomposition thus reducing the problems to bricks and braces.
- For example, pfaffian orientations are useful in counting the number of PMs.

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A cool result

A matching covered graph is pfaffian iff each of its bricks and braces is pfaffian.

Enough! How to implement these?

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The implementation problem

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The implementation problem

Given a matching covered graph; either

1. deduce that it is a brick or a brace, or
2. find a nontrivial tight cut.

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The implementation problem

Given a matching covered graph; either

1. deduce that it is a brick or a brace, or
2. find a nontrivial tight cut.

It turns out that finding arbitrary tight cuts (if any) in a matching covered graph is not an easy task.

How to do in polytime?

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Edmonds-Lovász-Pulleyblank 1982

If a matching covered graph has a nontrivial tight cut, then it has a non trivial tight cut that is either a barrier cut, or a two-separation cut.

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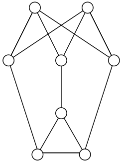

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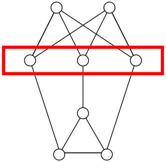

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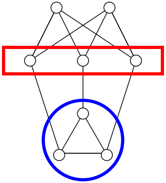

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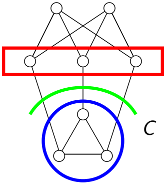

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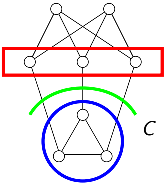

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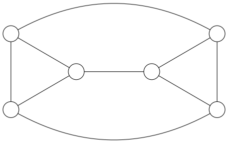

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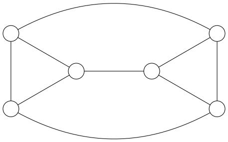

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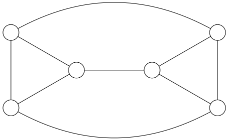

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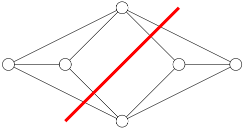

A bicritical graph that is not a brick

2-separation cut

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Bricks are free of 2 separation cuts as well. Morevoer, they are 3-connected.

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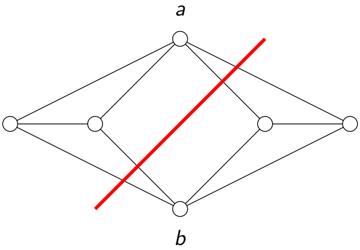

The illustration of a 2-sepration cut

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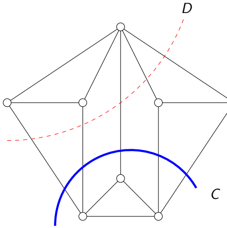

Figure: The cut C (shown in bold blue) is neither a barrier cut nor a 2-separation cut, but D (shown in dashed red) is a 2-separation cut

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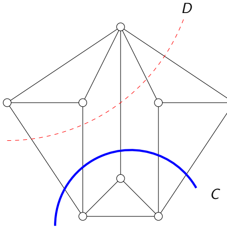

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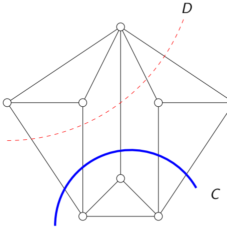

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- Finding an arbitrary tight cut is difficult, but finding a barrier cut or a 2-separation cut is polytime computable.
- The ELP theorem guarantees the existence of either a nontrivial barrier or a non trivial 2-separation cut, if at all the matching covered graph has a nontrivial tight cut.

Implementing tight cut decomposition procedure

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MCG G

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$$\begin{array}{ccc} & \text{MCG } G & \\ \swarrow & & \\ G \text{ is non-bipartite} & & \end{array}$$

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Implementing tight cut decomposition procedure

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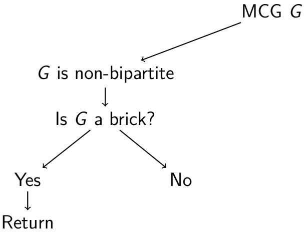
Return

Implementing tight cut decomposition procedure



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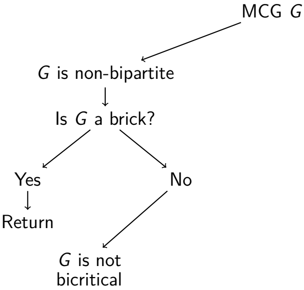

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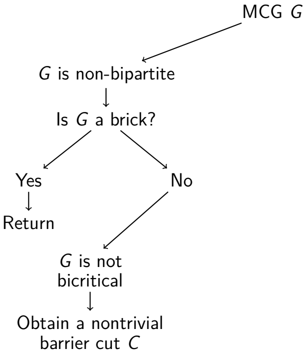

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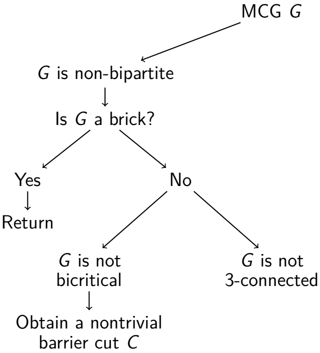

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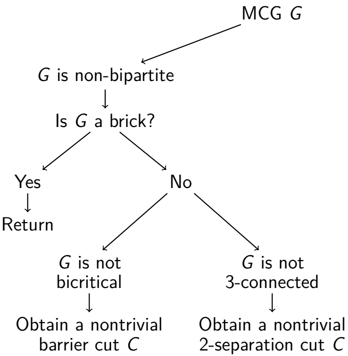

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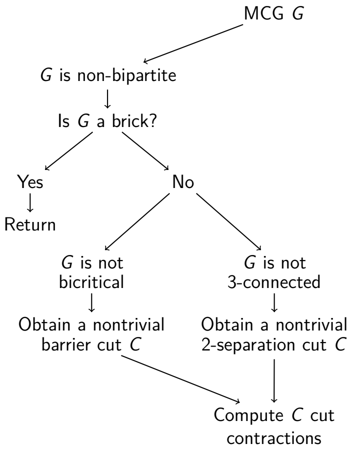

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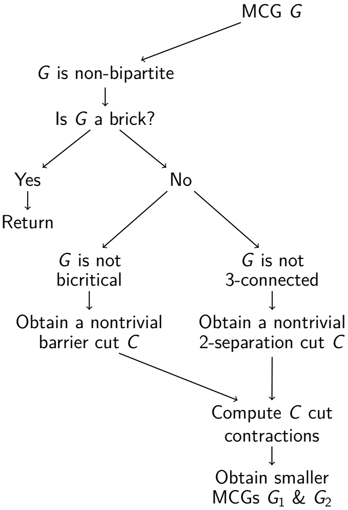

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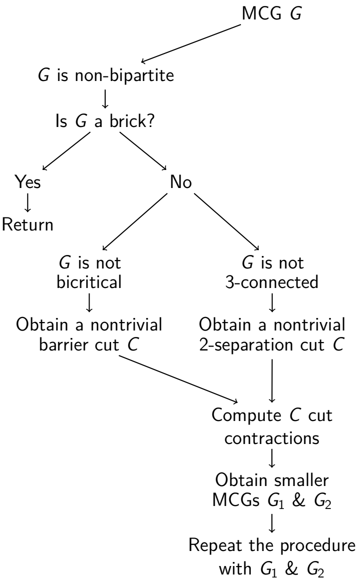

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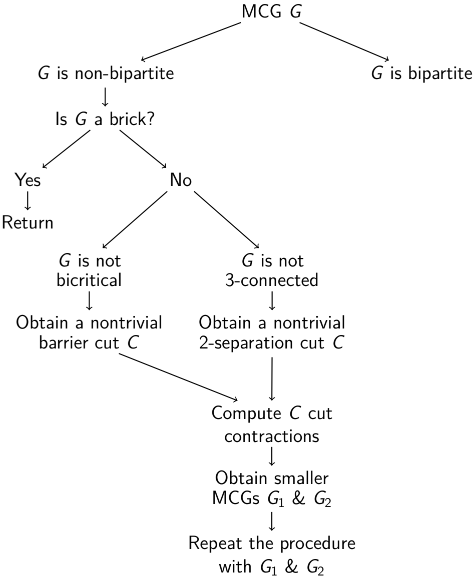

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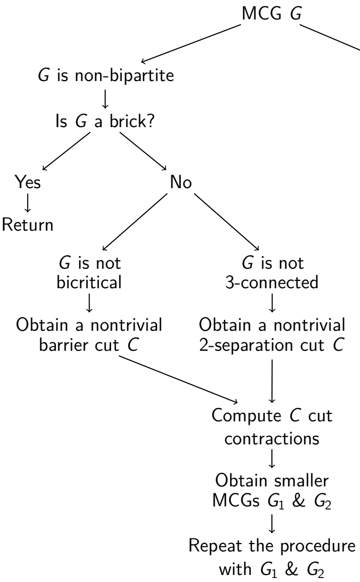

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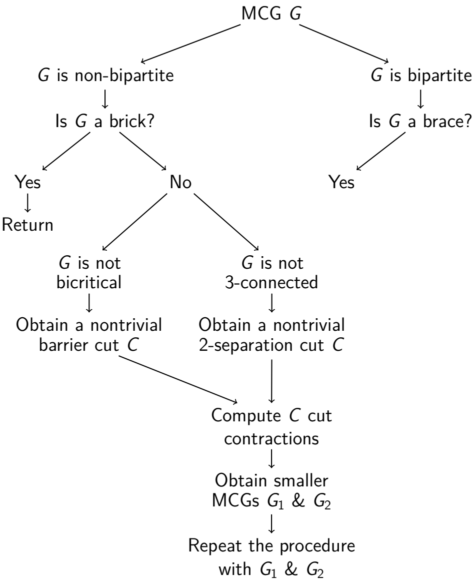

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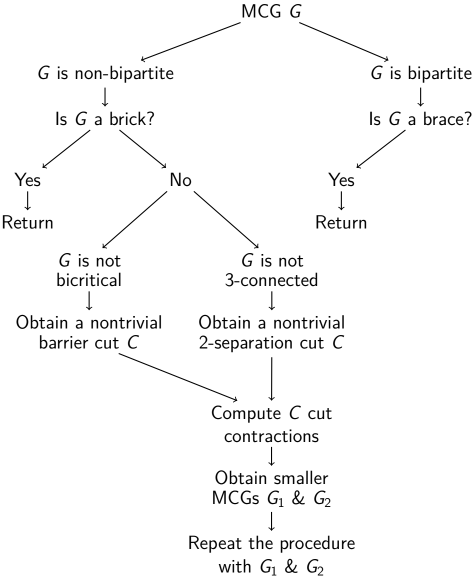

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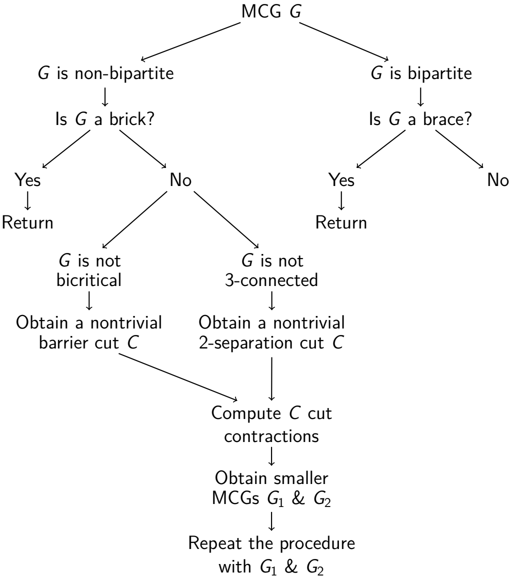

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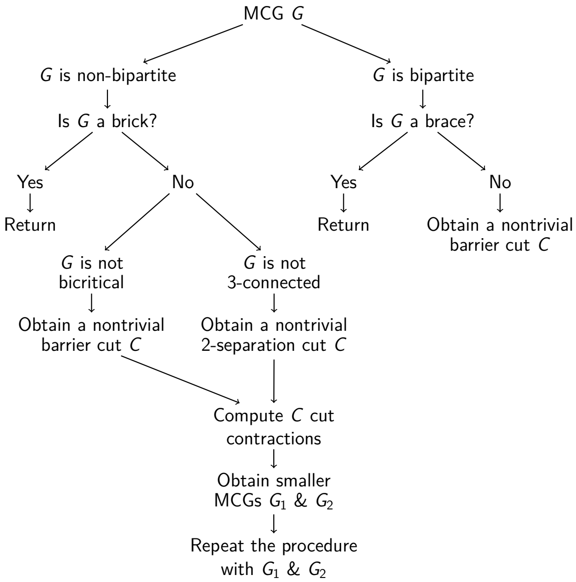

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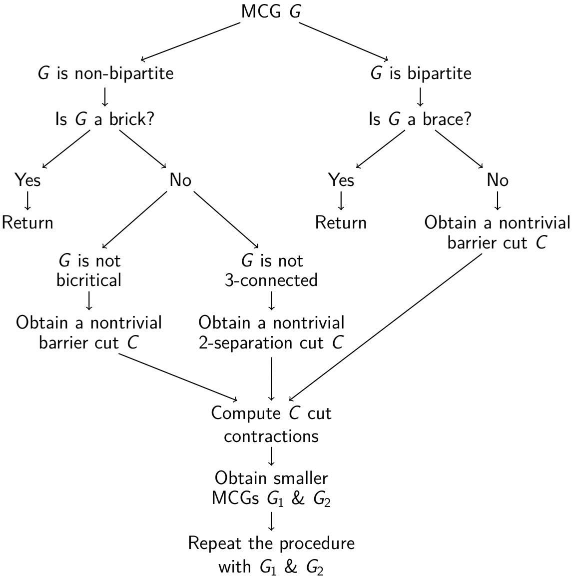

Notable families of bricks and braces

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These families play significant roles in the works of McCuaig and Norine and Thomas.

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The *Circular ladder graph*, aka *Prism graph* $\mathbb{P}_{2n}$, for $n \geqslant 3$, is the graph obtained from two disjoint cycles

$$u_1 u_2 u_3 \dots u_n u_1 \text{ and } v_1 v_2 \dots v_n v_1$$

of length n by the addition of the n edges $u_i v_i$, $i = 1, 2, \dots, n$.

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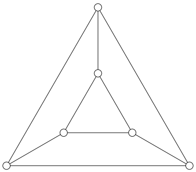

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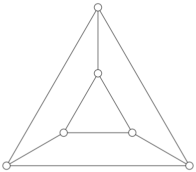

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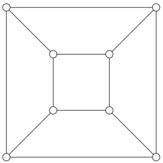

(b) CirclularLadderGraph(4)

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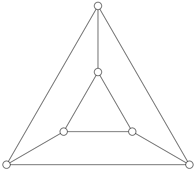

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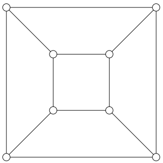

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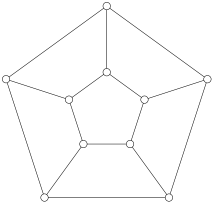

(c) CircularLadderGraph(5)

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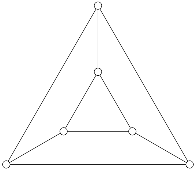

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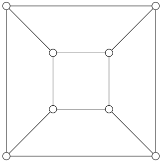

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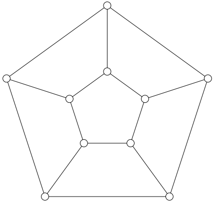

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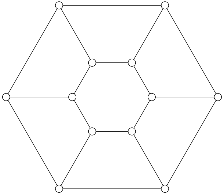

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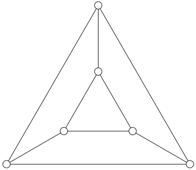
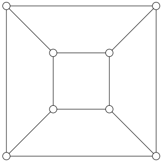
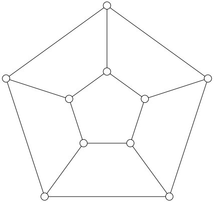


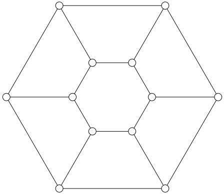
(a) CircularLadderGraph(3)



(b) CirclularLadderGraph(4)



(c) CircularLadderGraph(5)



(d) CirclularLadderGraph(6)

Figure: The family of circular ladder graphs

Notable families: Wheel graph

Notable families: Wheel graph

Wheel graph

Notable families: Wheel graph

Wheel graph

The *Wheel graph* $\mathbb{W}_n$, for $n \geqslant 4$, is the graph obtained from a cycle

$$v_1 v_2 \ldots v_{n-1} v_1$$

of length $n-1$, called the *rim* of $\mathbb{W}_n$, by the addition a universal vertex, called *hub* h.

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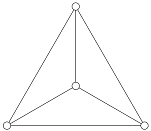
of length $n-1$, called the *rim* of $\mathbb{W}_n$, by the addition a universal vertex, called *hub* h. Note that wheels on an even number of vertices, aka W_{2k} for some $k \geqslant 2$, are matching covered — in fact, bricks.

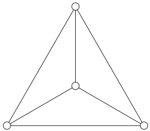
Notable families: Wheel graph

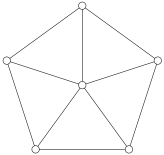
Notable families: Wheel graph



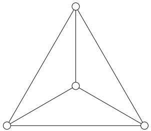
(a) WheelGraph(4)

Notable families: Wheel graph

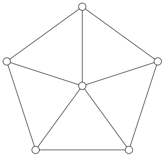

(a) WheelGraph(4)


(b) WheelGraph(6)

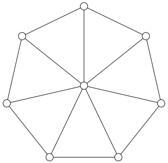
Notable families: Wheel graph



(a) WheelGraph(4)

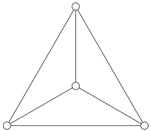


(b) WheelGraph(6)

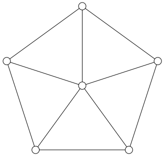


(c) WheelGraph(8)

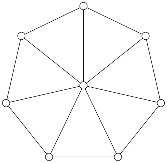
Notable families: Wheel graph



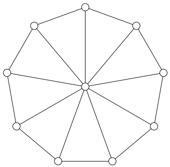
(a) WheelGraph(4)



(b) WheelGraph(6)

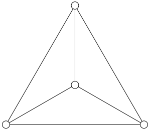


(c) WheelGraph(8)

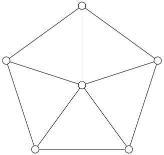


(d) WheelGraph(10)

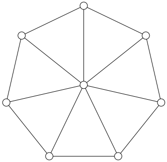
Notable families: Wheel graph



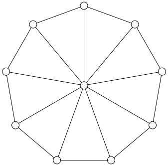
(a) WheelGraph(4)



(b) WheelGraph(6)



(c) WheelGraph(8)



(d) WheelGraph(10)

Figure: The family of wheel graphs, that are matching covered

Notable families: Möbius Ladder graph

Notable families: Möbius Ladder graph

Möbius Ladder graph

Notable families: Möbius Ladder graph

Möbius Ladder graph

The *Möbius ladder graph* $\mathbb{M}_{2n}$, for $n \geqslant 2$, is the graph obtained from a cycle

$$v_1 v_2 \dots v_{2n} v_1$$

of length $2n$ by the addition of the n chords $v_i v_{i+n}$, for $1 \leqslant i \leqslant n$, joining antipodal pairs of vertices of the cycle.

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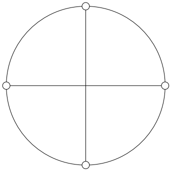
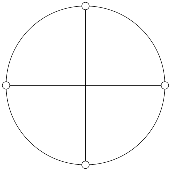


(a) MöbiusLadderGraph(2)

Notable families: Möbius Ladder graph



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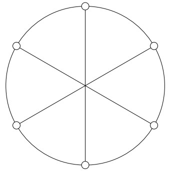
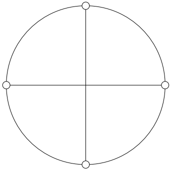
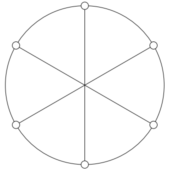


(b) MöbiusLadderGraph(3)

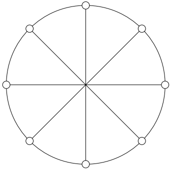
Notable families: Möbius Ladder graph



(a) MöbiusLadderGraph(2)

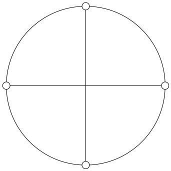


(b) MöbiusLadderGraph(3)



(c) MöbiusLadderGraph(4)

Notable families: Möbius Ladder graph



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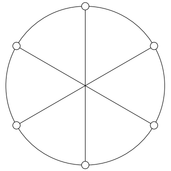
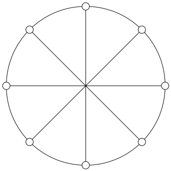
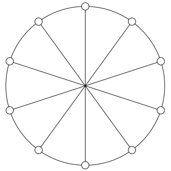


(b) MöbiusLadderGraph(3)

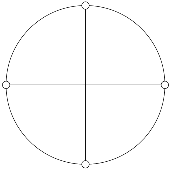


(c) MöbiusLadderGraph(4)



(d) MöbiusLadderGraph(5)

Notable families: Möbius Ladder graph



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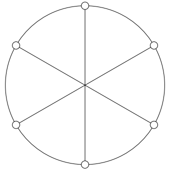
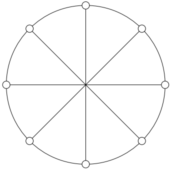
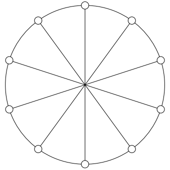


(b) MöbiusLadderGraph(3)



(c) MöbiusLadderGraph(4)



(d) MöbiusLadderGraph(5)

Figure: The family of möbius ladder graphs

Notable families: Biwheel graph

Notable families: Biwheel graph

Biwheel graph

Notable families: Biwheel graph

Biwheel graph

The *Biwheel graph* $\mathbb{B}_{2n}$, for $n \geqslant 4$, is the bipartite graph obtained from a cycle

$$v_1 v_2 \dots v_{2n-2} v_1$$

of length $2n-2$, called the *rim* of $\mathbb{B}_{2n}$, by the addition of two vertices, h_1 and h_2, called the *hubs* of $\mathbb{B}_{2n}$, and by the addition of edges $h_1 v_1, h_1 v_3, \dots, h_1 v_{2n-3}$ and edges $h_2 v_2, h_2 v_4, \dots, h_2 v_{2n-2}$.

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Notable families: Biwheel graph

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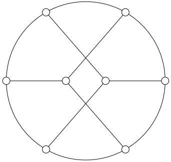
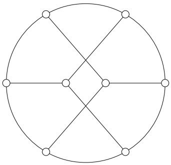
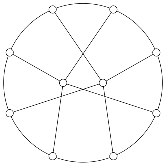


(a) BiwheelGraph(8)

Notable families: Biwheel graph

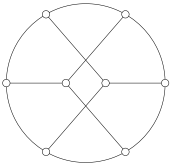


(a) BiwheelGraph(8)



(b) BiwheelGraph(10)

Notable families: Biwheel graph



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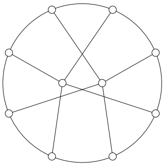
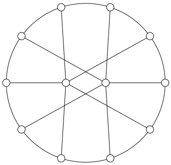
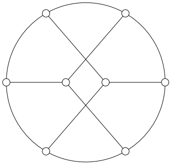


(b) BiwheelGraph(10)

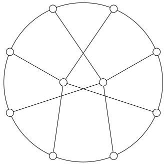


(c) BiwheelGraph(12)

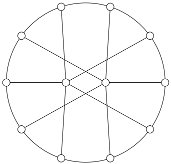
Notable families: Biwheel graph



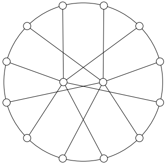
(a) BiwheelGraph(8)



(b) BiwheelGraph(10)

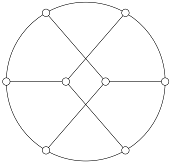


(c) BiwheelGraph(12)

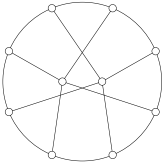


(d) BiwheelGraph(14)

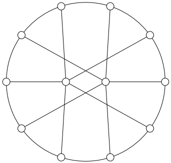
Notable families: Biwheel graph



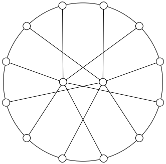
(a) BiwheelGraph(8)



(b) BiwheelGraph(10)



(c) BiwheelGraph(12)



(d) BiwheelGraph(14)

Figure: The family of biwheel graphs

Notable families: Truncated Biwheel graph

Notable families: Truncated Biwheel graph

Truncated Biwheel graph

Notable families: Truncated Biwheel graph

Truncated Biwheel graph

The *Truncated biwheel graph* $\mathbb{T}_{2n}$, for $n \geqslant 3$, is the graph obtained from a path $v_1 v_2 \ldots v_{2n-2}$ of length $2n-3$, by the addition of two vertices, h_1 and h_2, and by the addition of edges $h_1 v_1, h_1 v_3, \ldots, h_1 v_{2n-3}$, edges $h_2 v_2, h_2 v_4, \ldots, h_2 v_{2n-2}$ and edges $h_1 v_{2n-2}$ and $h_2 v_1$.

Notable families: Truncated Biwheel graph

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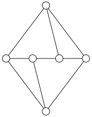
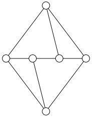
Notable families: Truncated Biwheel graph

Notable families: Truncated Biwheel graph



(a) TruncatedBiwheelGraph(3)

Notable families: Truncated Biwheel graph



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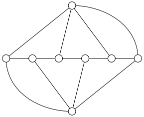
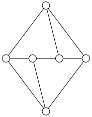


(b) TruncatedBiwheelGraph(4)

Notable families: Truncated Biwheel graph



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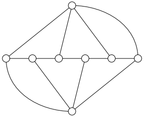
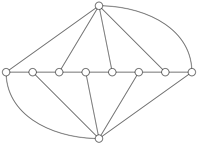
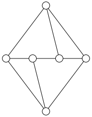


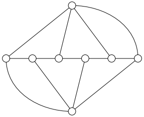
(b) TruncatedBiwheelGraph(4)

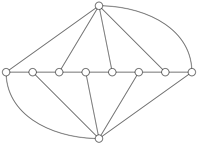


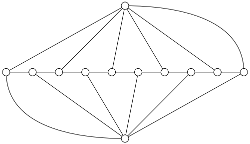
(c) TruncatedBiwheelGraph(5)

Notable families: Truncated Biwheel graph

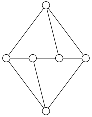

(a) TruncatedBiwheelGraph(3)


(b) TruncatedBiwheelGraph(4)

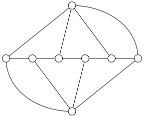

(c) TruncatedBiwheelGraph(5)


(d) TruncatedBiwheelGraph(6)

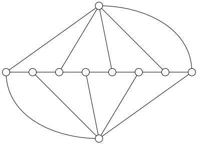
Notable families: Truncated Biwheel graph



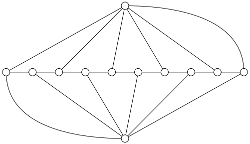
(a) TruncatedBiwheelGraph(3)



(b) TruncatedBiwheelGraph(4)



(c) TruncatedBiwheelGraph(5)



(d) TruncatedBiwheelGraph(6)

Figure: The family of truncated biwheel graphs

Notable families: Staircase graph

Notable families: Staircase graph

Staircase graph

Notable families: Staircase graph

Staircase graph

Consider the Ladder graph $\mathbb{L}_{2n-2}$ obtained from two disjoint paths $u_1 u_2 \ldots u_{n-1}$ and $v_1 v_2 \ldots v_{n-1}$ by adding, for $1 \leqslant i \leqslant n-1$, an edge joining u_i and v_i. For $n \geqslant 3$, the *Staircase graph* $\mathbb{S}_{2n}$ is the graph obtained from $\mathbb{L}_{2n-2}$ by adding two new vertices x and y, and then joining x to u_1 and v_1, the vertex y to u_{n-1} and v_{n-1}, and x and y to each other.

Staircase graph

Consider the Ladder graph $\mathbb{L}_{2n-2}$ obtained from two disjoint paths $u_1 u_2 \ldots u_{n-1}$ and $v_1 v_2 \ldots v_{n-1}$ by adding, for $1 \leqslant i \leqslant n-1$, an edge joining u_i and v_i. For $n \geqslant 3$, the *Staircase graph* $\mathbb{S}_{2n}$ is the graph obtained from $\mathbb{L}_{2n-2}$ by adding two new vertices x and y, and then joining x to u_1 and v_1, the vertex y to u_{n-1} and v_{n-1}, and x and y to each other.

Notable families: Staircase graph

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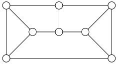
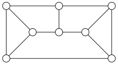
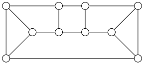


(a) StaircaseGraph(4)

Notable families: Staircase graph

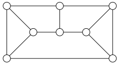


(a) StaircaseGraph(4)



(b) StaricaseGraph(5)

Notable families: Staircase graph



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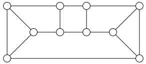
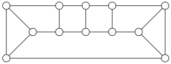
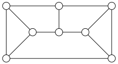


(b) StaricaseGraph(5)



(c) StaircaseGraph(6)

Notable families: Staircase graph



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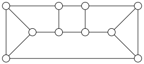

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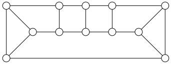
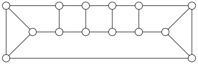
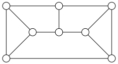


(c) StaircaseGraph(6)



(d) StaircaseGraph(7)

Notable families: Staircase graph



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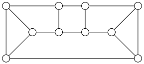
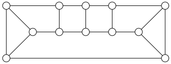


(b) StaricaseGraph(5)



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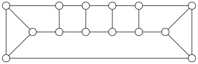

(d) StaircaseGraph(7)

Figure: The family of staircase graphs

An overview

An overview

Module	Method	Status

An overview

	Module	Method	Status
A	Fundamentals	M_alternating_tree_mark()	Implemented
		is_matching_covered()	Implemented
		is_bicritical()	Implemented

An overview

	Module	Method	Status
A	Fundamentals	M_alternating_tree_mark()	Implemented
		is_matching_covered()	Implemented
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		canonical_partition()	Implemented

An overview

	Module	Method	Status
A	Fundamentals	M_alternating_tree_mark()	Implemented
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		is_bicritical()	Implemented
B	Canonical Partition	maximal_barrier()	Implemented
		canonical_partition()	Implemented
C	Tight cut decomposition	is_brick()	Implemented
		is_brace()	Implemented
		tight_cut_decomposition()	Implemented
		bricks_and_braces()	Implemented
		number_of_bricks()	Implemented
		number_of_braces()	Implemented
		number_of_petersen_bricks()	Implemented

An overview (continued)

An overview (continued)

Module	Method	Status

An overview (continued)

	Module	Method	Status
D	Notable Families	MöbiusLadderGraph()	Implemented
		StaircaseGraph()	Implemented
		BiwheelGraph()	Ongoing
		TruncatedBiwheelGraph()	Implemented

An overview (continued)

	Module	Method	Status
D	Notable Families	MöbiusLadderGraph()	Implemented
		StaircaseGraph()	Implemented
		BiwheelGraph()	Ongoing
		TruncatedBiwheelGraph()	Implemented
E	Dependency relation	is_removable_edge()	Implemented
		removable_edges()	Implemented
		is_removable_doubleton()	Implemented
		removable_doubletons()	Implemented

An overview (continued)

An overview (continued)

Module	Method	Status

An overview (continued)

	Module	Method	Status
F	Ear decomposition	retract()	Ongoing
		matching_covered_ear_decomposition()	To be done
			To be done
		optimal_ear()	To be done
		optimal_ear_decomposition()	To be done

An overview (continued)

	Module	Method	Status
F	Ear decomposition	retract()	Ongoing
		matching_covered_ear_decomposition()	To be done
		optimal_ear()	To be done
		optimal_ear_decomposition()	To be done
G	Brick and brace generation	is_strictly_thin_edges()	To be done
		mccuaig_brace_decomposition()	To be done
		norine_thomas_brick_decomposition()	To be done

What's next?

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- One major goal is to make these implementations accessible for everyone.

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- Proposal to SageMath (via Google Summer of Code 2024).

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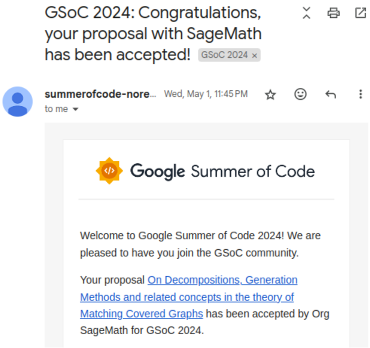

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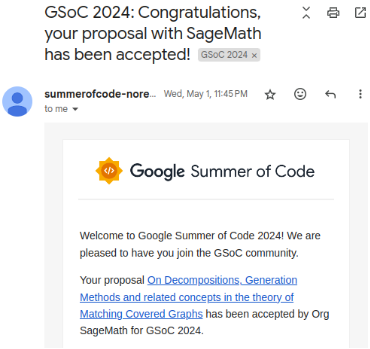
GSoC 2024: Congratulations, your proposal with SageMath has been accepted! GSoC 2024 ×

summerofcode-nore... Wed, May 1, 11:45 PM
to me

Google Summer of Code

Welcome to Google Summer of Code 2024! We are pleased to have you join the GSoC community.

Your proposal On Decompositions, Generation Methods and related concepts in the theory of Matching Covered Graphs has been accepted by Org SageMath for GSoC 2024.

- Should be accessible by everyone from 26th of August 2024.

Yes!

Yes! you folks made my day.

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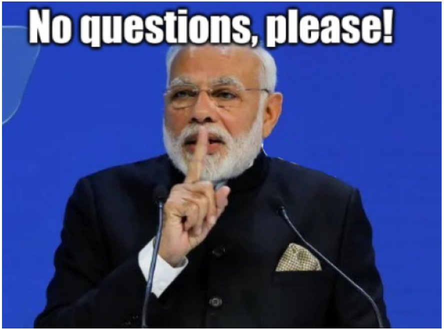

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