On Decompositions, Generation Methods, and related concepts in the theory of Matching Covered Graphs

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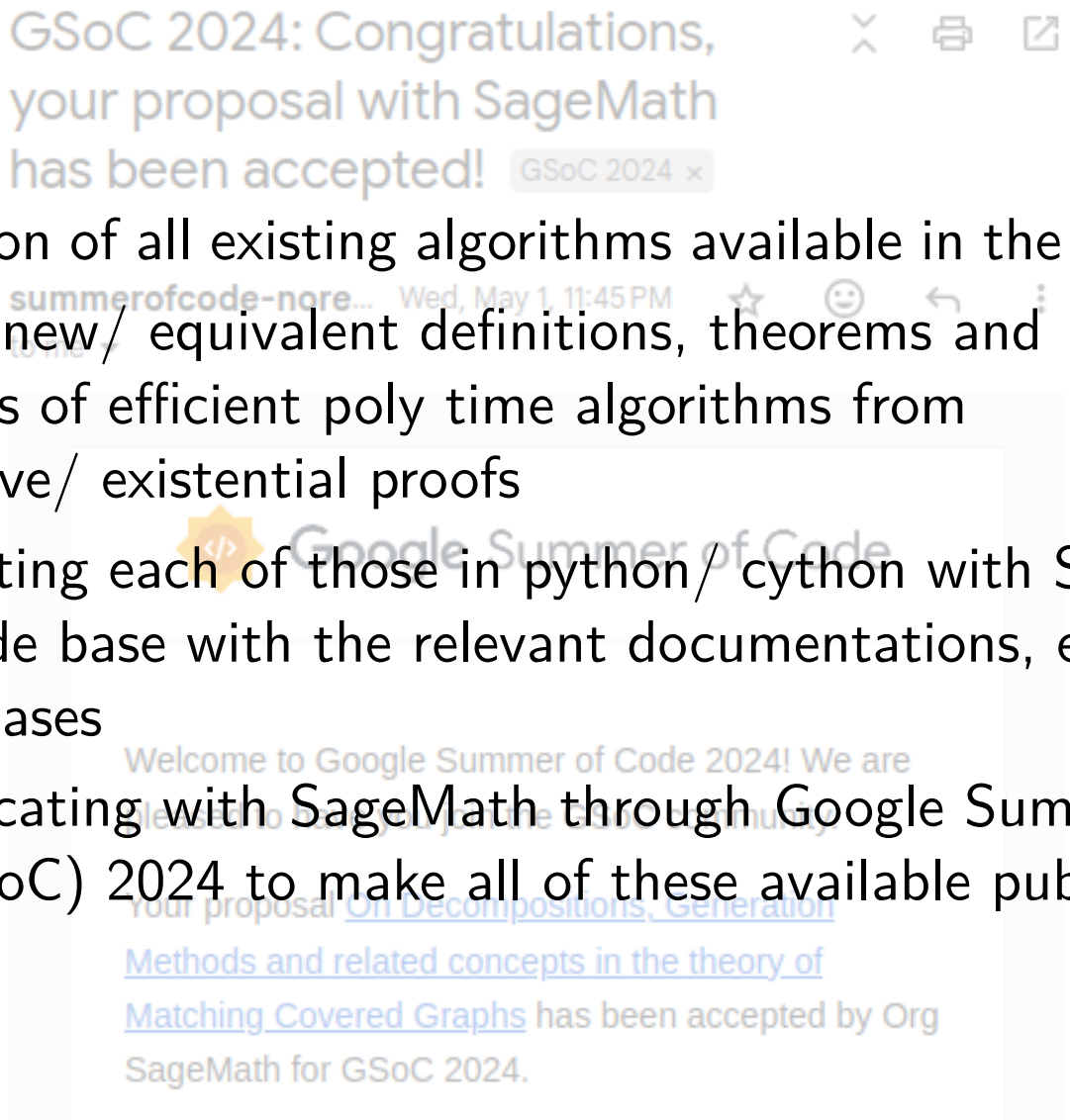
Problem Statement

Implementing efficient poly-time algorithms pertaining to

- the fundmental decision problems,
- the canonical partition,
- tight cut decomposition,
- dependency relations,
- (optimal) ear decomposition,
- brick and brace generation methods, and
- related concepts in the theory of matching covered graphs,

and to make all of these available freely to students, educators as well as researchers all across the world.

Methodology

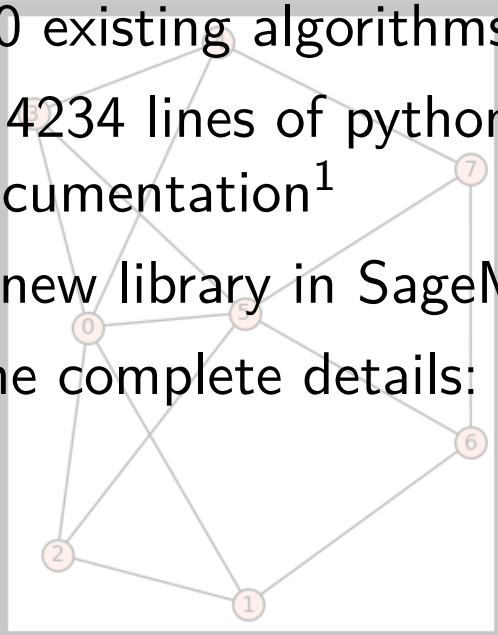


- Compilation of all existing algorithms available in the literature
- Devise of new/ equivalent definitions, theorems and derivations of efficient poly time algorithms from constructive/ existential proofs
- Implementing each of those in python/ cython with SageMath as the code base with the relevant documentations, examples and test cases
- Communicating with SageMath through Google Summer of Code (GSoC) 2024 to make all of these available publicly.

Novelty and Results

```
G = Graph()
G.add_edges([(0, 1), (0, 2), (0, 3), (0, 4), (0, 5),
             (1, 2), (1, 6), (2, 5), (3, 5), (3, 4),
             (4, 7), (5, 6), (5, 7), (6, 7)])
M = Graph(G.matching())
show(G)

try:
    answer = is_brick(G, M, matching_covered_check=True)
    print('Is G a brick? {}'.format(answer))
except ValueError as error:
    print('Error: {}'.format(error))
```




```
Is G a brick? True
```

- 31 functional methods, 12 graph generator methods and 21 algorithms
(10 existing algorithms, 11 formulated/ derived)
- $\approx$ 4234 lines of python/ cython code and $\approx$ 8807 lines of documentation[1]
- A new library in SageMath: Matching Covered Graphs
- The complete details: Link

[1] An estimation

Relevance

- Matchings and pefect matchings: A fundamental basis of graph theory, algorithms and optimization
- Existence of several open problems (Barnette's conjecture, Berge-Fulkerson conjecture, and so on): use matchings at their heart
- A free complete public efficient library/ module for Matching covered graphs in sage: a solution to numerical and computational aspect of research in matching theory