SDS 383D, Bayesian Inference in Simple Conjugate Families

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F: Marginalization of likelihood with Gamma Prior and Normal Likelihood

$$p(x) = \int_0^\infty p(x,\omega)d\omega = \int_0^\infty p(\omega)p(x|\omega)d\omega \tag{1}$$

$$p(\omega) = \frac{(b/2)^{a/2}}{\Gamma(a/2)}\omega^{a/2-1}\exp\left(-\frac{b}{2}\omega\right) \tag{2}$$

$$p(x|\omega) = \left(\frac{\omega}{2\pi}\right)^{1/2}\exp\left[-\frac{\omega}{2}(x-m)^2\right] \tag{3}$$

$$p(x,\omega) = \frac{(b/2)^{a/2}}{\Gamma(a/2)}\left(\frac{1}{2\pi}\right)^{1/2}(\omega)^{a/2+1/2-1}\exp\left[-\omega\left(\frac{b}{2}+\frac{1}{2}(x-m)^2\right)\right] \tag{4}$$

$$a' = \frac{a+1}{2}; \qquad b' = \frac{b}{2}+\frac{1}{2}(x-m)^2 \tag{5}$$

$$p(x) = \int_0^\infty p(x,\omega)d\omega = \frac{(b/2)^{a/2}}{\Gamma(a/2)\sqrt{2\pi}}\frac{\Gamma(a')}{b'^{a'}}\int_0^\infty \frac{b'^{a'}}{\Gamma(a')}\omega^{a'-1}\exp(-b'\omega)d\omega \tag{6}$$

$$p(x) = \frac{(b/2)^{a/2}}{\Gamma(a/2)\sqrt{2\pi}}\frac{\Gamma(\frac{a+1}{2})}{(b/2+(x-m)^2/2)^{(a+1)/2}}\cdot\frac{(b/2)^{1/2}}{(b/2)^{1/2}} \tag{7}$$

$$\left(\frac{b}{2}+\frac{1}{2}(x-m)^2\right)^{-\frac{a+1}{2}} = \left(\frac{b}{2}\right)^{-\frac{a+1}{2}}\left(1+\frac{(x-m)^2}{b}\right)^{-\frac{a+1}{2}} \tag{8}$$

$$p(x) = \frac{\Gamma\left(\frac{a+1}{2}\right)}{\Gamma(\frac{a}{2})\sqrt{\pi b}}\left(1+\frac{(x-m)^2}{b}\right)^{-\frac{a+1}{2}} \tag{9}$$