SDS 383D The Multivariate Normal Distribution

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C: Multivariate Normal for x

$$x = (x_1, ..., x_p)^T, \quad x \sim \mathrm{N}(\mu_x, \sigma_x^2) \tag{1}$$

$$z = a^T x \tag{2}$$

$$\mu_z = a^T \mu_x \tag{3}$$

$$\sigma_z^2 = a^T \sigma_x^2 a \tag{4}$$

$$y = \frac{z - \mu_z}{\sigma_z}, \quad z = \sigma_z y + \mu_z \tag{5}$$

$$f_Z(z) = \left(\frac{1}{\sqrt{2\pi\sigma_z^2}}\right) \exp\left(-\frac{1}{2}(z - \mu_z)^2\right) \tag{6}$$

$$f_Y(y) = f_Z(h(y)) \cdot |h'(y)| \tag{7}$$

$$\begin{aligned}
M_z(t) &= \int_{-\infty}^{\infty} \exp(t\sigma_z y + \mu_z t) \left(\frac{1}{\sqrt{2\pi\sigma_z^2}}\right) \exp\left(-\frac{1}{2\sigma_z^2}(\sigma_z y)^2\right) \left|\frac{dz}{dy}\right| dy && (8)\\
&= \exp(\mu_z t) \int_{-\infty}^{\infty} \exp(t\sigma_z y) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) dy && (9)\\
&= \exp(\mu_z t) \exp\left(\frac{1}{2}\sigma_z^2 t^2\right) && (10)\\
&= \exp\left(\mu_z t + \frac{1}{2}\sigma_z^2 t^2\right) && (11)\\
&= \exp\left(t a^T \mu_x + \frac{1}{2} t^2 a^T \sigma_x^2 a\right) && (12)
\end{aligned}$$