SDS 383D Conditionals and Marginals

Jan-Michael Cabrera, JC7858

February 5, 2019

B: Inverse of the Covariance Matrix

Let $\Omega = \Sigma^{-1}$, partition Ω like Σ

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^T & \Omega_{22} \end{bmatrix} \tag{1}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} \tag{2}$$

$$\Sigma\Omega = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3}$$

$$\Sigma\Omega = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^T & \Omega_{22} \end{bmatrix} \tag{4}$$

$$= \begin{bmatrix} \Sigma_{11}\Omega_{11} + \Sigma_{12}\Omega_{12}^T & \Sigma_{11}\Omega_{12} + \Sigma_{12}\Omega_{22} \\ \Sigma_{12}^T\Omega_{11} + \Sigma_{22}\Omega_{12}^T & \Sigma_{12}^T\Omega_{12} + \Sigma_{22}\Omega_{22} \end{bmatrix} \tag{5}$$

$$\Sigma_{11}\Omega_{11} + \Sigma_{12}\Omega_{12}^T = I \tag{6}$$

$$\Sigma_{11}\Omega_{12} + \Sigma_{12}\Omega_{22} = 0 \tag{7}$$

$$\Sigma_{12}^T\Omega_{11} + \Sigma_{22}\Omega_{12}^T = 0 \tag{8}$$

$$\Sigma_{12}^T\Omega_{12} + \Sigma_{22}\Omega_{22} = I \tag{9}$$

$$\Omega_{12} = -\Sigma_{11}^{-1}\Sigma_{12}\Omega_{22} \tag{10}$$

$$\Omega_{12}^T = -\Sigma_{22}^{-1}\Sigma_{12}^T\Omega_{11} \tag{11}$$

$$\Sigma_{11}\Omega_{11} + \Sigma_{12}(-\Sigma_{22}^{-1}\Sigma_{12}^T\Omega_{11}) = (\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T)\Omega_{11} = I \tag{12}$$

$$\Omega_{11} = (\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T)^{-1} \tag{13}$$

$$\Sigma_{12}^T(-\Sigma_{11}^{-1}\Sigma_{12}\Omega_{22}) + \Sigma_{22}\Omega_{22} = (\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12})\Omega_{22} = I \tag{14}$$

$$\Omega_{22} = (\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12})^{-1} \tag{15}$$

$$\Omega_{12} = -\Sigma_{11}^{-1}\Sigma_{12}(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12})^{-1} \tag{16}$$

$$\Omega_{12}^T = -\Sigma_{22}^{-1}\Sigma_{12}^T(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T)^{-1} \tag{17}$$

$$\Omega = \Sigma^{-1} = \begin{bmatrix} (\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T)^{-1} & -\Sigma_{11}^{-1}\Sigma_{12}(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12})^{-1} \\ -\Sigma_{22}^{-1}\Sigma_{12}^T(\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T)^{-1} & (\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12})^{-1} \end{bmatrix} \tag{18}$$