# SDS 383D, Bayesian Inference in Simple Conjugate Families

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## D: Normal prior and sampling distribution with known mean and unknown variance

$$p(\omega|x_1, ..., x_n) \propto p(\omega|a, b)p(x_1, ..., x_n|\omega) \tag{1}$$

$$p(\omega|a, b) = \frac{b^a}{\Gamma(a)}\omega^{a-1}\exp(-b\omega) \tag{2}$$

$$p(x_i|\omega) = \left(\frac{\omega}{2\pi}\right)^{1/2} \exp\left[-\frac{\omega}{2}(x_i - \theta)^2\right] \tag{3}$$

$$p(x_1, ..., x_n|\omega) = \prod_i^n p(x_i|\omega) = \left(\frac{\omega}{2\pi}\right)^{n/2} \exp\left[-\frac{\omega}{2}\sum_i^n (x_i - \theta)^2\right] \tag{4}$$

$$p(\omega|x_1, ..., x_n) \propto \omega^{a-1}\exp(-b\omega)\omega^{n/2}\exp\left[-\frac{\omega}{2}\sum_i^n (x_i - \theta)^2\right] \tag{5}$$

$$p(\omega|x_1, ..., x_n) \propto \omega^{a+\frac{n}{2}-1}\exp\left[-\omega\left(b + \frac{1}{2}\sum_i^n (x_i - \theta)^2\right)\right] \tag{6}$$

$$\omega|x_1, ..., x_n \sim \text{Ga}\left(a + \frac{n}{2}, b + \frac{1}{2}\sum_i^n (x_i - \theta)^2\right) \tag{7}$$

$$\sigma^2|x_1, ..., x_n \sim \text{IG}\left(a + \frac{n}{2}, b + \frac{1}{2}\sum_i^n (x_i - \theta)^2\right) \tag{8}$$