SDS 383D The Multivariate Normal Distribution

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G: Addition of Two Independent Multivariate Normals

$$x_1 \sim \mathrm{N}(\mu_1, \Sigma_1), \qquad x_2 \sim \mathrm{N}(\mu_2, \Sigma_2) \tag{1}$$

$$y = Ax_1 + Bx_2 \tag{2}$$

$$\begin{aligned} M_y(t) &= \mathrm{E}\left(\exp(t^T y)\right) && (3)\\ &= \mathrm{E}\left(\exp(t^T(Ax_1 + Bx_2))\right) && (4)\\ &= \mathrm{E}\left(\exp((A^T t)^T x_1\right) \mathrm{E}\left(\exp((B^T t)^T x_2\right) && (5) \end{aligned}$$

$$M_x(t) = \mathrm{E}(\exp(t^T x)) = \exp\left(t^T \mu + \frac{1}{2} t^T \Sigma t\right) \tag{6}$$

$$\begin{aligned} M_y(t) &= \exp\left((A^T t)^T \mu_1 + \frac{1}{2}(A^T t)^T \Sigma_1 (A^T t)\right) \exp\left((B^T t)^T \mu_2 + \frac{1}{2}(B^T t)^T \Sigma_2 (B^T t)\right) && (7)\\ &= \exp\left((A^T t)^T \mu_1 + \frac{1}{2}(A^T t)^T \Sigma_1 (A^T t) + (B^T t)^T \mu_2 + \frac{1}{2}(B^T t)^T \Sigma_2 (B^T t)\right) && (8)\\ &= \exp\left(t^T (A\mu_1 + B\mu_2) + \frac{1}{2} t^T (A\Sigma_1 A^T + B\Sigma_2 B^T) t\right) && (9) \end{aligned}$$

$$y \sim \mathrm{N}(A\mu_1 + B\mu_2, A\Sigma_1 A^T + B\Sigma_2 B^T) \tag{10}$$