

SDS 383D The Multivariate Normal Distribution

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F: Multivariate Normal PDF

$$f_Z(z) = \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left[-\frac{1}{2}z^T z\right] \quad (1)$$

$$x = Lz + \mu, \quad z = L^{-1}(x - \mu) \quad (2)$$

$$f_X(x) = f_Z(h(z)) \cdot |\det(\nabla h(z))| \quad (3)$$

$$= f_Z(L^{-1}(x - \mu)) \cdot |\det(L^{-1})| \quad (4)$$

$$= \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left[-\frac{1}{2}(L^{-1}(x - \mu))^T (L^{-1}(x - \mu))\right] \cdot |\det(L^{-1})| \quad (5)$$

$$= \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left[-\frac{1}{2}(x - \mu)^T (L^{-1})^T L^{-1}(x - \mu)\right] \cdot |\det(L^{-1})| \quad (6)$$

$$|\det(L^{-1})| = \frac{1}{\sqrt{\det L^T L}}, \quad (L^T)^{-1} = (L^{-1})^T, \quad L^T L = \Sigma \quad (7)$$

$$f_X(x) = \left(\frac{1}{\sqrt{2\pi}}\right)^n \frac{1}{\sqrt{\det \Sigma}} \exp\left[-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right] \quad (8)$$