SDS 383D The Multivariate Normal Distribution

Jan-Michael Cabrera, JC7858

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B: PDF and moment-generating function of z

$$z = (z_1, ..., z_p)^T; \qquad z_i \sim \mathrm{N}(0,1) \tag{1}$$

$$p(z) = \prod_i p(z_i) = \left(\frac{1}{\sqrt{2\pi}}\right)^{1/n} \exp\left[-\frac{1}{2}\sum_i z_i^2\right] \tag{2}$$

$$M_{z_i}(t_i) = \mathrm{E}[\exp(t_i z_i)] = \int_{-\infty}^{\infty} \exp(t_i z_i)\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z_i^2\right) dz_i \tag{3}$$

$$\exp(t_i z_i)\exp\left(-\frac{1}{2}z_i^2\right) = \exp\left(-\frac{1}{2}z_i^2 + z_i t_i\right) = \exp\left(-\frac{1}{2}(z_i - t_i)^2\right)\exp\left(\frac{1}{2}t_i^2\right) \tag{4}$$

$$M_{z_i}(t_i) = \exp\left(\frac{1}{2}t_i^2\right)\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(z_i - t_i)^2\right) dz_i = \exp\left(\frac{1}{2}t_i^2\right) \tag{5}$$

$$M_z(\mathbf{t}) = \mathrm{E}\left[\exp\left(\sum_i z_i t_i\right)\right] = \mathrm{E}\left[\prod_i \exp(z_i t_i)\right] \tag{6}$$

$$= \prod_i \mathrm{E}[\exp(z_i t_i)] = \prod_i M_{z_i}(t_i) \tag{7}$$

$$= \exp\left(\sum_i \frac{1}{2}t_i^2\right) = \exp\left(\frac{1}{2}\mathbf{t}^{\mathbf{T}}\mathbf{t}\right) \tag{8}$$