SDS 383D The Multivariate Normal Distribution

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A: Covariance of the multivariate normal

$$\begin{aligned}
\text{cov}(x) &= \text{E}[(x-\mu)(x-\mu)^T] && (1)\\
&= \text{E}[xx^T - x\mu^T - \mu x^T + \mu\mu^T] && (2)\\
&= \text{E}[xx^T] - \text{E}[x]\text{E}[\mu^T] - \text{E}[\mu]\text{E}[x^T] + \text{E}[\mu\mu^T] && (3)\\
& && (4)
\end{aligned}$$

$$\text{E}[x] = \mu; \qquad \text{E}[\mu] = \mu \tag{5}$$

$$\begin{aligned}
\text{cov}(x) &= \text{E}[xx^T] - 2\mu\mu^T + \mu\mu^T && (6)\\
&= \text{E}[xx^T] - \mu\mu^T && (7)
\end{aligned}$$

$$\text{cov}(X, Y) = 0; \quad \text{if } X \text{ and } Y \text{ are independent} \tag{8}$$

$$\begin{aligned}
\text{cov}(Ax+b) &= \text{cov}(Ax, Ax) + \text{cov}(Ax, b) + \text{cov}(b, Ax) + \text{cov}(b, b) && (9)\\
&= A\text{cov}(x)A^T && (10)
\end{aligned}$$