SDS 383D The Multivariate Normal Distribution

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D: Affine Transformation of Independent Normals

$$x = Lz + \mu; \qquad z \sim \mathrm{N}(0, 1) \tag{1}$$

$$M_z(t) = \mathrm{E}[\exp(t^T z)] = \exp\left(\frac{1}{2} t^T t\right) \tag{2}$$

$$\begin{aligned} M_x(t) &= \mathrm{E}[\exp(t^T(Lz + \mu))] && (3)\\ &= \mathrm{E}[\exp(t^T\mu)\exp(t^T(Lz))] && (4)\\ &= \exp(t^T\mu) M_z(L^T t) && (5)\\ &= \exp(t^T\mu)\exp\left(\frac{1}{2}(L^T t)^T(L^T t)\right) && (6)\\ &= \exp\left(t^T\mu + \frac{1}{2} t^T L L^T t\right) && (7) \end{aligned}$$

$$\mathrm{E}[x] = \mathrm{E}[Lz] + \mathrm{E}[\mu] = L\mathrm{E}[z] + \mu = \mu \tag{8}$$

$$\begin{aligned} \mathrm{cov}(x) &= \mathrm{cov}(Lz, Lz) + \mathrm{cov}(Lz, \mu) + \mathrm{cov}(\mu, Lz) + \mathrm{cov}(\mu, \mu) && (9)\\ &= L\mathrm{cov}(z, z)L^T = LL^T && (10) \end{aligned}$$