SDS 383D The Multivariate Normal Distribution

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F: Multivariate Normal PDF

$$f_Z(z) = \left(\frac{1}{\sqrt{2\pi}}\right)^{1/n} \exp\left[-\frac{1}{2}z^T z\right] \tag{1}$$

$$x = Lz + \mu, \qquad z = L^{-1}(x - \mu) \tag{2}$$

$$\begin{aligned}
f_X(x) &= f_Z(h(z)) \cdot |\det(\nabla h(z))| && (3)\\
&= f_Z(L^{-1}(x-\mu)) \cdot |\det(L^{-1})| && (4)\\
&= \left(\frac{1}{\sqrt{2\pi}}\right) \exp\left[-\frac{1}{2}(L^{-1}(x-\mu))^T(L^{-1}(x-\mu))\right] \cdot |\det(L^{-1})| && (5)\\
&= \left(\frac{1}{\sqrt{2\pi}}\right) \exp\left[-\frac{1}{2}(x-\mu)^T(L^{-1})^T L^{-1}(x-\mu)\right] \cdot |\det(L^{-1})| && (6)
\end{aligned}$$

$$|\det(L^{-1})| = \frac{1}{\sqrt{\det L^T L}}, \qquad (L^T)^{-1} = (L^{-1})^T, \qquad L^T L = \Sigma \tag{7}$$

$$f_X(x) = \left(\frac{1}{\sqrt{2\pi}}\right)^{1/n} \frac{1}{\sqrt{\det \Sigma}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right] \tag{8}$$