SDS 383D The Multivariate Normal Distribution

Jan-Michael Cabrera, JC7858

February 4, 2019

C: Multivariate Normal for x

$$x = (x_1, ..., x_p)^T; \qquad z_i \sim \mathrm{N}(\mu_i, \sigma_i^2) \tag{1}$$

$$z_i = \frac{x_i - \mu_i}{\sigma_i} \qquad x_i = z_i\sigma_i + \mu_i \tag{2}$$

$$f_Y(y) = f_X(h(y)) \cdot |h'(y)| \tag{3}$$

$$\begin{aligned}
M_{x_i}(t_i) &= \int_{-\infty}^{\infty} \exp(z_i\sigma_i t_i + \mu_i t_i) \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2\sigma_i^2}(z_i\sigma_i)^2\right) \left|\frac{dx_i}{dz_i}\right| dz_i && (4)\\
&= \exp(\mu_i t_i) \int_{-\infty}^{\infty} \exp(z_i\sigma_i t_i) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z_i^2\right) dz_i && (5)\\
&= \exp(\mu_i t_i) \exp\left(\frac{1}{2}\sigma_i^2 t_i^2\right) && (6)
\end{aligned}$$

$$\begin{aligned}
M_x(\mathbf{t}) &= \prod_i M_{x_i}(t_i) && (7)\\
&= \exp\left(\sum_i \mu_i t_i\right) \exp\left(\sum_i \frac{1}{2}\sigma_i^2 t_i^2\right) && (8)\\
&= \exp(\mathbf{t^T}\mu + \mathbf{\frac{1}{2}t^T\Sigma t}); \quad \mathrm{cov}(x) = \mathbf{\Sigma} && (9)
\end{aligned}$$