

# SDS 383D The Multivariate Normal Distribution

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## G: Addition of Two Independent Multivariate Normals

$$x_1 \sim N(\mu_1, \Sigma_1), \quad x_2 \sim N(\mu_2, \Sigma_2) \quad (1)$$

$$y = Ax_1 + Bx_2 \quad (2)$$

$$E[y] = E[Ax_1 + Bx_2] = AE[x_1] + BE[x_2] = A\mu_1 + B\mu_2 \quad (3)$$

$$\text{cov}(y) = \text{cov}(Ax_1 + Bx_2) = E[(Ax_1 + Bx_2)(Ax_1 + Bx_2)^T] \quad (4)$$

$$= E[Ax_1(Ax_1)^T + Ax_1(Bx_2)^T + Bx_2(Ax_1)^T + Bx_2(Bx_2)^T] \quad (5)$$

$$= \text{cov}(Ax_1, Ax_1) + \text{cov}(Ax_1, Bx_2) + \text{cov}(Bx_2, Ax_1) + \text{cov}(Bx_2, Bx_2) \quad (6)$$

$$= A\text{cov}(x_1, x_1)A^T + B\text{cov}(x_2, x_2)B^T \quad (7)$$

$$= A\Sigma_1A^T + B\Sigma_2B^T \quad (8)$$

$$y \sim N(A\mu_1 + B\mu_2, A\Sigma_1A^T + B\Sigma_2B^T) \quad (9)$$