SDS 383D, Bayesian Inference in Simple Conjugate Families

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E: Normal prior and sampling distribution with unknown mean and known idiosyncratic variance

$$p(\theta|x_1,...,x_n) \propto p(\theta|m,v)p(x_1,...,x_n|\theta) \tag{1}$$

$$p(\theta|m,v) = \left(\frac{1}{2\pi v}\right)^{1/2} \exp\left(-\frac{1}{2v}(\theta-m)^2\right) \tag{2}$$

$$p(x_i|\theta) = \left(\frac{\omega_i}{2\pi}\right)^{1/2} \exp\left(-\frac{\omega_i(x_i-\theta)^2}{2}\right) \tag{3}$$

$$p(x_1,...,x_n|\theta) = \prod_i^n p(x_i|\theta) \propto \exp\left(-\frac{1}{2}\sum_i^n \omega_i(x_i-\theta)^2\right) \tag{4}$$

$$p(\theta|x_1,...,x_n) \propto \exp\left(-\frac{1}{2v}(\theta-m)^2\right)\exp\left(-\frac{1}{2}\sum_i^n \omega_i(x_i-\theta)^2\right) \tag{5}$$

$$\propto \exp\left(-\frac{1}{2v}(\theta-m)^2 - \frac{1}{2}\sum_i^n \omega_i(x_i-\theta)^2\right) \tag{6}$$

$$-\frac{1}{2}\left[\left(\sum_i \omega_i + \frac{1}{v}\right)\theta^2 - 2\left(\sum_i x_i\omega_i + \frac{1}{v}m\right)\theta\right] \tag{7}$$

$$\theta|x_1,...,x_n \sim \mathrm{N}\left(\frac{\sum_i x_i w_i + \frac{1}{v}m}{\sum_i \omega_i + \frac{1}{v}}, \left[\sum_i \omega_i + \frac{1}{v}\right]^{-1}\right) \tag{8}$$