Poisson & Drift-Diffusion Equations Solver
for Semiconductor Device Modelling

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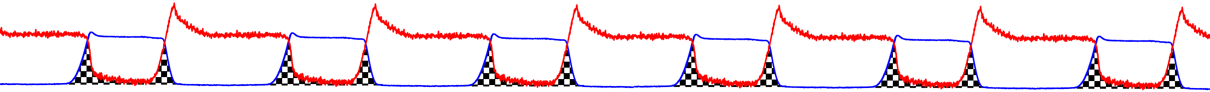
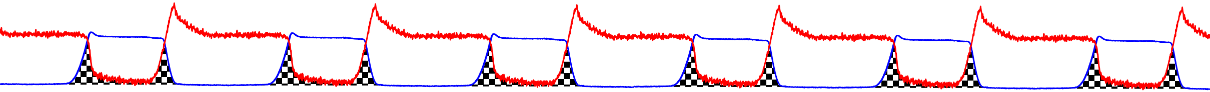
Motivation



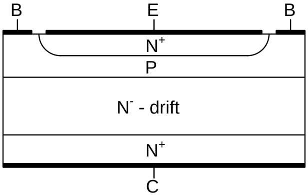
Switching loss analysis of power devices

Motivation



Switching loss analysis of power devices

- High voltage, high current



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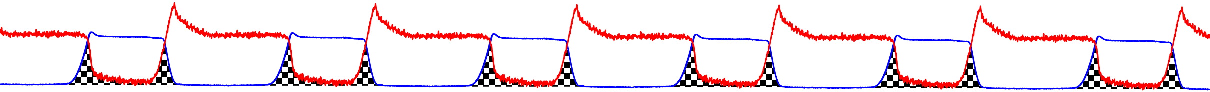
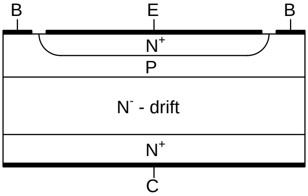


Switching loss analysis of power devices



- High voltage, high current
- Excess charge storage in bipolar transistors (BJT, IGBT) during on-state

-emiconductor Equations - Full Drift-Diffusion Model

Electrostatics (*Gauss's Law*): Poisson's Equation:

$$\nabla \cdot (\varepsilon \nabla \psi) = -q(p - n + N_D - N_A) \tag{1}$$

Carrier transport: drift-diffusion equations:

$$\mathbf{J}_p = \overbrace{qp\mu_p\mathbf{E}}^{\text{drift}} - \overbrace{qD_p\nabla p}^{\text{diffusion}} \tag{2}$$

$$\mathbf{J}_n = qn\mu_n\mathbf{E} + qD_n\nabla n \tag{3}$$

Continuity equations

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot \mathbf{J}_p - R \tag{4}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot \mathbf{J}_n - R \tag{5}$$

Shockley-Read-Hall (SRH) recombination

$$R = \frac{n \cdot p - n_i^2}{\tau_p(n + n_0) + \tau_n(p + p_0)} \tag{6}$$

Unknown variables $(\psi, p, n, \mathbf{J}_p, \mathbf{J}_n)$

Notation:

ε	Permittivity of material (silicon)
ψ	Electric potential
$\mathbf{E}$	Electric field intensity $\mathbf{E} = -\nabla\psi$
q	Elementary charge
p, n	Holes and electrons concentration
$\mathbf{J}_p, \mathbf{J}_n$	Hole and electron current density
$N_\mathrm{A}, N_\mathrm{D}$	Acceptors and donors concentration
μ_p, μ_n	hole and electron mobility
D_p, D_n	Diffusion constants (Fick's Law)
R	Recombination-generation rate
n_i	Intrinsic carrier concentration
p_0, n_0	thermal equillibrium concentrations
τ_p, τ_n	hole and electron recombination life-time

Semiconductor Equations - 3 equations system

- 3 independent variables (ψ, p, n)

$$\lambda_0 \nabla \cdot (\nabla \psi) = -(p - n + N_D - N_A) \tag{7}$$

$$\frac{1}{\lambda_1} \frac{\partial p}{\partial t} = 0 = \nabla \cdot (\mu_p p \nabla \psi + D_p \nabla p) - R(p, n) \tag{8}$$

$$\frac{1}{\lambda_1} \frac{\partial n}{\partial t} = 0 = \nabla \cdot (\underbrace{-\mu_n n \nabla \psi}_{\text{Drift}} + \underbrace{D_n \nabla n}_{\text{Diffusion}}) - \underbrace{R(p, n)}_{\text{Recombination}} \tag{9}$$

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- Scaled
- Coupled
- Nonlinear

Steady State Problem Statement

$$\lambda_0 \nabla \cdot (\nabla \psi) = -(p - n + N_D - N_A) \qquad \text{in } \Omega \tag{10}$$

$$0 = \nabla \cdot (p \mu_p \nabla \psi + D_p \nabla p) - R \qquad \text{in } \Omega \tag{11}$$

$$0 = \nabla \cdot (-n \mu_n \nabla \psi + D_n \nabla n) - R \qquad \text{in } \Omega \tag{12}$$

with “ohmic” boundary conditions

$$\psi = \psi_{\mathrm{BC}} \qquad \text{on } \Gamma_{\mathrm{D0}} \tag{13}$$

$$p = p_{\mathrm{BC}} \qquad \text{on } \Gamma_{\mathrm{D1}} \tag{14}$$

$$n = n_{\mathrm{BC}} \qquad \text{on } \Gamma_{\mathrm{D2}} \tag{15}$$

$$\mathbf{n} \cdot \nabla \psi = g \qquad \text{on } \Gamma_{\mathrm{N0}} \tag{16}$$

Variational Formulation

$$\lambda_0 \nabla \cdot (\nabla \psi) = -(p - n + N_D - N_A) \qquad \text{in } \Omega \tag{10}$$

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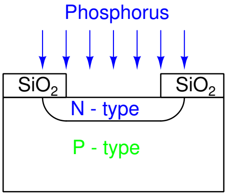
Find $(\psi, p, n) \in V_0 \times V_1 \times V_2$ such that

$$F\left((\psi, p, n); (v_0, v_1, v_2)\right) = 0 \tag{17}$$

for all $(v_0, v_1, v_2) \in \hat{V}_0 \times \hat{V}_1 \times \hat{V}_2$; with F given by:

$$\begin{aligned} F = &- \int_\Omega \lambda_0 \nabla \psi \cdot \nabla v_0 \, \mathrm{d}x + \int_\Omega (p - n + N_\mathrm{D} - N_\mathrm{A}) v_0 \, \mathrm{d}x + \int_{\Gamma_{N0}} g v_0 \, \mathrm{d}s \\ &- \int_\Omega D_p \nabla p \cdot \nabla v_1 \, \mathrm{d}x - \int_\Omega \mu_\mathrm{p} p \nabla \psi \cdot \nabla v_1 \, \mathrm{d}x - R v_1 \, \mathrm{d}x \\ &- \int_\Omega D_n \nabla n \cdot \nabla v_2 \, \mathrm{d}x + \int_\Omega \mu_\mathrm{n} n \nabla \psi \cdot \nabla v_2 \, \mathrm{d}x - R v_2 \, \mathrm{d}x \end{aligned} \tag{18}$$

Domain, Doping profiles



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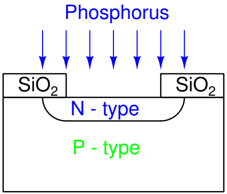

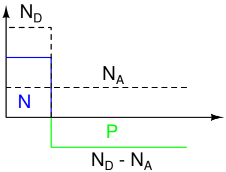

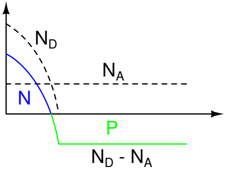

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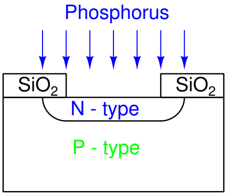
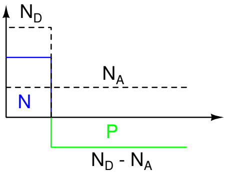
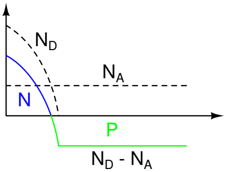
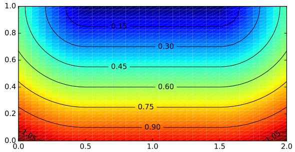







```python
def distance(x): ....
```



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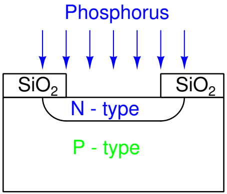


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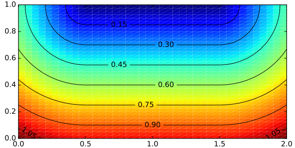

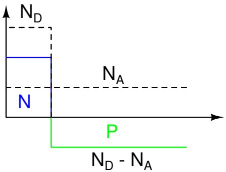


```
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Nd1.interpolate(doping_expr(distance(x)))
```
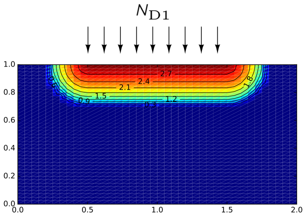
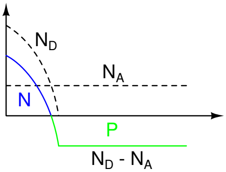
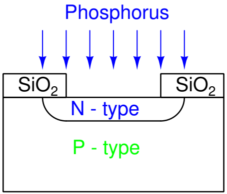
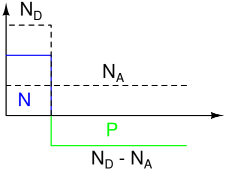
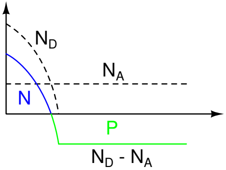





Domain, Doping profiles





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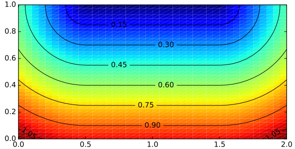


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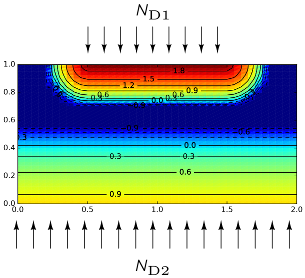
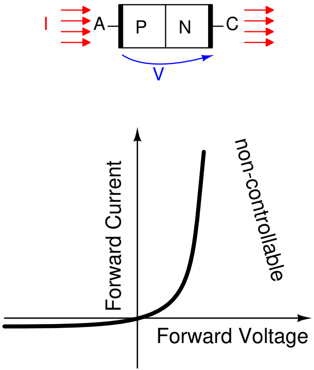



```
N = Nd1 + Nd2 - Na
```

Demonstration of Results

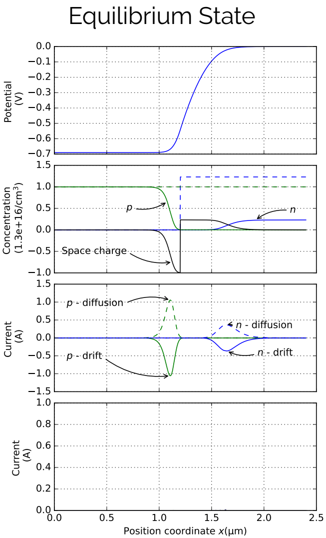
PN Junction Diode

Typical Electrical Output Characteristics



PN Junction Diode

Simulation Results - Step junction



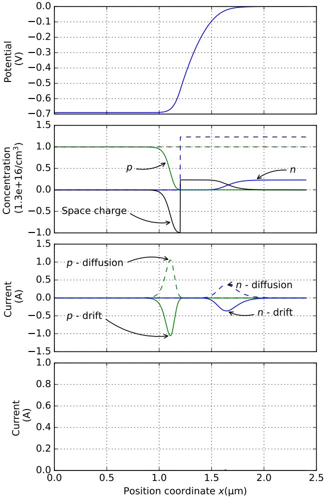
Reverse / Zero Bias:

- Depletion region formed
- Zero total current
- Drift compensated by diffusion

PN Junction Diode

Simulation Results - Step junction

Equilibrium State



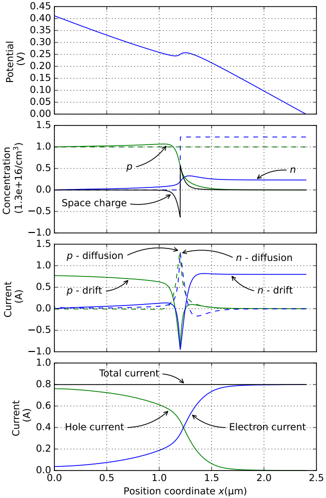
Reverse / Zero Bias:

- Depletion region formed
- Zero total current
- Drift compensated by diffusion

Forward Bias:

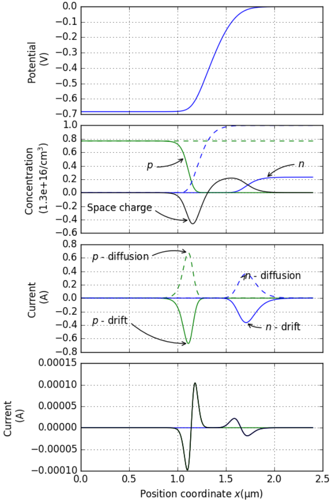
- Depletion region reduced
- Constant current across device
- minority carriers injected
- majority carriers exceeding doping concentration

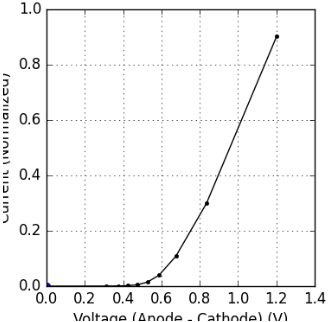
Forward Bias



PN Junction Diode

Simulation Results - Gaussian Doping Profile





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PN Junction Diode

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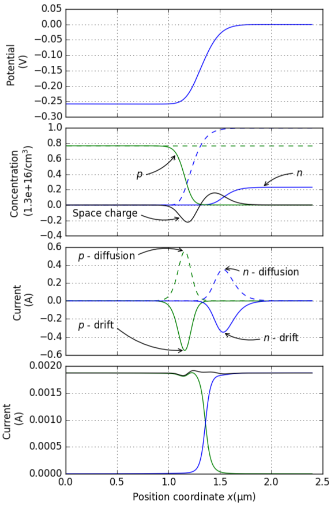

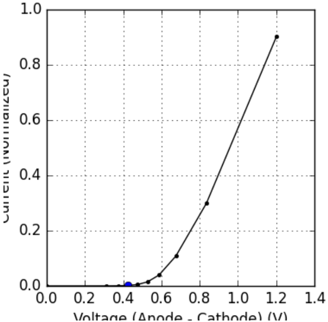

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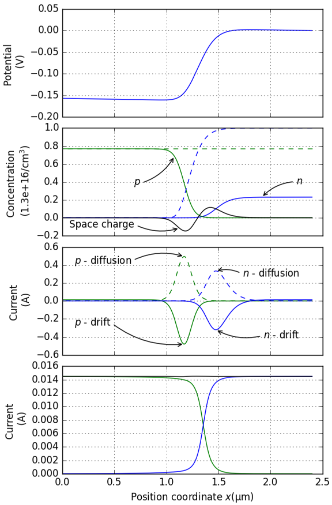

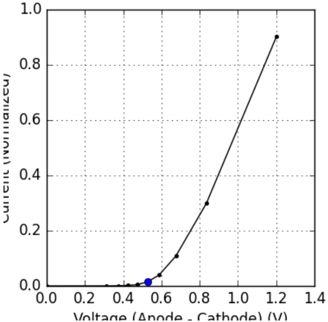

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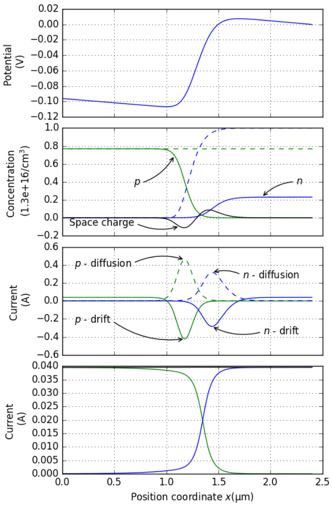

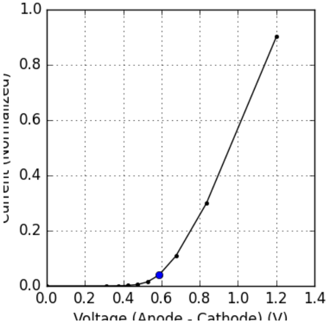

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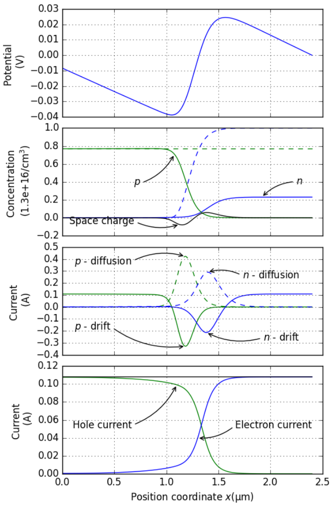

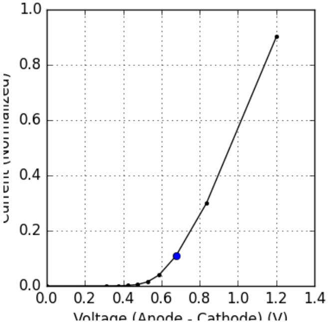

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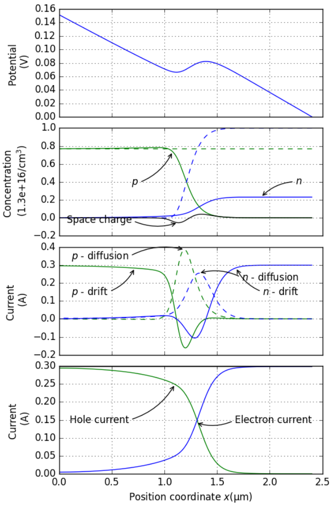

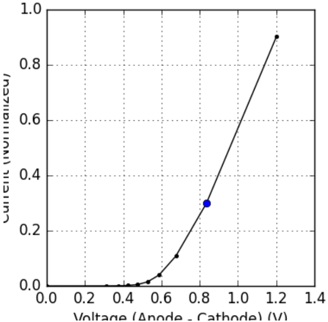

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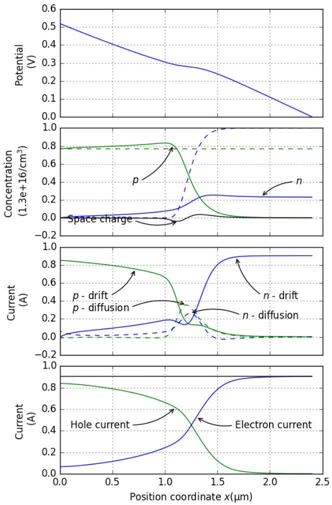

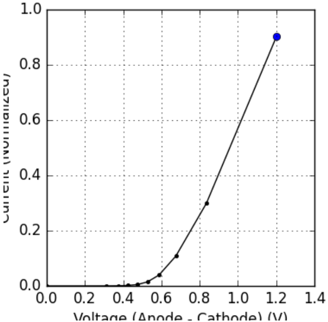

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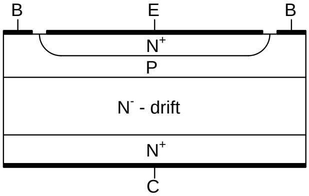
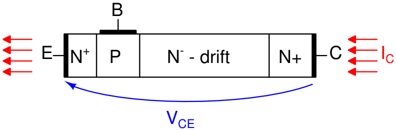
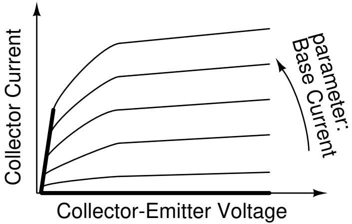
NPνN Power Transistor

Typical Electrical Output Characteristics



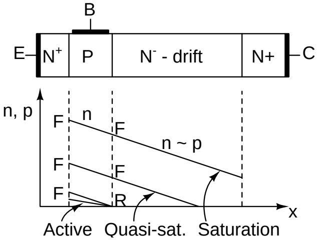


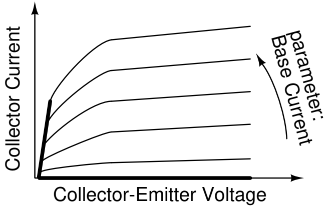


NPνN Power Transistor

1-D Lumped Charge Model

- Charge control + conductivity modulation
- "Rules":
  - **Zero mobile carriers** at reverse biased junction
  - **Some mobile charge** at forward biased junction
    - defined by applied voltage and doping concentrations
  - Diffusion current: defined by **concentration gradient**
  - Conductivity: defined by amount of mobile carriers





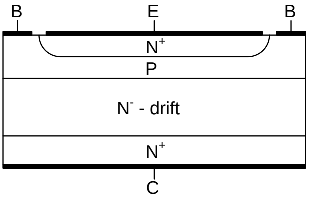
Equations (2),(3)

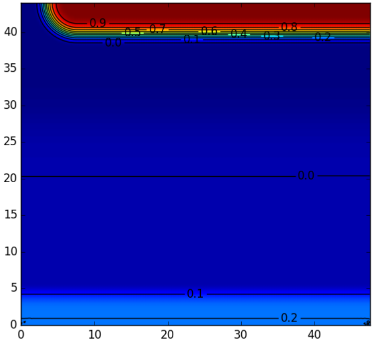
$$J_p = \overbrace{qp\mu_p \mathbf{E}}^{\text{drift}} - \overbrace{qD_p \nabla p}^{\text{diffusion}}$$

$$J_n = qn\mu_n \mathbf{E} + qD_n \nabla n$$

NPνN Power Transistor

Simulation Domain, BC, Doping

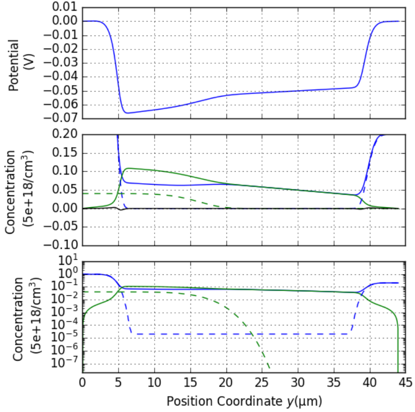




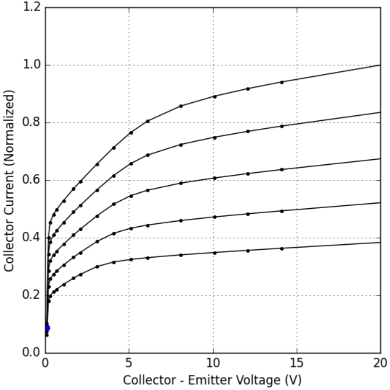
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Saturation



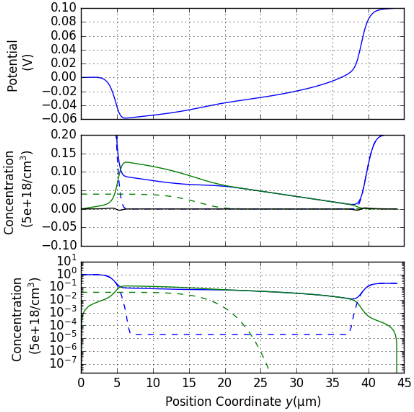
Saturation



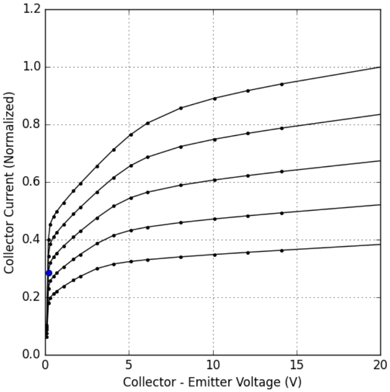
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Saturation



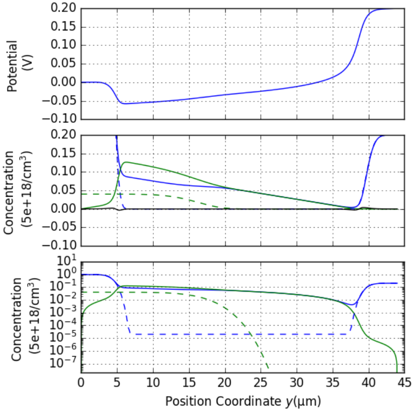
Saturation



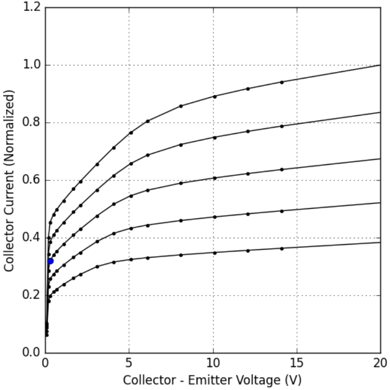
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Saturation



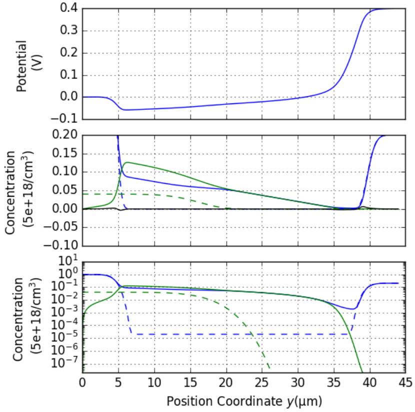
Saturation



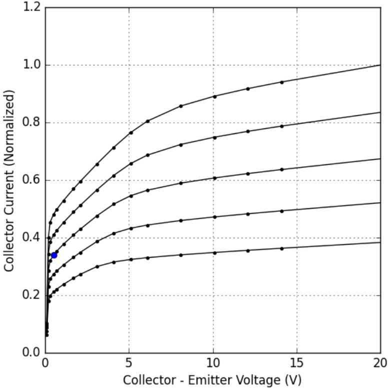
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Quasi-Saturation



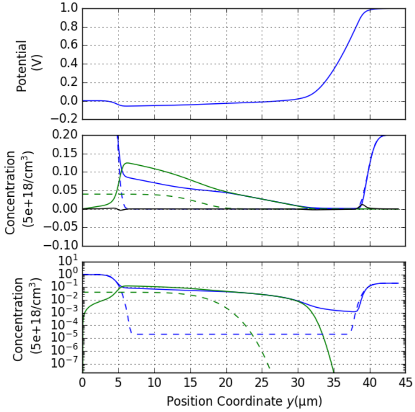
Quasi-Saturation



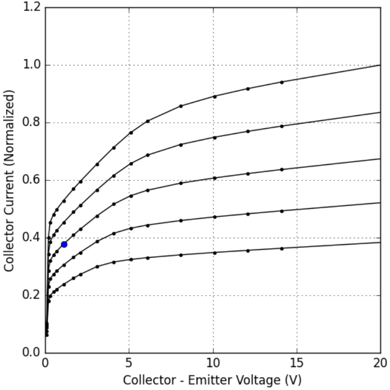
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Quasi-Saturation



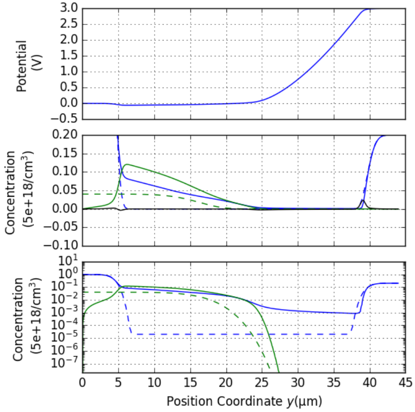
Quasi-Saturation



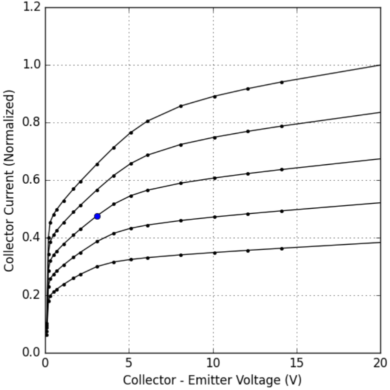
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Quasi-Saturation



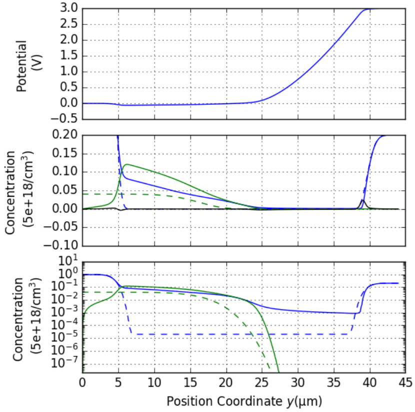
Quasi-Saturation



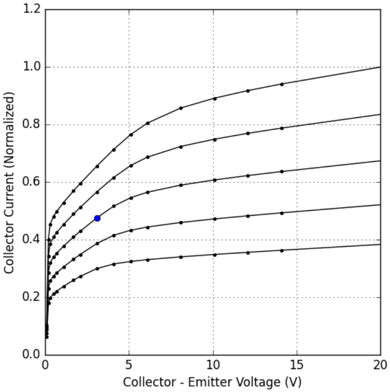
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Quasi-Saturation



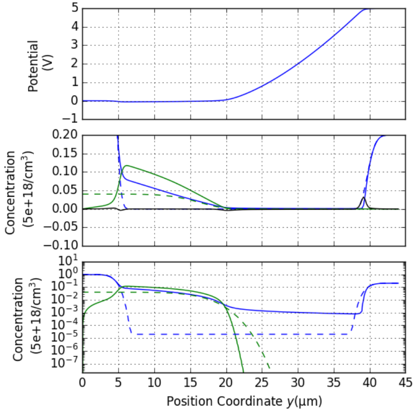
Quasi-Saturation



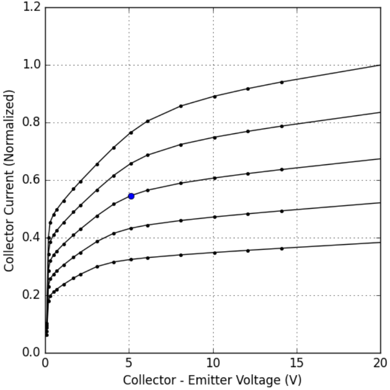
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Active Mode



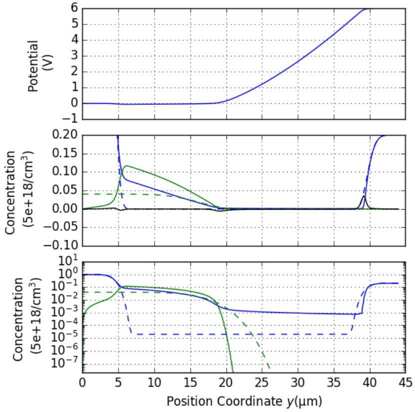
Active Mode



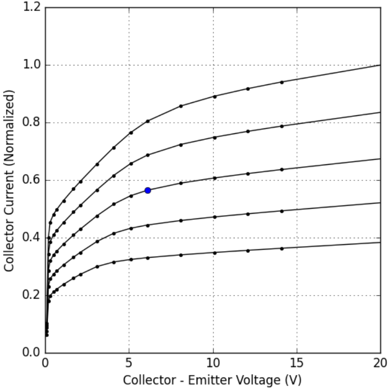
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Active Mode



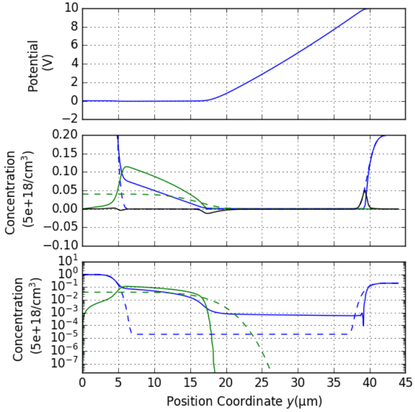
Active Mode



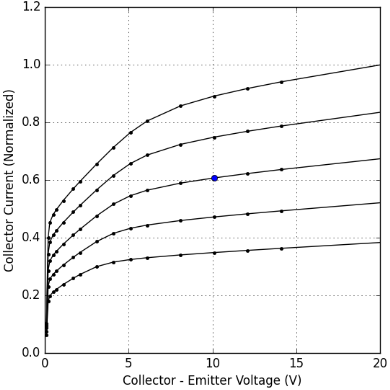
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Active Mode



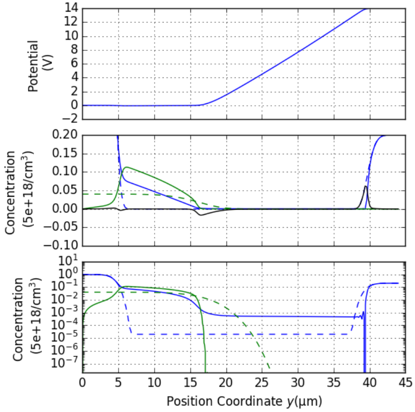
Active Mode



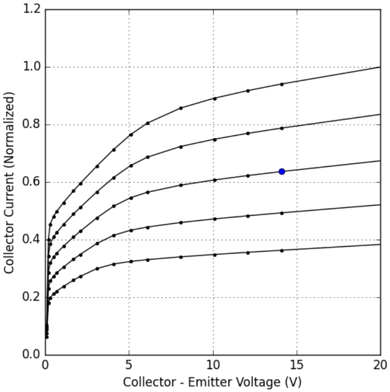
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Active Mode



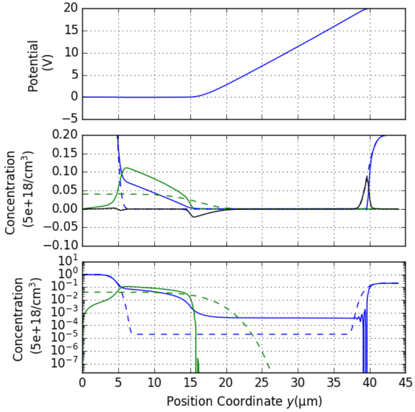
Active Mode



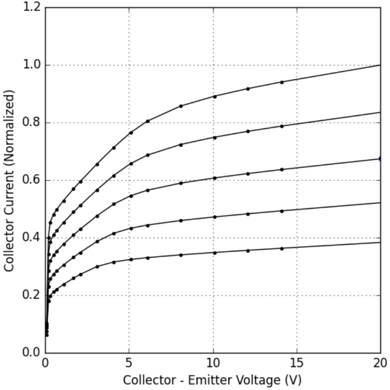
NPνN Power Transistor

Simulation Results - Operational Modes

Collector voltage sweep
Active Mode



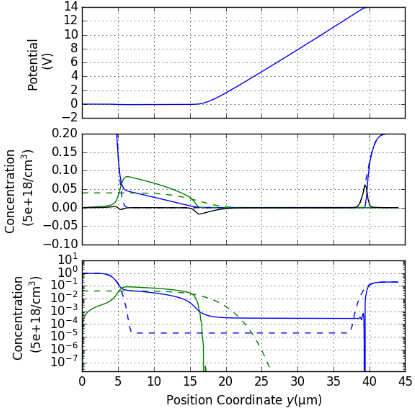
Active Mode

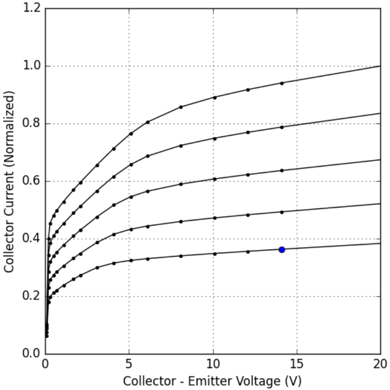


NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)

Base drive sweep

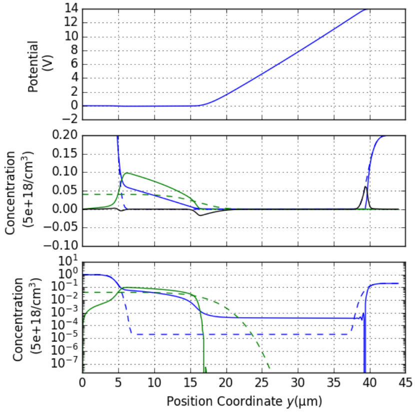


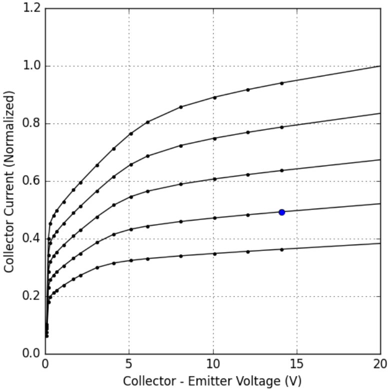


NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)

Base drive sweep

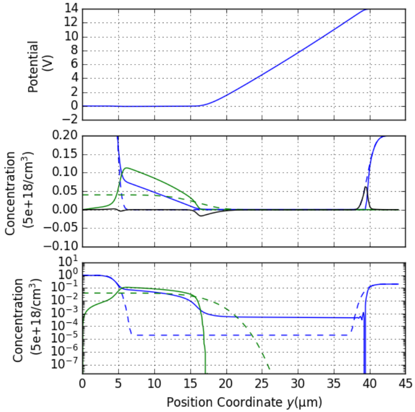


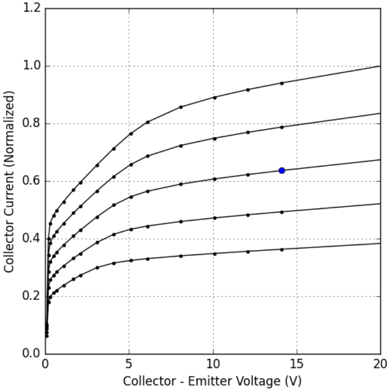


NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)

Base drive sweep

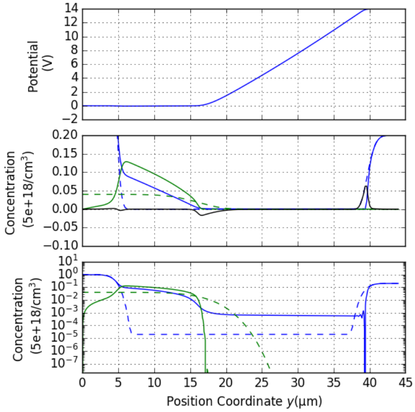


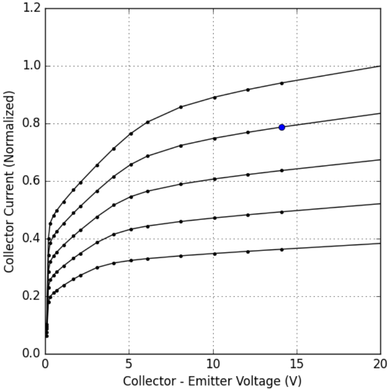


NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)

Base drive sweep

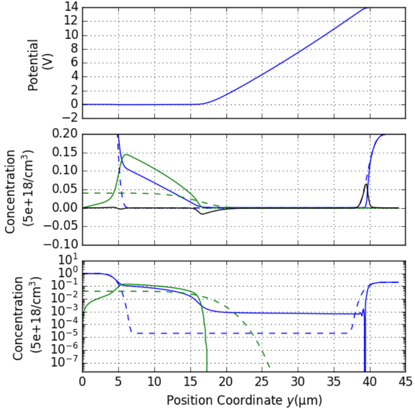


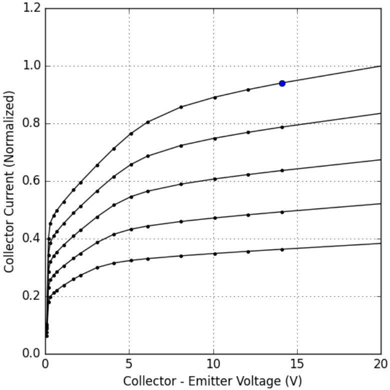


NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)

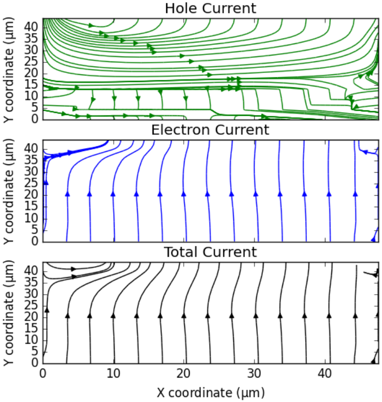
Base drive sweep





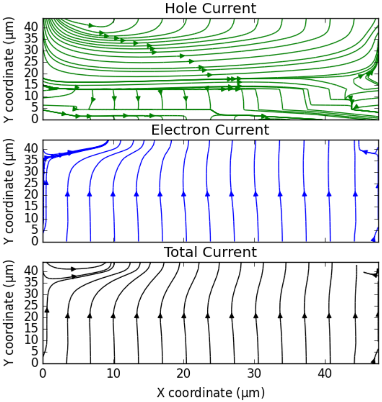
NPνN Power Transistor

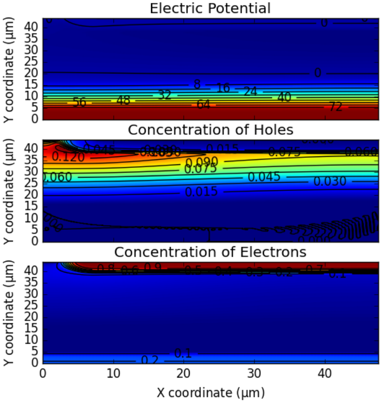
Simulation Results - Transistor Effect (Current Gain)



NPνN Power Transistor

Simulation Results - Transistor Effect (Current Gain)





Summary

- Numerical approach **validated and demonstrated** on basic structures
  - The equations and FEniCS seem to love each other
- Currently in “demo version” - qualitative explanation of device physics
- Good **validation tool** for analytical assumptions in simplified power BJT model

Outlook

Space for improvements:

- Time dependent problem
- Initial guess accuracy, BC for any bias
- **Nonlinear Solver**, Convergence, Stability

`https://github.com/janmiklas/fenicsx-semiconductor-eq`

Thank you for your kind attention!