Financial constraints, firm heterogeneity, and the cyclicality of employment growth

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Abstract

This paper examines the role of financial constraints for cross-sectional employment dynamics. First, I document that employment growth at more financially constrained firms is more sensitive to macroeconomic conditions. Second, I use a dynamic general equilibrium model to analyze the interconnection of financial constraints and employment elasticities under aggregate uncertainty. The key ingredients are (i) heterogeneity in debt capacity that allows firms to assume different levels of leverage, and (ii) borrowing costs that respond to expected default rates. As a result more constrained, high leverage firms undertake larger employment and investment reductions when economic conditions worsen. I assess the ability of the model to account for the different elasticities of employment growth in the data. The model matches well the higher elasticity of more constrained firms and can replicate the cyclicality of aggregate employment with a realistic degree of labor market rigidities. Other dimensions of firm heterogeneity, such as productivity or risk differences, fail to generate cyclical variation that is consistent with the employment dynamics from the data. In line with the previous literature, in my model more constrained firms also react stronger to monetary policy shocks.

1 Introduction

Credit constraints play a key role for the hiring and investment behavior of businesses. The loosening of constraints in periods of economic growth and their tightening in recessions has motivated a large literature on the amplifying effects of financial frictions: easier borrowing conditions reinforce booms, while more restrictive conditions aggravate and prolong downturns. However, much of this research has focused on aggregate outcomes. In this paper I explicitly account for the fact that the severity of financing constraints differs across firms, and I widen the focus to cross-sectional dynamics. How does a recession affect firms which are more and less constrained to begin with? Which of the many dimensions of firm heterogeneity can help understand heterogeneous employment dynamics in the data? And what role does leverage play in the transmission of aggregate shocks?

The contribution of my paper is twofold. On the empirical side I document that employment growth at more financially constrained firms is more sensitive to macroeconomic conditions. Specifically, I consider a sample of public firms with a debt rating and separate firms into two groups: those with an investment grade rating and those with a non-investment (speculative) grade rating. Non-investment grade firms fare worse on a number of indicators for financial constraints. Figure 1 shows the key cyclical pattern for these two groups of firms. The mean employment growth rate among more constrained, non-investment grade firms drops precipitously during the three recessions 1990-1991, 2001, and 2007-2009. Less constrained, investment grade firms see a much less pronounced decline. Moreover, non-investment grade firms appear to expand faster in expansions. More formally, I show that more constrained firms have a significantly higher elasticity of employment growth with respect to aggregate output growth. This result is not driven by industry effects or age-specific cyclical sensitivities.

The technical contribution is the introduction of persistent firm heterogeneity into an otherwise standard financial accelerator model. In my environment firms exhibit differences in their depreciation rates. Importantly, in the contracting problem between firms and finan-

Figure 1: Employment growth
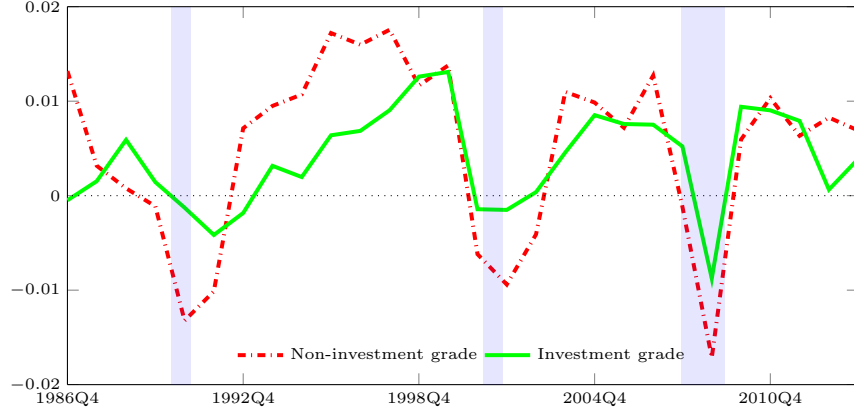
cial intermediaries - motivated by a costly state verification problem as in Bernanke et al. (1999) - the default risk premium on debt (or agency cost) depends on the depreciation rate. Namely, firms with lower depreciation rates have lower default incentives and thus lower agency costs for a given level of debt. Benefitting from cheaper access to external financing, in the stationary equilibrium these firms assume higher leverage, have higher expected default rates and pay higher interest rates. Thus, they correspond to the more constrained firms notwithstanding their cost advantage in borrowing. The idea behind this particular form of heterogeneity are differences in debt capacity that are not driven by productivity, risk, etc. As I argue below this feature is key to produce cyclical dynamics that are consistent with the micro data. In a recession the more constrained firms reduce debt financing, sell-off more capital, and hire fewer workers compared to less constrained firms. Because more constrained firms have a smaller equity buffer a negative aggregate shock increases their default risk and borrowing costs disproportionally. They respond by deleveraging while less constrained firms - for whom default risk rises only by little - actually increase leverage.

I assess the ability of the model to match the differential employment elasticities estimated from the data. Given the dynamics of US real GDP, the model matches well the higher elasticity of the more constrained firms. The elasticity of aggregate employment can be

replicated with a realistic degree of labor market rigidities. Both the endogenous debt choice and the default option are essential to these findings. Namely, if leverage was fixed at steady state levels and borrowing costs were not responding to default risk, then less constrained, low leverage firms would react more strongly to aggregate shocks. This indicates that high leverage by itself does not cause greater cyclicality. Rather, the endogenous amplification via default risk and borrowing costs plays a key role.

In addition, I argue that other dimensions of firm heterogeneity, such as heterogeneity in productivity or firm risk, do not produce cyclical variation that is consistent with the employment dynamics of rated public firms. Therefore, my results complement the analysis of heterogeneous firm models with credit frictions where higher employment elasticities are generally found among more productive firms[1]. The discrepancy to my findings emerges because the severity of constraints in that class of models is governed by life-cycle dynamics, such as age and size, an element absent in my framework. The model also has implications for the timing of deleveraging. Namely, more constrained firms reduce borrowing and investment immediately while less constrained firms delay deleveraging to absorb some of the capital. I provide suggestive evidence to support this prediction of the model.

I present extensions to the model to study the effects of monetary policy, financial shocks and changes in idiosyncratic firm risk. The disruption to financial intermediation during and after the financial crisis of 2007/ 2008 has arguably been a key element for recession dynamics. I introduce shocks to credit spreads to proxy for a breakdown in intermediation. Consistent with survey evidence on employment creation during the financial crisis (Campello et al. (2010)), I find that financially more constrained firms are hit the hardest by such shocks. The role of fluctuations in risk (or uncertainty) for investment dynamics has been discussed in policy circles and academia alike. While the aggregate effects are well documented, my model can break down the effects on more and less constrained firms. I document that in response to an economy-wide increase in idiosyncratic risk more (less) financially constrained firms

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[1]See Buera et al. (2015), Khan and Thomas (2013), Cui (2013).

experience a very persistent decrease (increase) in employment. This hints at ample risk-sharing opportunities between firms which are overlooked in models with representative firms. Finally, I consider monetary policy shocks and confirm earlier findings that more financially constrained firms react more strongly to contractionary monetary policy (see Gertler and Gilchrist (1994)).

Disproportional employment losses at more constrained firms may help explain why employment growth is more dispersed across firms in recessions. Along with the countercycliality of other cross-sectional dispersion measures - such as sales growth, equity returns, etc. - the literature has addressed this pattern either by appealing to time-variation in the volatility of idiosyncratic shocks, or by proposing real and financial frictions that in bad times generate more dispersed outcomes endogenously. This paper adds to the latter stream of work[2] and suggests that differences in debt capacity result in heterogeneous cyclical sensitivities and ultimately generate more dispersed employment growth in a downturn.

The paper is organized as follows. The remainder of this section summarizes the related literature. In section 2 I corroborate the evidence about differential elasticities. I present the model in section 3. Section 4 contains the quantitative evaluation. Section 5 concludes with some observations and directions for future research.

Related Literature: This paper connects to several lines of research: (i) macroeconomic models with financial frictions, in the context of both representative firms and heterogeneous firms, and (ii) the empirical work on financing constraints and the cyclical properties of firm-level employment.

The setup of my model borrows from Bernanke et al. (1999), in particular regarding the financial contract (costly state verification) and the amplification via asset prices. I introduce heterogeneity such that the equilibrium capital structure differs across firms while

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[2]Proposed mechanisms include endogenous risk taking as in Tian (2012) and Navarro (2014); Price experimentation as in Bachmann and Moscarini (2012); Learning as in Benhabib et al. (2015); Diversification as in D'Erasmo and Boedo (2013); Asymmetric hiring responses as in Ilut et al. (2014).

in Bernanke et al. (1999) it is symmetric. The amplification through asset prices - the financial accelerator - is key to the results of this paper. In their borrowing decisions firms do not take into account that in recessions the pressure to shed capital reduces asset prices and erodes their net worth. The financial accelerator framework has recently been used by Christiano et al. (2014) to study the importance of risk shocks. In an extension of my model I examine the consequences of such shocks to idiosyncratic firm risk for employment at more and less constrained firms.

Another line of papers builds on collateral constraints in the tradition of Kiyotaki and Moore (1997). For instance, Jermann and Quadrini (2012) consider the tightening of a collateral constraint due to financial shocks and analyze the cyclical dynamics of debt and equity financing. Brunnermeier et al. (2013) point out an important difference between the costly state verification framework and a Kiyotaki-Moore collateral constraint: "[W]ith costly state verification, the cost of external financing is increasing in the borrowing" while with a collateral constraint the "cost of external financing is constant [...] up to the constraint and then becomes infinite". This statement hints at differences between the two approaches when addressing heterogeneity in debt capacity. Note that in models without default in equilibrium, such as the representative firm model used by Jermann and Quadrini (2012), firms borrow at a common interest rate. This also holds if one were to assume that the tightness of collateral constraints differs across firms. Therefore, an aggregate tightening - think of a proportional drop in the multiplier for all firms - would hit firms symmetrically since their (marginal) cost of external financing is the same. On the contrary, in my framework equilibrium borrowing costs are different as a result of default risk. Therefore, an aggregate tightening provides stronger incentives to deleverage for firms with higher capital costs. Accordingly, these firms see a larger decline in capital and employment. Aside from differences with respect to heterogeneity a distinguishing feature of the costly state verification setup is that financing constraints tighten endogenously in response to a variety of shocks, including shocks to productivity, credit spreads, monetary policy, etc.

The above papers are aimed at capturing aggregate dynamics and they do not allow for persistent heterogeneity across firms. In contrast, there are a number of papers that share the focus on both financing frictions and firm-level heterogeneity. In Buera et al. (2015), Khan and Thomas (2013), and Cui (2013) firms differ in their idiosyncratic productivities, and capital is continuously reallocated from low productivity to high productivity firms, mitigated only by a collateral constraint. An exogenous tightening of this constraint prevents the transfer of capital from low to high productivity firms, thereby aggravating the misallocation of capital. Unlike in these papers, the severity of constraints here does not follow from productivity differences and there is no misallocation. Moreover, borrowing constraints become tighter in recessions due to higher default risk and the impact on more and less constrained firms reflect the incentives for deleveraging. Abstracting from life-cycle dynamics as in Buera et al. (2015) and Khan and Thomas (2013) has advantages and drawbacks. It allows me to calibrate the model to match interest rates of more and less constrained firms and to obtain implications for leverage that are consistent with the data. Models with firm dynamics usually imply that more constrained firms have lower leverage, which is contrary to the data.[3] On the other side, the heterogeneity in my model is not suitable to address the role of credit frictions for cyclical employment dynamics at small and young firms (see the evidence by Fort et al. (2013)).

I share with Gourio (2014) the importance of default risk for cross-sectional employment dynamics. However, the transmission mechanisms are very different. In Gourio (2014) default risk directly affects hiring behavior as workers require to be compensated for expected earnings losses due to default of their employer. In contrast, in my model default risk limits borrowing for the acquisition of capital and employment is in turn affected via the complementarity with capital. Allowing for capital, which is absent in Gourio (2014), generates important feedback through asset prices and the firm's balance sheet.

On the empirical side Giroud and Mueller (2015) document the importance of leverage for

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[3]For instance, to overcome this shortcoming Khan and Thomas (2013) introduce a class of completely unconstrained firms.

hiring during the Great Recession. Specifically, those firms which increased leverage prior to the crisis responded with larger employment cuts in response to household demand shocks. Their results are consistent with mine insofar as the firms which are more responsive to aggregate shocks are more financially constrained.

In earlier work Sharpe (1994) analyzes the cyclicality of employment in manufacturing firms between 1959 and 1985, and establishes that high leverage firms are more sensitive to aggregate demand and financial markets conditions. In line with Gertler and Gilchrist (1994) who focus on responses to monetary policy shocks, Sharpe (1994) also documents higher sensitivity of small firms.

Further evidence that financial constraints affected hiring during the downturn comes from Campello et al. (2010). In their survey of CFOs around the height of the financial crisis they find that the difference in investment and employment creation between constrained and unconstrained firms increases from the pre- to post-crisis period. Focusing on the worker side, Duygan-Bump et al. (2015) find that unemployment risk increased particularly for workers of smaller firms operating in industries that depend on external finance.

2   Employment elasticities

In this section I provide evidence on the heterogeneous elasticities among rated public firms. I begin by comparing characteristics of rated and non-rated firms. Then, I argue that the rated firms constitute a suitable sample for my study of financial constraints. Finally, I show the robustness of differential elasticities of employment growth that were suggested by Figure 1 in the introduction.

Firms can be categorized into those publicly held and those privately held. As comprehensive and reliable financial information (income statement, balance sheet) is only available for public firms, these corporations are the focus of my study.[4] Furthermore, among public

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[4]Asker et al. (2011) obtain financial information on a large sample of private firms. Comparing private and public firms they find that conditional on firm size and industry, private firms invest more, grow faster, are more profitable, hold less cash, and have higher leverage.

firms there is a subset of firms with access to the corporate bond market. These firms have been assigned a bond credit rating, usually from multiple major credit rating agencies, and can raise funds through bond issuance.[5] Ratings are expressed as letter grades, ranging from AAA for firms of the highest credit quality to D for firms that are in default on their debts.[6] For the purposes of this paper I only consider two broad categories: investment grade firms which have a rating of $BBB-$ or better, and non-investment (speculative) grade firms with a rating $BB+$ or worse.

Table 1 compares investment grade and non-investment grade firms in the Compustat database. As a comparison it also lists characteristics of public firms without a bond credit rating. The sample covers the years 1986 to 2014 and the table entries report median values from the pooled sample. Relatively few public firms have a rating and these firms are much larger than non-rated firms both in terms of assets and employees. Rated firms are also older than non-rated firms.[7] The relatively young age of non-investment grade firms is partially due to entry dynamics in the 1990s. I will return to this issue in the regression analysis below.

The financial indicators in Table 1 point to non-investment grade firms as the more financially constrained firms. Fewer non-investment grade firms pay dividends, they have higher leverage and they score higher on the KZ index measuring financial constraints. Moreover non-investment grade firms pay much higher interest rates as expressed in bond yields.[8] The comparison to non-rated public firms is not straightforward. These firms have lower leverage and pay dividends only infrequently, but with respect to the KZ index they rank in between the investment grade and non-investment grade firms.

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[5]Issuing bonds without a credit rating is possible, however non-rated bonds are rare in practice. Firms without a rating usually obtain debt financing through bank loans, private debt from other institutions, or - for larger corporations - via the syndicated loan market. Firms can pay fee for a rating agency to assess its credit quality and assign a credit rating.

[6]This paper uses rating grades from S&P. Other major credit rating agencies have equivalent rating categories.

[7]The age variable in Compustat refers to the first appearance in the database rather than the founding year of the company. Thus, it is confounded by the decision firms to go public.

[8]Figure A.2 in the appendix displays bond yields from an index of investment grade firms and non-investment grade firms.

Table 1: Descriptive Statistics

	Investment grade	Non-investment grade	No rating
Observations	529	690	4,950
Employment	21,800	3,980	290
Assets	6,680	1,180	62
Age	31	13	8
Leverage	0.58	0.69	0.42
KZ score	-1.36	0.90	-0.67
Dividend payer	0.87	0.43	0.28
Volatility	0.32	0.48	0.56
Return on assets	0.13	0.07	0.08
Return on equity	0.13	0.13	0.05

Notes: Compustat Fundamentals Annual (1986 to 2014), OptionMetrics (1996 to 2012). Financial firms (SIC codes 60 to 67) and utilities (SIC code 49) excluded. Assets are million deflated to 2009 dollars. Leverage is total liabilities divided by total assets. Return on assets/ equity is operating income before depreciation (OIBDP) minus interest expense (XINT) divided by book value of assets/ market value of equity. Equity volatility is the implied volatility for an at-the-money call option (50 delta), 30-day maturity. KZ score is the Kaplan-Zingales measure for financial constraints, constructed according to Lamont et al. (2001). Table entries are medians over the sample period. Dividend payer is the share of firms with positive dividends among total firms in a rating group.

Focusing on the group of rated firms has several advantages: First, as debt of rated firms is publicly traded the bond yields can be used to discipline the severity of financing constraints. Second, there is a clear ranking with regards to financial constraints between investment grade and non-investment grade firms and the differences are substantial. Third, the decision to obtain a rating may be driven by unobserved heterogeneity. Considering only rated firms likely reduces the influence of unobserved factors because these firms - both investment grade and non-investment grade - have decided for a rating while non-rated firms have decided against.

It remains to be shown that employment growth at firms denoted as more constrained is more elastic to changes in aggregate output. To corroborate the impression from Figure 1 I regress employment growth on real GDP growth, a dummy for non-investment grade status (NIG), the interaction between GDP growth and the NIG dummy, and the return on assets as a measure for productivity.

$$\Delta Employment_{i,t} = \beta_0 + \beta_1 \Delta Y_t + \beta_2 NIG_{i,t-1} + \beta_3 NIG_{i,t-1} \times \Delta Y_t + \beta_4 ROA_{i,t-1} + \cdots + \epsilon_{i,t}$$

Table 2: Employment growth cyclicality

	$\Delta Employment_{i,t}$			
	(1)	(2)	(3)	(4)
ΔY_t	0.096***	0.121***	0.141***	0.169***
	(4.24)	(5.75)	(6.12)	(6.84)
$NIG_{i,t-1}$	0.003***	0.002**	-0.001	0.001
	(3.93)	(2.35)	(-0.79)	(1.37)
$NIG_{i,t-1} \times \Delta Y_t$	0.252***	0.192***	0.132***	0.082**
	(6.81)	(5.79)	(3.82)	(2.20)
$Age_{i,t-1}$			-0.004***	
			(-7.89)	
$Age_{i,t-1} \times \Delta Y_t$			-0.083***	
			(-3.49)	
$Size_{i,t-1}$				-0.000
				(-1.32)
$Size_{i,t-1} \times \Delta Y_t$				-0.062***
				(-4.92)
$ROA_{i,t-1}$	0.074***	0.053***	0.053***	0.053***
	(14.01)	(11.13)	(11.32)	(11.27)
R^2	0.02	0.02	0.02	0.02
N	29,763	29,763	29,763	29,762
$Time \times Industry$	No	Yes	Yes	Yes

Notes: Compustat Fundamentals Annual, years 1986 to 2014. Financial firms (SIC codes 60 to 67) and utilities (SIC code 49) excluded. ΔY_t is the growth rate of real GDP from year $t-1$ to t. Employment is $\Delta Employment_{i,t} = 0.5(Emp_{i,t} - Emp_{i,t-1})/(Emp_{i,t} + Emp_{i,t-1})$. Firm size is measured as the log of total assets. Return on assets is operating income before depreciation (OIBDP) minus interest expense (XINT) divided by total assets. In columns (3) and (4), age and size are demeaned for better comparability of the coefficient on ΔY_t. Values in parentheses are t statistics computed using robust standard errors. (***/**/*) indicate significance at the (1/5/10) percent level.

A positive coefficient on the key interaction term, $NIG_{i,t-1} \times \Delta Y_t$, indicates greater cyclicality of non-investment grade firms. The specification is motivated by Sharpe's (1994) approach, however, he used leverage as the measure of financing constraints rather than the rating status. While employment data in Compustat is available only at yearly frequency, I follow Sharpe (1994) and take advantage of the fact that the fiscal year end differs across firms. I therefore match employment growth at firms whose fiscal year ends in quarter q with aggregate output growth from year $t-1$ quarter q to year t quarter q.[9]

In column (1) the coefficient on the interaction term is large and significant, but in relative terms the magnitude of the coefficient on output growth is small. This is surprising given that employment growth at both investment grade and non-investment grade dropped precipitously during the three recessions depicted in Figure 1. I suspect that cyclical dynamics at the sectoral level may not be well synchronized with aggregate output growth and bias the coefficient downward. Therefore, for columns (2)-(4) I include time-industry effects, with industry defined at the 3-digit NAICS level. Specifically, for each year t and quarter q I take out the average employment growth rate by 3-digit NAICS industry and add back the average overall employment growth rate.

Moreover, including time-industry effects helps to ascertain that the greater cyclicality of non-investment grade firms is not driven by composition effects. Namely, if non-investment grade firms were concentrated in industries that are more cyclical, for instance durable goods industries, then the greater cyclicality would emerge irrespective of differences in financing constraints. Column (2) shows that with time-industry effects the coefficient on output growth rises to 0.12 while the coefficient on the interaction term drops to 0.19. This implies that indeed some of the differences in cyclicality are driven by industry. However, a substantial difference in the employment elasticities remains.

In column (3) I include age as an explanatory variables and I allow for an age-specific elasticity of employment growth. The rationale behind this is twofold: First, evidence by

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[9]Sharpe (1994) used up to six semi-annuals lags of output growth (three years).

Fort et al. (2013) indicates that young firms have greater cyclicality of employment, possibly related to financing constraints. If the rating status merely captures firm age and severity of financing frictions that correlates with age, then the interacted rating status dummy should become insignificant. Second, there is concern that the different elasticities suggested by Figure 1 stem from composition effects, in particular in light of the high employment growth rates among non-investment grade firms during the 1990s. The boom period was characterized by a succinct increase in the number of non-investment grade firms, and the average age among these firms dropped from 19 years in 1991 to 14 years in 1999. In contrast the increase in the number of investment grade firms was smaller and the average age declined only by little. In column (3) I find that young firms grow faster, consistent with life-cycle behavior. Moreover, employment growth at young firms is more sensitive to changes in aggregate output while the interaction term on more constrained firms remains positive and significant.

Finally, in column (4) I add firm size measured as total assets and the respective interaction term with aggregate output growth. As was shown in Table 1 investment grade firms are much larger than non-investment grade firms. Similarly as with the addition of age, one can see that large firms grow slower and have a lower elasticity of employment growth. The main coefficient of interest remains positive and significant but drops in magnitude.

In summary, firms with a non-investment grade status grow more in expansions and drop stronger in recessions. This result is not driven by industry composition alone. Moreover, firm age may partially capture differences in financial constraints, but the rating status still remains important.

I conclude this section with two remarks. First, note that speculative grade firms have more volatile equity returns and a lower return on assets (see Table 1). While the model attempts to capture the heterogeneity of financing constraints directly via the firm's capital structure and associated borrowing costs, I will consider these alternative dimensions of heterogeneity in an extension of the model.

Second, the measures of financing constraints summarized in Table 1 do not reveal a clear picture on non-rated firms. Based on leverage they are less constrained than rated firms, while the low share of dividend payers among them lets them appear more constraint. Interestingly, according to the KZ index the median non-rated firm ranks somewhere in between the median investment grade and non-investment grade firms. I further explore this in Figure A.1 of the appendix where I plot the distribution of KZ scores among investment grade, non-investment grade, and non-rated firms. It supports an interpretation of investment grade and non-investment grade firms as extreme points with respect to the severity of financing constraints, while non-rated firms fall in between these two extremes. On the one hand many non-rated firms are as constrained as the majority of non-investment grade firms. But on the other hand there are also many non-rated firms which appear as unconstrained as investment grade firms.

3   Model

Time is discrete and the horizon is infinite. The economy consists of (i) a continuum of firms, (ii) a representative household, (iii) financial intermediaries, (iv) labor agencies, (v) capital goods producers, and (vi) a government sector.

3.1   Firms

There is a unit measure of firms, indexed by $i \in [0, 1]$. Firms produce output using a constant returns technology.

$$Y_t^i = (z_t^i K_t^i)^\alpha (A_t \ N_t^i)^{1-\alpha}$$

Aggregate productivity A_t follows a first order Markov process, while the idiosyncratic productivity/ return component z_t^i is an i.i.d. draw (across firms and across time) from a distribution F.

The timing of events for a firm i is displayed in Figure 2. The capital stock K_t^i is chosen

at the end of the previous period. The firm needs to finance capital purchases either through their own end-of-period net worth or through debt issuance B_t^i. Crucially, the firm is not allowed to raise new equity from household to purchase capital, or equivalently, it cannot pay a negative dividend. In the beginning of the period the exogenous state s is revealed, as well as the idiosyncratic component z_t^i. The firm chooses employment N_t^i and produces output. Then, it either repays its outstanding debt B_t^i or declares default. Upon default the firm exits the economy immediately and forfeits all claims to the firm's assets. Instead if it does repay its debt, next it is hit by an exogenous exit shock with probability σ. The share σ of non-defaulting firms that is hit by this shock is forced to exit after selling their capital and paying the proceeds to the household. Both defaulting and exiting (though non-defaulting) firms are replaced by new firms as to keep the measure of firms constant. Finally, continuing firms and new firms choose the capital stock for next period and the financing thereof. I assume that default is socially costly. Namely, a fraction θ of the output and capital stock of defaulting firms is destroyed.

Figure 2: Timeline for firm i
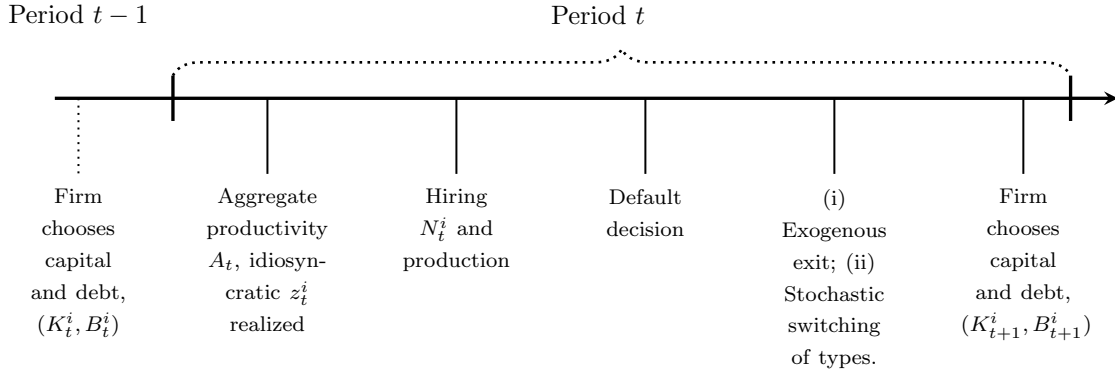
In departure from this standard setup, I assume that firms are heterogeneous with respect to their depreciation rates. The depreciation rate can be either high (δ^h) or low (δ^l), with equal shares in the population of firms.

I allow for stochastic switching of types. After the default decision, but before the choice of next period's capital stock and debt, firms with depreciation rate δ^h (δ^l) transition to

depreciation rate δ^l (δ^h) with probability p. Denote the transition probabilities as

$$\pi_\delta = \begin{cases} 1 - p, & \text{for } \delta^i_{t+1} = \delta^i_t \\[2mm] p, & \text{for } \delta^i_{t+1} \neq \delta^i_t \end{cases} \tag{1}$$

Depreciation rates can be observed by lenders. When a firm with a certain depreciation rate defaults or exits exogenously, it is replaced by a new firm with the same deprecation rate.

The draw from the idiosyncratic productivity distribution is independent of the depreciation rate. Notice that the share of firms with a particular depreciation rate is ex-ante homogeneous with respect to the idiosyncratic productivity shock z. That means that for each share of firms with a certain depreciation rate there will be a well-defined expected default rate.

To give a preview of the results, the idiosyncratic productivity component z introduces heterogeneity during the production stage and results in different profits across firms. However, the intertemporal decision regarding the capital structure will actually not depend on the realization of z. In fact, all firms (non-defaulting and new ones) will choose the same level of debt relative to the capital stock. In contrast, the depreciation rate matters for intertemporal decisions and introduces heterogeneity in the capital structure.

3.1.1 The firm's problem

One can separate the firm's problem into a static one - the optimal choice of employment -, and a dynamic one - the optimal choice of capital and the financing decision. Regarding the optimal choice of employment, consider a firm with capital stock K^i_t and idiosyncratic productivity z^i_t, and depreciation rate δ^i_t. It chooses employment N^i_t to maximize profits taking as given the wage W_t.

$$\max_{N^i_t} \ (z^i_t K^i_t)^\alpha (A_t N^i_t)^{1-\alpha} - W_t N^i_t$$

The optimal choice of employment equates the marginal product of labor with the wage and yields $N_t^{i*} = \left(\frac{A_t(1-\alpha)}{W_t}\right)^{\frac{1}{\alpha}} \frac{z_t^i K_t^i}{A_t}$.[10] The resulting profits are $\Xi_t z_t^i K_t^i$, where $\Xi_t = \alpha \left(\frac{A_t(1-\alpha)}{W_t}\right)^{\frac{1-\alpha}{\alpha}}$.

For studying the dynamic problem I define the net worth after production and debt repayment. As in Bernanke et al. (1999) and Christiano et al. (2014) the idiosyncratic component z_t^i also multiplies the residual value of capital.[11]

$$NW_t^i = \Xi_t z_t^i K_t^i + (1 - \delta_t^i) Q_t z_t^i K_t^i - B_t^i = (\Xi_t + (1 - \delta_t^i) Q_t) z_t^i K_t^i - B_t^i$$

Let $V_t(NW_t^i, \delta_{t+1}^i)$ denote the end-of-period value of a firm with net worth NW_t^i that has just learned that its depreciation rate for next period will be δ_{t+1}^i. It now chooses capital K_{t+1}^i and debt B_{t+1}^i to maximize the present discounted value of dividends, using the stochastic discount factor of the household, $m_{t,t+1}$, to price future cash flows.

$$V_t(NW_t^i, \delta_{t+1}^i) = \max_{\{K_{t+1}^i, B_{t+1}^i\}} \left\{ Div_t^i \right.$$
$$\left. + \mathbb{E}\left[m_{t,t+1} \sum_{\delta_{t+2}^i} \pi_\delta \int \max(0, (1-\sigma)V_{t+1}(NW_{t+1}^i, \delta_{t+2}^i) + \sigma NW_{t+1}^i) dF \right] \right\}$$

where the dividend paid today, Div_t^i, must be non-negative and is the sum of current net worth and new borrowing minus expenditures for capital purchases. The price of debt with face value B_{t+1}^i is $q_t(K_{t+1}^i, B_{t+1}^i, \delta_{t+1}^i)$ and is taken as given by firms.

$$Div_t^i = NW_t^i - Q_t K_{t+1}^i + q_t(K_{t+1}^i, B_{t+1}^i, \delta_{t+1}^i) B_{t+1}^i \geq 0. \tag{2}$$

In the following period depending on the aggregate state and the realization of idiosyn-

---

[10]On the balanced growth path aggregate productivity as well as capital and the wage grow at rate μ_A, therefore employment is constant.

[11]This feature gives the idiosyncratic component an alternative interpretation as a capital quality or return shock. If only current period profits were affected by the idiosyncratic shock then firms would not default at reasonable levels of leverage. The value of their capital stock would provide sufficient incentives to always repay their debts. An alternative approach would be the introduction of a fixed cost of operation (linear in capital).

cratic productivity the firm may decide to default, resulting in a value of zero. A non-defaulting firm may exit with probability σ in which case the net worth NW_{t+1}^i is paid as a dividend to the household. Otherwise the firm continues yielding value $V_{t+1}(NW_{t+1}^i, \delta_{t+2}^i)$. When the firm has the option to default it compares the value of continuing, $V_{t+1}(NW_{t+1}^i, \delta_{t+1}^i)$, against exiting and obtaining zero. Thus, it will exit whenever $V_{t+1}(NW_{t+1}^i, \delta_{t+2}^i) < 0$.

I now characterize the default decision and the optimal choice of capital and debt. I first guess that the firm's value function is linear in net worth NW_t^i and that there exists a non-negative $\lambda_t(\delta_{t+1}^i)$ such that

$$V_t(NW_t^i, \delta_{t+1}^i) = \lambda_t(\delta_{t+1}^i) NW_t^i$$

The multiplier $\lambda_t(\delta_{t+1}^i)$ is a forward-looking variable summarizing the value of a dollar net worth inside the firm. It does depend only on aggregate conditions and the firm-specific depreciation rate for next period, but not on current idiosyncratic variables, i.e., the depreciation rate δ_t^i and idiosyncratic productivity z_t^i.

Given this guess a firm defaults whenever $NW_t^i < 0$. Using the expression for net worth from above, one can derive a default threshold for the idiosyncratic productivity component z_t^i. A firm will default if

$$z_t^i < \bar{z}_t^i = \frac{\frac{B_t^i}{K_t^i}}{\Xi_t + (1 - \delta_t^i)Q_t} = \frac{l_t^i}{\Xi_t + (1 - \delta_t^i)Q_t}$$

where the second equality uses the definition of (book) leverage l_t^i as debt over assets $l_t^i = \frac{B_t^i}{K_t^i}$. For a given depreciation rate the likelihood of default, $F(\bar{z}_t^i)$, is increasing in leverage l_t^i. Likewise, for a given level of leverage l_t^i, the likelihood of default is increasing in the depreciation rate.

I denote net worth per unit of capital as $nw_t^i = \frac{NW_t^i}{K_t^i}$ and rewrite the dynamic problem of

the firm, substituting in the guess for the value function.

$$
\lambda_t(\delta^i_{t+1})\, nw^i_t K^i_t = nw^i_t\, K^i_t + \max_{\{K^i_{t+1}, B^i_{t+1}\}} \left\{ -Q_t K^i_{t+1} + q_t(K^i_{t+1}, B^i_{t+1}, \delta^i_{t+1}) B^i_{t+1} \right.
$$

$$
\left. + \mathbb{E}\left[m_{t,t+1} \int_{\bar{z}^i_{t+1}} nw^i_{t+1} K^i_{t+1} dF \sum_{\delta^i_{t+2}} \pi_\delta((1-\sigma)\lambda_{t+1}(\delta^i_{t+2}) + \sigma) \right] \right\}
$$

$$
\Leftrightarrow \lambda_t(\delta^i_{t+1})\, nw^i_t = nw^i_t + \max_{\{\iota^i_{t+1}, l^i_{t+1}\}} \iota^i_{t+1} \left\{ -Q_t + q_t(l^i_{t+1}, \delta^i_{t+1}) l^i_{t+1} + \right. \tag{3}
$$

$$
\left. \underbrace{\mathbb{E}\left[m_{t,t+1} \int_{\bar{z}^i_{t+1}} nw^i_{t+1} dF \sum_{\delta^i_{t+2}} \pi_\delta((1-\sigma)\lambda_{t+1}(\delta^i_{t+2}) + \sigma) \right]}_{v_{cont}} \right\}
$$

The second line expresses the problem per units of capital K^i_t, and uses the definition of capital growth $\iota^i_{t+1} = \frac{K^i_{t+1}}{K^i_t}$. The associated non-negativity constraint on dividends is

$$
nw^i_t - \iota^i_{t+1}(Q_t - q_t(l^i_{t+1}, \delta^i_{t+1}) l^i_{t+1}) \geq 0
$$

The problem of choosing $(\iota^i_{t+1}, l^i_{t+1})$ has a very intuitive solution. First, the capital structure choice l^i_{t+1} is determined by the first order condition

$$
[l^i_{t+1}:] \; \lambda_t(\delta^i_{t+1})\left(q_t(l^i_{t+1}, \delta^i_{t+1}) + \frac{\partial q_t(l^i_{t+1}, \delta^i_{t+1})}{\partial l^i_{t+1}} l^i_{t+1} \right)
$$

$$
- \mathbb{E}\left[m_{t,t+1}(1 - F(\bar{z}^i_{t+1})) \sum_{\delta^i_{t+2}} \pi_\delta((1-\sigma)\lambda_{t+1}(\delta^i_{t+2}) + \sigma) \right] = 0. \tag{4}
$$

This optimality condition balances the effect of leverage on the amount raised today against the change of the expected discounted repayment. In case the firm can continue beyond period $t+1$ the repayment is weighted by the multiplier $\lambda_{t+1}(\delta^i_{t+2})$. If the economy was to enter in a recession in period $t+1$, net worth would be scarce and its value $\lambda_{t+1}(\delta^i_{t+2})$ would be large. Accordingly, the repayment of the loan would be relatively costly to the firm.

Notice that the first order condition does not depend on the current realization of idiosyn-

cratic productivity z_t^i, but only the expectation over future values z_{t+1}^i. This means that the capital structure choice will be the same for all firms with the same next-period depreciation rate. However, firms with different depreciation rates may choose different capital structures.

Second, the term inside the max operator in equation (3), v_{cont}, is non-negative[12]. Therefore, the firm sets ι_{t+1}^i as high as possible, the only constraint being the requirement for non-negative dividends. Therefore, the growth rate will be set to

$$\iota_{t+1}^i = \frac{nw_t^i}{Q_t - q_t(l_{t+1}^i, \delta_{t+1}^i)l_{t+1}^i} \tag{5}$$

The firm retains all earnings to purchase more capital rather than paying out dividends to the household. Unlike in the case of the capital structure the acquisition of capital does depend on the realization z_t^i. Firms with a higher z_t^i and higher net worth nw_t^i and can buy more capital.

3.2 Financial intermediaries

Competitive financial intermediaries channel funds from savers (households) to borrowers (firms).

The borrowing arrangement is made in the end of period t and debt comes due in period $t+1$ after production has taken place. I assume that firms can borrow only in non-contingent debt. The repayment can be conditioned neither on aggregate productivity A_{t+1} nor on the idiosyncratic realization z_{t+1}^i. If the firm defaults in the subsequent period the intermediary receives zero while the defaulting firms' output and capital stock is seized by the government.

The financial contract can be motivated by a costly state verification problem in which lenders are unable to observe the idiosyncratic realization z_{t+1}^i unless they pay a monitoring cost. Under the assumption that lenders cannot recover the implicit monitoring cost is 1, notwithstanding the social cost of default is $\theta \le 1$.

---

[12]Formally, this can be shown by deriving the first-order condition for capital K_{t+1}^i and acknowledging that the Lagrange multiplier on the non-negativity constraint on dividends (2) is non-negative.

The price of debt with face value B_{t+1}^i issued by a firm with capital stock K_{t+1}^i and depreciation rate δ_{t+1}^i is $q_t(K_{t+1}^i, B_{t+1}^i, \delta_{t+1}^i)$. Therefore, the competitive pricing of debt simply reflects the default probability.

$$q_t(K_{t+1}^i, B_{t+1}^i, \delta_{t+1}^i)B_{t+1}^i = \mathbb{E}\left[m_{t,t+1}\left(1 - F\left(\bar{z}_{t+1}^i\right)\right)B_{t+1}^i\right]$$

Notice that the intermediary observes both the depreciation rate δ_{t+1}^i of the firm and its choice of capital K_{t+1}^i. Notice further that that capital K_{t+1}^i enter the price of debt only through the default threshold and the leverage ratio $\frac{B_{t+1}^i}{K_{t+1}^i}$. Therefore, the price of debt can be expressed as a function of leverage alone. $q_t(l_{t+1}^i, \delta_{t+1}^i) = \mathbb{E}\left[m_{t,t+1}(1 - F(\bar{z}_{t+1}^i))|S\right]$

Intermediaries raise deposits from household to fund their lending activity. Deposits are non-contingent debt contracts that are due in the end of period $t+1$ and that pay the risk-free interest rate r_t. The risk-free rate will be determined through the households Euler equation, $1 = \mathbb{E}[m_{t,t+1}(1 + r_t)]$, and it is taken as given by intermediaries.

Competition among intermediaries drives expected profits to zero, i.e., in equilibrium

$$\mathbb{E}\left[m_{t,t+1}\left\{(1 - F(\bar{z}_{t+1}^i))q_t(l_{t+1}^i, \delta_{t+1}^i)^{-1} - (1 + r_t)\right\}\right] = 0. \tag{6}$$

The interest rate on corporate debt $q_t(l_{t+1}^i, \delta_{t+1}^i)^{-1}$ is a spread over the risk-free rate that reflects expected default risk. It is increasing with leverage for a given next-period depreciation rate δ_{t+1}^i, and increasing in the depreciation rate for a given level of leverage. Therefore, firms with lower depreciation rates face a lower interest rate schedule.

While expected profits of intermediaries are always zero, realized profits are zero only in steady state. With aggregate uncertainty, profits can be positive or negative and will be disbursed to the household. Let D_t (for deposits) denote the amount of deposits raised in the previous period, then

$$\Pi_t^{Int} = \int_0^1 B_t^i \mathbb{I}_{\{z_t^i \geq \bar{z}_t^i\}} di - (1 + r_{t-1})D_t$$

where the first term reflects debt repayments from non-defaulting firms. The cash flows from newly raised deposits and new corporate debt always balance each other.

3.3 Labor agencies

Firms hire workers via monopolistically competitive labor agencies. In particular, each firm i uses a Dixit-Stiglitz composite of different labor varieties $N_t^i = \left(\int_0^1 (n_{g,t}^i)^{\frac{\epsilon_w - 1}{\epsilon_w}} dg \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$, where $n_{g,t}^i$ is the amount of the variety provided by agency $g \in [0,1]$. Labor agencies purchase raw labor services from the household at price $\tilde{W}_t$, then costlessly differentiate these into their respective variety g and sell it on to firms at price $W_{g,t}$.

The resulting labor demand from firm i for variety g is $n_{r,t}^i = \left(\frac{W_{g,t}}{W_t} \right)^{-\epsilon_w} N_t^i$. Aggregating over all firms yields

$$ n_{g,t} = \left(\frac{W_{g,t}}{W_t} \right)^{-\epsilon_w} N_t $$

where $n_{g,t} = \int_0^1 n_{g,t}^i di$ and $N_t = \int_0^1 N_t^i di$. The wage index for composite labor, W_t, is defined by $W_t N_t = \int_0^1 W_{g,t} n_{g,t} dg$. Plugging in labor demand this yields $W_t = \left(\int_0^1 W_{g,t}^{1-\epsilon_w} dg \right)^{\frac{1}{1-\epsilon_w}}$.

Labor agencies take as given labor demand from firms and choose the price of their labor variety to maximize the present discounted value of profits, using the household's stochastic discount factor between periods t and $t+j$, $m_{t,t+j}$. Labor agencies can adjust the price of the labor variety only infrequently. Every period a random fraction $1 - \lambda_w$ of agencies is allowed to freely reset their price $W_{g,t}$, while the complementary fraction λ_w has to retain the price from last period. When the agency can reset its price it takes into account all future states in which it is stuck with the current price. I assume that the wage mark-up over marginal costs is corrected by a subsidy ν_w.[13]

---

[13]Due to market power agencies set the price of their variety as a mark-up over marginal costs, i.e., the compensation $\tilde{W}_t$ paid to the household, implying that labor in equilibrium would be lower than under perfect competition. This inefficiency is not central to the analysis I assume that unions receive a subsidy that resolves the inefficiency and leads to optimal output in steady state. For every unit of labor purchases from the household the agency pays only $(1 - \nu_w)\tilde{W}_t$. The subsidy is financed through lump-sum taxes.

$$\max_{W_{g,t}^*} \mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_w^j m_{t,t+j}\left(W_{g,t}^* n_{g,t+j} - \tilde{W}_{t+j}(1-\nu_w)n_{g,t+j}\right)\bigg| S_t\right]$$

The first-order condition for the optimal reset price $W_{g,t}^*$ is

$$\mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_w^j m_{t,t+j} N_{t+j} W_{t+j}^{\epsilon_w}\left(W_{g,t}^* - \tilde{W}_{t+j}\right)\bigg| S_t\right] = 0$$

where the input subsidy ν_w has been set to $\nu_w = \frac{1}{\epsilon_w}$ as to eliminate the mark-up over marginal costs. The reset price is symmetric for all firms which are able to reset, such that $W_{g,t}^* = W_t^*$, $\forall g$. Finally, the wage index W_t writes as

$$W_t = \left[(1-\lambda_w)(W_t^*)^{1-\epsilon_w} + \lambda_w(W_{t-1})^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

If $\lambda_w = 0$ all labor agencies can reset the wage every period and the wage is fully flexible. This special case is equivalent to a centralized labor market in which households and firms interact directly and the wage clears the market. Finally, profits of labor agencies, $\Pi_t^{LA} = \left(W_t - (1-\nu_w)\tilde{W}_t\right)N_t$ are rebated to the household.

3.4 Representative household

The representative household maximizes the present discount utility $\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(C_\tau, L_\tau)$ by choosing consumption C_t, labor supply L_t, and savings in deposits D_{t+1}. The choice is subject to the following budget constraint.

$$\tilde{W}_t L_t + D_t(1 + r_{t-1}) + \Pi_t + Div_t \geq C_t + D_{t+1} + T_t$$

Labor is supplied to the employment agencies and compensated at rate $\tilde{W}_t$. Savings in deposits from last period pay the risk-free rate r_{t-1}. Besides, the household is the recipient of profits from capital producers, financial intermediaries, and labor agencies: $\Pi_t = \Pi_t^{CP} + \Pi_t^{Int} +$

Π^{LA}. Moreover, the household pays lump-sum taxes T_t and receives dividends of exiting firms Div_t. The optimality conditions for the household consists of the Euler equation for deposits, $1 = \mathbb{E}[m_{t,t+1}(1 + r_t)]$, where $m_{t,t+1} = \beta \frac{U_C(C_{t+1},L_{t+1})}{U_C(C_t,L_t)}$. The labor-consumption trade-off writes as $-\frac{U_L(C_t,L_t)}{U_C(C_t,L_t)} = \tilde{W}_t$.

Notice that while the household is the ultimate owner of firms, it cannot inject equity into them or otherwise interfere with the management. Therefore, firms (and firm specific variables) do not appear in the household's problem, and there is no asset pricing equation for equity.

3.5 Capital producers

Capital producers take the price of capital goods, Q_t, as given, and buy I_t units of the output good to produce new capital. Their production technology $\Phi\left(\frac{I_t}{K_t}\right)$ features positive, but decreasing returns ($\Phi' > 0, \Phi'' < 0$), and output of new capital goods is $\Phi\left(\frac{I_t}{K_t}\right)K_t$. The problem of capital producers is given by

$$\Pi_t^{CP} = \max_{I_t}\left\{Q_t\Phi\left(\frac{I_t}{K_t}\right)K_t - I_t\right\}.$$

The first order condition yields the optimal investment rate as a function of the price of capital.

$$Q_t = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}$$

3.6 Government

The government collects profits and capital net of depreciation from defaulting firms, whereof a fraction θ is lost as a deadweight cost of default. On the expenditure side the government pays the input subsidy ν_w to labor agencies. The government balances the budget period-by-period through lump-sum taxes T_t - or transfers if the balance is positive.

$$T_t = \nu_w \tilde{W}_t N_t - (1 - \theta) \int_0^1 (Y_t^i + (1 - \delta_t^i) Q_t K_t^i z_t^i) \mathbb{I}_{\{z_t^i \le \bar{z}_t^i\}} di$$

3.7 Aggregation and equilibrium

The resource constraint of this economy is

$$Y_t = C_t + I_t$$

where Y_t is output net of deadweight loss of defaulting firms.

$$Y_t = \int_0^1 Y_t^i \left\{ \mathbb{I}_{\{z_t^i \ge \bar{z}_t^i\}} + (1 - \theta) \mathbb{I}_{\{z_t^i < \bar{z}_t^i\}} \right\} di$$

Conceptually, the economy features two representative firms indexed by their depreciation rate. Each such stand-in firm represents the measure of firms that share the same depreciation rate (after transitions) and therefore chooses the same leverage each period. There is heterogeneity within each stand-in firm due to the idiosyncratic productivity component z_t^i and different levels of capital and net worth. However, the exact distribution of capital and net worth is irrelevant for equilibrium dynamics. I therefore denote as NW_t^δ the end-of-period net worth of firms with period $t + 1$ depreciation rate δ.

$$NW_t^\delta = (1 - \sigma) \int_0^1 NW_t^i \mathbb{I}_{\{\delta_{t+1}^i = \delta, z_t^i \ge \bar{z}_t^i\}}, \quad \delta = \delta^h, \delta^l$$

Notice that NW_t^δ encompasses net worth from all firms i for which either (i) $\delta_{t+1}^i = \delta_t^i = \delta$, or (ii) $\delta_t^i \ne \delta$, but $\delta_{t+1}^i = \delta$. The share of type-switching firms is given by the switching probability p. It therefore determines how changes in net worth in among one group of firms spills over to others.

Dividend payments to the household come from the share σ of non-defaulting firms that are forced to exit exogenously. Each of these firms has positive net worth NW_t^i and pays it

out to the household, $Div_t^i = NW_t^i$. Therefore, aggregate dividends are

$$Div_t = \sigma \int_0^1 NW_t^i \mathbb{I}_{\{NW_t^i \geq 0\}} di.$$

Denote total capital of firms with depreciation rate δ as K_t^δ, $\delta = \delta^h, \delta^l$. The law of motion for capital in the economy takes into account the deadweight loss on capital from defaulting firms.

$$K_{t+1} = \int (1 - \delta_t^i) K_t^i \left(\mathbb{I}_{\{z_t^i \geq \bar{z}_t^i\}} + (1 - \theta) \mathbb{I}_{\{z_t^i < \bar{z}_t^i\}} \right) di + \Phi \left(\frac{I_t}{K_t} \right)$$

$$= K_{t+1}^{\delta^h} + K_{t+1}^{\delta^l}$$

Moreover, the aggregation of equation (5) over all firms with period $t + 1$ depreciation rate δ provides transition dynamics for total capital with δ firms.

$$K_{t+1}^\delta = \frac{NW_t^\delta}{Q_t - q(l^\delta, \delta)l^\delta}, \quad \delta = \delta^h, \delta^l \tag{7}$$

The aggregate state S_t consists of (i) aggregate productivity, (ii) the distribution of debt and capital over firms with high and low depreciation rates, (iii) the wage W_{t-1}, and it is denoted as $S_t = \{A_t, \{l_t^\delta, K_t^\delta\}_{\delta = \delta^h, \delta^l}, W_{t-1}\}$. In equilibrium all agents behave optimally and the markets for labor, capital and deposits clear.

4   Quantitative analysis

In this section I calibrate the model using a sample of Compustat firms. More constrained, non-investment grade firms are represented by the set of firms with low depreciation rates. These firms assume higher steady state leverage and have a higher marginal value of net worth. I then evaluate the consequences of firm heterogeneity under aggregate uncertainty.

I assess the model fit with the employment dynamics in the data (recall Figure 1) and the heterogeneous elasticities with respect to output growth (recall Table 2). Finally, I consider extensions that allow me to study responses to other aggregate shocks (credit spreads, monetary policy, firm risk).

4.1   Functional forms and parameter choices

I first specify the subset of parameters that are set to values common in the literature. Subsequently, I discuss the joint calibration of parameters in the non-stochastic steady state and the respective data moments that are targeted.

The time period is a quarter. Aggregate productivity A_t follows an AR(1) process along a deterministic growth trend μ_A.

$$\log A_t = \mu_A + \log A_{t-1} + a_t$$

where $a_t = \rho_a a_{t-1} + \eta_{a,t}$ and $\eta_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$. The target for μ_A is the annual growth rate of per capital real GDP between 1966Q1 and 2012Q4, 1.63%. Given the trend growth rate I set the discount factor β to 0.994 to obtain an annual risk-free rate on the balance growth path of 4%.[14]

The household utility function is separable in consumption and labor, $U(C_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi e^{(1-\gamma)\mu_A} \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}$. I set the risk aversion parameter γ to 1, and the elasticity of labor supply η to 0.5. The disutility of labor, χ, scales the size of the economy and is set to obtain steady state output $\bar{Y}$ of 1.

The capital share in the production function of output is set to $\alpha = 0.33$. For the production function of new capital I follow Jermann (1998) and specify $\Phi\left(\frac{I_t}{K_t}\right) = \frac{\varphi_2}{1-\varphi_1}\left(\frac{I_t}{K_t}\right)^{1-\varphi_1} + \varphi_3$. The optimal investment rate under this specification satisfies $\frac{I_t}{K_t} = (Q_t\varphi_2)^{\frac{1}{\varphi_1}}$. Parameters (φ_2, φ_3) are set to yield a steady state investment rate $\frac{\bar{I}}{\bar{K}}$ of 2%, and a steady state price

---

[14]The assumption of trend growth in TFP implies that the following variables grow at rate μ_A: Y_t, C_t, I_t, $\{K^\delta\}_{\delta=\delta^h,\delta^l}$, $\{NW^\delta\}_{\delta=\delta^h,\delta^l}$, W_t, D_t, T_t. I redefine detrended variables as $\tilde{X}_t = X_t e^{-t\mu_A}$.

of capital $\bar{Q}$ of 1. The elasticity of the price of capital with respect to the investment rate, $\varphi_1 > 0$, regulates the importance of adjustment costs of capital. High adjustment costs imply less volatile investment, but a more volatile price of capital and therefore more net worth volatility. There is some disagreement in the literature about the admissible range of values. I follow Bernanke et al. (1999) with $\varphi_1 = 0.25$, noting that this value delivers a better fit for the relative volatility of investment than the value of 2.5 used by Guvenen (2009). The cost of default θ is set to 0.15 which is close to the value in Bernanke et al. (1999).

The distribution of the idiosyncratic productivity shock, $F(z_t^i)$, is assumed to be log-normal, $\log z_t^i \sim \mathcal{N}\left(-\frac{1}{2}\sigma_z^2, \sigma_z^2\right)$ $\forall i$. The dispersion parameter σ_z, together with the two depreciation rates (δ^h, δ^l) and the exit probability σ jointly determine (i) the average depreciation rate, (ii)/(iii) the credit spreads for non-investment grade/ investment grade firms, (iv) the average level of leverage. The average depreciation rate is chosen as 10% annually. The leverage target is the average level of leverage between 1986 and 2014 for investment grade and non-investment grade firms in Compustat, $\bar{l}^{av} = 0.63$. Bond yield indices from Bank of America Merrill Lynch (via FRED) are used to inform the credit spread targets. The average spread over a Treasury spot rate for investment grade corporate debt is 1.61%, while the speculative grade spread is 5.79% (1997-2014).[15][16]

The switching probability between high and low depreciation rates is related to the incentives for capital accumulation and the relative shares output and employment shares. I set it to 0.15, implying a steady state employment share of the more constrained firms of about 62%. I report sensitivity checks later in the paper.

The elasticity of substitution between different labor varieties is set to 6. The Calvo parameter λ_w is set to imply an average during of wages of one year. Thus, the reset probability is somewhat larger than the estimated value from Smets and Wouters (2007) and

---

[15]Investment grade series `BAMLC0A0CM`, Speculative grade series `BAMLH0A0HYM2`. See Figure A.2 in the appendix for the time series.

[16]I also experimented with the Baa bond yield relative to the yield on a 10-year Treasury security (series `BAA10Y`) and the corresponding Aaa bond yield (series `AAA10Y`). While the level of these spreads is lower the main implications of the model carry over since the difference between the two spreads is substantial.

Lawrence J. Christiano (2005), but smaller than the value by Christiano et al. (2014).

The set of jointly calibrated parameters for the baseline specification is presented in the lower section of Table 3.

Table 3: Parameters and targets

| Parameter	Value	Description	Target
β	0.994	Discount factor	Risk-free rate 4%
μ_A	0.004	Trend growth rate	Ann. GDP growth 1.63%
γ	1	Risk aversion	Standard
η	0.5	Elasticity of labor supply	Standard
α	0.33	Capital share	Standard
θ	0.15	Social cost of default	Bernanke et al. (1999)
φ_1	0.25	Elasticity of Q w.r.t. $\frac{I}{K}$	Bernanke et al. (1999)
χ	4.65	Disutility of labor	$\bar{Y} = 1$
φ_2, φ_3	0.40,-0.004	Investment technology	$\bar{Q} = 1$, $\bar{I}/\bar{K} = 0.02$
σ	0.13	Exogenous exit	$\bar{l}^{av} = 0.63$
σ_z	0.21	Disp idiosyncratic shock	I grade spread, 1.61% ann.
δ^l	0.60%	Depreciation rates	Non-I grade spread 5.79% ann.
δ^h	5.94%	Depreciation rates	$\delta^{av} = 10\%$ ann.
p	0.15	Switching probability	Relative employment shares
λ_w, ϵ_w	0.75, 6	Calvo, ES labor	Av. wage duration 1 year

4.2   Steady state properties

Figure 3 displays the interest rate schedules for more and less constrained firms as well as their choice of leverage in steady state. Firms with low depreciation rates can borrow at lower interest rates conditional on leverage since the higher residual value of their assets makes them less likely to default when the repayment comes due. Firms with low depreciation rates exploit this cost advantage by choosing higher leverage. Thus, in equilibrium they pay higher interest rates and have higher default probabilities.

Table 4 compares steady state moments that were not directly targeted with the relevant data counterpart. First, the targeting of credit spreads implies differences in leverage. The difference in the model of 0.10 is comparable to that in the data 0.11.

The return on assets is computed as profits after interest expense and before depreciation. Since productivity is the same for more and less constrained firms in the model, differences

Figure 3: Annualized interest rate $q(l, \delta)^{-1}$
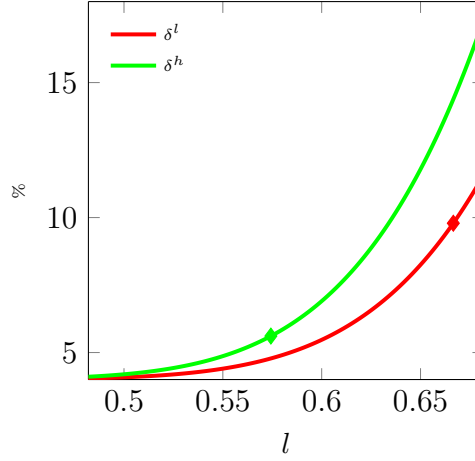
in profits emerge from the interest component. In line with the insight from Figure 3, the more constrained firms in the model pay higher interest rates and thus have lower profits. However, the profit differential is not as pronounced as in the data. This indicates the presence of technological differences between investment grade and non-investment grade firms.

Unlike return on assets, the return on equity in the model is higher for more constrained firms. This is synonymous with the net worth multiplier being higher for these firms. In the data, profits relative to the market value of equity do not appear to be different among investment grade and non-investment grade firms.

Default rates in the model map directly to credit spreads, therefore the difference displayed in Table 4 largely reflects the targeting of these spreads. In the data default rates of investment grade firms are much lower, indicating that the credit spreads contain a premium over the actual default risk. Moreover, the literature has concluded that these premia are time-varying and especially high in recessions (see Cochrane (2011)). In section 4.6 I introduce an extension with shocks to credit spreads that allow borrowing costs to rise over and beyond expected default rates.

Table 4: Steady state - Additional moments

	Model		Data	
	Low δ	High δ	I grade	Non-I grade
Leverage	0.57	0.67	0.58	0.69
Return on assets	0.08	0.07	0.13	0.07
Return on equity	0.21	0.19	0.13	0.13
Default rate (% ann)	1.59	5.65	0.14	4.62

Notes: In the model, leverage is debt relative to capital; Return on assets is profits before depreciation and after interest payments. In the Compustat data, leverage is total liabilities over total assets, return on assets is operating income before depreciation (OIBDP) minus interest expenses (XINT) over total assets. Default rates for investment grade and non-investment grade firms are average default rates on corporate bonds between 1981 and 2011 (see 2011 Annual US Corporate Default Study by S&P, the average includes financial firms).
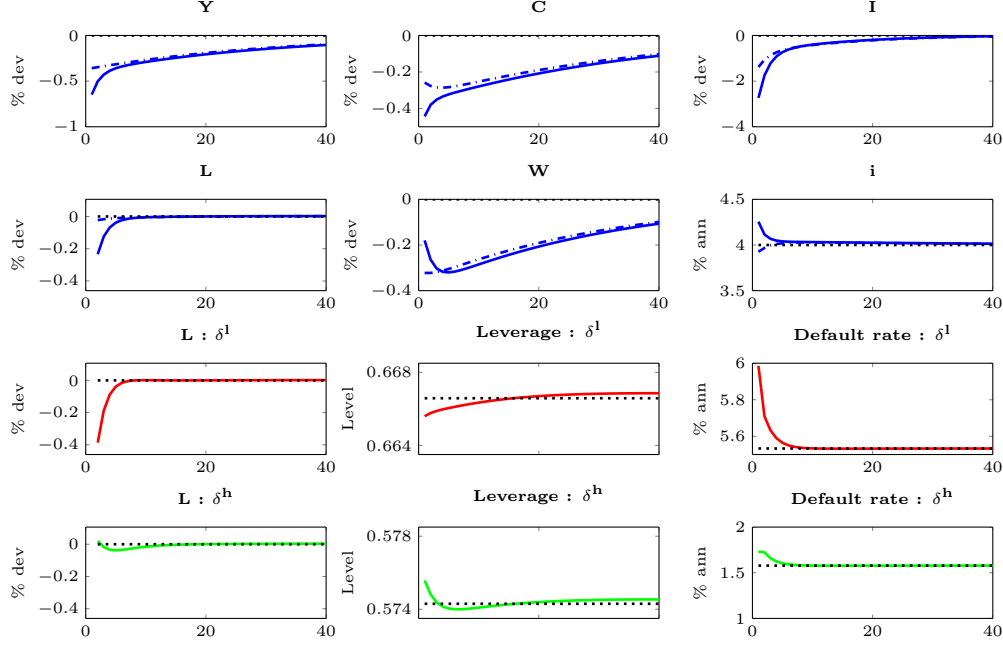
4.3 Impulse response to a productivity shock

This section studies the response of the baseline economy to a 0.5% aggregate productivity shock. The autocorrelation ρ_a is set to 0.95. The first two rows of Figure 4 show the response of aggregate variables, the third (fourth) row relates to the response of more (less) constrained firms. Additionally, the dynamics of aggregate variables are compared with the flexible wage economy ($\lambda_w = 0$, dash-dotted line).

Aggregate variables respond as usual, i.e., output, consumption, investment, employment, and wages fall, while the interest rate rises. Notice that under flexible wages employment does barely decline due to the wealth effect on labor supply. Wage rigidity undoes this effect by preventing the quick downward adjustment of the wage. Consequently, the fall in aggregate variables is more pronounced with rigid wages.

The responses of more and less constrained firms are in stark contrast. With the fall of investment and asset prices more constrained firms face the largest pressure to divest capital in order to rebalance their capital structure. Also, the net worth multiplier rises most for the more constrained (not shown), reflecting that financial constraints for these firms become disproportionally severe. Less constrained firms by themselves would also reduce investment. However, the deleveraging pressure among more constrained firms makes capital available at discount prices to them. Accordingly, they absorb some of the capital that is shed by

Figure 4: Impulse response for negative productivity shock (0.5%)

Note: Impulse response to a 0.5% productivity shock. Log productivity follows an AR(1) process with autocorrelation $\rho_a = 0.95$. Rows 1&2 show responses of aggregate variables. Rows 3&4 show responses for more and less constrained firms. In rows 1&2 solid line displays IRF with rigid wages, $\lambda_W = 0.75$. Dash-dotted line displays IRF with flexible wages (‐‐‐‐‐).

more constrained firms and finance these purchases through debt issuance. For this reason leverage of these firms actually rises in response to a negative aggregate shock.

These divergent responses are also reflected in the firms' employment choices. More constrained firms reduce employment precipitously, while less constrained firms slightly increase employment relative to steady state. Thus, both production factors, capital and labor, shift from financially more constrained to less constrained firms. Notice that the latter are not unconstrained, i.e., they are still subject to the dividend/ net worth constraint. Heterogeneity is also visible in the response of default rates. Despite their efforts to reduce leverage default rates rise much stronger for more constrained firms.

Moreover, market leverage (debt relative to the market value of assets, not shown) rises for all firms since the strong drop in asset prices outweighs the increase in debt issuance by less constrained firms.

4.4 Cyclicality of employment growth

Having confirmed that the model produces a consistent qualitative response I now ask whether the model can match the employment dynamics of investment grade and non-investment grade firms in the data. In particular, I ask whether the elasticities produced by the model line up with the data counterpart from Table 2.

For this exercise I infer aggregate productivity shocks such that the model replicates the dynamics of US real GDP.[17] I then consider the implied employment dynamics for more and less constrained firms from 1986 to 2012. Figure 5 compares employment growth of Compustat firms with the growth rates from the model implied series. Comparing the left and the right panel we see that non-investment grade, more constrained firms high growth in expansions and larger declines in downturns. It is striking that the model can replicate the differential drop of employment growth during the three recessions.

Figure 5: Model fit - Employment growth
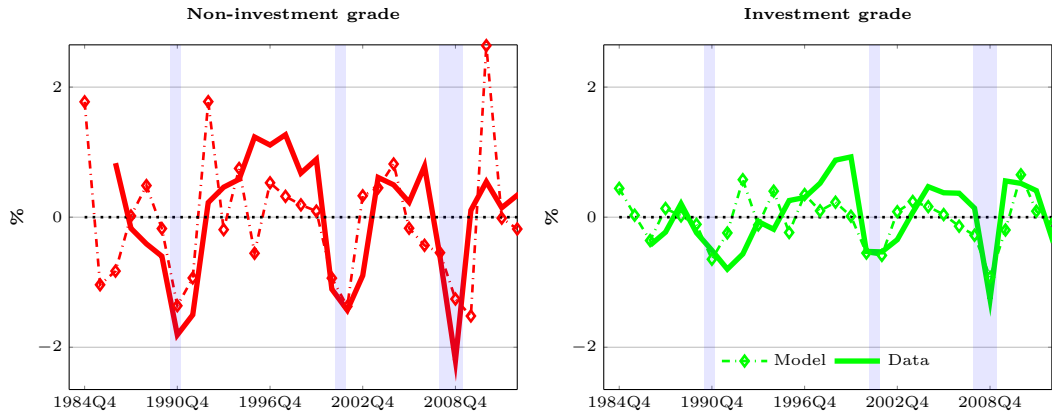
Note: Data series (solid) as in Figure 1, mean employment growth rate for firms with investment grade/ non-investment grade rating. Model series (dash-dotted).

Along two dimensions the model implied series differ from the data. First, there is a too fast recovery of employment growth after the 1990/ 1991 and 2008/2009 recessions, particularly among the more constrained firms. Second, the model misses the late stages of the booms in the 1990s and mid-2000s. Against the backdrop of the times series of US

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[17]The observable in this procedure is real GDP in deviations from a linear trend. See Figure B.1 and section B in the appendix for details.

Table 5: Employment growth cyclicality - Model comparison

	$\mathbf{\Delta Employment_{i,t}}$					
	Data	**Baseline**	$\mathbf{\lambda_w = 0}$	$\mathbf{p = 0.006}$	$\mathbf{p = 0.5}$	**No feedback**
ΔY_t	0.121***	0.120*	-0.045*	-0.019	0.154**	0.446***
	(5.75)	(1.75)	(-1.80)	(-0.34)	(2.21)	(6.51)
$NIG_{i,t-1}$	0.002**	0.000	0.000	0.000	0.000	0.000
	(2.35)	(0.24)	(0.51)	(0.45)	(0.11)	(0.33)
$NIG_{i,t-1} \times \Delta Y_t$	0.192***	0.227**	0.133***	0.218*	0.288***	-0.301***
	(5.79)	(2.33)	(3.80)	(3.67)	(2.02)	(-3.11)
$ROA_{i,t-1}$	0.053***					
	(11.13)					

Notes: Data column reports the regression results from Table 2, column (2). Model regressions use inferred series from the estimation, time span 1986Q1 to 2012Q4, transformed into annualized growth rates. NIG dummy is 1 for low δ firms. Values in parentheses are t statistics computed using robust standard errors. (***/**/*) indicate significance at the (1/5/10) percent level.

real GDP[18] that was used to infer these series these divergences are not too startling. US real GDP (in deviation from trend) peaked in the early 2000s and started declining while employment of Compustat firms increased up to 2007. In the late 1990s GDP was above trend and thus in the model output and employment are not very responsive to further positive shocks.

While the overall fit with the time series evidence for employment growth is encouraging, it remains to be shown that the model-implied employment series have different elasticities with respect to output growth as estimated from the data (see Table 2). For that matter, I regress employment growth at more and less constrained firms in the model (aggregated to yearly frequency) on output growth, a dummy that is equal to one for the more constrained firms, and the interaction of this dummy with output growth. Recall once more that the more constrained firms in the model represent the non-investment grade firms in the data. I compare the estimated elasticities against column (2) of Table 2 which also controlled for time-industry effects.

Table 2 shows that the baseline specification does a good job in matching the employment

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[18]See Figure B.1 in the appendix.

elasticities. Both the coefficients on output growth and on the interaction with the dummy for more constrained firms are positive and significant. The coefficient on the dummy for more constrained firms is not significant since both types of firms have zero employment growth in the long run.[19]

Next, I show that while the greater cyclicality of more constrained firms is very robust in the model, the overall cyclicality of employment hinges on the degree of labor market rigidities. To do so, I repeat the inference exercise with flexible wages, $\lambda_w = 0$, and re-estimate the elasticities from the model. The third column of Table 5 shows that the coefficient on output growth becomes negative and significant, while more constrained firms still have substantial higher cyclicality. Thus, wage rigidity primarily affects the response of aggregate employment but leaves the response patterns of more and less constrained firms unchanged. Notice that the volatility of wages (relative to the volatility of productivity) is 0.83 under rigid wages (with $\lambda_w = 0.75$), and therefore in the range of values that can be derived from aggregate data.[20] To summarize, this exercise shows that the model can reproduce both the aggregate as well as the cross-sectional responses of employment growth with a realistic degree of labor market friction.

Next, I explore the role of the transition probability p in the calibration. A more persistent depreciation rate (lower p) increases the incentives for more constrained firms to accumulate capital. They are more likely to retain their high debt capacity and to reap the resulting high returns on equity. Accordingly, more constrained firms will grow larger with respect to capital and labor. For the baseline economy I have set this parameter to $p = 0.15$ which resulted in an employment share of the more constrained firms of about 64%.

The relative employment shares are not just a steady state feature, but have important

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[19]The trend growth rate μ_A does not affect the regression since employment is constant along the balanced growth path.

[20]Labor productivity is real output per person in the nonfarm business sector (BLS series PRS85006163). Wages are hourly compensation in the nonfarm business sector (BLS series PRS85006103) deflated by the consumer price index (series CPIAUCSL via FRED). Sample period 1966Q1 to 2012Q4. Linear detrending/ HP filtering yields relative volatility of 0.76/ 0.88. The latter value is close to the relative volatility reported in Gertler and Trigari (2009).

implications for the dynamic responses of the more and less constrained firms. Namely, the larger the more constrained firms are the more capital will be shed in a recession and shifted to the smaller, less constrained firms. Taken to the extreme, these firms may actually grow in a recession rather than shrink.

To investigate this concern I set the transition probability p to the actual transition rate between investment grade and non-investment grade firms from the data, about 0.6% quarterly.[21] In the newly calibrated steady state, more constrained firms have an employment share of 96% and thus dominate the movement of aggregate employment.[22] The estimated elasticities reflect this and attribute the cyclicality of employment growth to these firms, while the coefficient on output growth becomes insignificant. In fact, employment growth between more and less constrained firms becomes negatively correlated, while it is positively correlated in the data.

As a robustness check, I examine a version of the model in which depreciation rates are i.i.d., the transition probability is set to $p = 0.5$. More constrained firms then account for 55% of total steady state employment. The estimation of the elasticities displays only small differences to the baseline economy. I conclude that using weak persistence of depreciation rates as in the baseline economy ensures consistent dynamic responses. It is reassuring that the results do not hinge on the exact value of p unless the persistence becomes very strong.

If one seeks to calibrate the model to the actual transition rates the heterogeneity of depreciation rates would need to be complemented with a second dimension of heterogeneity such that the weight of less constrained firms increases. For instance, if firms with higher depreciation rates had higher productivity they would accumulate more capital and increase their relative share in the economy.

Finally, the last column of Table 5 refers to an economy in which the financial accelerator

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[21]The share of investment grade rated (non-investment grade rated) firms which transition into the non-investment grade (investment grade) category is 3.0% (2.1%) annually. On average, i.e., maintaining equal shares of types, this require a quarterly transition probability of 0.6%.

[22]The re-calibrated steady state still has an equal number of firms of either type, and the same average leverage and average depreciation rate as the baseline economy.

is switched off. To be precise, capital purchases still have to be financed through net worth and borrowings, and firms with low realizations of the idiosyncratic productivity shock will find it desirable to default. However, there is no feedback from default rates to borrowing costs. Without this feedback the firm's first order condition for the capital structure choice does no longer have a well-defined solution. I assume that firms maintain leverage as in the steady state level of the baseline economy.

While the elasticity of employment growth with respect to output growth is still positive and significant, the coefficient on the interaction term representing more constrained firms turns negative. In a recession less constrained firms actually reduce employment by more than the highly constrained firms. This finding shows the importance of the endogenous debt choice to match the heterogeneous cyclicalities in the data. To better appreciate this finding, consider the log-linearized version of equation (7), i.e., the transition equation for capital. The approximation around the non-stochastic steady state writes as

$$\hat{K}^{\delta}_{t+1} = \widehat{NW}^{\delta}_t - \frac{1}{\bar{Q} - \bar{q}(\bar{l}^{\delta}, \delta)\bar{l}^{\delta}} \left(\bar{Q}\hat{Q}_t - \left(\bar{q}(\bar{l}^{\delta}, \delta) + \frac{\partial \bar{q}(\bar{l}^{\delta}, \delta)}{\partial \bar{l}^{\delta}} \right) \bar{l}^{\delta}\hat{l}^{\delta}_t \right), \quad \delta = \delta^h, \delta^l \qquad (8)$$

where variables with a hat denote percentage deviation from steady state, $\hat{K}^{\delta}_{t+1} = \frac{\hat{K}^{\delta}_{t+1} - \bar{K}^{\delta}}{\bar{K}^{\delta}}$, etc.

Deviations of capital from steady state are positively related to deviations in net worth and leverage, and negatively related to deviations of asset prices. Notice, however, that a fall in assets prices has a direct and an indirect effect. The indirect effect emerges because asset prices reduce net worth and therefore act to depress investment. On the contrary, the direct effect makes capital more affordable and has a positive effect on investment. The strength of the direct effect depends on the multiple $\frac{1}{\bar{Q} - \bar{q}(\bar{l}^{\delta}, \delta)\bar{l}^{\delta}}$ which is larger for more constrained (δ^l) firms. This means that these more leveraged firms have a larger direct benefit from a fall in asset prices.

In the baseline economy, the multiple also determines how strong investment reacts to changes in leverage. In a recession firms want to decrease leverage since the fall in productivity has increased their expected default risk. A given reduction of leverage now translate into a larger drop of capital among more constrained (δ^l) firms. Importantly, this effect outweighs the positive effect from lower asset prices such that overall more constrained firms experience a larger drop in capital.

When the financial accelerator is switched off the effect of leverage changes on capital is absent. Therefore, the positive effect from lower asset prices dominates and more constrained firms have a smaller fall in capital. Finally, through the capital-labor complementarity the differential effect on firms is also reflected in employment.

This result implies that the endogenous debt choice together with default in equilibrium is key for matching heterogeneous elasticities. Put differently, high leverage by itself does not imply higher sensitivity to aggregate shocks. The key question is whether firms can affect borrowing costs by lowering leverage. If they can - as is the case in the baseline economy - then more constrained, high leverage firms cut investment and employment by more. The reason is that their borrowing costs are very sensitive to leverage changes, therefore the gains from deleveraging are large. On the contrary, at less constrained, low leverage firms borrowing costs are less sensitive, therefore gains are smaller. If firms cannot actively influence the cost of borrowing - in case the financial accelerator is switched off - then these effects are absent and the effect of asset price dynamics dictates the magnitude of employment changes.

This result has broader relevance in the discussion of leverage as an indicator of financial constraints. Namely, it is striking that the distributions of leverage among investment grade and non-investment grade firms have considerable overlap. For instance, about 25% of investment grade firms have higher leverage than the median non-investment grade firm. One may ask: If these firms have high leverage, why does one not observe strong employment adjustments in recessions at these firms? In light of the above result the answer would be that the cost of borrowing at these firms is probably not very sensitive to changes in leverage.

Thus, the gains from decreasing debt, capital, and employment are limited, and the firm does better keeping its high level of debt.

One implication derived from the model is that less constrained firms delay deleveraging in a recession. While all firms want to reduce debt in a downturn, the pressure to do so is much stronger for more constrained firms. Therefore, more constrained firms will immediately reduce debt while less constrained firms initially increase debt as a share of assets. Figure 6 shows the leverage dynamics implied by the inference procedure during the Great Recession episode. The movements of leverage are very benign, nevertheless one can discern this pattern in the left panel of Figure 6. More constrained firms start to delever earlier in the recession and reach the trough earlier, too. In the Compustat sample the deleveraging among firms without an investment grade rating is more pronounced throughout the recession while leverage at investment grade firms increases initially. The quick recovery of leverage among firms with access to bond markets may partially be due to expansive monetary policy that compressed risk premia and allowed rated firms to tap credit markets so shortly after the recession.

Figure 6: Model fit - Leverage dynamics
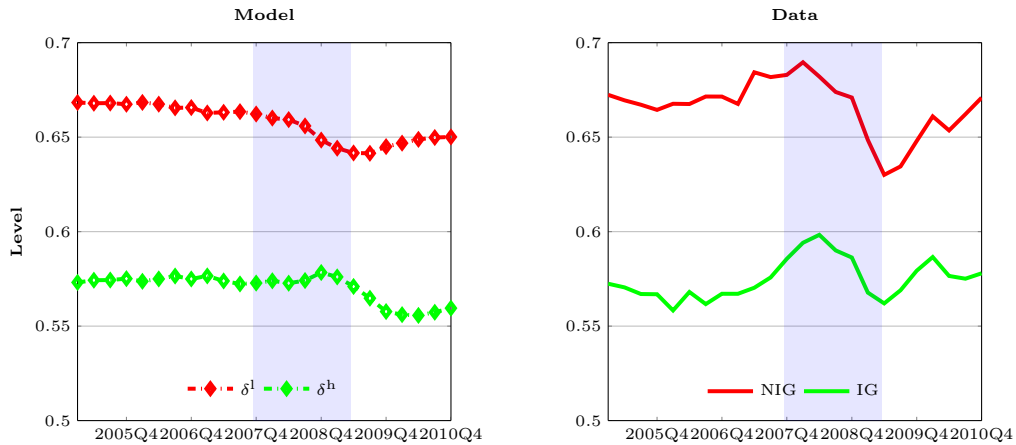
Note: Leverage is debt in year t divided by the average of year t and year $t-1$ capital (all of the same quarter). The data measure of debt/ assets is total liabilities (`LTQ`)/ total assets (`ATQ`) in Compustat Fundamentals Quarterly.

4.5 Alternative dimensions of heterogeneity

Arguably, the assumption of different depreciation rates is a particular form of heterogeneity. Table 1 showed that investment grade and non-investment grade firms differ along many other dimensions. Most notably, investment grade firms are more productive (return on assets) and less risky (implied equity volatilities). In this section, I will discuss these two dimensions of heterogeneity and their implications for the cyclical sensitivities.

In the data the differences in returns on assets (before depreciation, after interest expense) were large and Table 4 illustrated that the model cannot generate such large difference through borrowing costs alone. This means that potential technological differences should be taken into consideration.

Persistent differences in productivity have important consequences for capital accumulation as well as for the choice of the capital structure. From heterogeneous firm models with collateral constraints such as Buera et al. (2015), Khan and Thomas (2013), or Cui (2013), we know that more productive firms are more severely constrained in their growth opportunities. Less productive firms, often large firms that have grown in the past but have since declined in productivity, are less constrained and take on less leverage.

I calibrate a version of my model to compare it to these findings. I dispense of heterogeneity in depreciation rates and instead assume that there are persistent differences in productivities. One half of firms has mean productivity μ_a^h, the other half mean productivity $\mu_a^l < \mu_a^h$. While I do not report the full calibration here[23], the qualitative results are very clear. More productive firms are larger, assume more leverage, have higher default probabilities and a higher marginal value of net worth, i.e., they can be considered more constrained. Squaring this with the characteristics of rated firms in Compustat is challenging. Firms with an investment grade rating are more productive, but they are less leveraged and have lower default probabilities. Relatedly, a negative aggregate productivity shock produces a larger drop of employment among firms with high productivity (see Figure C.1 in the appendix).

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[23]Section C in the appendix contains the calibration details and impulse responses described here.

This can be attributed to their more vulnerable balance sheet, as expressed in the high level of leverage. This experiment shows that productivity heterogeneity by itself does not produce responses which are consistent with the cross-sectional evidence. One needs to recognize that the capital structure of firms plays a key role in the transmission of aggregate shocks into firm-level dynamics.

Introducing risk heterogeneity leads to similar counterfactual implications. Suppose that firms differ in the variance of the idiosyncratic productivity shock. The more risky measure of firms draws its productivity from a log-normal distribution with dispersion parameter $\sigma_z^h > \sigma_z^l$. For a given choice of leverage, more risky firms have a higher default probability and therefore face a higher interest rate schedule. In equilibrium these firms then choose lower leverage, they have a lower marginal value of net worth, but higher default probabilities. In response to a negative aggregate productivity shock the less risky firms deleverage while high risk firms would take on more debt. This is at odds with the leverage dynamics of investment grade and non-investment grade firms in Figure 6. With regards to the relative strength of the employment responses, I show in the appendix (Figure C.2) that low risk firms react stronger to the aggregate shock. However, it appears that the response pattern depends to some extent on the employment shares of high and low risk firms.

To summarize, focusing on productivity and risk differences between investment grade and non-investment grade firms does not generate consistent cyclical responses. In the model more productive and less risky firms opt for more debt financing such that they suffer most from a tightening of financial constraints. In the data, however, the stronger response to aggregate shocks comes from the less productive and more risky non-investment grade firms. Therefore, I conclude that that one needs to model a dimension of heterogeneity that produces consistent differences in the capital structure of firms. Heterogeneity in depreciation rates, or differences in the ability to borrow, is a promising approach since it allows matching credit spreads and implies realistic differences in leverage.

4.6 Alternative sources of aggregate uncertainty

Aggregate productivity risk is a useful benchmark to study the model mechanism and to assess the elasticities of employment growth. Yet, other sources of uncertainty arguably contribute to economic activity over the business cycle. In this section I first introduce nominal rigidities into the model and subsequently study three alternative sources of aggregate uncertainty: Shocks to credit spreads, monetary policy shocks, and shocks to idiosyncratic risk. I relegate the details to section D in the appendix and focus on the description of the responses of more and less constrained firms.

4.6.1 Additions to the baseline economy

Evidently, nominal rigidities are important to study the effects of monetary policy. But they are equally important for aggregate effects in response to shocks to credit spreads and idiosyncratic risk. As we will see below these shocks primarily affect investment. Under flexible prices the reduction of investment would be accompanied by an increase in consumption with only little effect on aggregate output and employment. With price rigidities a reduction in investment demand cannot be compensated by higher consumption since the real interest rate does not drop sufficiently.

I introduce monopolistically competitive intermediate good firms which purchase the homogeneous output from δ^h and δ^l firms, differentiate it and sell it on to competitive final goods producers. Final goods producers package the different varieties into a CES aggregate with elasticity of substitution ϵ_p. Intermediate goods producers are subject to price stickiness à la Calvo and can reset prices only infrequently (with probability $1 - \lambda_p$).[24] Let P_t denote the price level in the economy and define inflation as $\bar{\pi}_t = \frac{P_t}{P_{t-1}}$.

The nominal interest rate is set by the central bank according to a Taylor rule that

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[24]As in Bernanke et al. (1999) the price setting problem is set at the level of intermediate good firms rather than at the level of the firms which are subject to financing frictions. If these firms were to set prices by themselves there would be non-trivial distribution of leverage within types.

responds to contemporaneous inflation.

$$r_t = \bar{r} + \alpha_\pi \log \bar{\pi}_t + \epsilon_{r,t}$$

The deviation from the interest rate rule, $\epsilon_{r,t}$, follows an AR(1) process $\epsilon_{r,t} = \rho_r \epsilon_{r,t-1} + \eta_{r,t}$ where $\eta_{r,t} \sim \mathcal{N}(0, \sigma_r^2)$ is the interest rate shock.

In the baseline economy credit spreads are determined by expected default rates. I introduce a "credit spread shock" which distorts the stochastic discount factor of intermediaries' and thus creates a wedge corporate borrowing costs and expected default rates. More precisely, I assume that the stochastic discount factor of financial intermediaries is the household's discount factor twisted by some persistent component ζ_t:

$$\tilde{m}_{t,t+1} = e^{\zeta_t} m_{t,t+1}$$

where $\zeta_t = \rho_\zeta \zeta_{t-1} + \sigma_\zeta \eta_{\zeta,t}$, and $\eta_{\zeta,t} \sim \mathcal{N}(0, \sigma_\zeta^2)$.

If ζ_t is negative, the intermediary is more impatient than the household, and the spread between the interest rate on corporate debt and the risk-free rate will increase (see equation (6)). This reduced form shock is similar to the risk premium shock in Smets and Wouters (2007). Some papers explicitly model financial constraints at the intermediary level. In such models the drop in the stochastic discount factor of can be obtained endogenously through a fall in bank capital (see Gertler and Kiyotaki (2010), for instance).

While the "credit spread shock" raises borrowing costs for all firms, I also consider a variant that I call "flight to quality shock". Such a shock raises only spreads of the more constrained firms and is motivated by the precipitous increase of yields for non-investment grade corporates relative to yields of investment grade firms during the Financial Crisis.[25]

Finally, I add time-variation in idiosyncratic firm risk to the model. A large literature has addressed the role of time-varying micro uncertainty for macroeconomic dynamics.[26] I

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[25]See right panel in Figure A.2 in the appendix.
[26]See Bloom (2009), Bloom et al. (2012), Arellano et al. (2012), Gilchrist et al. (2014), Christiano et al.

follow Christiano et al. (2014) and capture changes in risk through changes in the dispersion of the idiosyncratic productivity distribution. Dispersion is no longer constant, but it follows an AR(1) process around its mean $\bar{\sigma}_z$:

$$\sigma_{z,t} = (1 - \rho_\sigma)\bar{\sigma}_z + \rho_\sigma \sigma_{z,t-1} + \eta_{\sigma,t}$$

where $\eta_{\sigma,t} \sim \mathcal{N}(0, \sigma_\sigma^2)$. The distribution of idiosyncratic productivity then writes as $\log z_t^i \sim \mathcal{N}\left(-\frac{1}{2}\sigma_{z,t-1}^2, \sigma_{z,t-1}^2\right)$. Notice that the dispersion of productivity in period t is already known in period $t-1$.

4.6.2 Results

For the calibration I choose the probability of price adjustment $1 - \lambda_p = 0.25$, such that prices are reset on average every year. The elasticity of substitution between varities is set to $\epsilon_p = 6$. The Taylor rule coefficient on inflation is set to $\alpha_\pi = 1.5$. All autocorrelation are set to $\rho_r = \rho_\zeta = \rho_\sigma = 0.9$. The addition of nominal rigidities and aggregate uncertainty with respect to credit spreads and idiosyncratic risk does not change the steady state of the economy, provided that the inefficiency emerging from mark-up pricing is undone through an input subsidy for intermediate goods producers.

Figure 7 shows the response of employment (in deviations from steady state) to four different shocks, all scaled such that the output drop on impact is 0.5%.

The credit spread shock raises borrowing costs for all firms. Employment drop precipitously among more constrained firms. At less constrained firms it increases initially but subsequently falls below its steady state level as well. The responses are reminiscent of the aggregate productivity impulse response (see Figure 4). The need for deleveraging is stronger at more constrained firms and aggregate capital does not adjust quickly such that less constrained firms buy up capital by issuing debt. However, deleveraging at these firms is only delayed and begins when more constrained firms start to recover. Therefore, the drop of

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(2014), and others.

Figure 7: Employment response to various shocks
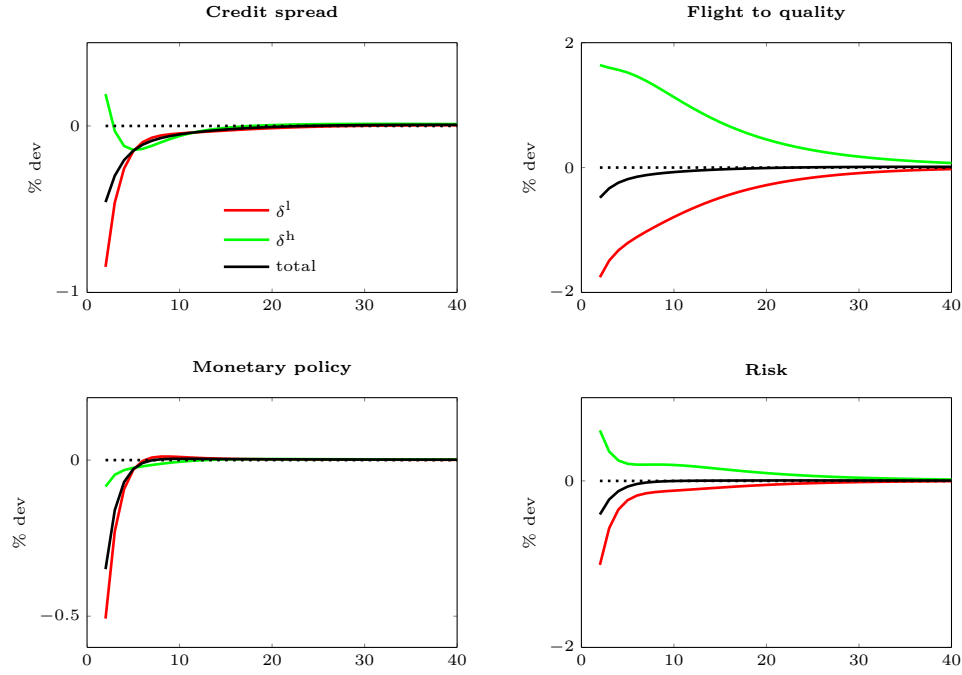
employment at less constrained becomes larger than the drop at more constrained firms a few quarters after the shock.

The flight to quality shock leaves borrowing costs at less constrained firms unaffected. Thus, operating capital at more constrained firms becomes more costly and capital shifts to less constrained firms, accompanied by a rise in employment at these firms.

Similarly to the credit spread shock, the contractionary monetary policy shock raises borrowing costs for firms. Again, it leads to a bigger decline of employment among more constrained firms. This result is reminiscent of earlier findings by Gertler and Gilchrist (1994) on the relation between firm size and the responsiveness to tight monetary policy.

In increase in idiosyncratic risk is followed by a persistent increase (decrease) of employment at less (more) constrained firms. Expected default rates of more constrained firms are already higher in steady state. This means that the additional increase of default risk due to more dispersed idiosyncratic shocks is more worrisome for more constrained firms. They have stronger interest in reducing their default risk. Therefore, they divest and deleverage

while less constrained firms absorb some of that capital and finance it through debt issuance.

5   Conclusion

In summary, this paper analyzes financial constraints with a particular focus on the interconnection of time series and cross section. I introduce heterogeneity in the debt capacity across firms and I find that this feature allows matching the greater employment cyclicality of more financially constrained firms. Other dimensions of firm heterogeneity, such as productivity or risk differences, fail to generate cyclical variation that is consistent with the data.

My results show that leverage is a key factor for the employment cyclicality of firms provided that borrowing costs are sensitive to the debt choice. Put differently, a firm with high leverage does not necessarily have greater employment cyclicality. This hints at challenges for policymakers when using leverage data to assess vulnerabilities in the nonfinancial sector.

The results of this paper are applicable to a variety of settings. For instance, in emerging economies only few firms access foreign debt markets. Insofar as this increases their debt capacity, the model would predict greater employment cyclicality at these firms. Additionally, domestic currency depreciation would inflate debts and add to their woes.

Moreover, the mechanism presented in this paper can help explain the countercylicality of cross-sectional dispersion measures. In fact, the model features an endogenous dispersion channel via heterogeneous employment sensitivities while it also allows incorporating exogenous variation in the magnitude of idiosyncratic shocks. Accordingly, this framework may serve to assess the contributions of the endogenous and exogenous margins to the cross-sectional dispersion dynamics. A quantitative assessment of these channels would require a more fine-grained representation of firm-level heterogeneity than is possible in the current setup. I leave this as a perspective for future research.

Figure A.1: KZ score and bond ratings
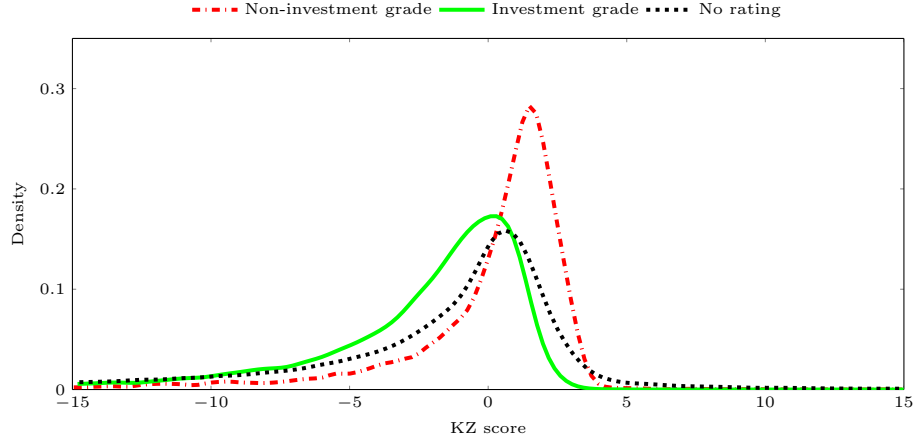
Sources: Compustat Fundamentals Annual, 1986 to 2014. Financial firms (SIC codes 60 to 67) and utilities (SIC code 49) excluded. Rating is S&P long term credit rating coded from AAA (=1) to D (=22). KZ score is the Kaplan-Zingales measure for financial constraints, constructed according to Lamont et al. (2001). Graph shows density of KZ score among investment grade firms (AAA to $BBB-$ rating, green solid line), and non-investment grade firms ($BB+$ to D, red dash-dotted line), and firms without rating (black dotted line).

Figure A.2: Data - Bond yields
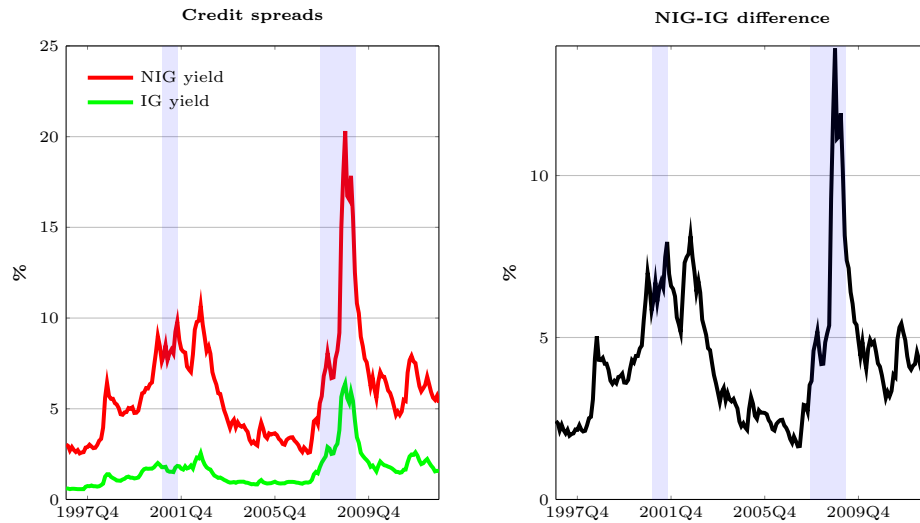
Note: Investment grade (IG) series BAMLC0A0CM, and Speculative grade (NIG) series BAMLH0A0HYM2 from Bank of America Merrill Lynch via FRED. Option-adjusted spread between index of all bonds of investment grade/ below-investment grade firms and a spot Treasury curve.

A Data appendix

B Model inference

Figure B.1 displays US real GDP per capita in levels and in deviations from a linear trend. The series of trend deviations is used as an observable to trace out productivity shocks and to infer the employment dynamics in section 4.4. More precisely, I estimate the autocorrelation and standard deviation of the productivity process with Bayesian methods. In the first step, the algorithm finds the parameter vector that maximizes the sum of log likelihood and priors on parameters. In the second step, a Metropolis-Hastings algorithm is run to simulate the posterior distribution of the parameters vector. Table B.1 reports the priors and moments of the posterior distribution.

For each model specification listed in Table 5 the estimation has to be repeated since the change of parameters affects the responsiveness of variables to aggregate shocks. The results for the posterior distribution are very similar, however.

The smoothed shocks from the estimation imply dynamics for all other model variables. I take the yearly average of the implied employment series to be consistent with the yearly frequency of Compustat data. Then I compute year-on-year growth rates that are used for Figure 5 and Table 5.

Table B.1: Estimated parameters

Parameter	Posterior		Prior		
	Mean	[5%,95%]	Distribution	Mean	Std
ρ_a	0.9796	[0.9633, 0.9995]	Beta	0.90	0.10
σ_a	0.0073	[0.0067, 0.0080]	Inverse Gamma	0.005	0.005

Figure B.1: Real GDP
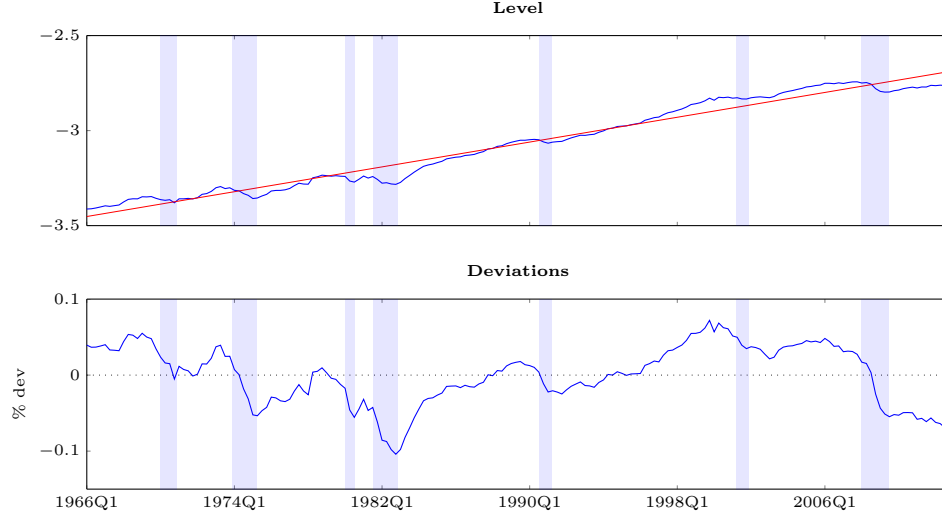
Level

Deviations

C   Alternative sources of heterogeneity

C.1   Heterogeneity in productivity

I assume that the stationary component of aggregate productivity has a type dependent mean:

$$a_t^j = (1 - \rho_a)\mu_a^j + \rho_a a_{t-1}^j + \sigma_a \eta_{a,t}, \quad j = h, l$$

Notice that the innovation to productivity, $\eta_{a,t}$, is common to all firms. As in the baseline economy firms transition between types in the end of the period, right before the investment and capital structure decision. The depreciation rate δ is common to all firms and is set to 10% annually. With some slight adjustments the calibration targets are as in Table 3. Instead of targeting a credit spread, I choose the dispersion of idiosyncratic productivity σ_z to match a 3% annual default rate. This is about $\frac{2}{3}$ of the default rate of non-investment grade firms (see Table 4); Bernanke et al. (1999) also use this value. Finally, I calibrate $\tilde{\mu}_a = \mu_a^h = -\mu_a^l$ to match the difference of return on assets between investment grade and non-investment grade firms. The required difference in productivities is rather large, $\tilde{\mu}_a = 0.26$. This is

even larger than the standard deviation of idiosyncratic productivity $\sigma_z = 0.20$. Table C.2 compares high and low productivity firms in the steady state. High productivity firms have more debt relative to assets, are larger, and have markedly higher probabilities. They also operate at a lower capital-labor ratio.

Table C.1: Steady state - Productivity heterogeneity

	High productivity	Low productivity
Leverage	0.68	0.53
Employment share	0.85	0.15
Return on assets	0.09	0.03
Default rate (% ann)	5.68%	0.38%
Capital-labor ratio	6.04	16.19

Figure C.1: IRF - Productivity heterogeneity
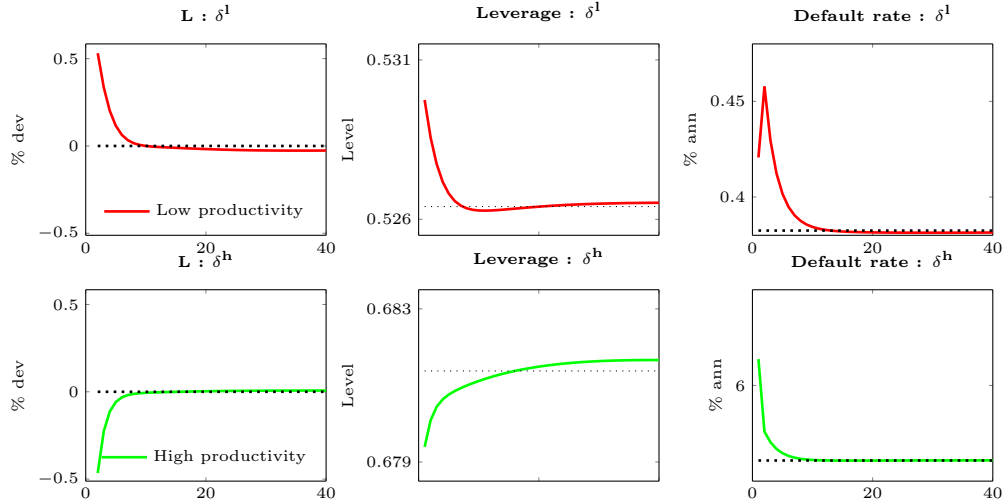
Note: Response of employment, leverage and default rates among high (low) productivity firms in lower (upper) panel. Negative 0.5% aggregate productivity shock.

C.2   Heterogeneity in risk

I assume that dispersion of the idiosyncratic productivity distribution is type dependent. One half of firms has productivity dispersion $\sigma_z^h > \sigma_z^l$, and are called the "more risky firms". As in the baseline economy firms transition between types in the end of the period, right

before the investment and capital structure decision. The depreciation rate δ is common to all firms and is set to 10% annually. With some slight adjustments the calibration targets are as in Table 3. Recall from Table 1 that non-investment grade firms are about 1.5 times as volatile as investment grade firms. Therefore, I set $\sigma_z^h = 1.5 \times \sigma_z^l$, and calibrate σ_z^l to match a 3% annual default rate. The resulting volatilities are $\sigma_z^l = 0.17$ and $\sigma_z^h = 0.26$, and surround the value in the baseline calibration ($\sigma_z = 0.21$ in see Table 3).

Table C.2: Steady state - Risk heterogeneity

	Low risk	High risk
Leverage	0.69	0.55
Employment share	0.60	0.40
Return on assets	0.06	0.06
Default rate (% ann)	2.84%	3.22%
Capital-labor ratio	7.91	7.91

Figure C.2: IRF - Risk heterogeneity
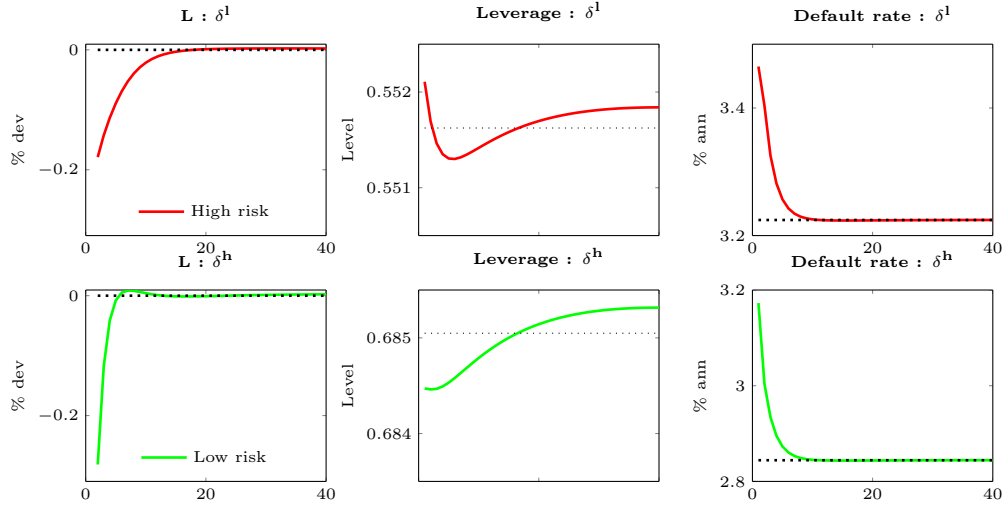
Note: Response of employment, leverage and default rates among low (risk) risk firms in lower (upper) panel. Negative 0.5% aggregate productivity shock.

D Model extensions

D.1 Nominal rigidities

Recall that δ^h and δ^l firms produce the same homogeneous good. I relabel this good now as "wholesale good", thereby indicating that it is no longer the final output/ consumption good. Rather the wholesale good is purchased by intermediate goods producers at price P_t^w. Intermediate good producers are monopolistically competitive, indexed by $r \in [0,1]$ (r for retailers), each intermediate good firms transforms the wholesale into its respective variety r. Finally, a CES aggregate of these different varieties is assembled by competitive final good produces. The price of the final good, used for consumption by households and in the production of new capital, is denoted as P_t. Inflation is define as $\bar{\pi}_t = \frac{P_t}{P_{t+1}}$.

D.1.1 Final goods producers

The final output Y_t is a CES aggregate of intermediate goods, where ϵ_p is the elasticity of substitution between the different varieties.

$$Y_t = \left(\int y_{r,t}^{\frac{\epsilon_p - 1}{\epsilon_p}} dr \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

Final goods producers choose the amount of each variety r, taken as given the price of output P_t and the price of intermediate goods, $P_{r,t}$, $r \in [0,1]$.

$$\max_{\{y_{r,t}\}} P_t Y_t - \int P_{r,t} y_{r,t} dr$$

Demand for variety r is

$$y_{r,t} = \left(\frac{P_{r,t}}{P_t} \right)^{-\epsilon} Y_t$$

Plugging the demand equation back into the firm's problem and imposing the zero profit condition yields the prices index $P_t = \left(\int P_{r,t}^{1-\epsilon} dr \right)^{\frac{1}{1-\epsilon}}$.

D.1.2 Intermediate good firms

Intermediate good firms take demand from final goods producers as given, and choose their price $P_{r,t}$ to maximize the presented discount value of profits. However, they can reset prices on infrequently. Every period a random fraction $1 - \lambda_p$ is selected and can freely reset its price. The remaining fraction λ_p retains the price from the previous period. An intermediate good firm r that is allowed to adjust its price in period t choose its price taking into account expected profits in all future states in which it is not allowed to adjust its price.

Due to market power intermediate good producers set the price as a mark-up over marginal costs, implying that the quantity in equilibrium would be lower than under perfect competition. This inefficiency is not central to the analysis, therefore I assume that intermediate good producers receive an input subsidy that eliminates the inefficiency. The effective price per unit of wholesale good is $(1 - \nu_p)P_t^w$. The subsidy is financed through lump-sum taxes.

$$
\max_{P_{r,t}^*} \mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_p^j m_{t,t+j} \frac{P_t}{P_{t+j}} \left(P_{r,t}^* y_{r,t+j} - P_{t+j}^w (1 - \nu) y_{r,t+j} \right) \right]
$$

$$
\Leftrightarrow \max_{P_{r,t}^*} \mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_p^j m_{t,t+j} \frac{P_t}{P_{t+j}} \left(P_{r,t}^* \left(\frac{P_{r,t}^*}{P_{t+j}} \right)^{-\epsilon_p} Y_{t+j} - P_{t+j}^w \left(\frac{P_{r,t}^*}{P_{t+j}} \right)^{-\epsilon_p} Y_{t+j} (1 - \nu_p) \right) \right]
$$

$p_t^w = \frac{P_t^w}{P_t}$, $p_{r,t} = \frac{P_{r,t}}{P_t}$.

The first-order condition to the firm's problem writes as follows.

$$
\mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_p^j m_{t,t+j} Y_{t+j} P_{t+j}^{\epsilon_p} \left(p_{r,t}^* \frac{P_t}{P_{t+j}} - \frac{\epsilon_p}{\epsilon_p - 1} p_{t+j}^w (1 - \nu_p) \right) \right] = 0
$$

$$
\Leftrightarrow \mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_p^j m_{t,t+j} Y_{t+j} X_{t,j}^{-\epsilon_p} \left(p_{r,t}^* X_{t,j} - p_{t+j}^w \right) \right] = 0
$$

In the last line the input subsidy ν_p has been set to $\nu_p = \frac{1}{\epsilon_p}$ as to eliminate the mark-up over marginal costs, i.e., $\bar{p}_r^* = \bar{p}^w$. Under this subsidy intermediate good producers

make positive profits, but the output is efficient in steady state. Moreover, I define $P_{t+j} = P_t \cdot \bar{\pi}_{t+1} \cdots \bar{\pi}_{t+j-1} + \bar{\pi}_{t+j}$, and $X_{t,j} = \frac{1}{\bar{\pi}_{t+1} \cdots \bar{\pi}_{t+j-1} \bar{\pi}_{t+j}}$ if $j > 0$. $X_{t,j} = 1$ if $j = 0$.

All firms which are allowed to adjust their price will choose the same targets P_t^*, or $p_t^* = \frac{P_t^*}{P_t}$.

The aggregate price index writes as

$$P_t = \left[(1 - \lambda_p) P_t^{*(1-\epsilon_p)} + \lambda_p P_{t-1}^{1-\epsilon_p} \right]^{\frac{1}{1-\epsilon_p}}$$

$$1 = (1 - \lambda_p) p_t^{*(1-\epsilon_p)} + \lambda_p \bar{\pi}_t^{\epsilon_p - 1}$$

D.1.3   Central bank

The nominal interest rate is set by the central bank according to a Taylor rule that responds to contemporaneous inflation.

$$r_t = \bar{r} + \alpha_\pi \log \bar{\pi}_t + \epsilon_{r,t}$$

The deviation from the interest rate rule, $\epsilon_{r,t}$, follows an AR(1) process $\epsilon_{r,t} = \rho_r \epsilon_{r,t-1} + \eta_{r,t}$ where $\eta_{r,t} \sim \mathcal{N}(0, \sigma_r^2)$ is the interest rate shock.

D.1.4   Household

The household's Euler equation for deposits now also takes into account inflation:

$$1 = \mathbb{E} \left[m_{t,t+1} \frac{1 + r_t}{\pi_{t+1}} \right]$$

D.1.5   Wage rigidity revisited

In the baseline economy labor agencies were essentially setting real wages. With price rigidities labor agencies are now setting nominal wages. This means that wages are not indexed to inflation, and thus unexpected inflation does affect labor costs because not all labor agencies can reset wages.

The equation characterizing the nominal wage target relative to the price level, w_t^*, is given by:

$$\mathbb{E}\left[\sum_{j=0}^{\infty} \lambda_w^j m_{t,t+j} N_{t+j} w_{t+j}^{\epsilon_w} X_{t,j}^{-\epsilon_w} \left(w_t^* X_{t,j} - \tilde{w}_{t+j}\right) \bigg| S_t\right] = 0$$

where nominal variables W_t^*, W_t and $\tilde{W}_t$, have been normalized by the price level $w_t^* = W_t^*/P_t$, $w_t = W_t/P_t$ and $\tilde{w}_t = \tilde{W}_t/P_t$.

The nominal and real wage indices are

$$W_t = \left[(1-\lambda_w)(W_t^*)^{1-\epsilon_w} + \lambda_w W_{t-1}^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

$$w_t = \left[(1-\lambda_w)(w_t^*)^{1-\epsilon_w} + \lambda_w (w_{t-1})^{1-\epsilon_w}\bar{\pi}_t^{\epsilon_w-1}\right]^{\frac{1}{1-\epsilon_w}}$$

D.2 Financial intermediaries and discount factor shocks

Section 4.6 introduces a distortion to the stochastic discount factor of financial intermediaries:

$$\tilde{m}_{t,t+1} = e^{\zeta_t} m_{t,t+1}$$

where $\zeta_t = \rho_\zeta \zeta_{t-1} + \sigma_\zeta \eta_{\zeta,t}$, and $\eta_{\zeta,t} \sim \mathcal{N}(0, \sigma_\zeta^2)$.

Recall the zero profit condition for intermediaries (6) and plug in the stochastic discount factor $\tilde{m}_{t,t+1}$.

$$\mathbb{E}\left[m_{t,t+1}\left\{e^{\zeta_t}(1 - F(\bar{z}_{t+1}^i))q_t(l_{t+1}^i, \delta_{t+1}^i)^{-1} - \frac{1+r_t}{\pi_{t+1}}\right\}\right] = 0. \tag{9}$$

The interest rate on corporate debt, $q_t(l_{t+1}^i, \delta_{t+1}^i)^{-1}$, is a spread over the risk-free rate that reflects not only expected default risk, but also the relative impatience of financial intermediaries.

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