EE241 SPRING 2015: TUTORIAL #5

Friday, Feb. 13, 2015

PROBLEM 1: Fill in the following table with 0, 1, or ∞ solutions possible (multiple answer possible in one cell), or write DNA for "does not apply".

System	RREF	Pivots	$m < n$	$m = n$	$m > n$
$A\vec{x} = \vec{b}$	A has row(s) of all 0 in RREF	$[A, b]$ has a pivot in the last column			
		$[A, b]$ has no pivots in the last column			
	A no row(s) of all 0 in RREF	$[A, b]$ has a pivot in the last column			
		$[A, b]$ has no pivots in the last column			
$A\vec{x} = \vec{0}$	A has row(s) of all 0 in RREF				
	A no row(s) of all 0 in RREF				

PROBLEM 2 (Fitting): The first three *physicists' Hermite polynomials* are the following

$$H_0(x) = 1,$$
$$H_1(x) = 2x,$$
$$H_2(x) = 4x^2 - 2.$$

Find a linear combination of these polynomials such that the resulting function passes through $(0, 0)$ $(1, 0)$, and the first derivative at $x = 1$ is -1.

PROBLEM 3 (Permutations): Group the following permutations into even and odd,

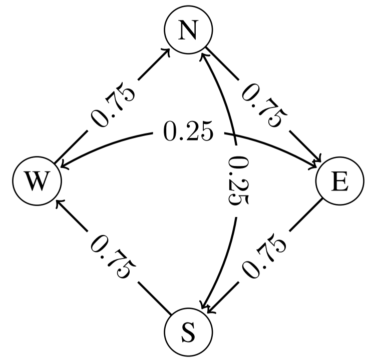
$$\{1, 2, 4, 3\}\ ,\ \{1, 4, 2, 3\}\ ,\ \{4, 1, 2, 3\}\ ,\ \{4, 1, 3, 2\}\ ,\ \{4, 3, 1, 2\}\ ,\ \{4, 3, 2, 1\}$$

PROBLEM 4 (Vector lengths): Which is longer: a n-dimensional vector of 1's? Or a $2n$-dimensional vector of $1/2$'s?

PROBLEM 5 (Determinants): Find the determinant of the following matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

PROBLEM 6: The graph below represents the following process: You are steering a ship probabilistically. Your first mate flips two fair coins every half-an-hour. If he gets two heads, you turn the ship around. Otherwise, you turn the ship $90°$ "starboard" (right). This graph contains no self-loops, i.e.: a "stay" is not a valid move.



(a) What is the matrix M representing this Markov process? This matrix should transform the vector $\vec{p} = [p_N, p_E, p_S, p_W]$ to new probabilities $\vec{p}'$ according to the rules of the coin flips. (To check your answer, you can apply the matrix to $\vec{p} = [1, 0, 0, 0], [0, 1, 0, 0], \ldots$).

(b) You have no idea which direction you initially set sail in ($\vec{p}$). However, after an hour and a half, you estimate the following probabilities for your direction $\vec{p}' = [9/64, 27/64, 27/64, 1/64]$. Which direction did you set sail in?