EE241 SPRING 2015: TUTORIAL #2

Friday, Jan. 23, 2015

PROBLEM 1 (Balancing chemical equations): We can use linear systems to balance chemical equations. Consider the "skeleton" equation below

$$\mathbf{HgS} + \mathbf{Cl}^- + \mathbf{NO_3^-} + \mathbf{H}^+ \to \mathbf{HgCl_4^{2-}} + \mathbf{NO_2} + \mathbf{S} + \mathbf{H_2O}$$

In order to *balance* this equation, we must choose coefficient for each of the molecules such that the number of each atoms of each element on either side of the equation is the same. First, let's add the coefficients in the equation above,

$$a\mathbf{HgS} + b\mathbf{Cl}^- + c\mathbf{NO_3^-} + d\mathbf{H}^+ \to w\mathbf{HgCl_4^{2-}} + x\mathbf{NO_2} + y\mathbf{S} + z\mathbf{H_2O}.$$

Construct a linear system with one equation per element (Hg, S, Cl, N, O, and H) and find a solution such that a, b, c, d, w, x, y, and z are integers. Recall that the subscript on an element symbol is the number of atoms of that element in the molecule. Additionally, the number of charges on both sides of the equation must be equal. You can count the charges by looking at the superscripts on each atom.

Solution. Counting each element we can create the six element equations and one charge equation

$$\left\{\begin{array}{lrl} (\mathbf{Hg}) & a & = w \\ (\mathbf{S}) & a & = y \\ (\mathbf{Cl}) & b & = 4w \\ (\mathbf{N}) & c & = x \\ (\mathbf{O}) & 3c & = 2x + z \\ (\mathbf{H}) & d & = 2z \\ (\pm) & -b-c+d & = -2w \end{array}\right. \Rightarrow \left\{\begin{array}{lrl} (\mathbf{Hg}) & a-w & = 0 \\ (\mathbf{S}) & a-y & = 0 \\ (\mathbf{Cl}) & b-4w & = 0 \\ (\mathbf{N}) & c-x & = 0 \\ (\mathbf{O}) & 3c-2x-z & = 0 \\ (\mathbf{H}) & d-2z & = 0 \\ (\pm) & -b-c+d+2w & = 0 \end{array}\right.$$

We can start to solve this system by using a few substitutions. First, we can eliminate w and y by replacing them by a and we can eliminate x by replacing it by c to get

$$\left\{\begin{array}{lrl} (\mathbf{Cl}) & b-4a & = 0 \\ (\mathbf{O}) & 3c-2c-z & = 0 \\ (\mathbf{H}) & d-2z & = 0 \\ (\pm) & -b-c+d+2a & = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{lrl} (\mathbf{Cl}) & b-4a & = 0 \\ (\mathbf{O}) & c-z & = 0 \\ (\mathbf{H}) & d-2z & = 0 \\ (\pm) & -b-c+d+2a & = 0 \end{array}\right.$$

We can replace z by c and eliminate the $(\mathbf{O})$ equation.

$$\left\{\begin{array}{lrl} (\mathbf{Cl}) & b-4a & = 0 \\ (\mathbf{H}) & d-2c & = 0 \\ (\pm) & -b-c+d+2a & = 0 \end{array}\right.$$

Now we can subtract $(\mathbf{H})$ from $(\pm)$ to get

$$\left\{\begin{array}{rl} b-4a & = 0 \\ d-2c & = 0 \\ -b+c+2a & = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{rl} b-4a & = 0 \\ d-2c & = 0 \\ c-2a & = 0 \end{array}\right. \Rightarrow \left\{\begin{array}{rl} b-4a & = 0 \\ c-2a & = 0 \\ d-4a & = 0 \end{array}\right.$$

We now have our linear system in a much better form. We can simply choose a value for a such that b, c, and d are integers and then substitute the appropriate values into w, x, y, and z as well. For the purpose of keeping our chemical equation legible, we should choose the smallest possible a such that all the other coefficients are still integers. Thus, choose $a = 1$ then,

$$a = 1\ ,\ b = 4\ ,\ c = 2\ ,\ d = 4\ ,\ w = 1\ ,\ x = 2\ ,\ y = 1\ ,\ z = 2\ ,$$

Altogether, the balanced chemical equation is

$$\mathbf{HgS} + 4\mathbf{Cl}^- + 2\mathbf{NO_3^-} + 4\mathbf{H}^+ \to \mathbf{HgCl_4^{2-}} + 2\mathbf{NO_2} + \mathbf{S} + 2\mathbf{H_2O}.$$

$\square$

PROBLEM 2: solve the following linear system

$$\left\{\begin{array}{rcrcrl} 2x & + & 3y & + & z & = 6 \\ 6x & + & 10y & + & 5z & = 21 \\ 4x & + & 8y & + & 9z & = 21 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

Solution. First, let's subtract 3 times the first equation from the second and 2 times the first equation from the third. We can denote this operation as

$$\left\{\begin{array}{rcrcrl} 2x & + & 3y & + & z & = 6 \\ & & y & + & 2z & = 3 \\ & & 2y & + & 7z & = 9 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \to (2) - 3 \cdot (1) \\ (3) \to (3) - 2 \cdot (1) \end{array}$$

Now we can eliminate y from (1),

$$\left\{\begin{array}{rcrcrl} 2x & + & 3y & + & z & = 6 \\ & & y & + & 2z & = 3 \\ & & & & 3z & = 3 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \\ (3) \to (3) - 2 \cdot (2) \end{array}$$

Our equations are now in a form where we can use *back-substitution* to solve for x, y, z,

$$\left\{\begin{array}{rcrcrl} 2x & + & 3y & + & z & = 6 \\ & & y & + & 2z & = 3 \\ & & & & z & = 1 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\left\{\begin{array}{rcrcrl} 2x & + & 3y & + & 1 & = 6 \\ & & y & + & 2 & = 3 \\ & & & & z & = 1 \end{array}\right. \qquad \begin{array}{l} (1) \Leftarrow (z = 1) \\ (2) \Leftarrow (z = 1) \\ (3) \end{array}$$

$$\left\{\begin{array}{rcrcl} 2x & + & 3y & & = 5 \\ & & y & & = 1 \\ & & & z & = 1 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\left\{\begin{array}{rcrcl} 2x & + & 3 & & = 5 \\ & & y & & = 1 \\ & & & z & = 1 \end{array}\right. \qquad \begin{array}{l} (1) \Leftarrow (y = 1) \\ (2) \\ (3) \end{array}$$

$$\left\{\begin{array}{cccl} x & & & = 1 \\ & y & & = 1 \\ & & z & = 1 \end{array}\right. \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

Thus $x = y = z = 1$. □

PROBLEM 3: solve the following linear system

Solution. □

PROBLEM 4:

Solution. □

PROBLEM 5:

Solution. □

PROBLEM 6:

Solution. □