WELLESLEY COLLEGE

Study Packet for the

Quantitative Reasoning Assessment

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Dear Incoming Wellesley Student:

Hello! We have prepared this study packet to help you prepare for the Quantitative Reasoning (QR) Assessment that you will take soon after arriving on campus.

What is the QR Assessment for?

The QR Assessment tests your quantitative skills, including your ability to read and understand information presented in formulas, tables, and graphs; to interpret information and draw appropriate inferences; and to solve real world problems that deal with numbers or data. The mathematical skills you will apply on this test span arithmetic, algebra, graph reading, geometry, and linear and exponential modeling. Logical and statistical skills are core "QR skills." You need to demonstrate adequate basic QR skills before enrolling in a wide array of first-year courses at the College. Additionally, strong QR skills are needed to ensure that you can explore any academic major, pursue any career, and address the wide array of quantitative problems that arise in everyday life.

What happens if I do not do well on the QR Assessment?

If you do not pass the QR Assessment, you will need to enroll in "Introduction to Quantitative Reasoning" in your first year at the College. After successfully completing this course, you should have the quantitative skills that are a prerequisite for any introductory level course that emphasizes quantitative analyses – from science courses to economic courses.

What is included in this study packet?

This study packet includes a set of 24 review problems (starting on page 3) and two QR Assessments actually given to students in past years (starting on page 27). Short answers are provided for each problem. In addition, complete, worked-out solutions are provided for all but the last actual assessment. We suggest you work through these problems without using a calculator, as calculators are NOT allowed when taking the 90-minute QR Assessment. If there are particular types of problems that you find challenging, we recommend reviewing notes or books on these topics.

If I have questions, where should I turn?

For more information about Wellesley's two-part QR requirement or about the QR Assessment itself, visit the QR web site at www.Wellesley.edu/QR/. The web site lists recommended readings, etc. You may also contact Corri Taylor, Director of the QR Program, by e-mail at ctaylor1@wellesley.edu or by phone at 781-283-2152.

Review Problems

1. Officials estimate that 320,000 Boston-area partygoers attended last year's Independence Day celebration on the banks of the Charles River. They also estimate that the partygoers left behind 40 tons of garbage. Given that a ton equals 2,000 pounds, how many pounds of garbage did the average partygoer leave behind?

2. Subway tokens cost 85 cents. How many can you buy with $20?

3. A student's grade depends on her score on four equally-weighted exams. Her average on the first three exams is 92. What must she score on the fourth exam in order to guarantee a final average of at least 90?

4. In 2000, a person places $1,000 in an investment that earned 10% annual interest, compounded annually. Calculate the value of the investment for the years 2001, 2002, and 2003.

5. According to *The New York Times*, scientists studying the atmosphere have recently detected a decrease in the level of methyl chloroform, a man-made industrial solvent that is harmful to the ozone layer. In 1990, the level of methyl chloroform was 150 parts per trillion (150 ppt), but by 1994 the level had fallen to 120 ppt. By what percentage did the level of methyl chloroform decrease between 1990 and 1994?

6. In 1990, according to Census data, one in four Americans over 18 years of age had never married, as compared to one in six in 1970. What is the ratio of the fraction of never married Americans in 1990 to the fraction of never married Americans in 1970? Simplify your answer.

7. One year ago, a person invested $6,000 in a certain stock. Today, the value of the investment has risen to $7,200. If, instead, the person had invested $15,000 one year ago instead of $6,000, what would the investment's value be today? (Assume that the investment would increase by the same proportion.)

8. Evaluate the following expressions given that $v = -2$ and $w = 3$

   (a) $3(v - 2w)$ (b) $v^2 + w^2$

9. Figure 1 gives weight charts for baby boys and girls from birth to 18 months. Each chart gives weights for three different sizes of babies: 5th percentile (small babies), 50th percentile (average babies), and 95th percentile (large babies). For example, according to the chart, the 50th percentile weight for a six-month-old girl is about 15 pounds, while the 95th percentile weight for a six-month-old girl is about 18 pounds.

Figure 1:

*Weight charts for baby boys and girls, age birth to 18 months*
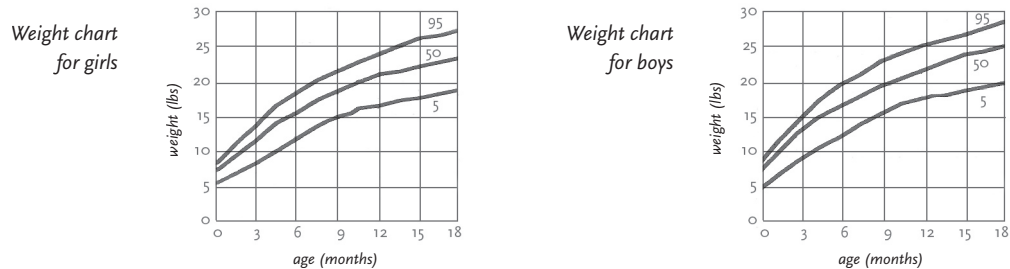
(a) About how much more does an average eighteen-month-old boy weigh than an average eighteen-month-old girl?

(b) Consider two 6-month-old boys, one in the 5th percentile and one in the 95th percentile. About how old will the smaller boy be when he weighs as much as the larger boy does now? (You should assume that the smaller boy remains in the 5th percentile as he grows.)
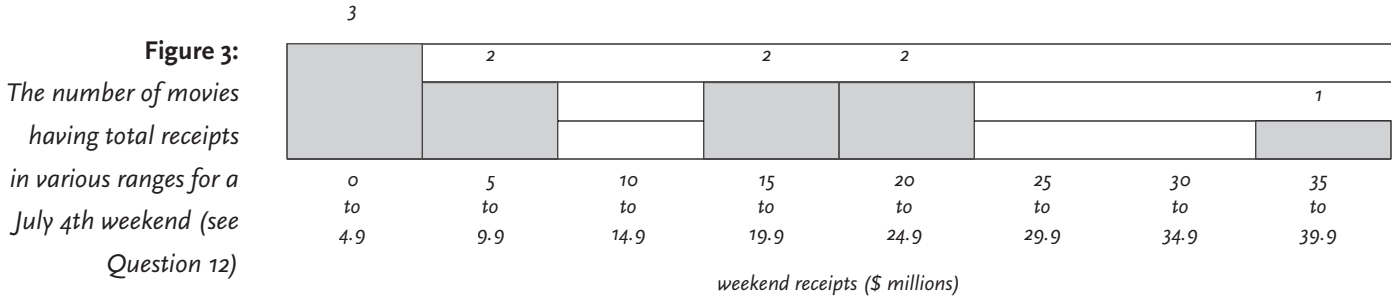
10. In 2003, there were 80 turtles living in a wetland. That year, the population began to grow by 12 turtles per year. Find a formula for P, the number of turtles in terms of t, the number of years since 2003.

11. Find (a) the perimeter and (b) the area of the shape in Figure 2.

Figure 2:

*For Question 11, find the perimeter and area of this shape.*
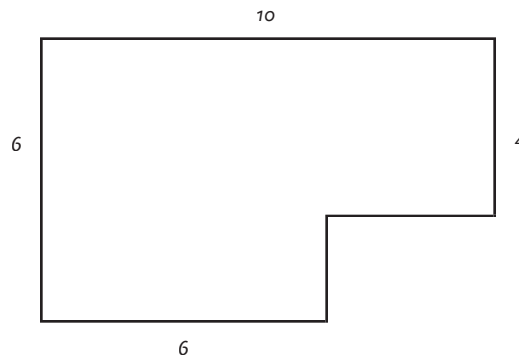
12. Figure 3 shows the number of movies with weekend receipts in different dollar ranges for a July 4th holiday weekend. For example, according to the chart, two movies earned at least $15 million but less than $20 million. According to Figure 3, how many movies earned more than $10 million?

Figure 3:

The number of movies having total receipts in various ranges for a July 4th weekend (see Question 12)

weekend receipts ($ millions)

13. Suppose you need to rent a car for one day and that you compare that cost at two different agencies. The cost (in $) at agency A is given by $C_A = 30 + 0.22n$, where n is the number of miles you drive. Similarly, the cost at agency B is given by $C_B = 12 + 0.40n$.

(a) If you drive only a few miles (say, less than 20 miles), which agency costs less, A or B?

(b) How far would you need to drive in order for the other agency to become less expensive?

14. According to the Cable News Network, the number of injured in-line skaters (or "roller-bladers") was 184% larger this year than it was last year. Did the number of injured skaters almost double, almost triple, or more than triple?

15. For a certain flight out of Chicago, let P_A be the price of a first-class seat and P_c be the price of a coach-class seat. Furthermore, let N_A be the number of first-class seats and N_c be the number of coach-class seats available on the flight. Assuming that every available seat is sold, write an expression in terms of these constants that gives the value of R, the total amount of money brought in by the airline for this flight.

16. Solve the equation $\dfrac{Z_1}{Z_2} = \dfrac{K_1}{K_2}$ for K_2.

17. There are 0.6 grams of powder in a dish. One-fifth of the powder spills out of the dish. How many grams of powder are left in the dish?

18. Read the values at the two pointers shown in Figure 4.
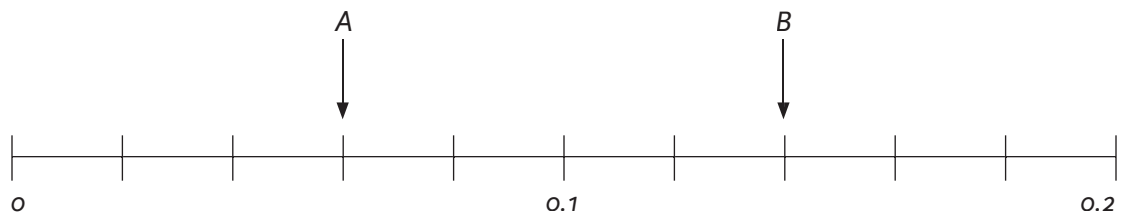
19. A six-foot tall man is walking home. His shadow on the ground is eight feet (ft) long. At the same time, a tree next to him casts a shadow that is 28 ft long. How tall is the tree?

20. Graph the equation $5s + 15 - t = 0$ on the set of axes provided in Figure 5. Label the s- and t- intercepts.
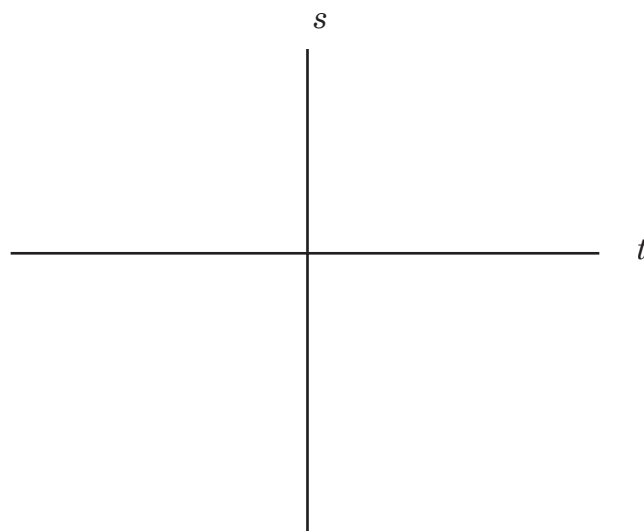
21. Express the following values in simplified form, in scientific notation.

(a) $(2 \times 10^{-4}) (3 \times 10^{5})$

(b) $\dfrac{(3 \times 10^{-4})(8 \times 10^{5})}{4 \times 10^{-7}}$

22. The equations below describe several different animal populations over a period of time. In the equations, P stands for the size of the population and t stands for the year. Match the appropriate equation or equations to the verbal descriptions that follow.

(i) $P = 1000 - 50t$

(ii) $P = 8000(0.95)^{t}$

(iii) $P = 1000 + 70t$

(iv) $P = 5000 + 2000\sin(2\pi t)$

(a) This population decreases by 5% each year.

(b) This population increases by the same number of animals each year.

(c) This population rises and falls over the course of the year.

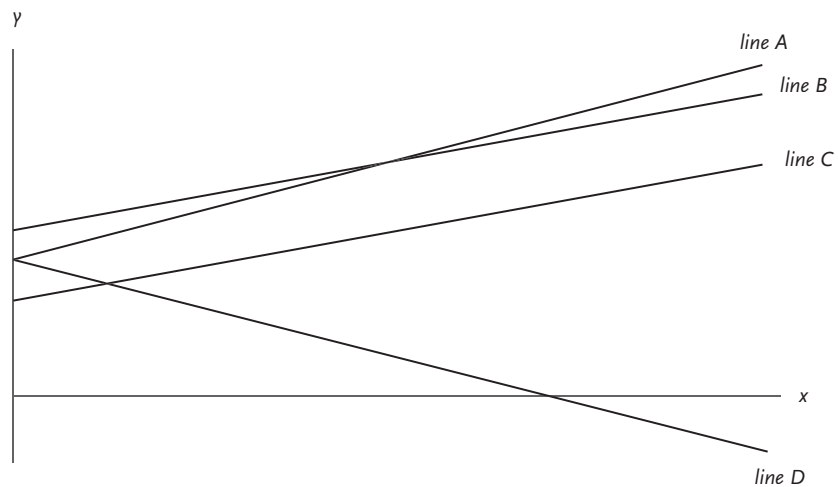(d) In year $t = 0$, these populations are at the same level.

23. Match the following equations to the graphs shown in Figure 6.

(a) $y = 6 - 0.8x$

(b) $y = 6 + 0.9x$

(c) $y = 4 + 0.5x$

(d) $y = 8 + 0.5x$

Figure 6:
Match the equations given in Question 23 on these graphs.

24. Figure 7 gives the rate (in thousands of gallons per minute) that water is entering or leaving a reservoir over a certain period of time. A positive rate indicates that water is entering the reservoir and a negative rate indicates that water is leaving the reservoir. State all time intervals on which the *volume* of water in the reservoir is increasing.

Figure 7:
The rate that water enters (positive values) or leaves (negative values) a reservoir (see Question 24).

rate (1000 gallons per minute)
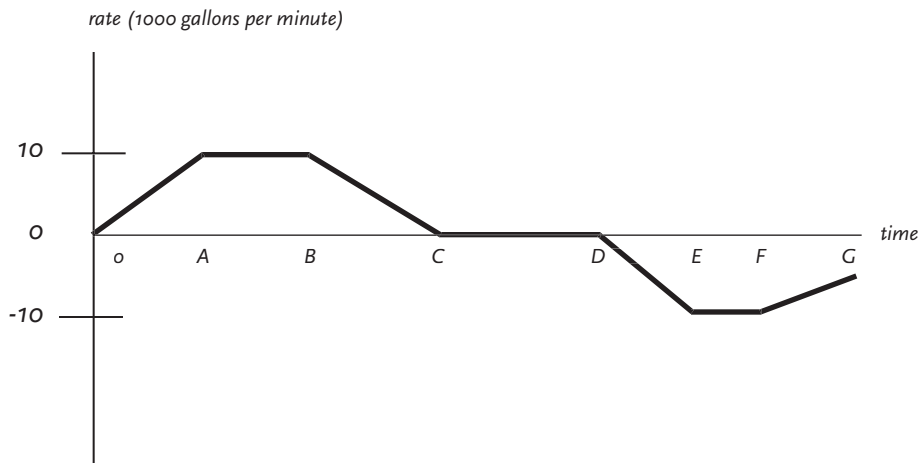
STOP!!!

- Don't read these solutions until you have tried the problems on your own!

- Short answers are provided on page 11. Check these first.

- Worked-out solutions have also been provided, starting on page 13.

- *Tip*. Work the problems and then check the short answers. If you miss a question, try to figure out how to solve it on your own before you read the complete, worked-out solution. You may feel the urge to read the worked-out solutions right away, but discovering the answer to a problem on your own is usually a much more valuable learning experience.

Short Answers to Review Problems

1. 1/4 pound

2. 23 tokens

3. 84

4. $1,100 in 2001, $1,210 in 2002, $1,331 in 2003

5. 20%

6. 3/2 or 1.5

7. $18,000

8. (a) -24   **(b)** 13

9. (a) about 2 lbs more   **(b)** about 18 months old

10. $P = 80 + 12t$

11. (a) Perimeter is 32 units.   **(b)** Area is 52 squared units.

12. five movies

13. (a) agency B **(b)** more than 100 miles

14. almost triple

15. $R = N_f P_f + N_c P_c$

16. $K_2 = K_1 Z_2 / Z_1$

17. 0.48 grams

18. Pointer A reads 0.06 and pointer B reads 0.14.

19. 21 feet tall

20. The s-intercept is -3 and the t intercept is 15.
See Figure 7 on page 22.

21.   **(a)** 6.0×10^{1} **(b)** 6.0×10^{8}

22.   **(a)** ii   **(b)** iii   **(c)** iv   **(d)** i and iii

23.   **(a)** line D **(b)** line A **(c)** line C **(d)** line B

24. from time 0 to time C

Worked Solutions to Review Problems

1. The partygoers left behind 40 tons of garbage, each ton weighing 2000 pounds (lbs). That makes

$$40 \text{ tons} \quad \times \quad \frac{2{,}000 \text{ lbs.}}{\text{ton}} \quad = \quad 80{,}000 \text{ lbs.}$$

We can find the average amount of garbage that each person left behind by dividing the 80,000 pounds of garbage among the 320,000 partygoers:

$$\frac{80{,}000 \text{ lbs.}}{320{,}000 \text{ people}} \quad = \quad 1/4 \text{ lb. per person.}$$

Thus, the average partygoer left behind one quarter of a pound of garbage.

2. Since $\$20 \div 0.85 = 23.5$ (to one decimal of accuracy), this means you can buy 23 tokens with $20 and expect some change. Another way to work this problem is to think of buying tokens in sets of 10. One set of 10 tokens costs $10 \times \$0.85 = \$8.50$. This means that two sets of 10 tokens would cost $2 \times \$8.50 = \$17$. So if you buy 20 tokens for $17, you still have $3 left. This is only enough to buy 3 more tokens, because $3 \times \$0.85 = \$2.55$. In conclusion, $20 will buy 23 tokens and leave you with some change ($0.45, to be exact).

3. Think about how you calculate an exam average: the point total of all of your exams is divided by the number of exams. Since this student's average for her first 3 exams is 92 points, this means that her point total for these 3 exams is $3 \times 92 = 276$ points. If you're not sure about this step, notice that

$$\text{Exam average} \quad = \quad \frac{\text{total score}}{\text{no. exams}}$$

$$\text{Total score} \quad = \quad \text{exam average} \times \text{no. exams}$$

$$= \quad 92 \times 3 = 276.$$

Now, if the student wants her final average for all 4 exams to be at least 90 points, then her point total for all 4 exams must at least $4 \times 90 = 360$. Since she already has 276 points, she only needs $360 - 276 = 84$ points more. Thus, she must score at least an 84.

4. Table 1 shows the bank balance for 1990, 1991, 1992, and 1993.

Year	Balance
2000	$1,000
2001	$1,100
2002	$1,210
2003	$1,331

To find the balance for 2001, we begin with the 2000 balance of $1,000 and then add 10%:

Balance in 2001	=	balance in 2000 + 10% of balance in 2000
	=	$1,000 + 10% of $1,000
	=	$1,000 + 0.10 × $1,000 because 10% is 0.10
	=	$1,000 + $100 = $1,100.

Similarly, to find the balance for 2002, we begin with the 2001 balance of $1,100 and then add 10%. Notice that 10% of $1,100 is not the same as 10% of $1,000, and so this time the balance goes up by a different amount:

Balance in 2002	=	balance in 2001 + 10% of balance in 2001
	=	$1,100 + 10% of $1,100
	=	$1,100 + 0.10 × $1,100
	=	$1,100 + $110 = $1,210.

Finally, to find the balance for 2003, we begin with the 2002 balance of $1,210 and then add 10%:

Balance in 2003	=	balance in 2002 + 10% of balance in 2002
	=	$1,210 + 10% of $1,210
	=	$1,210 + 0.10 × $1,210
	=	$1,210 + $121 = $1,331.

5. The formula for percent change – a *very* useful formula to know – is

$$\underline{\text{Percent change}} = \frac{\text{amount of change}}{\text{original amount}}.$$

Here, the level of methyl chloroform changes by 30 ppt (parts per trillion), dropping from its original level of 150 ppt to its current level of 120 ppt. Using our formula, we see that

$$\underline{\text{Percent decrease}} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$= \frac{30 \text{ ppt}}{150 \text{ ppt}} = 0.20 = 20\%.$$

Note that in general, to convert a decimal to a percentage, we shift the decimal point two places to the right. (Likewise, to convert a percentage to a decimal, we shift the decimal point two places to the left.) Here, we converted the fraction 30/150 to the decimal 0.20 by dividing:

$$\frac{30}{150} = \frac{1}{5} = 0.20 = 20\%.$$

6. The fraction of never married Americans in 1990 is 1/4, and the fraction of never married Americans in 1970 is 1/6. The ratio of the first fraction to the second is given by

$$\underline{\text{Ratio}} = \frac{1/4}{1/6}$$

$$= \frac{1}{4} \times \frac{6}{1} \qquad \text{multiply by reciprocal}$$

$$= 6/4 \qquad \text{must be reduced}$$

$$= 3/2 \text{ or } 1.5.$$

Recall that to divide by a fraction like 1/6, we must multiply by the fraction's reciprocal, which means that we flip the fraction "upside-down" and then multiply.

7. Thinking in terms of proportionality, we see that the ratio of the stock's value today to its value one year ago ($15,000) should equal the ratio of $7,200 to $6,000:

$$\frac{\text{value today}}{\$15,000} = \frac{\$7,200}{\$6,000}$$

$$\text{value today} = \frac{\$7,200}{\$6,000} \times \$15,000 \qquad \text{multiplying}$$

$$= \frac{6}{5} \times \$15,000 \qquad \text{reducing fraction}$$

$$= 6 \times \$3,000 \qquad \text{simplifying}$$

$$= \$18,000.$$

Thus, the investment would be worth $18,000. Another way to solve this problem is to use our formula for percent change (see Question 5):

$$\underline{\text{Percent change}} = \frac{\text{amount of change}}{\text{original amount}}$$

$$= \frac{\$7,200 - \$6,000}{\$6,000}$$

$$= \frac{\$1,200}{\$6,000} = 0.20 = 20\%.$$

Thus, if the person had invested $15,000, it would grow by 20%:

$$\text{Value now} = \$15,000 + 20\% \times \$15,000$$

$$= \$15,000 + 0.20 \times \$15,000 \qquad \text{because 20\% = 0.20}$$

$$= \$15,000 + \$3,000 = \$18,000,$$

which is the same answer that we got before.

8. To *evaluate* these expressions, we find their values by "plugging in" the values of v and w.

(a) We have

$$3(v - 2w) = 3((\text{-}2) - 2(3))$$

$$= 3(\text{-}2 - 6)$$

$$= 3(\text{-}8) = \text{-}24.$$

(b) We have

$$v^2 + w^2 = (-2)^2 + (3)^2$$

$$= (-2)(-2) + (3)(3)$$

$$= 4 + 9 = 13.$$

9. (a) The average 18-month-old girl weighs 23 pounds (lbs). The average 18-month-old boy weighs 25 lbs, or 2 lbs more than the girl.

Figure 1:

For part (a) of Question 9, we see that the average 18-month-old girl weighs 23 lbs and that the average 18-month-old boy weighs 25 lbs.

Weight chart for girls

Weight chart for boys
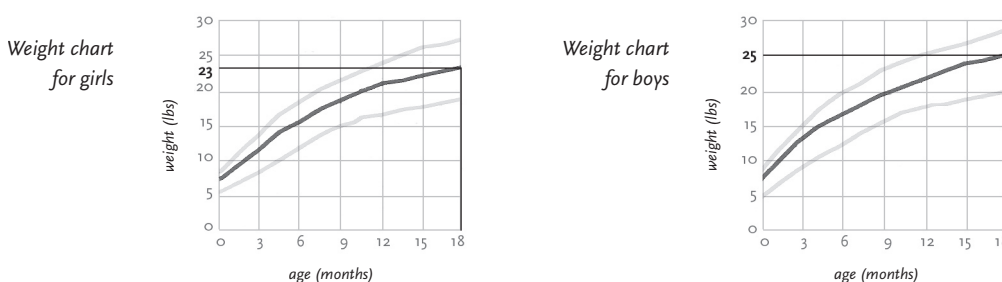
(b) The larger boy weighs 21 lbs at 6 months of age, and the small boy won't weigh this much until he is 18 months of age.

Figure 2:

For part (b) of Question 9, we see that a 5th percentile, 18- month-old boy weighs the same as a 95th percentile, 6-month-old boy.

Weight chart for boys
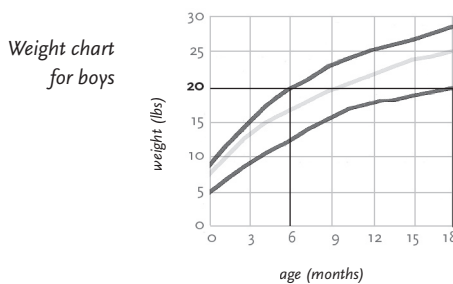
10. We have

$$\text{no. turtles after } t \text{ years} = \text{no. turtles in 2003} + \text{additional turtles since 2003}$$

$$= \underbrace{80 + 12 + 12 + \dots + 12}_{12 \text{ more turtles each year}}$$

$$80 + 12 \times \text{no. years since 2003}$$

$$= 80 + 12t.$$

Thus, a formula for P is $P = 80 + 12t$.

Another way to work this problem is to notice that the number of turtles is growing at a constant rate over time, which means that the equation for P will be linear, so that $P = b + mt$ where b and m are constants. Here, b is the initial value, or 80, and m is the growth rate, or 12. This gives us $P = 80 + 12t$, the same answer that we got before.

11. (a) The perimeter of a shape is the distance around its border. The given shape has two unlabeled sides. From Figure 3, we see that these sides measure 2 and 4. Thus, adding up all the sides, we see that the perimeter of the shape is given by

Perimeter = 6 + 10 + 4 + 4 + 2 + 6 = 32 units.

(b) From Figure 4, we see that the shape can be broken into two different squares, one of side 6 and one of side 4. The area of a square is given by

Area of square = (side)^2 = side × side.

Figure 3:

Finding the perimeter for part (a) of Question 11
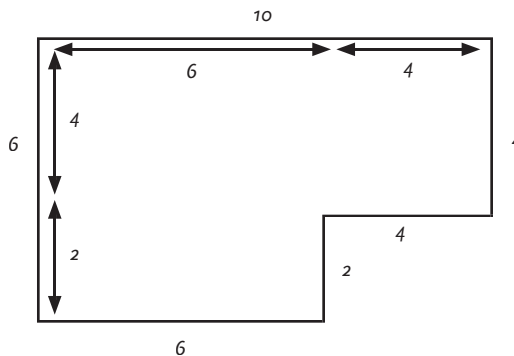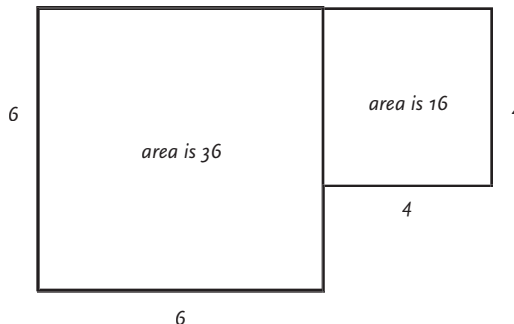

This means that the area of the square of side 6 is 6 × 6 = 36 squared units, and the area of the square of side 4 is 4 × 4 = 16 squared units, and so

area of shape = 36 + 16 = 52 squared units.

If the units were feet, the perimeter (a length) would be 32 ft. and the area would be 52 ft^2.

Figure 4:

Finding the area for part (b) of Question 11

12. From Figure 5, we see that 5 of the top 10 movies made more than $10 million on the July 4th holiday weekend.

Figure 5:

For Question 12, we see that 5 of the top 10 movies (darkly shaded) made more than $10 million.
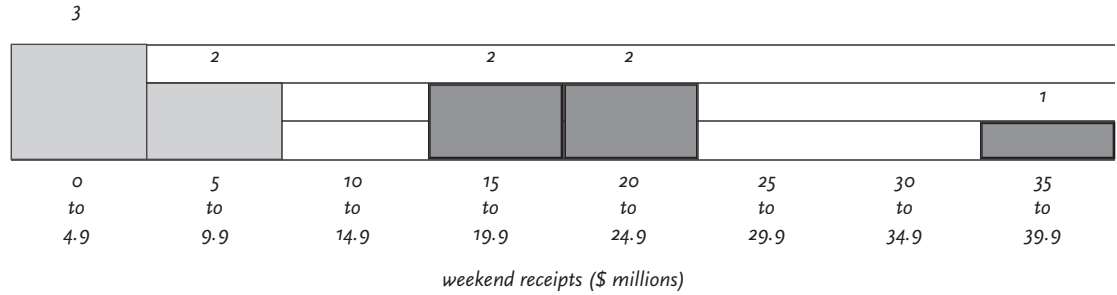
13. The formula for both agencies is *linear*. This means that for each agency a graph of cost versus distance will be a straight line.

(a) Suppose we imagine driving only 5 miles. In this case, the value of n would be 5, and we would have

$$C_A = 30 + 0.22 \times 5 = 31.10$$

$$C_B = 12 + 0.40 \times 5 = 14.00.$$

Thus, the cost for driving 5 miles is $31.10 at agency A but only $14.00 at agency B. Agency B is cheaper for a short distance like 5 miles. On the other hand if we imagine driving 200 miles, the costs work out differently:

$$C_A = 30 + 0.22 \times 200 = 74$$

$$C_B = 12 + 0.40 \times 100 = 92.$$

We see that to drive 200 miles would cost $74 at agency A and $92 at agency B. This means that to drive a long distance, agency A is cheaper.

(b) At what distance should we switch agencies? In other words, at what distance does agency A cost no more than agency B? We can answer this by solving the equation $C_A = C_B$. Setting the formulas for these two costs equal to each other gives:

$$
\begin{aligned}
12 + 0.40n &= 30 + 0.22n \\
0.40n - 0.22n &= 30 - 12 \\
0.18n &= 18 \\
n &= 18/0.18 = 100.
\end{aligned}
$$

Thus, agency A will cost the same as agency B at n = 100 miles. This means that if we drive farther than 100 miles, we would save money by renting from agency A instead of agency B.

14. If a quantity increases by 100% it doubles in size. If it goes up by 200%, it triples in size; if it goes up by 300%, it quadruples in size; and so on. Since the number of injured skaters increased by 184%, this means that the number of injured skaters more than doubled – and almost tripled – in size.

15. We have

Total amount of money = amount for 1st class + amount for coach.

Now, suppose (just for sake of argument) that 20 first-class tickets are sold for $1,000 each. This would mean that

$$\text{Amount of money for 1st class} = \text{\$1,000 per seat} \times \text{20 seats} = \text{\$20,000.}$$

We see that to calculate the money brought in, we multiply the cost per seat by the number of seats. Since we aren't told the number of first-class seats or how much they cost, we must use the symbols N_f and P_f instead of 20 and $1,000, but the reasoning is the same:

$$\text{Amount of money for 1st class} = \text{\$}P_f \text{ per seat} \times N_f \text{ seats} = N_f P_f.$$

Similarly, the amount of money for coach is given by

$$\text{Amount of money for coach} = \text{\$}P_c \text{ per seat} \times N_c \text{ seats} = N_c P_c.$$

Adding these two amounts together gives a formula for R, the total amount of money brought in by the airline for this flight:

$$R = \text{amount for 1st class} + \text{amount for coach}$$
$$= N_f P_f + N_c P_c.$$

16. One way to work this problem is to flip or invert both sides of the equation and then multiply:

$$\frac{K_1}{K_2} = \frac{Z_1}{Z_2} \qquad \text{original equation}$$

$$\frac{K_2}{K_1} = \frac{Z_2}{Z_1} \qquad \text{flip both sides}$$

$$K_2 = K_1 \frac{Z_2}{Z_1} \quad or \quad = \frac{K_1 Z_2}{Z_1} \qquad \text{multiply by } K_1.$$

Another approach is more direct. First, because we are solving for K_2, we will multiply both sides by K_2 in order to clear it from the denominator:

$$\frac{Z_1}{Z_2} = \frac{K_1}{K_2} \qquad \text{original equation}$$

$$K_2 \frac{Z_1}{Z_2} = K_1 \qquad \text{multiply by } K_2.$$

Next, divide both sides by the fraction Z_1 / Z_2 in order to isolate K_2:

$$K_2 = \frac{K_1}{Z_1 / Z_2} .$$

To simplify this result, we recall that to divide by a fraction, we must multiply by its reciprocal. Since the reciprocal of Z_1/Z_2 is Z_2/Z_1, we have:

$$K_2 = K_1 \frac{Z_2}{Z_1} .$$

17. Since one-fifth of the 0.6 grams (g) is given by

$$\frac{1}{5} \times 0.6 \text{ g} = \frac{0.6 \text{ g}}{5} = 0.12 \text{ g},$$

we see that 0.12 grams of powder spill from the dish. Thus, there are $0.60 - 0.12 = 0.48$ grams left in the dish. Alternatively, since one-fifth of the 0.6 grams spills out, four-fifths of the 0.6 grams remain. This means that there are

$$\frac{4}{5} \times 0.6 \text{ g} = \frac{2.5 \text{ g}}{5} = 0.48 \text{ grams left.}$$

18. The scale is divided into 10 evenly spaced tick marks. This means that each small tick mark measures 0.20/10 = 0.02 units. Pointer A is at the third tick mark, which means it reads 3 × 0.02 = 0.06. Pointer B is at the seventh tick mark which means that it reads 7 × 0.02 = 0.14.

19. From Figure 6, we see that the man, the tree, and the shadow form two similar triangles. In the figure, x stands for the height of the tree, which is what we would like to determine. Since the triangles are similar, the ratios of corresponding sides are equal. In other words, the ratio of the tree's height to its shadow's length, $x/28$, equals the ratio of the man's height to his shadow's length, 6/8. This gives us the equation $x/28 = 6/8$, which we can solve for x:

$$\frac{x}{28} = \frac{6}{8} \qquad \text{by similar triangles}$$

$$x = \frac{6}{8} \times 28 \qquad \text{solving for x}$$

$$= \frac{3}{4} \times 28 \qquad \text{reducing}$$

$$= 21$$

This means that the tree is 21 feet tall.

Figure 6:

The man and tree from Question 19 form two similar triangles.

6 foot man

8 foot shadow

28 foot shadow

x
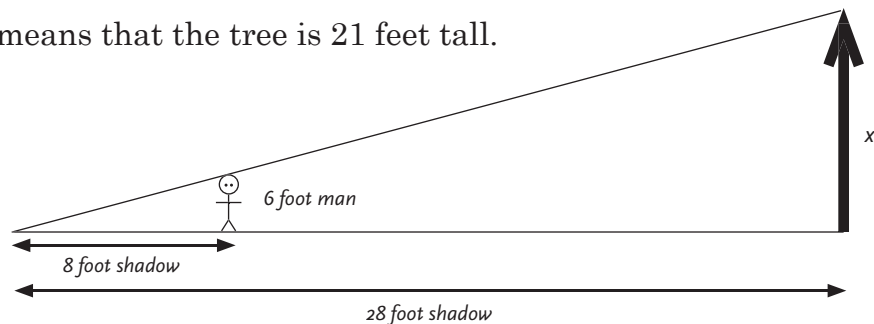
20. If we recognize that this equation is linear, then we know that its graph will be a straight line. We can find the s-intercept by setting $t = 0$, which gives:

$$5s + 15 - 0 = 0 \qquad \text{setting } t = 0$$

$$5s \quad = \quad -15$$

$$s \quad = \quad -3.$$

Similarly, we can find the t-intercept by setting $s = 0$, which gives:

$$5 \times 0 + 15 - t \quad = \quad 0 \qquad \text{setting } s = 0$$

$$15 - t \quad = \quad 0$$

$$t \quad = \quad 15.$$

Thus, the s-intercept is the point $(t, s) = (0, -3)$ and the t-intercept is the point $(t, s) = (15, 0)$. Plotting these two points, we can draw a line passing through them to find the graph of the equation. (See Figure 7.)

Figure 7:

The graph of the equation from Question 20 is a straight line.
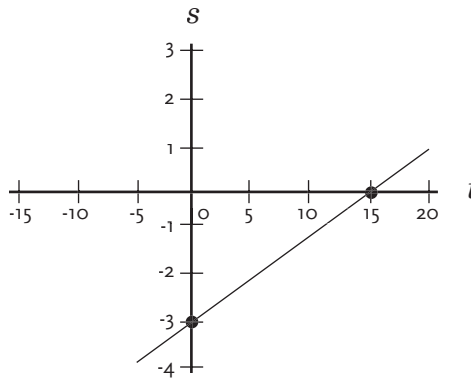
Another approach is to first place this equation into slope-intercept form – in other words, to write it as $s = mt + b$ where m is the slope and b is the s-intercept (the vertical intercept). Solving for s gives

$$5s + 15 - t \quad = \quad 0$$

$$5s + 15 \quad = \quad t$$

$$5s \quad = \quad t - 15$$

$$s \quad = \quad \frac{1}{5}(t - 15)$$

$$= \quad \frac{1}{5}t - 3 \,.$$

Thus (as we have already seen) the s-intercept is $b = -3$. The slope is $m = \dfrac{1}{5}$.

To find the t-intercept, we set $s = 0$ and solve. We obtain $t = 15$, as before.

21. (a) We have $(2 \times 10^{-4})(3 \times 10^5)$ $= 2 \times 3 \times 10^{-4} \times 10^5$

$$= 6 \times 10^{-4+5}$$

$$= 6 \times 10^1,$$

which is the same as 60.

(b) We have $\dfrac{(3 \times 10^{-4})(8 \times 10^5)}{4 \times 10^{-7}}$ $= \dfrac{3 \times 8 \times 10^{-4} \times 10^5}{4 \times 10^{-7}}$

$$= \dfrac{24}{4} \times \dfrac{10^{-4+5}}{10^{-7}}$$

$$= 8 \times \dfrac{10^{-1}}{10^{-7}}$$

$$= 6 \times 10^{1-(-7)}$$

$$= 6 \times 10^8,$$

which equals 600,000,000 or 600 million.

22. Equations (i) and (iii) are both linear. The slope of (i) is -50, so this population decreases (goes down) by 50 animals per year. The slope of (iii) is +70, so this population increases by 70 animals per year. Both populations start out at the same level: 1,000. We can see this by setting t equal to 0 in these equations.

On the other hand, equations (ii) and (iv) are not linear. Equation (ii) is exponential, and equation (iv) is sinusoidal. If we try several different values for t in equation (ii), such as $t = 0, 1, 2,$ and 3, we see that this population goes down by 5% each year:

$$8000(0.95)^0 = 8000$$

$$8000(0.95)^1 = 7600 \quad \text{a 5\% decrease}$$

$$8000(0.95)^2 = 7220 \quad \text{a 5\% decrease}$$

$$8000(0.95)^3 = 6859 \quad \text{a 5\% decrease.}$$

By process of elimination, we see that (d) must go with equation (iv). This makes sense, because sinusoidal quantities rise and fall over time.

Putting all of this together, we see that statement (a) goes with equation (ii), statement (b) goes with equation (iii), statement (c) goes with equation (iv), and statement (d) goes with equations (i) and (iii).

23. The key to this problem is to understand what the values of m and b tell us about the graph of the linear equation $y = mx + b$. A positive value of the slope m corresponds to a line that rises when read from left to right, while a negative value of m corresponds to a line that falls. The larger the value of m, either positive or negative, the steeper the line. The value of the y-intercept, b, determines where the line crosses the y-axis.

Since equations (a) and (b) have the same y-intercept ($b = 6$), they must cross the y-axis at the same point. Moreover, equation (a) has a negative slope (-0.8) while equation (b) has a positive slope (+0.6). Notice that lines A and D have the same y-intercept and that line A climbs while line D falls. Thus line A corresponds to equation (b) while line D corresponds to equation (a).

On the other hand, equations (c) and (d) have the same slope (+0.5), and thus they describe lines of the same steepness, which is to say they describe parallel lines. Moreover, the y-intercept of equation (c) is less than 6, while the y-intercept of equation (d) is more than 6. This is significant because lines A and D cross the y-axis at 6. Thus, we know that equation (c) describes a line that crosses the y-axis below lines A and D. Similarly, equation (d) describes a parallel line that crosses the y-axis above lines A and D. Thus, line C corresponds to equation (c) while line B corresponds to equation (d).

24. It is important to realize that the graph does not show the amount of water in the reservoir; rather, it shows the *rate* that water is entering or leaving the reservoir. We are told that water is entering the reservoir when the rate is positive, which means that the volume is increasing when the rate is positive. Thus, the answer is time 0 to time C, because on this interval (and at no other times) the rate is positive.

S T O P ! ! !

These are actual QR Assessments, given to students in 1995 and 1999. They are similar to the one you will take upon arrival at the College.

- Don't start either assessment until you are ready.

- First, solve the study problems (starting on page 3).

- Allow 90 minutes to finish each practice assessment.

- **Do not use notes or a calculator.** (Calculators may *not* be used during the actual assessment exam.)

- Short answers for both practice assessments have been provided starting on page 43. Check these first.

- Worked-out solutions for the 1995 assessment have also been provided, starting on page 47.

- For each of these assessments, a student who answered **more than** 9 out of 18 questions correctly was considered to have passed.

- *Tip:* Work each practice assessment and then check the short answers (starting on page 43). If you miss a question, try to figure out how to work it on your own before you read the solution. You may feel the urge to read the solutions right away, but discovering the answer to a problem on your own is usually a much more valuable learning experience.

## Practice Exam: QR Assessment for 1995

1. The number of tuberculosis cases in New York state dropped from 4,000 in 1993 to 3,600 in 1994. By what percent did the number of tuberculosis cases drop during this time period?

2. A rectangular swimming pool is 30 meters long, 10 meters wide, and 3 meters deep. Assuming that 1,000 cubic centimeters of water weighs two pounds, how much does the water in a full pool weigh?

3. Evaluate the expression $5(3q^2 - 4p)$ assuming that $p = $ -3 and $q = $ -5.

4. Graph the equation $3h + 5c = 30$ on the set of axes provided. Label the h- and c- intercepts.

Figure 1:

Use these axes to answer Question 4.
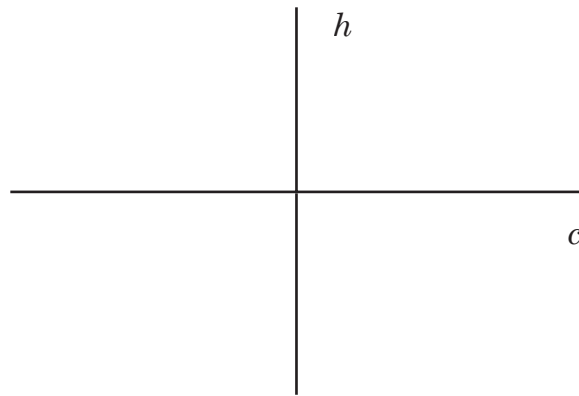
5. The figure shows a 1/4-inch long segment of a ruler. How many inches long is the object next to the ruler?

Figure 2:

For Question 5, measure the length (in inches) of the object.

measure this mark

0 in   1/4 in
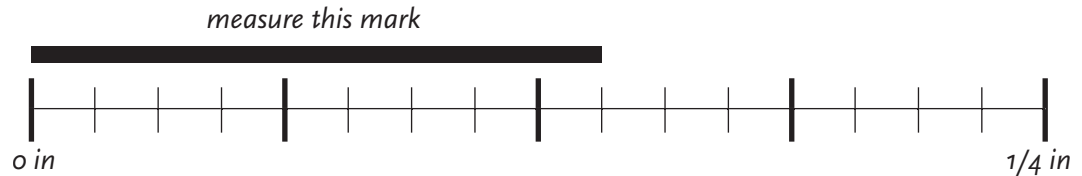
6. A patient is given an injection of a therapeutic drug. Over time, the level of the drug in the body falls as the drug is metabolized. Figure 3 shows the level of the drug in the body over a 12-hour period. During which of the three 4-hour periods shown:

Figure 3:

The drug level over a 12-hour period (see Question 6)

Hours elapsed since injection
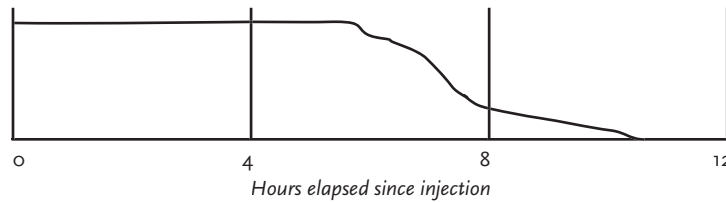
(a) does the drug level drop by the greatest amount?

(b) is the drug level most nearly constant?

7. The total cost of planting a crop depends on two things: the cost of the equipment, which is fixed, and the cost of seed, fertilizer, irrigation, and labor, which depends on the number of acres sown. Suppose a farmer must spend $120,000 on equipment plus $3,500 for every acre sown. Find a formula for C, the farmer's total cost, in terms of n, the number of acres sown.

8. A beaker contains 83.2 milliliters (ml) of liquid. The liquid is heated until one-fourth of it has evaporated. How much liquid (in ml) remains in the beaker?

9. Let $U_{A'} = \dfrac{r}{1 - U_B}$. Solve this equation for U_B in terms of $U_{A'}$ and r.

10. Figure 4 gives the number of U.S. presidents who cast a given number of vetoes during their terms in office. For example, we see from the figure that only one president cast between 60 and 69 vetoes during his term in office. (It happened to be Gerald Ford with 66 vetoes.) Based on the chart, how many presidents cast fewer than 50 vetoes during their terms in office?

Number of presidents

Figure 4:

For Question 10, how many presidents cast fewer than 50 vetoes?

Number of vetoes
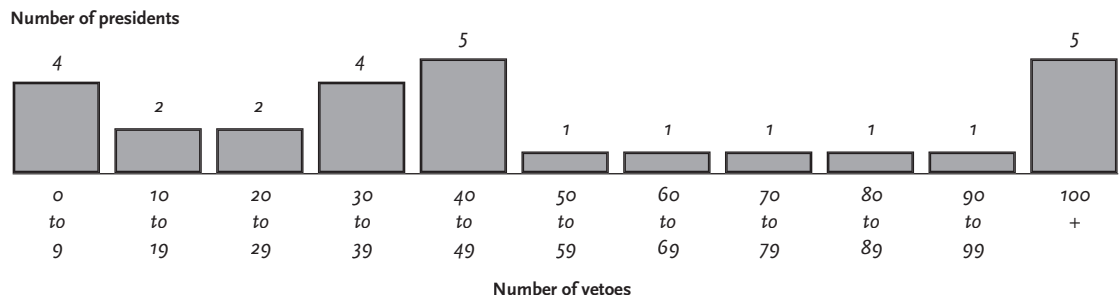
11. The *hate-crime rate* of a state is the ratio of hate or bias-related crimes per year to the number of people living in the state. According to *The New York Times,* in 1994, New Jersey's hate-crime rate was 13 hate crimes per 100,000 residents. Assuming that in 1994 the population of New Jersey was eight million people, how many hate-related crimes were committed there?

12. A parcel of land measures 2 3/4 acres. A developer wishes to divide the land into lots for houses for each lot measuring 1/7 of an acre. Into how many complete lots can the acres of the parcel of land be divided?

13. Let *W* represent the number of female employees and *M* the number of male employees at a certain company. Write an expression in terms of *W* and *M* that represents the fraction of employees that are women.

14. The graphs in Figure 5 describe the growth of four different populations (labeled *A*, *B*, *C*, and *D*) over an extended period of time. For each of the verbal statements following the figure, indicate the two populations that it best describes. Note that some of the populations shown in the figure may correspond to no matching verbal statement.

Figure 5:

*Match these graphs to the statements below.*
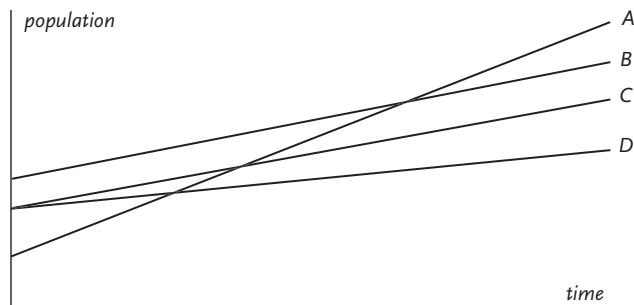
(a)   These two populations begin with the same number of members.

(b)   These two populations grow at the same rate.

15. As shown in Figure 6, a man whose eyes are six feet above the ground stands next to a round pit dug into the ground. The pit measures five feet cross. How deep is the pit (in feet)?

6 ft

2 ft

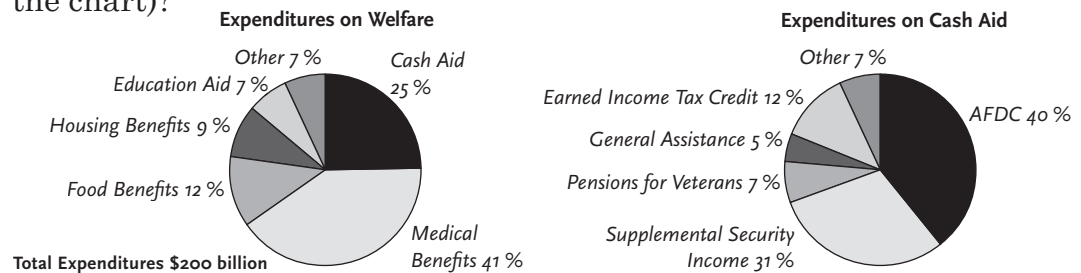Note: figure **NOT** drawn to scale

pit

5 ft
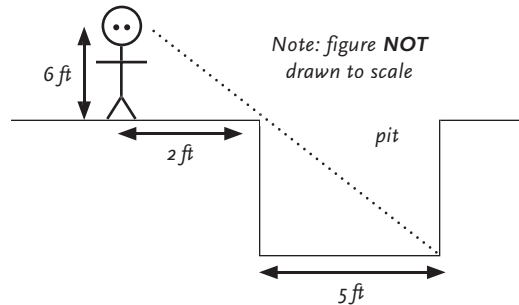
16. Based on the charts shown in Figure 7, how much money (in dollars) was spent on Aid to Families with Dependent Children (labeled AFDC on the chart)?

Expenditures on Welfare

Other 7 %
Education Aid 7 %
Housing Benefits 9 %
Food Benefits 12 %
Cash Aid 25 %
Medical Benefits 41 %

Total Expenditures $200 billion

Expenditures on Cash Aid

Other 7 %
Earned Income Tax Credit 12 %
General Assistance 5 %
Pensions for Veterans 7 %
Supplemental Security Income 31 %
AFDC 40 %

17. A commercial artist needs to rent some high quality photographic equipment to reproduce her artwork. She considers two different types of equipment. The cost of renting and using the first type is $400 plus $2.50 per copy. The cost of renting and using the second type is $150 plus $5.00 per copy.

(a) Which type of equipment is less expensive if the artist only needs to make a small number of copies, the first or the second?

(b) How many copies would the artist need to make before the other type of equipment becomes less expensive?

18. Find the perimeter and area of the shape in Figure 8.

3

8

6

5

19
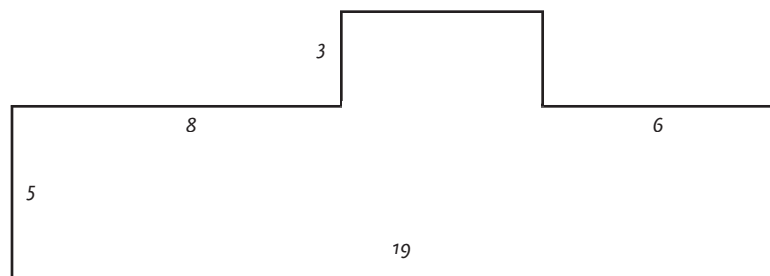
Practice Exam: QR Assessment for 1999

1. Figure 1 shows the states with the highest and lowest rates of prisoners.[1] The figure shows both the total number of prisoners (in 1,000s) in each state and the prison rate per million people in the population. For instance, we see from the figure that in Vermont (VT), there is a total of 1,300 prisoners, which works out to 14 prisoners for every million people in the population.

Figure 1:

States with the highest rates (top chart) and lowest rates (bottom chart) of prisoners in 1997
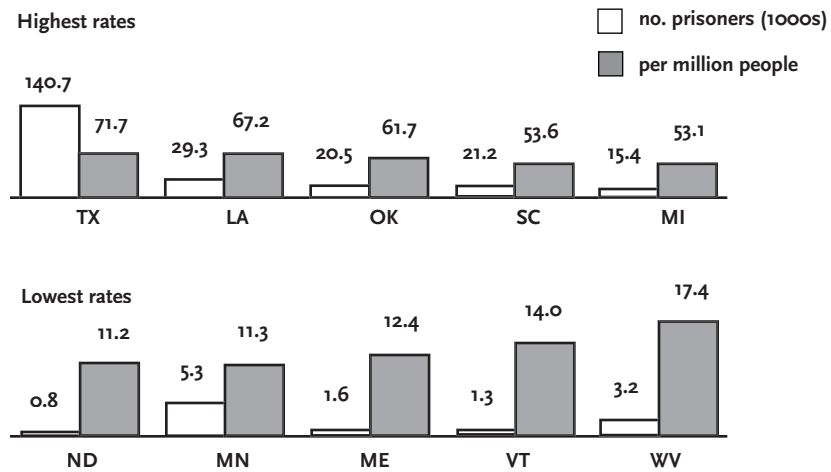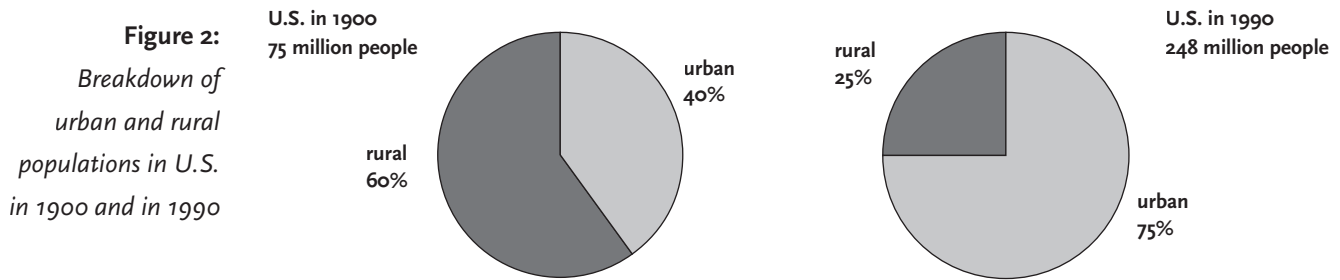

One of the statements (a)-(d) is not supported by Figure 1. Which statement is it?

(a) There are more prisoners in Texas (TX) than the combined total of the other nine states shown by the figure.

(b) The percentage of the population that is in prison in Minnesota (MN) is more than five times the percentage of the population of North Dakota (ND) that is in prison.

(c) Although there are twice as many prisoners in West Virginia (WV) as there are in Maine (ME), the rate (per million) in West Virginia is less than twice the rate (per million) in Maine.

(d) The state with the smallest number of prisoners is North Dakota (ND), and the state with the next smallest number of prisoners is Vermont (VT).

---

[1]Data are from *The New York Times, August* 9, 1998. Data are for 1997.

2. Figure 2 shows the breakdown of urban and rural populations in the U.S. in 1900 and in 1990.

Figure 2:

Breakdown of urban and rural populations in U.S. in 1900 and in 1990

U.S. in 1900
75 million people

urban
40%

rural
60%

rural
25%

U.S. in 1990
248 million people

urban
75%

(a) How large was the rural population of the U.S. in 1900?

(b) The rural population of the U.S. actually grew in size between 1900 and 1990, even though it went down in proportion to the overall population. By how many people did the rural population grow?

3. Figure 3 gives an enlarged view of a flat piece of metal.

Figure 3:

A flat piece of metal

1 in

length

flat piece of metal

width

0 in

0 in   1 in   2 in   3 in
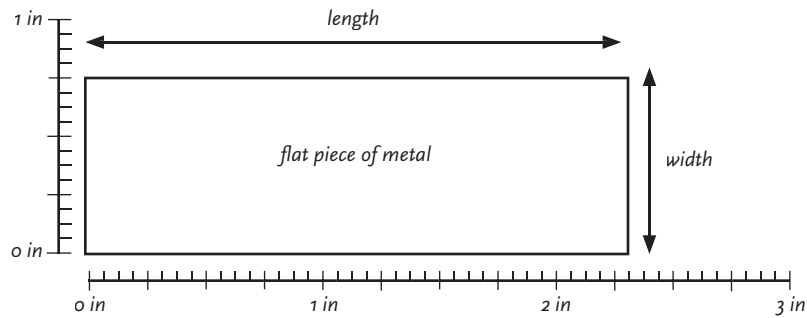
(a) What is the length of the piece of metal in inches (in)?

(b) What is the area of the piece of metal in square inches (in²)?

4. A *hectare* is a metric unit of area and an acre is a U.S. unit of area. Both are often used to measure the sizes of large plots of land.

(a) A hectare is equivalent in area to a square plot of land measuring 100 meters by 100 meters. Suppose a certain piece of land measures 4 km by 8 km. What is its area in hectares? **Note**: Recall that 1 kilometer (km) equals 1,000 meters.

(b) An acre is approximately equivalent to 1/640 of a square mile. One hectare equals two and a half acres. A certain piece of land measures 5 miles by 5 miles. What is its area in hectares?

5. The *national debt* is the amount of money owed by the U.S. government. As of July 1998, the U.S. national debt amounted to $5.54 trillion.

(a) Suppose the debt were reduced by $120 billion. What would the new debt be? **Note:** In the U.S., 1 trillion equals 1,000 billion.

(b) As of July 1998, the size of the U.S. population was 270 million. Suppose every person in America contributed $5,000 towards paying off the national debt. What would the remaining balance be? **Note:** In the U.S., 1 billion equals 1,000 million.

6. A busy corporate executive divides her time at the office each week between meetings with staff and meetings with clients. Each staff meeting takes 4 hours, and each client meeting takes two hours. The number of staff meetings changes from week to week, and she must attend all of them. She works a full 60 hours every week, and every hour not spent at a staff meeting is spent meeting with clients. Table 1 shows the number of staff and client meetings each week over a six-week period. For example, we see that during week 1 the executive has six staff meetings. In a 60-hour week, this leaves her enough time for 18 client meetings.

Table 1:
Weekly meeting schedule of a corporate executive

week	week 1	week 2	week 3	week 4	week 5	week 6
s, no. staff meetings	6	5	9	4	7	15
c, no. client meetings	18	20	12	22	16	0

Let c stand for the number of client meetings and s stand for the number of staff meetings. Find a *simplified* formula for c in terms of s.

7. According to the *logistic model* of population growth, an initially small population will grow rapidly at first but then will level off at a maximum value (the so-called *carrying capacity*). The size of the population in year t is given by the formula

$$\text{Population in year } t = \frac{L}{1 + k\,(1/2)^{t/\tau}}$$

where L is the carrying capacity and where k and τ are constants. Suppose that for a certain population of insects, $L = 4500$, $k = 8$, and $\tau = 3$.

(b) How large is this population initially (in year $t = 0$)?

(c) How large is this population 9 years later (in year $t = 9$)?

8. When studying sedimentary deposits, geologists are often interested in the distribution of *grain size* – that is, the proportions of different sizes of grains there are in a given deposit. To measure grain size, geologists use what is known as the ϕ grade scale, where ϕ is a Greek letter pronounced either *fee* or *fie*. Numbers on this scale range from -12 for very coarse material (like boulders) to +14 for very fine material (like clay). A zero (0) on the scale represents coarse sand. Figure 4 shows grain-size distributions for four different samples of sediment, numbered (i) through (iv).

Figure 4:
Grain-size distributions for four different samples. The horizontal axis shows grain size using the ϕ grade scale. The columns represent the proportion of each sample consisting of a given grain size.
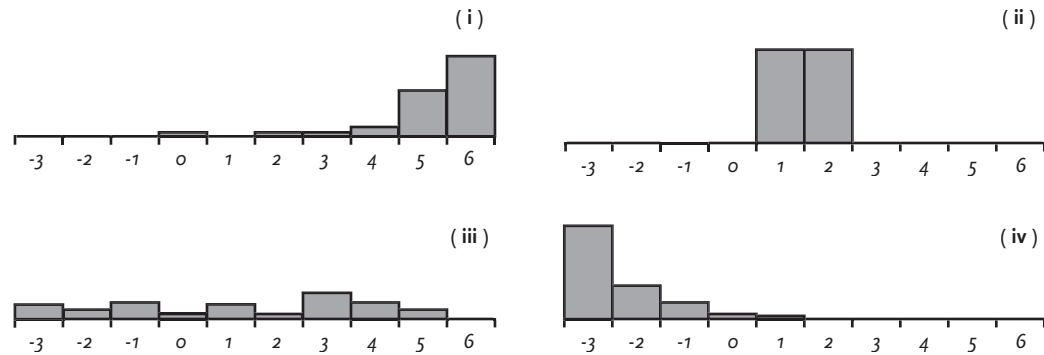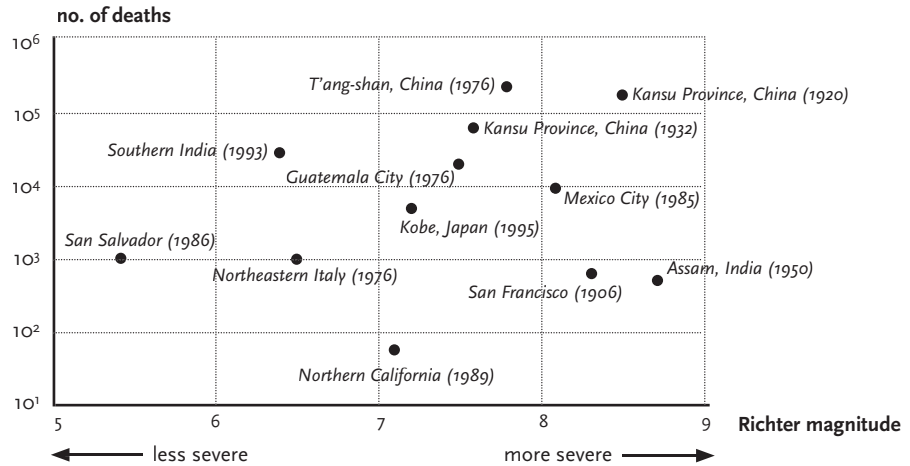

Match each of the following statements to the grain-size distribution from Figure 4 it best describes.

(a) This sample is taken from a sandy beach with no rocks. The sand grains range in size from coarse to fine.

(b) This sample is taken from a laboratory sieve. All of the coarsest and finest grains have been filtered out, leaving a very uniform mix.

9. Figure 5 describes twelve earthquakes that occurred in different parts of the world. The horizontal axis gives the magnitude of the earthquake using the Richter scale. The vertical axis gives the number of people killed by the quake.

Figure 5:

Earthquakes this century from around the world

no. of deaths

10^6

10^5

10^4

10^3

10^2

10^1

T'ang-shan, China (1976) ●

● Kansu Province, China (1920)

● Kansu Province, China (1932)

Southern India (1993) ●

Guatemala City (1976)

● Mexico City (1985)

San Salvador (1986) ●

Kobe, Japan (1995)

Northeastern Italy (1976)

San Francisco (1906)

● Assam, India (1950)

Northern California (1989)

5   6   7   8   9   **Richter magnitude**

← less severe   more severe →

(a) How many earthquakes were less severe than the Mexico City earthquake in 1985, and yet killed more people?

(b) How many of the earthquakes killed at least 100 times as many people as the San Salvador earthquake in 1986?

10. A general contractor is hired to complete an addition to a single-family house. The contractor does much of the work herself, but hires a plumber and electrician for certain parts of the job. The contractor charges $\$p_1$ per hour and bills a total of n_1 hours. She pays the plumber $\$p_2$ for n_2 hours of work and the electrician $\$p_3$ per hour for n_3 hours of work. How much money does the contractor keep after paying the plumber and the electrician? **Note:** Your answer will be an algebraic expression.

11. A patient is given an injection of a therapeutic drug. Over time, the level of the drug in the body falls as the drug is metabolized. Figure 6 shows the level of the drug in the body (in milligrams, or mg) over a 30-hour time period.

Figure 6:

The drug level in a patient's body
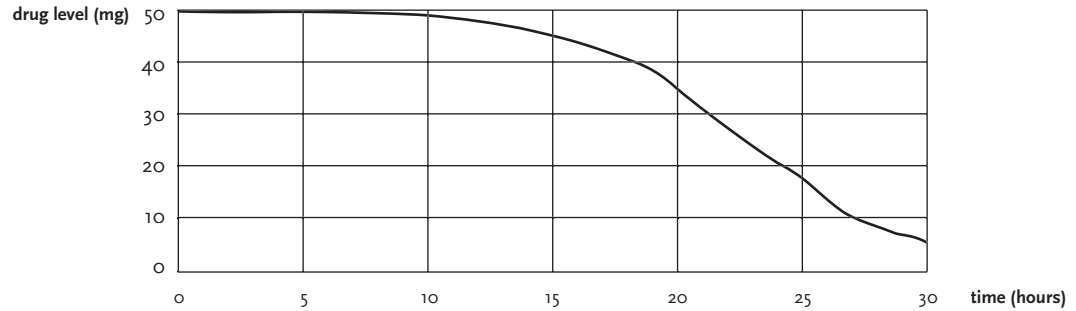
(a) Which of the following drops in drug level takes the most time?

From 50 mg to 45 mg	From 45 mg to 40 mg	From 40 mg to 30 mg	From 30 mg to 20 mg	From 20 mg to 10 mg

(b) On average, about how fast (in mg/hr) does the drug level drop between hour 20 and hour 30?

1   2   3   4   5   10   20   30   40   50

12. Nantucket, a small island off the coast of Massachusetts, is the smallest county in the state. Wellesley is in Norfolk County, the fifth largest county in the state.

(a) Over the past few years the population of Nantucket has grown rapidly. In 1990, its population was 6,000, but by 1996 its population had grown to 7,200. Suppose the population of Nantucket goes on to increase by the same proportion between 1996 and 2002 as it did between 1990 and 1996. How large will it be in 2002? **Note:** The population is assumed to increase by the same *proportion* – not by the same number of persons.

(b) In 1996, the population of Norfolk County was 639,000. Suppose the population of Norfolk County had increased by the same proportion between 1990 and 1996 as Nantucket's population did. How large would Norfolk County's population have been in 1990?

13. Let E stand for the fuel efficiency of a car, in miles per gallon (mpg).

(a) Suppose the car travels for 4 hours at a rate of 60 mph, and that it burns 8 gallons of gas during this time. Find the value of E, the car's fuel efficiency.

(b) How far must the car travel on 8 gallons of gas in order for the value of E to be 42.5?

14. If 1 calorie of heat energy is added to a 1 gram (g) piece of copper, the temperature of the copper will go up by 11°C. The temperature of a 1 g piece of copper is 25°C. Find a formula for its temperature, T, after Q calories of heat energy have been added.

15. Four different animal populations are increasing in size as time goes by. Each population grows at a constant annual rate. Let P_1, P_2, P_3, and P_4 stand for the sizes of the four populations in year t. Formulas for these four populations are

$$P_1 = L + rt$$
$$P_2 = 3L + 2rt$$
$$P_3 = L/2 + (r + 250)\, t$$
$$P_4 = L + 250 + rt$$

where L and r are positive constants. For each of the following verbal descriptions, determine which of the populations of the statement could describe. **Note:** Some statements may describe none of the populations, and some may describe more than one.

(a) This population begins (in year $t = 0$) at the same level as P_1.

P_2 P_3 P_4 No match

(b) This population grows at the same annual rate as P_1.

P_2 P_3 P_4 No match

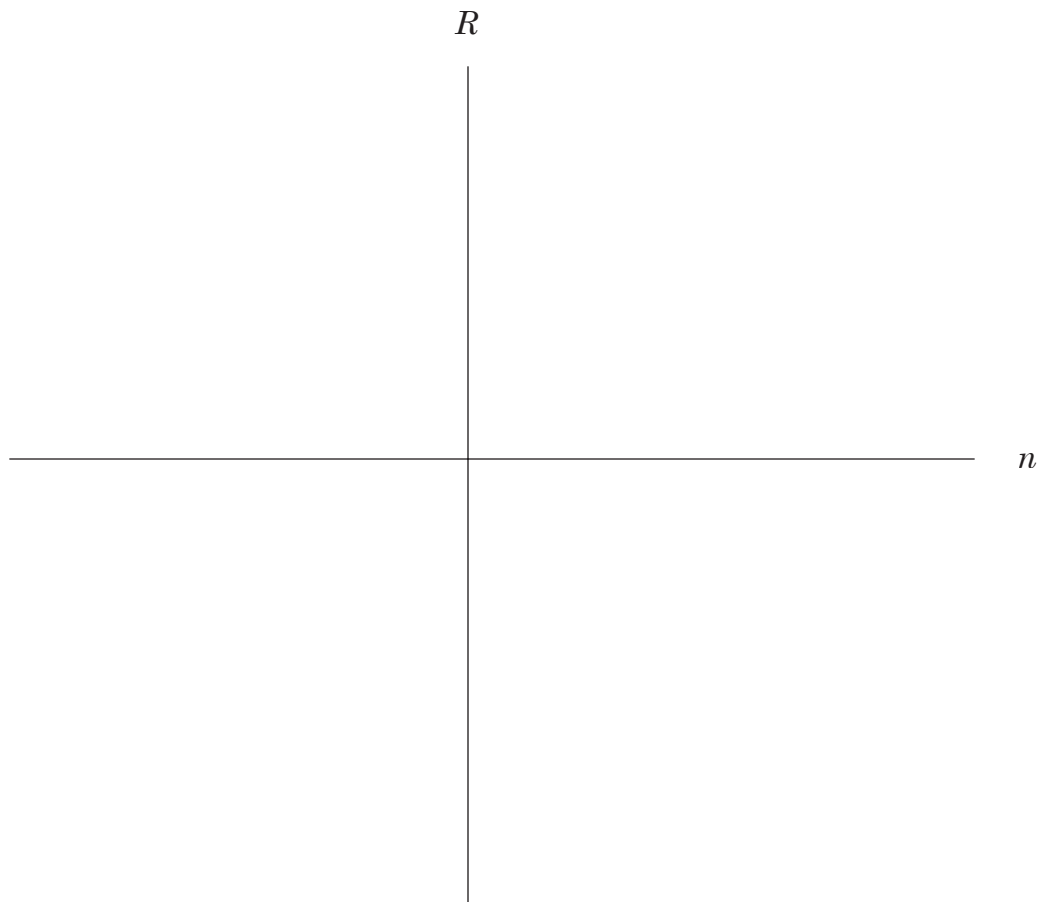(c) This population grows by 250 more animals per year than P_1.

P_2 $\qquad\qquad$ P_3 $\qquad\qquad$ P_4 $\qquad\qquad$ No match

(d) This population begins (in year $t = 0$) with more animals than P_1, and it grows at a faster rate.

P_2 $\qquad\qquad$ P_3 $\qquad\qquad$ P_4 $\qquad\qquad$ No match

16. Sketch a graph of the equation

$$5R - 12n = 60$$

on the set of axes in Figure 7. Label the coordinates of the n- and R-intercepts.

R

n

Figure 7:
Use these axes to sketch your graph.

17. Children with elevated lead levels in their blood are typically given high doses of iron (in the form of iron sulfide), because iron displaces lead from vulnerable receptors in a child's developing brain. Iron sulfide is sold without a prescription as a liquid to be taken orally. The concentration of iron sulfide in this form is 15 mg for every 0.6 ml of liquid.

   (a) If a child takes 1.8 ml of this liquid, how much iron sulfide (in mg) is she receiving?

   (b) Another child needs 75 mg of iron sulfide per day. How much of the liquid (in ml) should he be given?

18. Figure 8 shows two polygons labeled A and B. As illustrated in Figure 8 these polygons fit together to form a square.

Figure 8:
Polygons A and B fit together to form a square.
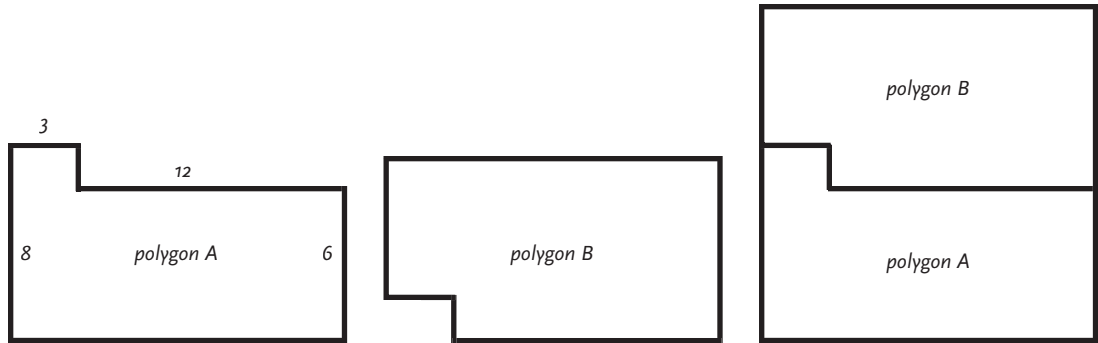
   (a) What is the perimeter of polygon A?

   (b) What is the perimeter of polygon B?

STOP!!!

- Don't read these solutions until you have tried the problems on your own!

- Short answers for both practice assessments are provided beginning on the next page.

- Worked-out solutions have also been provided for the 1995 assessment, starting on page 47.

- *Tip:* Solve the study problems and then check the short answers. If you miss a question, try to figure out how to work it on your own before you read the answer. You may feel the urge to read the answers right away, but discovering how to solve a problem on your own is usually a much more valuable learning experience.

Short Answers to QR Assessment for 1995

1. 10%

2. 1,800,000 lbs

3. 435

4. The c-intercept is 6 and the h-intercept is 10.

5. 9/64 inches

6. **(a)** 2nd 4-hour period   **(b)** 1st 4-hour period

7. $Cl = 120,000 + 3,500n$

8. 62.4 ml

9. $U_B = 1 - r/U_A$

10. 17 presidents

11. 1,040 crimes

12. 19 complete lots

13. $W/(W+M)$

14. **(a)** Cl and D **(b)** B and C

15. 15 feet

16. $20 billion ($20,000,000,000)

17. **(a)** 2nd type   **(b)** more than 100 copies

18. **(a)** Perimeter is 54 units.   **(b)** Area is 110 squared units.

Short Answers to QR Assessment for 1999

1. statement (b)

2. **(a)** 45 million   **(b)** 17 million

3. **(a)** 2 5/16 inches, or 37/16 inches   **(b)** 1 47/64 in², or 111/64 in²

4. **(a)** 3,200   **(b)** 6,400

5. **(a)** $5,420 billion or $5.42 trillion   **(b)** $4,190 billion or $4.19 trillion

6. $c = 30 - 2s$

7. **(a)** 500   **(b)** 2,250

8. **(a)** histogram (i)   **(b)** histogram (ii)

9. **(a)** 4   **(b)** 2

10. $p_1 n_1 - p_2 n_2 - p_3 n_3$

11. **(a)** from 50 mg to 45 mg   **(b)** 3

12. **(a)** 8,640   **(b)** 532,500

13. **(a)** 30   **(b)** 340

14. $Tl = 25 + 11Q$

15. **(a)** no match   **(b)** P_4 **(c)** P_3 **(d)** P_2

16. See Figure 1: solution to Question 16.
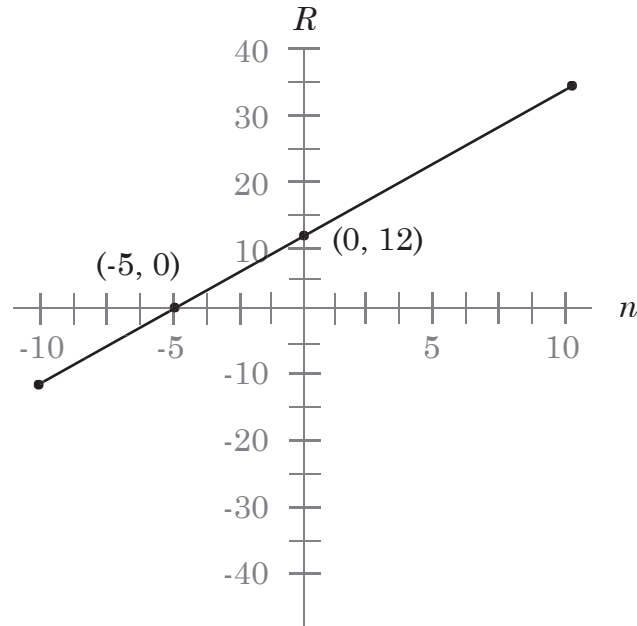
17. (a) 45 **(b)** 3

18. (a) 46 **(b)** 48

Worked Solutions to QR Assessments for 1995

1. The formula for percent change – a good formula to memorize – is

$$\underline{\text{Percent change}} \quad = \quad \frac{\text{amount of change}}{\text{original amount}}.$$

Here, the number of tuberculosis cases dropped by 400, from 4,000 to 3,600. This means that the number of tuberculosis cases dropped by

$$\text{Percent decrease} \quad = \quad \frac{\text{amount of decrease}}{\text{original amount}} \quad = \quad \frac{400}{4000} \quad = \quad 10\%.$$

2. The volume of the pool is given by

$$\begin{aligned} \text{Volume} \quad &= \quad \text{length} \times \text{width} \times \text{depth} \\ &= \quad 30 \text{ m} \times 10 \text{ m} \times 3 \text{ m} \quad = \quad 900 \text{ m}^3. \end{aligned}$$

We aren't told how much a cubic meter (m^3) of water weighs. However, we know that 1000 cubic centimeters (cm^3) weigh about 2 pounds (lbs). This means that we should convert our volume from m^3 to cm^3:

$$\underline{\text{Volume (in cm}^3)} \quad = \quad 900 \text{ m}^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3.$$

Notice that we must multiply by *three* factors of 100 to convert from m^3 to cm^3.

This gives (counting zeros)

$$\text{Volume} \quad = \quad 900 \times 100 \times 100 \times 100 = 900,000,000 \text{ cm}^3.$$

Since 1,000 cm^3 weighs about 2 lbs, we have

$$\underline{\text{Weight}} \quad = \quad \underline{900,000,000 \text{ cm}^3} \quad \times \quad \frac{2 \text{ lbs}}{1000 \text{ cm}^3} \quad = \quad 1,800,000 \text{ lbs.}$$

3. We have
$$\begin{aligned} 5(3q^2 - 4p) \quad &= \quad 5(3(\text{-}5)^2 - 4(\text{-}3)) \\ &= \quad 5(3(25) + 12) \\ &= \quad 5(87) \quad = \quad 435. \end{aligned}$$

4. This equation is linear, so its graph will be a line. One way to work this problem is to find the axis intercepts and use them to draw the line. To find the h-intercept, we set $c = 0$ and solve:
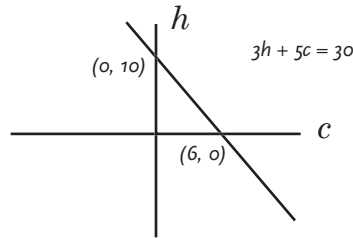
$$3h + 5(0) = 30$$
$$3h = 30$$
$$h = 10.$$

Thus, the point $(c,h) = (0, 10)$ is on the line. We now find the c-intercept:

$$3(0) + 5c = 30$$
$$5c = 30$$
$$c = 6.$$

Thus, the point $(c, h) = (6, 0)$ is on the line. (See Figure 1.)
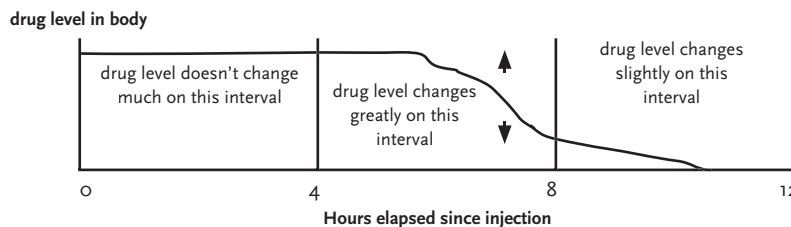
5. The ruler shows a 1/4-inch length divided into 16 smaller units. The length of each of these units is

$$\underline{\text{Length}} = \underline{\text{1/4 inch}} \div \underline{16} = \frac{1/4}{16} = 1/64 \text{ in.}$$

The length of the mark is 9 of these units. Thus, the length of the mark is $9 \times 1/64 = 9/64$ inches.

6. See Figure 2.

(a) The drug level drops by the greatest amount during the second 4-hour period, so it is falling most rapidly on this time interval.

(b) The drug level hardly drops at all during the first 4-hour period, so it is almost constant on this time interval.

7. We have

$$\text{Total cost} = \text{cost for equipment} + \text{cost per acre} \times \text{no. acres}$$

which gives

$$C = 120,000 + 3,500n.$$

8. One-fourth of the liquid, or one-fourth of 83.2 ml, evaporates. This amounts to $(1/4) \times 83.2 = 20.8$ ml of liquid. Since this is the amount that evaporates, the amount remaining is $83.2 - 20.8 = 62.4$ ml.

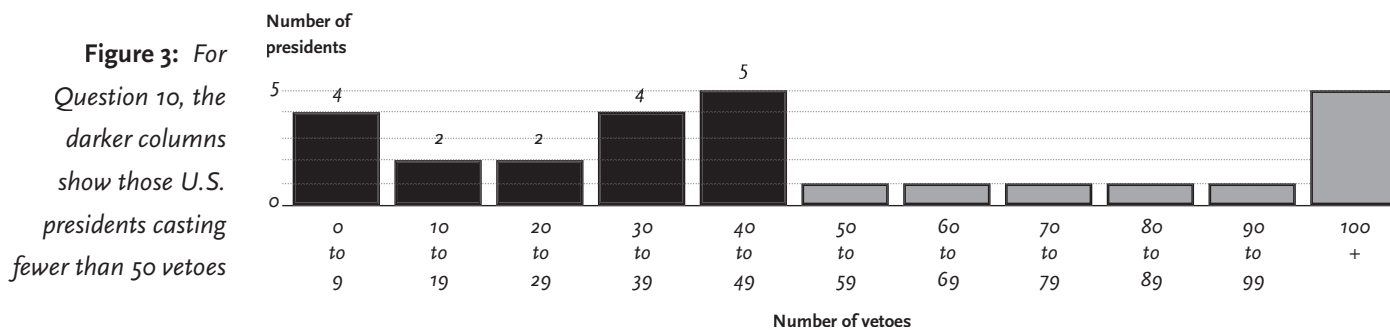9. We have

$$U_A = \frac{r}{1 - U_B}$$

$$U_A (1 - U_B) = r \qquad \text{multiply by } 1 - U_B$$

$$1 - U_B = r/U_A \qquad \text{divide by } U_A$$

$$-U_B = r/U_A - 1 \qquad \text{subtract 1}$$

$$U_B = 1 - r/U_A \qquad \text{change signs (multiply by -1).}$$

10. The darker columns in Figure 3 indicate those presidents who cast fewer than 50 vetoes. From Figure 3, we see that in total there are $4 + 2 + 2 + 4 + 5 = 17$ presidents in this category.

Figure 3: *For Question 10, the darker columns show those U.S. presidents casting fewer than 50 vetoes*

Number of presidents vs Number of vetoes

11. We have $\dfrac{\text{no. hate crimes in NJ}}{\text{no. people in NJ}} = \dfrac{13 \text{ crimes}}{100{,}000 \text{ persons}}$.

Since there are 8 million people in New Jersey, we have

$$\frac{\text{no. hate crimes in NJ}}{8{,}000{,}000 \text{ persons}} = \frac{13 \text{ crimes}}{100{,}000 \text{ persons}}$$

$$\text{no. hate crimes in NJ} = \frac{8{,}000{,}000 \text{ persons} \times 13 \text{ crimes}}{100{,}000 \text{ persons}}$$

$$= 13 \text{ crimes} \times 80 = 1040 \text{ crimes.}$$

12. There are 2 3/4 acres, which can be written as 11/4 acres. To find out how many 1/7-acre lots will fit on this land, we must divide 11/4 by 1/7:

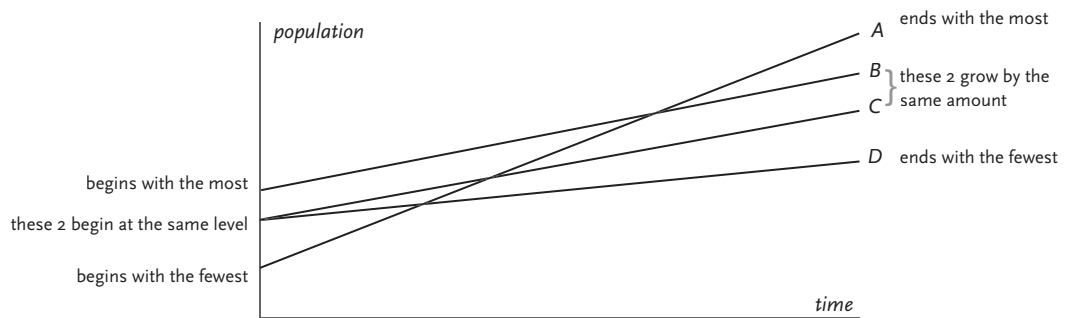$$\frac{11/4}{1/7} = \frac{11}{4} \times \frac{7}{1} = \frac{77}{4} = 19.25.$$

This means that 19 complete lots can be built, leaving 0.25 (1/4 acre) undeveloped.

13. The total number of women is W and the total number of employees is $W + M$. We have

$$\text{Fraction of employees that are women} = \frac{\text{no. of women}}{\text{no. employees}} = \frac{W}{W + M} .$$

14. See Figure 4.

Figure 4:
Solution to
Question 14

(a) From the figure, we see that populations C and D begin with the same number of members, because lines C and D start at the same point on the vertical axis.

(b) We also see that populations B and C grow at the same rate, because lines B and C are parallel.

15. Letting x stand for the depth of the pit, we have (by similar triangles)

$$\frac{x}{5} = \frac{6}{2}$$

$$x = 15.$$

16. From the chart, we see that $200 billion was spent on welfare. Of this $200 billion, 25%, or $50 billion, was spent on cash aid. And of this $50 billion, 40% was spent on Aid to Families with Dependent Children (AFDC). Since 40% of $50 billion is $20 billion, this means that $20 billion was spent on AFDC.

17. (a) To make 1 copy using the first type of equipment costs $400 + $2.50 = $402.50. To make 1 copy using the second type of equipment costs $150 + $5 = $155. Clearly, the second type of equipment is cheaper to use than the first type, provided that only a few copies are needed.

(b) We can answer this question by finding the value of n where the cost for the first method equals the cost for the second method. This gives
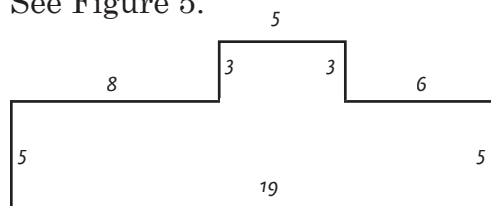
$$400 + 2.50n = 150 + 5.00n$$
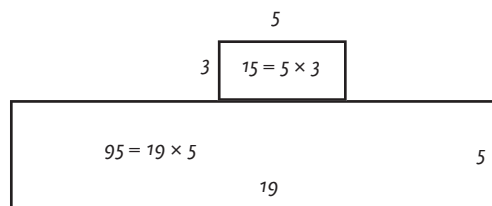
$$250 = 2.50n$$

$$n = 100.$$

This means that both methods cost the same amount for $n = 100$. We know from part (a) that the second method is cheaper for just a few copies. Here we see that if $n > 100$, then the first method becomes less expensive.

18. See Figure 5.

Figure 5:
Finding the perimeter and area for Question 18

(a) The perimeter is 54 units. **(b)** The area is 110 squared units.