1.) $T(n) = 1 + 2T(n-1)$, $T(0) = 1$

$T(n) = 1 + 2[1 + 2T(n-2)] = 1+2+4T(n-2)$

$T(n) = 1 + 2 + 4[1+2T(n-3)] = 1+2+4+8T(n-3)$

$T(n) = 1+2+4+8[1+2T(n-4)] = 1+2+4+8+16T(n-4)$

$T(n) = 2^0 + 2^1 + 2^2 + 2^3 + \ldots + 2^k T(n-k)$

$T(0) = 1$ when $n-k=0$ or $n=k$

$T(n) = \sum_{i=0}^{n-1} 2^i + 2^n(T(0)) = \sum_{i=0}^{n-1} 2^i + 2^n = \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

$T(n) = 2^{n+1} - 1 \leq c \cdot 2^n$ when $n \geq 1$ and $c \geq 2$ $\quad$ $T(n) = 2^{n+1} - 1 \geq 2^n$ when $n \geq 1$

$\boxed{T(n) \text{ is } O(2^n)} \longrightarrow \boxed{T(n) = \Theta(2^n)} \longleftarrow \boxed{T(n) \text{ is } \Omega(2^n)}$

2.) $T(n) = n^2 + T(n-2)$, $T(0) = 1$ $\quad$ $n = 2k$ for some integer k

$T(n) = n^2 + [(n-2)^2 + T(n-4)]$

$T(n) = n^2 + (n-2)^2 + [(n-4)^2 + T(n-6)]$

$T(n) = n^2 + (n-2)^2 + (n-4)^2 + [(n-6)^2 + T(n-8)]$

$T(n) = T(n-2j) + j(n - \text{"value"})^2$ $\quad$ $T(0) = 1$ when $n - 2j = 0$

$\hookrightarrow$ some integer $\quad$ $\frac{n}{2} = j$

$T(n) = T(0) + \frac{n}{2}(n - \text{"value"})^2$ $\quad$ $n \geq n - \text{"value"}$ for "value" ≥ 0

$T(n) \leq 1 + \frac{n}{2}(n)^2 = 1 + \frac{n^3}{2}$ $\quad$ $\boxed{T(n) = O(n^3)}$

3.) $T(n) = T(n-1) + \frac{1}{n}$, $T(1) = 1$

$T(n) = \frac{1}{n} + \left[\frac{1}{n-1} + T(n-2)\right]$

$T(n) = \frac{1}{n} + \frac{1}{n-1} + \left[\frac{1}{n-2} + T(n-3)\right]$

$T(n) = \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \left[\frac{1}{n-3} + T(n-4)\right]$ $\quad$ $T(1) = 1$ when $n-k=1$

$n = k+1$

$T(n) = \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \ldots \frac{1}{n-k+1} + T(n-k)$

$T(n) = \sum_{i=n-k+1}^{n} \frac{1}{i} + T(1) = \sum_{i=2}^{n} \frac{1}{i} + 1 = \sum_{i=1}^{n} \frac{1}{i} \approx \ln(n) + \gamma$

$\downarrow$ $\quad$ $\hookrightarrow$ Euler constant

$i = (k+1) - k + 1 = 2$

$T(n) \approx \ln(n) + \gamma \leq \ln(n) + \gamma \ln(n)$ when $n \geq e$

$T(n) \leq c_0 \ln(n) \implies \boxed{T(n) = O(\ln(n))}$

4.) $T(n) = 2T(\frac{n}{4}) + 1, \; T(0) = 1$

$T(n) = 2T(\frac{n}{4}) + n^0$

$a = 2 \quad b = 4 \quad d = 0$

$2 > 4^0$

$a > b^d$

$\boxed{T(n) = O(n^{\log_4 2}) = O(n^{\frac{1}{2}})}$

5.) $T(n) = 2T(\frac{n}{4}) + n^{\frac{1}{2}}, \; T(0) = 1$

$a = 2 \quad b = 4 \quad d = \frac{1}{2}$

$2 = 4^{\frac{1}{2}}$

$a = b^d$

$\boxed{T(n) = O(n^{\frac{1}{2}} \cdot \lg n)}$

6.) $T(n) = 2T(\frac{n}{4}) + n^2$

$a = 2 \quad b = 4 \quad d = 2$

$2 < 4^2$

$a < b^d$

$\boxed{T(n) = O(n^2)}$

7.) $T(n) = 10T(\frac{n}{3}) + n^2, \; T(0) = 1$

$a = 10 \quad b = 3 \quad d = 2$

$10 > 3^2$

$a > b^d$

$\boxed{T(n) = O(n^{\log_3 10}) = O(n^{2.10})}$

8.) $T(n) = 2T(\frac{n}{3/2}) + 1, \; T(0) = 1$

$a = 2 \quad b = \frac{3}{2} \quad d = 0$

$2 > (\frac{3}{2})^0$

$a > b^d$

$T(n) = O(n^{\log_{\frac{3}{2}} 2})$

$\boxed{T(n) = O(n^{1.71})}$