

CSC 6013 – Summer 2022

Week 5 – Recursive algorithms and recurrence relations

Submit your work as a docx or pdf file through Blackboard.

Coding a recursive function; recurrence relation; back substitution or master method

1) a) Write python code for a recursive algorithm that will calculate the number of digits in the binary expansion/representation of a positive integer n . The logic of the recursive algorithm should be something like:

if $n = 1$, the answer is 1;

if $n > 1$, the answer is 1 more than the number of digits in the binary representation of $n/2$.

You will need to use the python function `math.floor()` in your code.

b) Run your code on the problem instances $n = 32$ and $n = 75$.

c) Create a recurrence relation that gives the work performed by the algorithm in the worst-case for a problem of size n . In your recurrence relation, count the number of recursive calls to the function as the fundamental unit of work. In your asymptotic analysis, you can assume that n is an integer power of 2.

d) Perform the asymptotic analysis with either the back-substitution method OR the master method to solve the recurrence relation and determine the algorithm's asymptotic class Big-Oh.

2) a) Write python code for a recursive algorithm that will calculate the sum of the squares of the positive integers $1^2 + 2^2 + 3^2 + \dots + n^2$ when supplied with a positive integer n .

The logic of the recursive algorithm should be something like:

if $n = 1$, the answer is 1;

if $n > 1$, the answer is (the sum of the squares of the integers from 1 to $n-1$) + n^2 .

Do not find a closed form formula for the summation; make your algorithm do all the arithmetic.

b) Run your code on the problem instances $n = 5$ and $n = 10$.

c) Create a recurrence relation that gives the work performed by the algorithm in the worst-case for a problem of size n . In your recurrence relation, count the number of recursive calls to the function as the fundamental unit of work. Use the back-substitution method to solve the recurrence relation.