In[1]:=
```
M = 5;
beta = 0.1;
g = 0.;
alpha = 0.5;
l = 0.9;
rz = 0.5;
L = 3.0;
epsilon = Exp[-2 * L];
bVal = 1 - 2 * epsilon;
r = 2 * L * rz;

xi[z_, Delta_, OmegaA_, sign_] = M * (sign * Delta * z + OmegaA)

betaA = alpha + beta - 1 - xi^2

betaZ = Together[
  beta + 2 * alpha + 2 * alpha^2 * (xi^4 + 2 * g * xi^3 + 2 * g^2 * xi^2 - 5 * xi^2 - 6 * g * xi + 3) /
    ((xi^2 - 1) * (xi^4 - 6 * xi^2 - 4 * g * xi + 3))]

nBetaZ = Numerator[betaZ]

dBetaZ = Denominator[betaZ]

q[z_, Delta_, OmegaA_] = ((1 - l) * nBetaZ + l * betaA * dBetaZ) / nBetaZ
F[z_, Delta_, OmegaA_] = (Sqrt[-q[z, Delta, OmegaA] /. xi → xi[z, Delta, OmegaA, 1]] +
    Sqrt[-q[z, Delta, OmegaA] /. xi → xi[z, Delta, OmegaA, -1]]) / (-z^2 + 1)
```

Out[11]=
$$5\,(\text{OmegaA} + \text{Delta}\,\text{sign}\,z)$$

Out[12]=
$$-0.4 - \text{xi}^2$$

Out[13]=
$$\frac{1.1\left(-1.63636 + 6.72727\,\text{xi}^2 - 6.54545\,\text{xi}^4 + 1.\,\text{xi}^6\right)}{\left(-1. + \text{xi}^2\right)\left(3. - 6.\,\text{xi}^2 + \text{xi}^4\right)}$$

Out[14]=
$$1.1\left(-1.63636 + 6.72727\,\text{xi}^2 - 6.54545\,\text{xi}^4 + 1.\,\text{xi}^6\right)$$

Out[15]=
$$\left(-1. + \text{xi}^2\right)\left(3. - 6.\,\text{xi}^2 + \text{xi}^4\right)$$

Out[16]=
$$\left(0.909091\left(0.9\left(-0.4 - \text{xi}^2\right)\left(-1. + \text{xi}^2\right)\left(3. - 6.\,\text{xi}^2 + \text{xi}^4\right) + 0.11\left(-1.63636 + 6.72727\,\text{xi}^2 - 6.54545\,\text{xi}^4 + 1.\,\text{xi}^6\right)\right)\right) / \left(-1.63636 + 6.72727\,\text{xi}^2 - 6.54545\,\text{xi}^4 + 1.\,\text{xi}^6\right)$$

Out[17]=

$$\frac{1}{1-z^2}\left(0.953463\sqrt{-\left(\left(0.9\left(-0.4-25\,(\text{OmegaA}-\text{Delta z})^2\right)\left(-1.+25\,(\text{OmegaA}-\text{Delta z})^2\right)\left(3.-150.\,(\text{OmegaA}-\text{Delta z})^2+625\,(\text{OmegaA}-\text{Delta z})^4\right)+0.11\left(-1.63636+168.182\,(\text{OmegaA}-\text{Delta z})^2-4090.91\,(\text{OmegaA}-\text{Delta z})^4+15\,625.\,(\text{OmegaA}-\text{Delta z})^6\right)\right)\Big/\left(-1.63636+168.182\,(\text{OmegaA}-\text{Delta z})^2-4090.91\,(\text{OmegaA}-\text{Delta z})^4+15\,625.\,(\text{OmegaA}-\text{Delta z})^6\right)\right)}+0.953463\sqrt{-\left(\left(0.9\left(-0.4-25\,(\text{OmegaA}+\text{Delta z})^2\right)\left(-1.+25\,(\text{OmegaA}+\text{Delta z})^2\right)\left(3.-150.\,(\text{OmegaA}+\text{Delta z})^2+625\,(\text{OmegaA}+\text{Delta z})^4\right)+0.11\left(-1.63636+168.182\,(\text{OmegaA}+\text{Delta z})^2-4090.91\,(\text{OmegaA}+\text{Delta z})^4+15\,625.\,(\text{OmegaA}+\text{Delta z})^6\right)\right)\Big/\left(-1.63636+168.182\,(\text{OmegaA}+\text{Delta z})^2-4090.91\,(\text{OmegaA}+\text{Delta z})^4+15\,625.\,(\text{OmegaA}+\text{Delta z})^6\right)\right)}\right)$$

In[18]:=
```
LogSpace[start_, end_, steps_] :=
 Exp[Range[Log[start], Log[end], (Log[end] - Log[start]) / steps]]

deltaValues = LogSpace[0.01, 1, 99]
initialOmega = 0.` + 0.07464607515065497` i; (*Initial guess for the first Delta*)
omegaValues =
 Table[initialOmega = OmegaA /. FindRoot[NIntegrate[F[z, Delta, OmegaA], {z, 0, bVal}] == r,
     {OmegaA, initialOmega}], {Delta, deltaValues}]
```

Out[19]=
```
{0.01, 0.0104762, 0.010975, 0.0114976, 0.012045, 0.0126186, 0.0132194, 0.0138489,
 0.0145083, 0.0151991, 0.0159228, 0.016681, 0.0174753, 0.0183074, 0.0191791,
 0.0200923, 0.021049, 0.0220513, 0.0231013, 0.0242013, 0.0253536, 0.0265609,
 0.0278256, 0.0291505, 0.0305386, 0.0319927, 0.033516, 0.0351119, 0.0367838,
 0.0385353, 0.0403702, 0.0422924, 0.0443062, 0.0464159, 0.048626, 0.0509414, 0.053367,
 0.0559081, 0.0585702, 0.0613591, 0.0642807, 0.0673415, 0.070548, 0.0739072,
 0.0774264, 0.0811131, 0.0849753, 0.0890215, 0.0932603, 0.097701, 0.102353,
 0.107227, 0.112332, 0.117681, 0.123285, 0.129155, 0.135305, 0.141747, 0.148497,
 0.155568, 0.162975, 0.170735, 0.178865, 0.187382, 0.196304, 0.205651, 0.215443,
 0.225702, 0.236449, 0.247708, 0.259502, 0.271859, 0.284804, 0.298365, 0.312572,
 0.327455, 0.343047, 0.359381, 0.376494, 0.394421, 0.413201, 0.432876, 0.453488,
 0.475081, 0.497702, 0.521401, 0.546228, 0.572237, 0.599484, 0.628029, 0.657933,
 0.689261, 0.722081, 0.756463, 0.792483, 0.830218, 0.869749, 0.911163, 0.954548, 1.}
```

NIntegrate: The integrand

$$\frac{0.953463\sqrt{-\frac{0.9\ \text{Plus[<<2>>]}\ \text{Plus[<<2>>]}\ \text{Plus[<<3>>]}+0.11\ \text{Plus[<<4>>]}}{-1.63636+\text{Times[<<2>>]}+\text{Times[<<2>>]}+\text{Times[<<2>>]}}}+0.953463\sqrt{-\frac{0.9\ \text{Plus[<<2>>]}\ \text{Plus[<<2>>]}\ \text{Plus[<<3>>]}+\text{<<20>>}\ \text{<<1>>}}{-1.63636+\text{Times[<<2>>]}+\text{<<1>>}+\text{Times[<<2>>]}}}}{1-z^2}$$

has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}.

NIntegrate: The integrand $\frac{0.476731\left(\frac{(0.9\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 3\gg]+\ll 20\gg\ \ll 1\gg)\ \ll 1\gg}{(-\ll 19\gg+\ll 2\gg+\text{Times}[\ll 2\gg])^2}-\frac{\ll 1\gg}{-\ll 19\gg+\ll 2\gg+\ll 1\gg}\right)}{\sqrt{-\frac{0.9\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 3\gg]+0.11\ \ll 1\gg}{-1.63636+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]}}}+\frac{\ll 19\gg\left(\frac{\ll 1\gg}{\ll 1\gg}-\ll 1\gg\right)}{\sqrt{-\frac{\ll 1\gg\ \ll 1\gg\ \ll 1\gg}{\ll 1\gg}}}}{1-z^2}$ has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}.

NIntegrate: The integrand $\frac{0.476731\left(\frac{(0.9\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 3\gg]+\ll 20\gg\ \ll 1\gg)\ \ll 1\gg}{(-\ll 19\gg+\ll 2\gg+\text{Times}[\ll 2\gg])^2}-\frac{\ll 1\gg}{-\ll 19\gg+\ll 2\gg+\ll 1\gg}\right)}{\sqrt{-\frac{0.9\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 2\gg]\ \text{Plus}[\ll 3\gg]+0.11\ \ll 1\gg}{-1.63636+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]}}}+\frac{\ll 19\gg\left(\frac{\ll 1\gg}{\ll 1\gg}-\ll 1\gg\right)}{\sqrt{-\frac{\ll 1\gg\ \ll 1\gg\ \ll 1\gg}{\ll 1\gg}}}}{1-z^2}$ has evaluated to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}.

General: Further output of NIntegrate::inumr will be suppressed during this calculation.

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

General: Further output of FindRoot::lstol will be suppressed during this calculation.

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.888138}. NIntegrate obtained 10.6698 + 0. i and 0.01957520718121196` for the integral and error estimates.

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small.

General: Further output of NIntegrate::slwcon will be suppressed during this calculation.

NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.233198}. NIntegrate obtained 10.6787 + 0. i and 0.0001408379572213836` for the integral and error estimates.

NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.888138}. NIntegrate obtained 10.6782 + 0. i and 0.0015477527461221145` for the integral and error estimates.

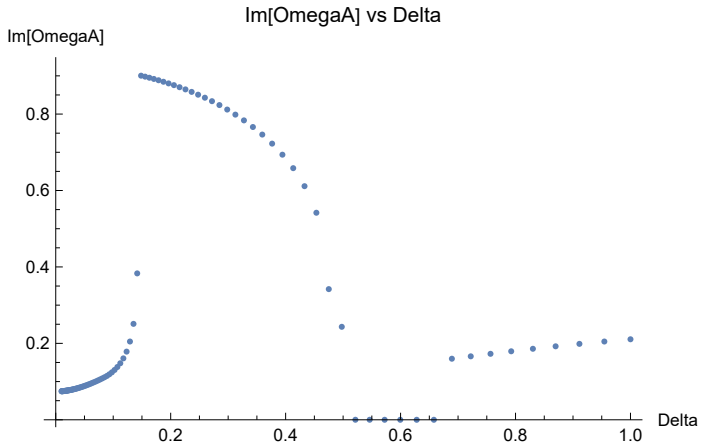
General: Further output of NIntegrate::ncvb will be suppressed during this calculation.

Out[21]=

$$\{0. + 0.0746461\,i,\ 0. + 0.0747066\,i,\ 0. + 0.074773\,i,\ 0. + 0.0748458\,i,$$
$$0. + 0.0749257\,i,\ 0. + 0.0750133\,i,\ 0. + 0.0751093\,i,\ 0. + 0.0752145\,i,$$
$$0. + 0.0753299\,i,\ 0. + 0.0754564\,i,\ 0. + 0.075595\,i,\ 0. + 0.0757469\,i,\ 0. + 0.0759133\,i,$$
$$0. + 0.0760956\,i,\ 0. + 0.0762953\,i,\ 0. + 0.0765139\,i,\ 0. + 0.0767533\,i,\ 0. + 0.0770153\,i,$$
$$0. + 0.077302\,i,\ 0. + 0.0776156\,i,\ 0. + 0.0779585\,i,\ 0. + 0.0783334\,i,\ 0. + 0.078743\,i,$$
$$0. + 0.0791905\,i,\ 0. + 0.079679\,i,\ 0. + 0.0802121\,i,\ 0. + 0.0807936\,i,\ 0. + 0.0814274\,i,$$
$$0. + 0.0821178\,i,\ 0. + 0.0828694\,i,\ 0. + 0.083687\,i,\ 0. + 0.0845758\,i,\ 0. + 0.0855412\,i,$$
$$0. + 0.0865891\,i,\ 0. + 0.0877255\,i,\ 0. + 0.0889569\,i,\ 0. + 0.0902903\,i,\ 0. + 0.0917332\,i,$$
$$0. + 0.0932937\,i,\ 0. + 0.0949807\,i,\ 0. + 0.0968039\,i,\ 0. + 0.0987747\,i,\ 0. + 0.100906\,i,$$
$$0. + 0.103212\,i,\ 0. + 0.10571\,i,\ 0. + 0.108423\,i,\ 0. + 0.111374\,i,\ 0. + 0.114766\,i,$$
$$0. + 0.119005\,i,\ 0. + 0.124143\,i,\ 0. + 0.130394\,i,\ 0. + 0.138118\,i,\ 0. + 0.147896\,i,$$
$$0. + 0.16071\,i,\ 0. + 0.178378\,i,\ 0. + 0.204801\,i,\ 0. + 0.251007\,i,\ 0. + 0.383093\,i,$$
$$0. + 0.900455\,i,\ 0. + 0.897904\,i,\ 0. + 0.895049\,i,\ 0. + 0.891902\,i,\ 0. + 0.888468\,i,$$
$$0. + 0.884536\,i,\ 0. + 0.880253\,i,\ 0. + 0.875691\,i,\ 0. + 0.870265\,i,\ 0. + 0.864568\,i,$$
$$0. + 0.858039\,i,\ 0. + 0.850693\,i,\ 0. + 0.842815\,i,\ 0. + 0.833716\,i,\ 0. + 0.823531\,i,$$
$$0. + 0.811813\,i,\ 0. + 0.79855\,i,\ 0. + 0.783483\,i,\ 0. + 0.766129\,i,\ 0. + 0.746344\,i,$$
$$0. + 0.722634\,i,\ 0. + 0.693711\,i,\ 0. + 0.658456\,i,\ 0. + 0.61131\,i,\ 0. + 0.541777\,i,$$
$$0. - 0.341935\,i,\ 0. - 0.243314\,i,\ 0. - 0.000090422\,i,\ 0. - 0.0000880294\,i,$$
$$0. - 1.60255 \times 10^{-15}\,i,\ 0. + 1.59918 \times 10^{-6}\,i,\ 0. + 1.69809 \times 10^{-15}\,i,\ 0. + 1.69809 \times 10^{-15}\,i,$$
$$0. - 0.159793\,i,\ 0. - 0.166066\,i,\ 0. - 0.172638\,i,\ 0. - 0.179249\,i,\ 0. - 0.185811\,i,$$
$$0. - 0.192254\,i,\ 0. - 0.198668\,i,\ 0. - 0.204799\,i,\ -1.74967 \times 10^{-15} - 0.210689\,i\}$$

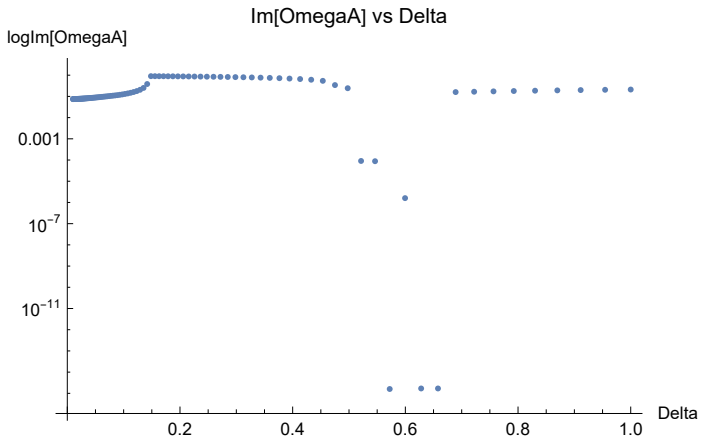
```
combinedValues = Transpose[{deltaValues, omegaValues}];
ListPlot[Transpose[{deltaValues, Abs[Im[omegaValues]]}],
 PlotStyle → PointSize[0.01], Joined → False, PlotRange → All,
 AxesLabel → {"Delta", "Im[OmegaA]"}, PlotLabel → "Im[OmegaA] vs Delta"]
```

Out[23]=



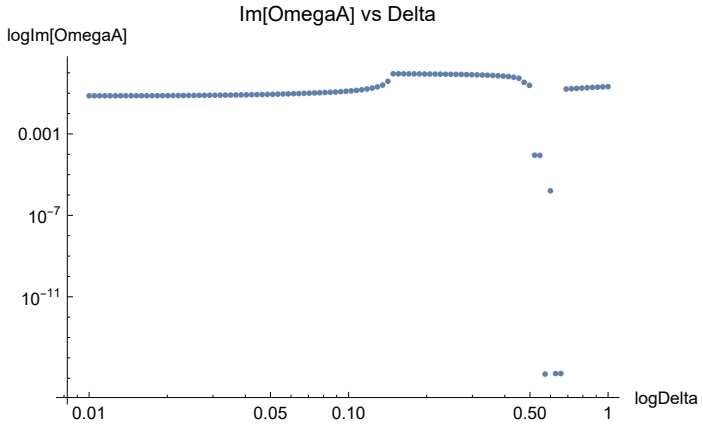
In[28]:=
```
ListLogPlot[Transpose[{deltaValues, Abs[Im[omegaValues]]}],
 PlotStyle → PointSize[0.01], Joined → False, PlotRange → All,
 AxesLabel → {"Delta", "logIm[OmegaA]"}, PlotLabel → "Im[OmegaA] vs Delta"]
```

Out[28]=



In[27]:=
```
ListLogLogPlot[Transpose[{deltaValues, Abs[Im[omegaValues]]}],
 PlotStyle → PointSize[0.01], Joined → False, PlotRange → All,
 AxesLabel → {"logDelta", "logIm[OmegaA]"}, PlotLabel → "Im[OmegaA] vs Delta"]
```

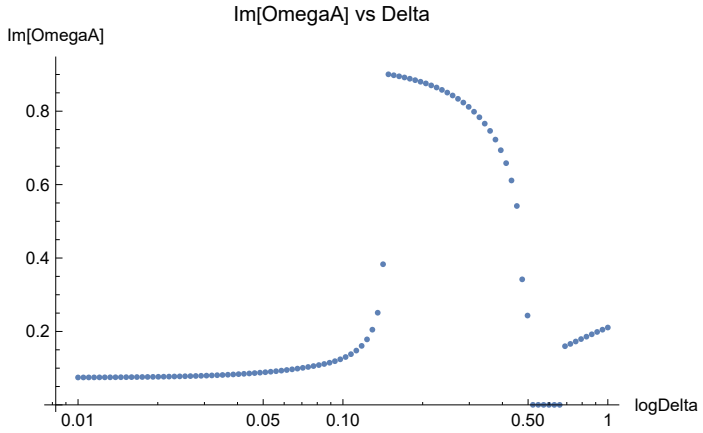
Out[27]=



In[34]:=

```
ListLogLinearPlot[Transpose[{deltaValues, Abs[Im[omegaValues]]}],
 PlotStyle → PointSize[0.01], Joined → False, PlotRange → All,
 AxesLabel → {"logDelta", "Im[OmegaA]"}, PlotLabel → "Im[OmegaA] vs Delta"]
```

Out[34]=



In[31]:=

```
Do[Print[combinedValues[[i]]], {i, Length[combinedValues]}]
```

```
{0.01, 0. + 0.0746461 i}
{0.0104762, 0. + 0.0747066 i}
{0.010975, 0. + 0.074773 i}
{0.0114976, 0. + 0.0748458 i}
{0.012045, 0. + 0.0749257 i}
{0.0126186, 0. + 0.0750133 i}
{0.0132194, 0. + 0.0751093 i}
{0.0138489, 0. + 0.0752145 i}
{0.0145083, 0. + 0.0753299 i}
{0.0151991, 0. + 0.0754564 i}
{0.0159228, 0. + 0.075595 i}
{0.016681, 0. + 0.0757469 i}
{0.0174753, 0. + 0.0759133 i}
{0.0183074, 0. + 0.0760956 i}
{0.0191791, 0. + 0.0762953 i}
{0.0200923, 0. + 0.0765139 i}
{0.021049, 0. + 0.0767533 i}
{0.0220513, 0. + 0.0770153 i}
{0.0231013, 0. + 0.077302 i}
{0.0242013, 0. + 0.0776156 i}
{0.0253536, 0. + 0.0779585 i}
```

{0.0265609, 0. + 0.0783334 i}

{0.0278256, 0. + 0.078743 i}

{0.0291505, 0. + 0.0791905 i}

{0.0305386, 0. + 0.079679 i}

{0.0319927, 0. + 0.0802121 i}

{0.033516, 0. + 0.0807936 i}

{0.0351119, 0. + 0.0814274 i}

{0.0367838, 0. + 0.0821178 i}

{0.0385353, 0. + 0.0828694 i}

{0.0403702, 0. + 0.083687 i}

{0.0422924, 0. + 0.0845758 i}

{0.0443062, 0. + 0.0855412 i}

{0.0464159, 0. + 0.0865891 i}

{0.048626, 0. + 0.0877255 i}

{0.0509414, 0. + 0.0889569 i}

{0.053367, 0. + 0.0902903 i}

{0.0559081, 0. + 0.0917332 i}

{0.0585702, 0. + 0.0932937 i}

{0.0613591, 0. + 0.0949807 i}

{0.0642807, 0. + 0.0968039 i}

{0.0673415, 0. + 0.0987747 i}

{0.070548, 0. + 0.100906 i}

{0.0739072, 0. + 0.103212 i}

{0.0774264, 0. + 0.10571 i}

{0.0811131, 0. + 0.108423 i}

{0.0849753, 0. + 0.111374 i}

{0.0890215, 0. + 0.114766 i}

{0.0932603, 0. + 0.119005 i}

{0.097701, 0. + 0.124143 i}

{0.102353, 0. + 0.130394 i}

{0.107227, 0. + 0.138118 i}

{0.112332, 0. + 0.147896 i}

{0.117681, 0. + 0.16071 i}

{0.123285, 0. + 0.178378 i}

{0.129155, 0. + 0.204801 i}

{0.135305, 0. + 0.251007 i}

$$\{0.141747, 0. + 0.383093\,i\}$$

$$\{0.148497, 0. + 0.900455\,i\}$$

$$\{0.155568, 0. + 0.897904\,i\}$$

$$\{0.162975, 0. + 0.895049\,i\}$$

$$\{0.170735, 0. + 0.891902\,i\}$$

$$\{0.178865, 0. + 0.888468\,i\}$$

$$\{0.187382, 0. + 0.884536\,i\}$$

$$\{0.196304, 0. + 0.880253\,i\}$$

$$\{0.205651, 0. + 0.875691\,i\}$$

$$\{0.215443, 0. + 0.870265\,i\}$$

$$\{0.225702, 0. + 0.864568\,i\}$$

$$\{0.236449, 0. + 0.858039\,i\}$$

$$\{0.247708, 0. + 0.850693\,i\}$$

$$\{0.259502, 0. + 0.842815\,i\}$$

$$\{0.271859, 0. + 0.833716\,i\}$$

$$\{0.284804, 0. + 0.823531\,i\}$$

$$\{0.298365, 0. + 0.811813\,i\}$$

$$\{0.312572, 0. + 0.79855\,i\}$$

$$\{0.327455, 0. + 0.783483\,i\}$$

$$\{0.343047, 0. + 0.766129\,i\}$$

$$\{0.359381, 0. + 0.746344\,i\}$$

$$\{0.376494, 0. + 0.722634\,i\}$$

$$\{0.394421, 0. + 0.693711\,i\}$$

$$\{0.413201, 0. + 0.658456\,i\}$$

$$\{0.432876, 0. + 0.61131\,i\}$$

$$\{0.453488, 0. + 0.541777\,i\}$$

$$\{0.475081, 0. - 0.341935\,i\}$$

$$\{0.497702, 0. - 0.243314\,i\}$$

$$\{0.521401, 0. - 0.000090422\,i\}$$

$$\{0.546228, 0. - 0.0000880294\,i\}$$

$$\left\{0.572237, 0. - 1.60255\times10^{-15}\,i\right\}$$

$$\left\{0.599484, 0. + 1.59918\times10^{-6}\,i\right\}$$

$$\left\{0.628029, 0. + 1.69809\times10^{-15}\,i\right\}$$

$$\left\{0.657933, 0. + 1.69809\times10^{-15}\,i\right\}$$

$$\{0.689261, 0. - 0.159793\,i\}$$

$$\{0.722081, 0. - 0.166066\,i\}$$

$$\{0.756463,\ 0.-0.172638\,i\}$$

$$\{0.792483,\ 0.-0.179249\,i\}$$

$$\{0.830218,\ 0.-0.185811\,i\}$$

$$\{0.869749,\ 0.-0.192254\,i\}$$

$$\{0.911163,\ 0.-0.198668\,i\}$$

$$\{0.954548,\ 0.-0.204799\,i\}$$

$$\left\{1.,\ -1.74967\times10^{-15}-0.210689\,i\right\}$$