```
M = 5; β = 0.1; g = 0.; δ = 0.; α = 0.5; l = 0.9; r_z = 1;
ξ[ω_] := ω * M;
β_a[ω_] := α + β - 1 - ξ[ω]^2;
```

$$\beta_z[\omega\_] = \frac{\beta + 2*\alpha + 2*\alpha^2 * \left(\xi[\omega]^4 + 2*g*\xi[\omega]^3 + 2*g^2*\xi[\omega]^2 - 5*\xi[\omega]^2 - 6*g*\xi[\omega] + 3\right)}{\left(\left(\xi[\omega]^2 - 1\right) * \left(\xi[\omega]^4 - 6*\xi[\omega]^2 - 4*g*\xi[\omega] + 3\right)\right)}$$

```
nβ_z[ω_] := Numerator[β_z[ω]];
dβ_z[ω_] := Denominator[β_z[ω]];
q[ω_] := ((1 - l) * nβ_z[ω] + l * β_a[ω] * dβ_z[ω]) / nβ_z[ω]
```

$$\text{omegaRoots} = \text{NSolve}\left[-q[\omega] == \left(\frac{r_z}{2}\right)^2, \omega\right]$$

```
{{ω → -0.4672683793806899`}, {ω → 0.4672683793806904`}, {ω → -0.20956642019603966`},
 {ω → 0.20956642019603974`}, {ω → -0.15600624830012666`}, {ω → 0.15600624830012672`},
 {ω → 0.` - 0.07405849877231427` i}, {ω → 0.` + 0.07405849877231427` i}}
```