```
In[1]:= M = 5;
        beta = 0.1;
        g = 0.;
        alpha = 0.5;
        l = 0.9;
        rz = 0.5;
        L = 3.0;
        epsilon = Exp[-2 * L];
        bVal = 1 - 2 * epsilon;
        r = 2 * L * rz;

        xi[z_, Delta_, OmegaA_, sign_] = M * (sign * Delta * z + OmegaA)

        betaA = alpha + beta - 1 - xi^2

        betaZ = Together[
          beta + 2 * alpha + 2 * alpha^2 * (xi^4 + 2 * g * xi^3 + 2 * g^2 * xi^2 - 5 * xi^2 - 6 * g * xi + 3) /
            ((xi^2 - 1) * (xi^4 - 6 * xi^2 - 4 * g * xi + 3))]

        nBetaZ = Numerator[betaZ]

        dBetaZ = Denominator[betaZ]

        q[z_, Delta_, OmegaA_] = ((1 - l) * nBetaZ + l * betaA * dBetaZ) / nBetaZ
        F[z_, Delta_, OmegaA_] = (Sqrt[-q[z, Delta, OmegaA] /. xi → xi[z, Delta, OmegaA, 1]] +
            Sqrt[-q[z, Delta, OmegaA] /. xi → xi[z, Delta, OmegaA, -1]]) / (-z^2 + 1)
```

Out[11]=
$5 \, (\text{OmegaA} + \text{Delta sign } z)$

Out[12]=
$-0.4 - \text{xi}^2$

Out[13]=
$$\frac{1.1 \left(-1.63636 + 6.72727 \, \text{xi}^2 - 6.54545 \, \text{xi}^4 + 1. \, \text{xi}^6\right)}{\left(-1. + \text{xi}^2\right) \left(3. - 6. \, \text{xi}^2 + \text{xi}^4\right)}$$

Out[14]=
$1.1 \left(-1.63636 + 6.72727 \, \text{xi}^2 - 6.54545 \, \text{xi}^4 + 1. \, \text{xi}^6\right)$

Out[15]=
$\left(-1. + \text{xi}^2\right) \left(3. - 6. \, \text{xi}^2 + \text{xi}^4\right)$

Out[16]=
$$\left(0.909091 \left(0.9 \left(-0.4 - \text{xi}^2\right) \left(-1. + \text{xi}^2\right) \left(3. - 6. \, \text{xi}^2 + \text{xi}^4\right) + \right.\right.$$
$$\left.\left. 0.11 \left(-1.63636 + 6.72727 \, \text{xi}^2 - 6.54545 \, \text{xi}^4 + 1. \, \text{xi}^6\right)\right)\right) \Big/$$
$$\left(-1.63636 + 6.72727 \, \text{xi}^2 - 6.54545 \, \text{xi}^4 + 1. \, \text{xi}^6\right)$$

Out[17]=

$$\frac{1}{1-z^2}$$
$$\left(0.953463 \sqrt{\left(-\left(\left(0.9\left(-0.4-25\,(OmegaA-Delta\,z)^2\right)\right)\left(-1.+25\,(OmegaA-Delta\,z)^2\right)\left(3.-150.\right.\right.}\right.$$
$$(OmegaA-Delta\,z)^2+625\,(OmegaA-Delta\,z)^4\Big)+0.11\left(-1.63636+168.182\right.$$
$$(OmegaA-Delta\,z)^2-4090.91\,(OmegaA-Delta\,z)^4+15\,625.\,(OmegaA-Delta\,z)^6\Big)\Big)\Big/$$
$$\left(-1.63636+168.182\,(OmegaA-Delta\,z)^2-4090.91\,(OmegaA-Delta\,z)^4+\right.$$
$$15\,625.\,(OmegaA-Delta\,z)^6\Big)\Big)\Big)+$$
$$0.953463 \sqrt{\left(-\left(\left(0.9\left(-0.4-25\,(OmegaA+Delta\,z)^2\right)\right)\left(-1.+25\,(OmegaA+Delta\,z)^2\right)\right.\right.}$$
$$\left(3.-150.\,(OmegaA+Delta\,z)^2+625\,(OmegaA+Delta\,z)^4\right)+0.11\left(-1.63636+168.182\right.$$
$$(OmegaA+Delta\,z)^2-4090.91\,(OmegaA+Delta\,z)^4+15\,625.\,(OmegaA+Delta\,z)^6\Big)\Big)\Big/$$
$$\left(-1.63636+168.182\,(OmegaA+Delta\,z)^2-4090.91\,(OmegaA+Delta\,z)^4+\right.$$
$$15\,625.\,(OmegaA+Delta\,z)^6\Big)\Big)\Big)\Big)$$

In[18]:=
```
deltaValues = Range[0, 1, 0.01];
initialOmega = 0.` + 0.0740584987723142` I; (*Initial guess for the first Delta*)
omegaValues =
 Table[initialOmega = OmegaA /. FindRoot[NIntegrate[F[z, Delta, OmegaA], {z, 0, bVal}] == r,
   {OmegaA, initialOmega}], {Delta, deltaValues}]
```

⋯ NIntegrate: The integrand $\dfrac{1.90693 \sqrt{-\frac{0.9\,(-0.4-25\,Power[\ll2\gg])\,(-1.+25\,Power[\ll2\gg])\,(3.-150.\,Power[\ll2\gg]+625\,Power[\ll2\gg])+0.11\,(\ll1\gg)}{-1.63636+168.182\,Plus[\ll2\gg]^2-4090.91\,Plus[\ll1\gg]^4+15625.\,Plus[\ll2\gg]^6}}}{1-z^2}$ has evaluated

to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}. ⓘ

⋯ NIntegrate: The integrand

$\dfrac{0.953463\left(-\frac{0.9\,(-0.4-25\,Power[\ll2\gg])\,(-1.+25\,Power[\ll2\gg])\,(-300.\,Plus[\ll2\gg]+2500\,Power[\ll2\gg])+\ll1\gg-\ll1\gg+0.11\,(\ll1\gg)}{-1.63636+168.182\,Plus[\ll2\gg]^2-4090.91\,Plus[\ll1\gg]^4+15625.\,Plus[\ll2\gg]^6}+\frac{(\ll1\gg)\,\ll1\gg}{\ll1\gg^2}\right)}{\sqrt{-\frac{0.9\,(-0.4-25\,Power[\ll2\gg])\,(-1.+25\,Power[\ll2\gg])\,(3.-150.\,Power[\ll2\gg]+625\,Power[\ll2\gg])+0.11\,(\ll1\gg)}{-1.63636+168.182\,Plus[\ll2\gg]^2-4090.91\,Plus[\ll2\gg]^4+15625.\,Plus[\ll2\gg]^6}}\,(1-z^2)}$ has evaluated

to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}. ⓘ

⋯ NIntegrate: The integrand

$\dfrac{0.953463\left(-\frac{0.9\,(-0.4-25\,Power[\ll2\gg])\,(-1.+25\,Power[\ll2\gg])\,(-300.\,Plus[\ll2\gg]+2500\,Power[\ll2\gg])+\ll1\gg-\ll1\gg+0.11\,(\ll1\gg)}{-1.63636+168.182\,Plus[\ll2\gg]^2-4090.91\,Plus[\ll1\gg]^4+15625.\,Plus[\ll2\gg]^6}+\frac{(\ll1\gg)\,\ll1\gg}{\ll1\gg^2}\right)}{\sqrt{-\frac{0.9\,(-0.4-25\,Power[\ll2\gg])\,(-1.+25\,Power[\ll2\gg])\,(3.-150.\,Power[\ll2\gg]+625\,Power[\ll2\gg])+0.11\,(\ll1\gg)}{-1.63636+168.182\,Plus[\ll2\gg]^2-4090.91\,Plus[\ll2\gg]^4+15625.\,Plus[\ll2\gg]^6}}\,(1-z^2)}$ has evaluated

to non-numerical values for all sampling points in the region with boundaries {{0, 0.995042}}. ⓘ

⋯ General: Further output of NIntegrate::inumr will be suppressed during this calculation. ⓘ

⋯ NIntegrate: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. ⓘ

⋯ NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.808457}. NIntegrate obtained 3.06433 + 0. *i* and 0.06123879194993142` for the integral and error estimates. ⓘ

⋯ NIntegrate: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.808457}. NIntegrate obtained 3.07691 + 0. *i* and 7.591718593403812`*^-6 for the integral and error estimates. ⓘ

⋯ **NIntegrate**: NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in z near {z} = {0.808457}.

NIntegrate obtained $3.05954 + 0. i$ and 0.002258254785605767` for the integral and error estimates. ⓘ

⋯ **General**: Further output of NIntegrate::ncvb will be suppressed during this calculation. ⓘ

⋯ **NIntegrate**: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. ⓘ

⋯ **NIntegrate**: Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. ⓘ

⋯ **General**: Further output of NIntegrate::slwcon will be suppressed during this calculation. ⓘ

⋯ **FindRoot**: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ⓘ

⋯ **FindRoot**: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ⓘ

⋯ **FindRoot**: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ⓘ

⋯ **General**: Further output of FindRoot::lstol will be suppressed during this calculation. ⓘ

Out[20]=

$\{0. + 0.0740229\, i, 0. + 0.0746461\, i, 0. + 0.0764914\, i, 0. + 0.0794871\, i, 0. + 0.0835196\, i,$

$0. + 0.0884513\, i, 0. + 0.094152\, i, 0. + 0.100537\, i, 0. + 0.107593\, i, 0. + 0.115688\, i,$

$0. + 0.127111\, i, 0. + 0.143181\, i, 0. + 0.167377\, i, 0. + 0.209649\, i, 0. + 0.324728\, i,$

$0. + 0.0000232302\, i, 0. + 1.09882 \times 10^{-15}\, i, 0. + 0.892252\, i, 0. + 0.88796\, i, 0. + 0.883387\, i,$

$0. + 0.878513\, i, 0. + 0.873364\, i, 0. + 0.86773\, i, 0. + 0.86204\, i, 0. + 0.855801\, i,$

$0. + 0.84916\, i, 0. + 0.842407\, i, 0. + 0.83512\, i, 0. + 0.827306\, i, 0. + 0.818972\, i,$

$0. + 0.810129\, i, 0. + 0.80129\, i, 0. + 0.791352\, i, 0. + 0.780926\, i, 0. + 0.770022\, i,$

$0. + 0.75811\, i, 0. + 0.745238\, i, 0. + 0.731933\, i, 0. + 0.717165\, i, 0. + 0.701489\, i,$

$0. + 0.684097\, i, 0. + 0.664851\, i, 0. + 0.643306\, i, 0. + 0.619164\, i, 0. + 0.591033\, i,$

$0. + 0.556022\, i, 0. + 0.509099\, i, 0. - 0.424335\, i, 0. - 0.300741\, i, 0. - 0.26227\, i,$

$0. - 0.238492\, i, 0. + 1.32592 \times 10^{-15}\, i, 0. + 1.32592 \times 10^{-15}\, i, 0. - 3.51887 \times 10^{-6}\, i,$

$0. - 0.0000113994\, i, 0. - 0.0000113994\, i, 0. - 1.73954 \times 10^{-15}\, i, 0. + 0.0000296398\, i,$

$0. + 3.9661 \times 10^{-6}\, i, 0. + 0.0000336621\, i, 0. + 0.0000104365\, i, 0. - 0.000026947\, i,$

$0. - 0.000026947\, i, 0. + 0.149812\, i, 0. + 0.151203\, i, 0. + 0.152767\, i, 0. + 0.154426\, i,$

$0. + 0.156213\, i, 0. + 0.158047\, i, 0. + 0.159949\, i, 0. + 0.161855\, i, 0. + 0.163781\, i,$

$0. + 0.165705\, i, 0. + 0.167621\, i, 0. + 0.169541\, i, 0. + 0.171397\, i, 0. + 0.173323\, i,$

$0. + 0.175142\, i, 0. + 0.176963\, i, 0. + 0.178788\, i, 0. + 0.180571\, i, 0. + 0.182357\, i,$

$0. + 0.184124\, i, 0. + 0.185826\, i, 0. + 0.18744\, i, 0. + 0.189147\, i, 0. + 0.190767\, i,$

$0. + 0.192299\, i, 0. + 0.193923\, i, 0. + 0.195413\, i, 0. + 0.196952\, i, 0. + 0.198401\, i,$

$0. + 0.199863\, i, 0. + 0.20132\, i, 0. + 0.202779\, i, 0. + 0.204228\, i, 0. + 0.205598\, i,$

$0. + 0.206879\, i, 0. + 0.208253\, i, 0. + 0.209537\, i, 7.25157 \times 10^{-16} + 0.210823\, i\}$

In[21]:= `combinedValues = Transpose[{deltaValues, omegaValues}];`
`ListPlot[Transpose[{deltaValues, Abs[Im[omegaValues]]}],`
` PlotStyle → PointSize[0.01], Joined → False, PlotRange → All,`
` AxesLabel → {"Delta", "Im[OmegaA]"}, PlotLabel → "Im[OmegaA] vs Delta"]`
`Do[Print[combinedValues[[i]]], {i, Length[combinedValues]}]`

Out[22]=
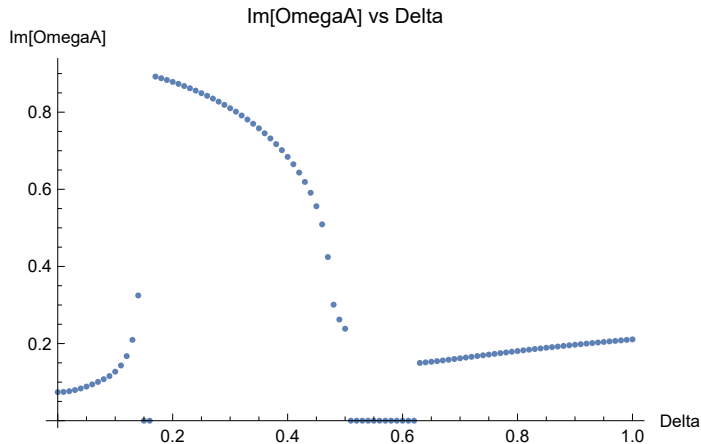
{0., 0. + 0.0740229 $\mathbb{i}$}

{0.01, 0. + 0.0746461 $\mathbb{i}$}

{0.02, 0. + 0.0764914 $\mathbb{i}$}

{0.03, 0. + 0.0794871 $\mathbb{i}$}

{0.04, 0. + 0.0835196 $\mathbb{i}$}

{0.05, 0. + 0.0884513 $\mathbb{i}$}

{0.06, 0. + 0.094152 $\mathbb{i}$}

{0.07, 0. + 0.100537 $\mathbb{i}$}

{0.08, 0. + 0.107593 $\mathbb{i}$}

{0.09, 0. + 0.115688 $\mathbb{i}$}

{0.1, 0. + 0.127111 $\mathbb{i}$}

{0.11, 0. + 0.143181 $\mathbb{i}$}

{0.12, 0. + 0.167377 $\mathbb{i}$}

{0.13, 0. + 0.209649 $\mathbb{i}$}

{0.14, 0. + 0.324728 $\mathbb{i}$}

{0.15, 0. + 0.0000232302 $\mathbb{i}$}

{0.16, 0. + 1.09882×10^{-15} $\mathbb{i}$}

{0.17, 0. + 0.892252 $\mathbb{i}$}

{0.18, 0. + 0.88796 $\mathbb{i}$}

{0.19, 0. + 0.883387 $\mathbb{i}$}

{0.2, 0. + 0.878513 $\mathbb{i}$}

{0.21, 0. + 0.873364 i}

{0.22, 0. + 0.86773 i}

{0.23, 0. + 0.86204 i}

{0.24, 0. + 0.855801 i}

{0.25, 0. + 0.84916 i}

{0.26, 0. + 0.842407 i}

{0.27, 0. + 0.83512 i}

{0.28, 0. + 0.827306 i}

{0.29, 0. + 0.818972 i}

{0.3, 0. + 0.810129 i}

{0.31, 0. + 0.80129 i}

{0.32, 0. + 0.791352 i}

{0.33, 0. + 0.780926 i}

{0.34, 0. + 0.770022 i}

{0.35, 0. + 0.75811 i}

{0.36, 0. + 0.745238 i}

{0.37, 0. + 0.731933 i}

{0.38, 0. + 0.717165 i}

{0.39, 0. + 0.701489 i}

{0.4, 0. + 0.684097 i}

{0.41, 0. + 0.664851 i}

{0.42, 0. + 0.643306 i}

{0.43, 0. + 0.619164 i}

{0.44, 0. + 0.591033 i}

{0.45, 0. + 0.556022 i}

{0.46, 0. + 0.509099 i}

{0.47, 0. − 0.424335 i}

{0.48, 0. − 0.300741 i}

{0.49, 0. − 0.26227 i}

{0.5, 0. − 0.238492 i}

$\{0.51, 0. + 1.32592 \times 10^{-15} \text{ i}\}$

$\{0.52, 0. + 1.32592 \times 10^{-15} \text{ i}\}$

$\{0.53, 0. − 3.51887 \times 10^{-6} \text{ i}\}$

{0.54, 0. − 0.0000113994 i}

{0.55, 0. − 0.0000113994 i}

$\{0.56, 0. − 1.73954 \times 10^{-15} \text{ i}\}$

{0.57, 0. + 0.0000296398 i}

{0.58, 0. + 3.9661×10⁻⁶ i}

{0.59, 0. + 0.0000336621 i}

{0.6, 0. + 0.0000104365 i}

{0.61, 0. − 0.000026947 i}

{0.62, 0. − 0.000026947 i}

{0.63, 0. + 0.149812 i}

{0.64, 0. + 0.151203 i}

{0.65, 0. + 0.152767 i}

{0.66, 0. + 0.154426 i}

{0.67, 0. + 0.156213 i}

{0.68, 0. + 0.158047 i}

{0.69, 0. + 0.159949 i}

{0.7, 0. + 0.161855 i}

{0.71, 0. + 0.163781 i}

{0.72, 0. + 0.165705 i}

{0.73, 0. + 0.167621 i}

{0.74, 0. + 0.169541 i}

{0.75, 0. + 0.171397 i}

{0.76, 0. + 0.173323 i}

{0.77, 0. + 0.175142 i}

{0.78, 0. + 0.176963 i}

{0.79, 0. + 0.178788 i}

{0.8, 0. + 0.180571 i}

{0.81, 0. + 0.182357 i}

{0.82, 0. + 0.184124 i}

{0.83, 0. + 0.185826 i}

{0.84, 0. + 0.18744 i}

{0.85, 0. + 0.189147 i}

{0.86, 0. + 0.190767 i}

{0.87, 0. + 0.192299 i}

{0.88, 0. + 0.193923 i}

{0.89, 0. + 0.195413 i}

{0.9, 0. + 0.196952 i}

{0.91, 0. + 0.198401 i}

{0.92, 0. + 0.199863 i}

{0.93, 0. + 0.20132 i}

{0.94, 0. + 0.202779 i}

{0.95, 0. + 0.204228 i}

{0.96, 0. + 0.205598 i}

{0.97, 0. + 0.206879 i}

{0.98, 0. + 0.208253 i}

{0.99, 0. + 0.209537 i}

$\{1., 7.25157 \times 10^{-16} + 0.210823 \, i\}$