```
% Cross Entropy Gaussian Process Model Double Well
% 1. Sampling of biased trajectories and evaluation of path functional
% 2. Build Matrix for solving regularized linear equation


% Sampling
% a bias can be included here
V=@(x) 1/2.*x.^4-x.^2;
gradV = @(x) 2*x.*(x.^2-1);
dt = 0.01;
sdt = sqrt(dt);
beta = 3;
sigma = sqrt(2/beta);

nvs = 1;
ntrjs = 100; %number of trajectories
opt_steps=5;
nsteps = 150;
n_pred=100;
sk=1;
l=1;
pathfunc =  ones(ntrjs,1);
c_old = zeros(opt_steps,n_pred);
c_pred=0;

% the considers path functional is the moment generating function of
 the
% stopping time

bias=0;

for opt = 1:opt_steps


        %Eta= randn(ntrjs,nsteps-1);
        time=zeros(1,ntrjs);
        X = zeros(ntrjs,nsteps);
        X(:,1)=-1;
        X_nonbias = zeros(ntrjs,nsteps);
        X_nonbias(:,1) = -1;
    for i = 1:ntrjs

        Is=0;
        Id=0;

        x = -1;

        for j = 2:nsteps
            eta=randn(1);


            if opt==1
```

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                x = x + (bias - gradV(x)) * dt + eta * sigma*sdt;
            else
                bias=0;
                for t=1:ntrjs
                    K_pred = sk/sqrt(2*pi)*l.^2.*exp(-0.5*(x-
X(t,:)).^2/l.^2);
                    bias =  bias - pathfunc(t)/
(2*beta*ntrjs)*( K_pred*c*dt + K_pred*(X_nonbias(t,:)'));
                end

                x = x + (bias-gradV(x) ) * dt + eta * sigma*sdt;
            end
            X(i,j) = x;
            X_nonbias(i,j) = x-bias*dt;
            %Eta(i,j)= eta;

            Is = Is - bias * eta/ sigma * sdt;
            Id = Id - bias.^2 / sigma^2 *dt;


             if  x > 0.9 && x < 1.1
                 time(i) = j;
                 pathfunc(i) = exp(-beta*j*dt)*exp(Is
+0.5*Id); %weighted path functional
                 X(i,j:end)=x;
                 X_nonbias(i,j:end) = x-bias*dt;
                 break;
             else
                 pathfunc(i) = exp(0.1)*exp(Is+0.5*Id);
             end
        end

    end

    K=zeros(nsteps,nsteps);
    A=zeros(nsteps,nsteps);
    b=zeros(nsteps,1);

    for t=1:ntrjs
        for i=1:length(X(t,:))
            for j=1:length(X(t,:))
                K(i,j) = sk/sqrt(2*pi)*l.^2.*exp(-0.5*(X(t,i)-
X(t,j)).^2/l.^2);
            end
        end
        A = A + pathfunc(t)*K*dt + 2*beta*eye(nsteps,nsteps);
        b = b + pathfunc(t)*(K*(X_nonbias(t,:)'));
    end
    c = A\-b;

    c_old(opt,:) = c_pred;
    % plot zur Kontolle
    x_pred = linspace(-2,2,n_pred);
    K_pred = zeros(length(x_pred),nsteps);
    c_pred=0;

    for t=1:ntrjs
        for i=1:length(x_pred)
            for j=1:length(X(t,:))
                K_pred(i,j) = sk/sqrt(2*pi)*l.^2.*exp(-0.5*(x_pred(i)-
X(t,j)).^2/l.^2);
            end
        end
        c_pred =  c_pred - pathfunc(t)/(2*beta*ntrjs)*( K_pred*c*dt +
 K_pred*(X_nonbias(t,:)'));
    end


    figure(opt)
    plot(x_pred, c_pred)

    fprintf('Trajectories in T %d \n', sum(time>0) )
    fprintf('|c_new-c_old|_2 = %f \n', norm(c_pred'-c_old(opt,:)))

end

figure(6)
plot(x_pred,  -gradV(x_pred)); hold on
plot(x_pred,   c_pred -gradV(x_pred)'  )
legend('-gradV','-gradV+cPred')
title('Gradients')
hold off

dx=x_pred(2)-x_pred(1);
per_pot = zeros(1,n_pred+1);
control = zeros(1,n_pred+1);

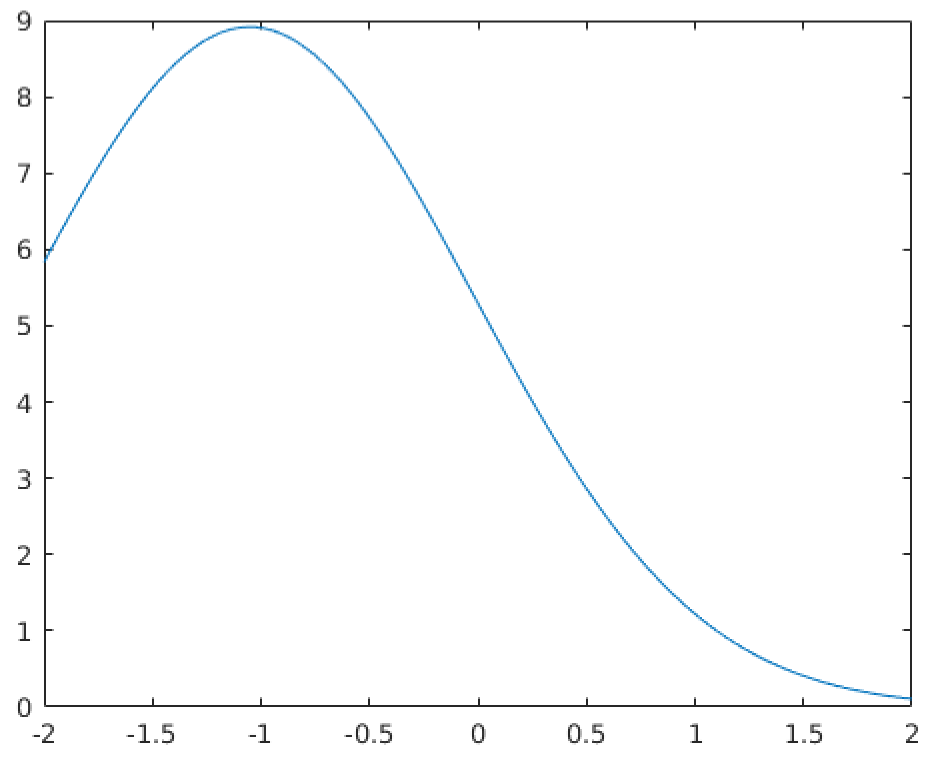
for i=2:n_pred+1
    per_pot(i) = per_pot(i-1) + (-c_pred(i-1) +
 gradV(x_pred(i-1)) )*dx;
    control(i) = control(i-1) -c_pred(i-1)*dx;
end

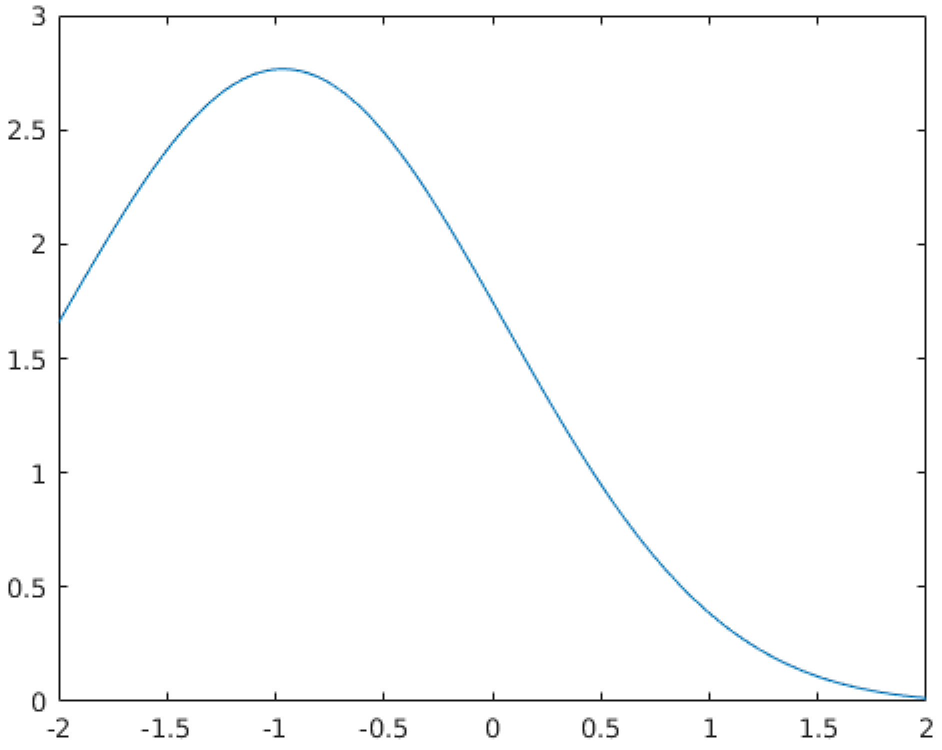
figure(7)
plot(x_pred,V(x_pred)); hold on
plot(x_pred,per_pot(2:end)+9); hold off
title('Perturbed Potential')

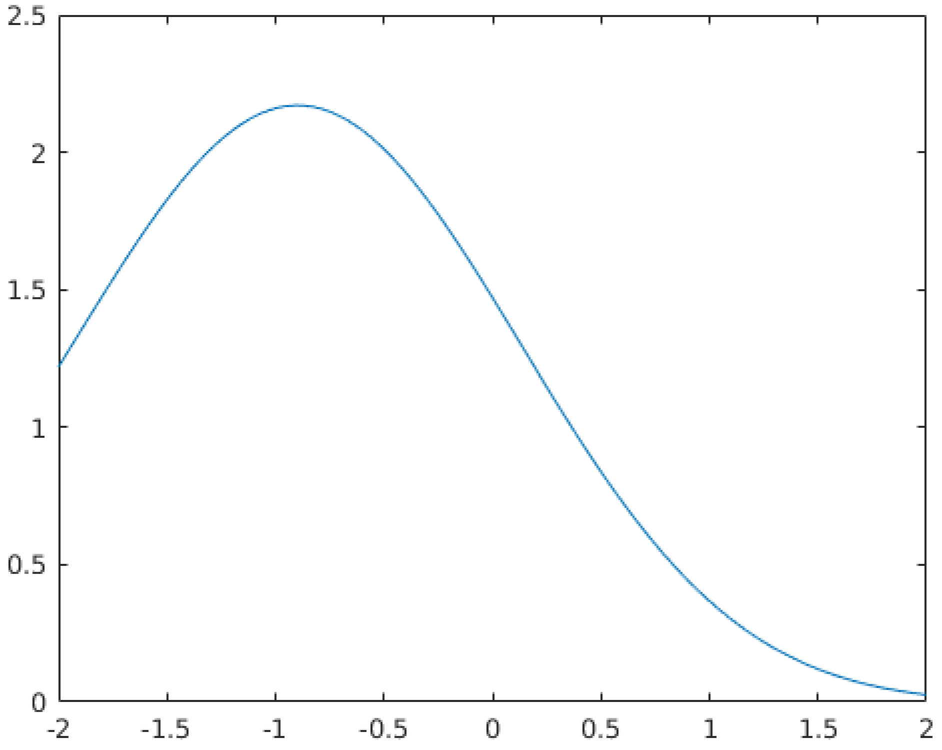
figure(8)
plot(x_pred,V(x_pred)); hold on
plot(x_pred,V(x_pred)+control(2:end)+6); hold off
```

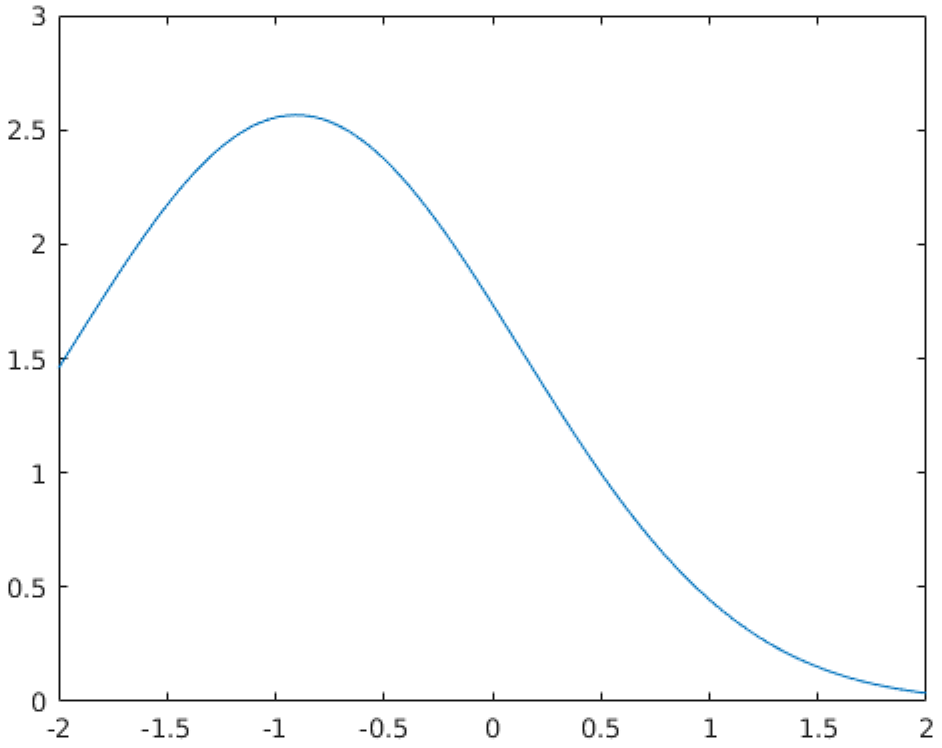
```
Trajectories in T 2
|c_new-c_old|_2 = 57.113905
Trajectories in T 41
|c_new-c_old|_2 = 39.380363
Trajectories in T 48
|c_new-c_old|_2 = 3.758936
```

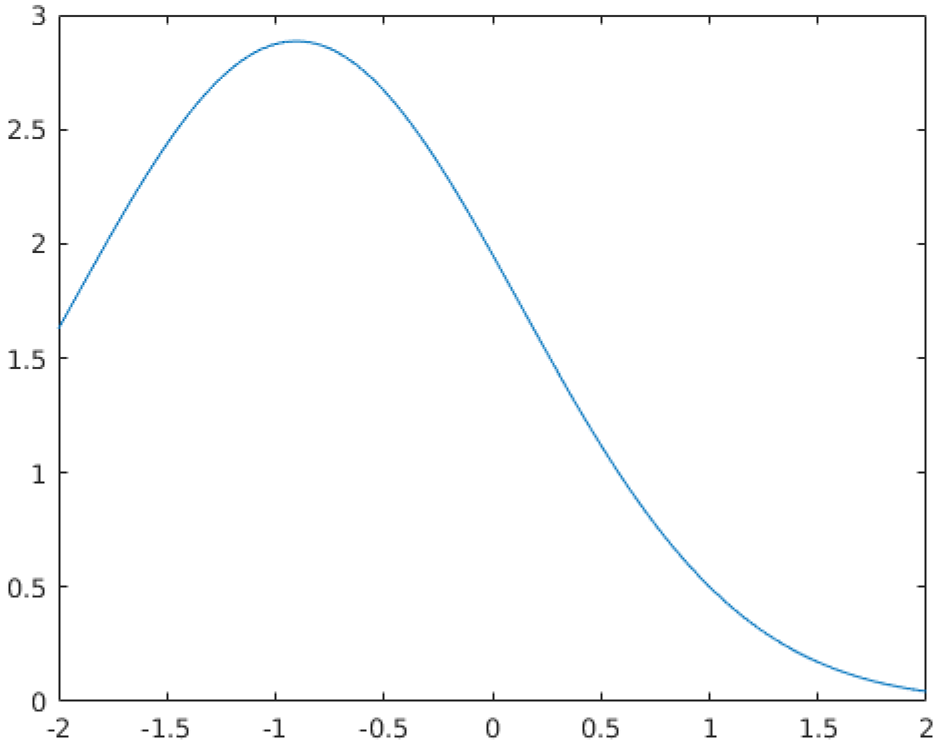
```
Trajectories in T 56
|c_new-c_old|_2 = 2.589878
Trajectories in T 64
|c_new-c_old|_2 = 2.064010
```

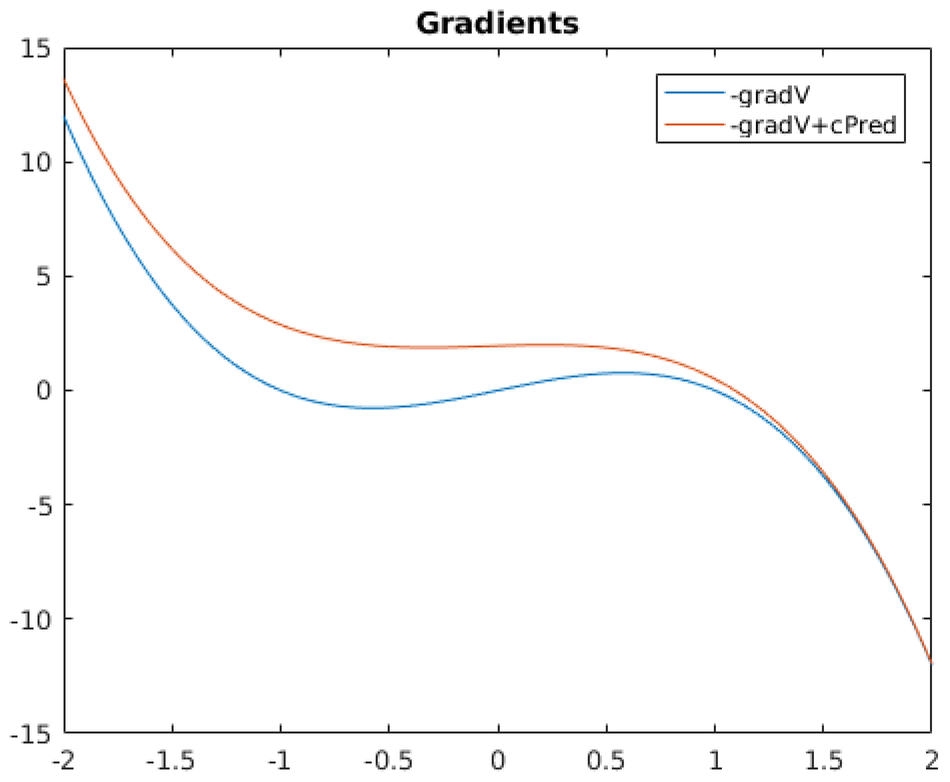


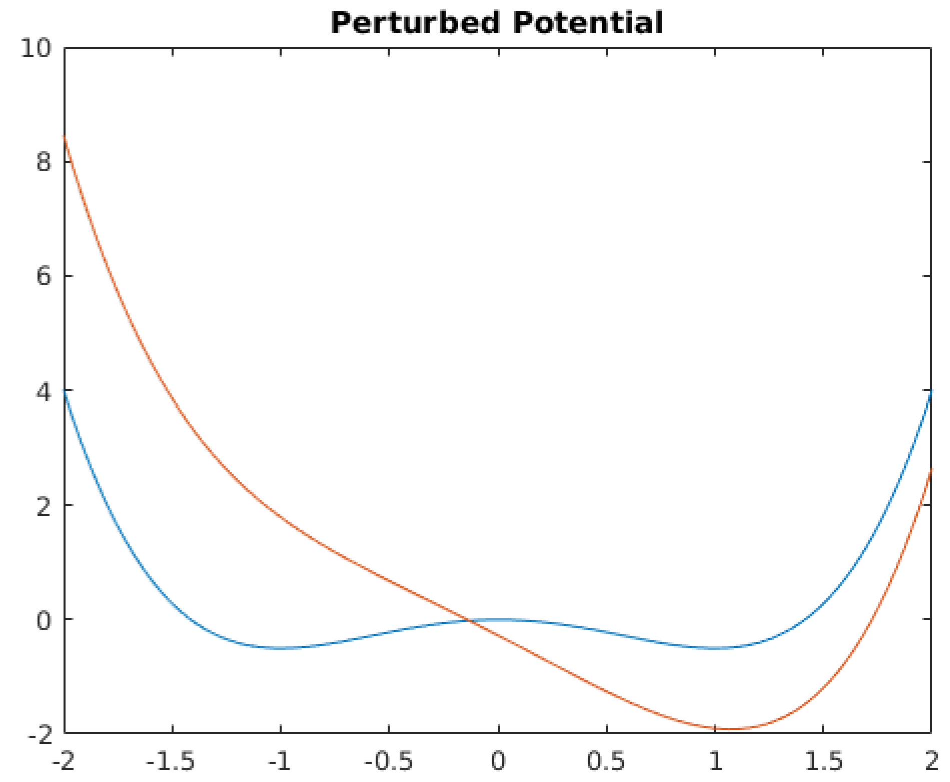

3
2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

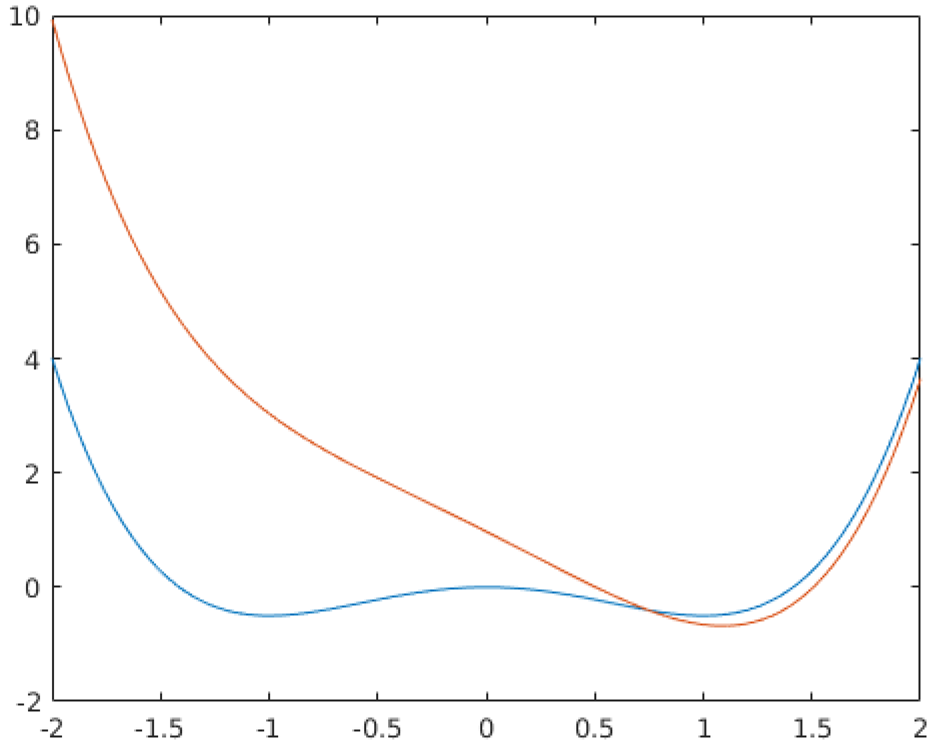

2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


3
2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


3
2.5
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Gradients
-gradV
-gradV+cPred
15
10
5
0
-5
-10
-15
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


Perturbed Potential
10
8
6
4
2
0
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



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