

Databases Autumn 2025

Hand-In Exercise 3

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Total Points	
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Task 1

- a) $AHV \rightarrow (FullName, Birthday, Zip, country)$

$FullName \rightarrow (FirstName, LastName)$

$Birthday \rightarrow YearOfBirth$

$(County, Zip) \rightarrow City$

The AHV uniquely identifies each resident. From these attributes, all others can be derived transitively: $FullName$ determines $(FirstName, LastName)$, $Birthday$ determines $YearOfBirth$, and $(Country, Zip)$ determines $City$. We thought about including $(Country, City) \rightarrow Zip$ but there are cities with multiple zips (Zürich for example) and that's why we did not include this dependency.

- b) With this functional dependencies we compute the attribute closure for AHV. F^+ is $(AHV, FullName, Birthday, Zip, Country, FirstName, LastName, YearOfBirth, City)$ = $sch(Resident)$. Since no subset of AHV determines all attributes, AHV is minimal and therefore the only candidate key.

- c) The relation Resident is in 2NF, since the only candidate key is AHV and therefore no partial dependencies on a subset of a composite key can exist. It is not in Third Normal Form, because there are several transitive dependencies. For example: $AHV \rightarrow (Zip, country)$ and $(Country, Zip) \rightarrow City$, hence $City$ is transitively dependent on AHV. Version in 3NF:

Resident(AHV, FullName, Birthday, Zip, Country)

Name(FullName, FirstName, LastName)

Birthday(Birthday, YearOfBirth)

Location(Zip, Country, City)

Task 2

- a) $\{\{A, B, C\}, \{A, C, D\}\}$

- b) i Because for FD2 B on the left is not a superkey, R is not in BCNF.
ii Because for FD3 E on the right which is not part of a primary key, R is not in 3NF.
iii Because in FD3, E (non-prime attribute) is dependent on C,D which is a subset of a candidate key, R is not in 2NF

R is only in 1NF.

Task 3

- a) $\{\{A, C\}, \{A, B\}\}$

- b) i S is in 2NF because in FD1, neither A nor B alone determine D and C is in the candidate keys. Also in FD2, although C is a proper subset of a candidate key, B is a primary key and therefore all rules for 2NF hold true.

- ii S is also in 2NF because in FD1, A, B is a candidate key and therefore a superkey and therefore 2NF is satisfied. Because C alone is not a superkey, but B is a prime key, this FD also satisfies 3NF.
- iii However, because C alone is not a superkey, it does not satisfy FD2 and therefore is not a BCNF.

Task 4

a) Candidate keys:

$A^+ = A, B, C, D, U$ not all attributes, so not candidate key alone.

$(A, E)^+ = A, B, C, D, E, U, V, W$ all attributes, so candidate key A, E .

$(A, W)^+ = A, B, C, D, E, U, V, W$ all attributes, so candidate key A, W .

because W and E also not candidate key alone, $\{A, E\}$ and $\{A, W\}$ both minimal candidate keys. Because A only on left side, A needs to be part of all candidate keys.

b) All possible functional dependencies:

- $A \rightarrow B$
- $A \rightarrow C$
- $A \rightarrow D$
- $A \rightarrow U$
- $A \rightarrow \{B, C, D, U\}$ (Union)
- $A, E \rightarrow V, W$ (Pseudotransitivity from $A \rightarrow D$ and $D, E \rightarrow V, W$)
- $A, E \rightarrow V$
- $A, E \rightarrow W$
- $(D, W \rightarrow V, W$ (Pseudotransitivity from $W \rightarrow E$ and $D, E \rightarrow V, W$))
- $A, W \rightarrow W, V$ (Pseudotransitivity from $A \rightarrow D$ and $D, W \rightarrow V, W$)
- $A, W \rightarrow W$
- $A, W \rightarrow V$
- $A, W \rightarrow E$ (Transitivity from $A, W \rightarrow W$ and $W \rightarrow E$)

Task 5

a) **Task 5(a) – Candidate Keys of Relation U Given:** Functional dependencies:

$$\{ CU \rightarrow D, D \rightarrow E, V, W, U, A, U \rightarrow B, V \rightarrow A \}.$$

Identify attributes that must be in every key No functional dependency determines C , therefore C must be included in every candidate key.

Determine attribute closures

- For $\{C, U\}$: From $CU \rightarrow D$ we obtain D . Then, $D \rightarrow E, V, W, U, A$ and $U \rightarrow B$. Hence,

$$(CU)^+ = \{A, B, C, D, E, U, V, W\}.$$

$\Rightarrow \{C, U\}$ is a superkey. Since neither C^+ nor U^+ contains all attributes, $\{C, U\}$ is **minimal** and therefore a **candidate key**.

- For $\{C, D\}$: From $D \rightarrow U, A, E, V, W$ and $U \rightarrow B$ we obtain

$$(CD)^+ = \{A, B, C, D, E, U, V, W\}.$$

$\Rightarrow \{C, D\}$ is a superkey. Since neither C nor D alone determines all attributes, it is also **minimal** and thus a **candidate key**.

Check other combinations Other sets such as $\{C, V\}$ or $\{C, A\}$ do not determine U or D , and therefore cannot be superkeys. Supersets of $\{C, U\}$ or $\{C, D\}$ are not minimal.

Result:

The candidate keys of U are $\{C, U\}$ and $\{C, D\}$.

b) **Given:** Relation $U(A, B, C, D, E, U, V, W)$ with functional dependencies

$$\{CU \rightarrow D, D \rightarrow E, V, W, U, A, U \rightarrow B, V \rightarrow A\}.$$

Check 1NF All attribute domains are atomic.

$$\Rightarrow U \text{ is in 1NF.}$$

Check 2NF A relation is in 2NF if every non-prime attribute is fully functionally dependent on each candidate key. Candidate keys are $\{C, U\}$ and $\{C, D\}$.

- $U \rightarrow B$: U is only part of key $\{C, U\} \Rightarrow$ partial dependency.
- $D \rightarrow E, V, W$: D is only part of key $\{C, D\} \Rightarrow$ partial dependency.

\Rightarrow *Violates 2NF.*

Check 3NF A relation is in 3NF if, for every FD $X \rightarrow A$, either X is a superkey or A is a key attribute. Since $D \rightarrow E, V, W$ and $U \rightarrow B$ hold, and neither D nor U are superkeys, these dependencies violate 3NF.

$$\Rightarrow U \text{ is not in 3NF.}$$

Check BCNF In BCNF, every determinant must be a superkey. Dependencies $D \rightarrow E, V, W$, $U \rightarrow B$, and $V \rightarrow A$ violate this rule.

$$\Rightarrow U \text{ is not in BCNF.}$$

Result: Since U violates 2NF (and therefore 3NF and BCNF), it is only in:

First Normal Form (1NF).

c) **Minimal Cover (Canonical Form)**

Given functional dependencies (already minimal on both sides):

$$\{ CU \rightarrow D, D \rightarrow E, V, W, U, A, U \rightarrow B, V \rightarrow A \}.$$

No attribute on the left-hand side can be removed (neither C nor U in $CU \rightarrow D$), and none of the dependencies is implied by the others. Hence, this is already the **minimal cover**.

3NF Synthesis

For each determinant, create one relation that includes all attributes from its FD group. Since one of them (R_{CU}) contains the candidate key $\{C, U\}$, the resulting decomposition will be lossless and dependency-preserving.

$$\begin{aligned} R_{CU}(C, U, D) & \text{ from } CU \rightarrow D \\ R_D(D, U, A, E, V, W) & \text{ from } D \rightarrow U, A, E, V, W \\ R_U(U, B) & \text{ from } U \rightarrow B \\ R_V(V, A) & \text{ from } V \rightarrow A \end{aligned}$$

Since R_{CU} includes the key $\{C, U\}$, the natural join of all projections is **lossless**, and all dependencies are **preserved** (property of the 3NF-synthesis algorithm).

Normal Forms of the Resulting Relations

- $R_{CU}(C, U, D)$ – Contains FDs $CU \rightarrow D$ and $D \rightarrow U$. – Satisfies 3NF (each non-trivial FD has either a key or a key attribute on the RHS). – Not in BCNF because $D \rightarrow U$ holds while D is not a key.
- $R_D(D, U, A, E, V, W)$ – D is a key for all its dependencies \rightarrow BCNF. – However, $V \rightarrow A$ also holds globally and both attributes occur here, so to maintain BCNF we separate $R_V(V, A)$.
- $R_U(U, B)$ – U is key \rightarrow BCNF.
- $R_V(V, A)$ – V is key \rightarrow BCNF.

Variant B (cleaner BCNF form): Split R_D into individual relations for each RHS attribute:

$$R_{DU}(D, U), R_{DA}(D, A), R_{DE}(D, E), R_{DV}(D, V), R_{DW}(D, W).$$

Each of these is in BCNF because D is the key in its relation. $R_{CU}(C, U, D)$ remains in 3NF (not BCNF). $R_U(U, B)$ and $R_V(V, A)$ are BCNF.

Final Normal Forms

$$\begin{aligned} R_{CU}(C, U, D) & \rightarrow 3NF \text{ (not BCNF)} \\ R_{DU}(D, U), R_{DA}(D, A), R_{DE}(D, E), R_{DV}(D, V), R_{DW}(D, W) & \rightarrow BCNF \\ R_U(U, B), R_V(V, A) & \rightarrow BCNF \end{aligned}$$

Result: The decomposition is **lossless** (a key relation is retained) and **dependency-preserving**. All relations are in BCNF except R_{CU} , which remains in 3NF to ensure dependency preservation.

Highest possible normal form with dependency preservation: 3NF.

- d) **Goal:** Find a lossless BCNF decomposition of U and compare it to the 3NF decomposition from (c).

Observation. In the 3NF design of (c), the only non-BCNF relation is $R_{CU}(C, U, D)$ due to the dependency $D \rightarrow U$ with D not being a key of R_{CU} .

BCNF Step. Decompose $R_{CU}(C, U, D)$ using $D \rightarrow U$:

$$R_{CU}(C, U, D) \Rightarrow R_{CD}(C, D) \quad \text{and} \quad R_{DU}(D, U).$$

This split is **lossless** because the common attribute D is a key in R_{DU} .

Final BCNF schema. Starting from the 3NF set in (c) and replacing R_{CU} by its BCNF split, we obtain:

$$R_{CD}(C, D), R_{DU}(D, U), R_{DA}(D, A), R_{DE}(D, E), R_{DV}(D, V), R_{DW}(D, W), \\ R_U(U, B), R_V(V, A).$$

All these relations are in **BCNF**:

- Each R_{D*} has determinant D which is a key in that relation.
- $R_U(U, B)$ and $R_V(V, A)$ have keys U and V , respectively.
- $R_{CD}(C, D)$ contains no nontrivial projected FD with a non-key LHS, hence BCNF.

Losslessness. Each BCNF split is lossless (decomposition by a dependency $X \rightarrow Y$ is lossless when the common part contains a key of one component). Since we only refined R_{CU} by such a split, the overall join remains **lossless**.

Dependency preservation and comparison to (c).

- In (c), the 3NF design preserves all original dependencies, notably $CU \rightarrow D$ in R_{CU} .
- In the BCNF design above, $CU \rightarrow D$ is *not* preserved in any single relation (it can only be checked via a join), while the other dependencies ($D \rightarrow U, A, E, V, W, U \rightarrow B, V \rightarrow A$) remain preserved in their respective relations.

Answer. The BCNF decomposition differs from (c) by replacing $R_{CU}(C, U, D)$ with $R_{CD}(C, D)$ and $R_{DU}(D, U)$. It is **lossless** but not fully **dependency-preserving** (the FD $CU \rightarrow D$ is lost as a directly enforceable dependency). All resulting relations are in **BCNF**.