Computer Assignment #2 Report

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How my function works:

1) **convex/concave:**
Because we got 1000 discrete points, we can calculate the vectors from a point to its next point to be as the slope of the points, then take the outer products of each vector with its next vector to stand for the concavity. If the result is positive then these original three points are concave upward, and if it's negative then these original three points are concave downward.
And then check all the results of the outer products. If all of them are 0, then it's a linear function. If all of them greater than or equal to 0, then it's a convex function. If less than or equal to 0, then it's a concave function.
2) **superconvex/superconcave:**
We can just take the log of y, which is $f(x)$, and then do the same process as in convex/concave part. Then we can verify whether if $log(f(x))$ is convex or concave, Thus is to say $f(x)$ is superconvex or superconcave or neither.
However, the domain of $log(\cdot)$ need to be positive, thus we can just check the minimum value of y if it's non-negative. If there's a value less than zero, then it's neither superconcave or superconvex.
3) **quasiconvex/quasiconcave:**
According to Fact 3.3, if $f(x)$ is nondecreasing or nonincreasing, then it's both quasiconvex and quasiconcave. If there's a point $c \in$ dom f and it's a minimum, for $x \leq c$, f is nonincreasing, and for $x \geq c$, f is nonincreasing, then it's quasiconvex. The oppisite case applies to quasiconcave functions.
Then we can just use the same method as in convex case to find approximated slope of $f(x)$, then check if before some points, it's slope are all non-positive, and after that points the slope are all non-negative, then it's a quasiconvex function. Same as above, the oppisite case applies to quasiconcave functions.
I actually times the slopes with their next slope, then if there exists negative value means that it's a local minimum or maximum. If the number of negative value ¿ 1, then it's neither a quasiconcave or quasiconvex function, of course slope = 0 points won't participate in this process because they won't affect the quasiconcavity.

The result of testing data:

From in_data.mat import x and y, then use fcn_checker to determine those 6 properties, the results are shown below:

problems	1	2	3	4	5	6	7	8	9	10	11
convex	1	1	0	0	1	0	0	0	1	0	0
concave	1	0	0	1	0	1	0	0	0	0	1
superconvex	0	0	0	0	1	0	0	0	0	0	0
superconcave	0	0	0	0	0	1	0	0	0	0	1
quasiconvex	1	1	1	1	1	0	0	1	1	1	1
quasiconcave	1	0	1	1	1	1	0	1	0	0	1

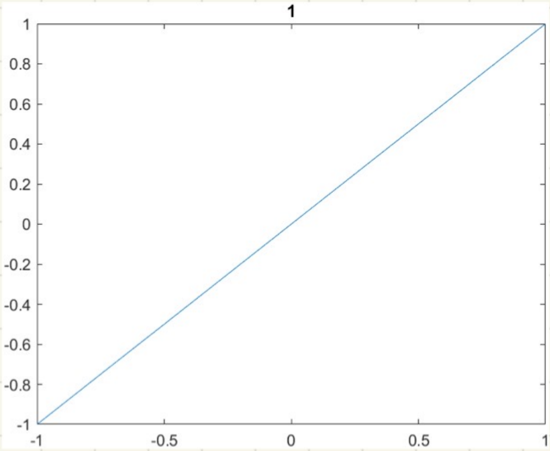
These results are the same as baseline.mat, and consistent with expectation.

Handwriting proof of the original continious $f(x)$

In the following pages, I find the oringinal functions of the first 5 questions and use the theorems to prove if they satisfy the 6 properties. Because the interval of each data points of x are all the same, we can define that dom f are line segments, which are convex sets.

All of the domain f is defined to be a segment $\Rightarrow$ dom$f$ is convex.

Problem 1 :



(1) let $f(x) = x$, then $f(\theta a + (1-\theta)b) = \theta a + (1-\theta) b$

$\Rightarrow$ it's both convex and concave

(2) $f(x)$ might be negative

$\Rightarrow$ it's neither superconvex or superconcave

(3) from (1), we know it's both convex and concave,

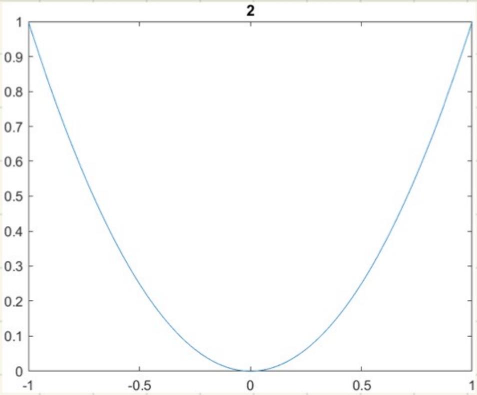
so that $f(x)$ is also both quasi-convex

and quasi-concave

$\Rightarrow$ it's consistent with the result from fcn_checker

```
convex : yes
concave : yes
superconvex : no
superconcave : no
quasiconvex : yes
quasiconcave : yes
```

Problem 2:



let $f(x)$ to be x^2

(1) $f(\theta a + (1-\theta)b) = [\theta a + (1-\theta)b]^2 = \theta^2 a^2 + 2\theta(1-\theta)ab + (1-\theta)^2 b^2$,

$\theta^2 a^2 + 2\theta(1-\theta)ab + (1-\theta)^2 b^2 - \theta a^2 - (1-\theta)b^2$

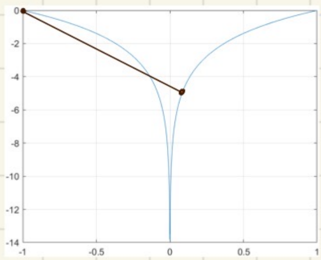
$= 2\theta(1-\theta)ab + (\theta-1)\theta a^2 + (1-\theta-1)(1-\theta)b^2$

$= 2\theta(1-\theta)ab - (1-\theta)\theta a^2 - (1-\theta)\theta b^2$

$= \theta(1-\theta)[2ab - a^2 - b^2] = -\theta(1-\theta)(a-b)^2 \le 0$

$\Rightarrow$ it's a convex function

(2) $\log(f(x)) = \log x^2 = \begin{cases} 2\log x, & x > 0 \\ 2\log(-x), & x < 0 \end{cases}$



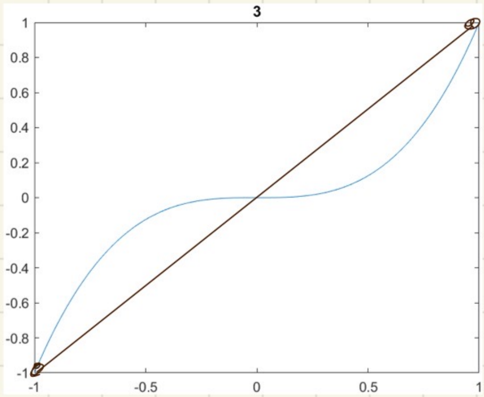
if we choose $a = -1$, $b \in (0,1)$, then draw a line between these 2 points, part of $\log(f(\theta a + (1-\theta)b))$ is above this line, but another part is below it $\Rightarrow$ it's neither superconvex or concave

(3) it's convex but not concave $\Rightarrow$ it's quasi-convex

```
convex : yes
concave : no
superconvex : no
superconcave : no
quasiconvex : yes
quasiconcave : no
```

$\Rightarrow$ consisent with fcn_checker

Problem 3 :



Let $f(x)$ to be x^3

(1) draw a line from $a=-1$ to $b=1$, first part of $f(\theta a+(1-\theta)b)$ above this line but another part is below the line $\Rightarrow$ it's nither convex of concave

(2) $f(x)$ might be negative

$\Rightarrow$ it's neither superconvex or superconcave

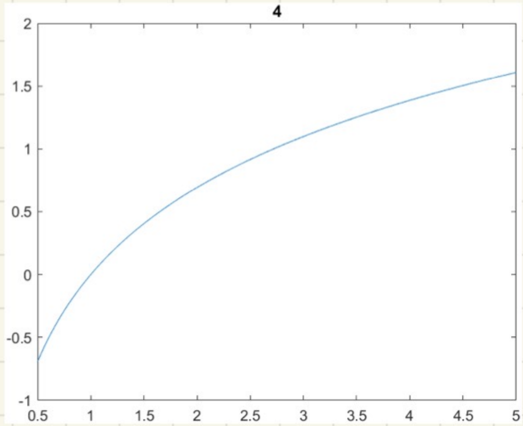
(3) $f(x) = x^3$ is non-decreasing

$\Rightarrow$ it's both quasi-convex and quasi-concave

$\Rightarrow$ it's consistent with the result from fcn-checker

```
convex : no
concave : no
superconvex : no
superconcave : no
quasiconvex : yes
quasiconcave : yes
```

Problem 4:



let $f(x) = \log(x)$, $x \in \mathbb{R}_{++}$

(1) $f''(x) = \frac{-1}{x^2} < 0$

$\Rightarrow$ it's a <u>concave</u> func.

(2) $f(x)$ might be negative

$\Rightarrow$ it's <u>neither superconvex or superconcave</u>

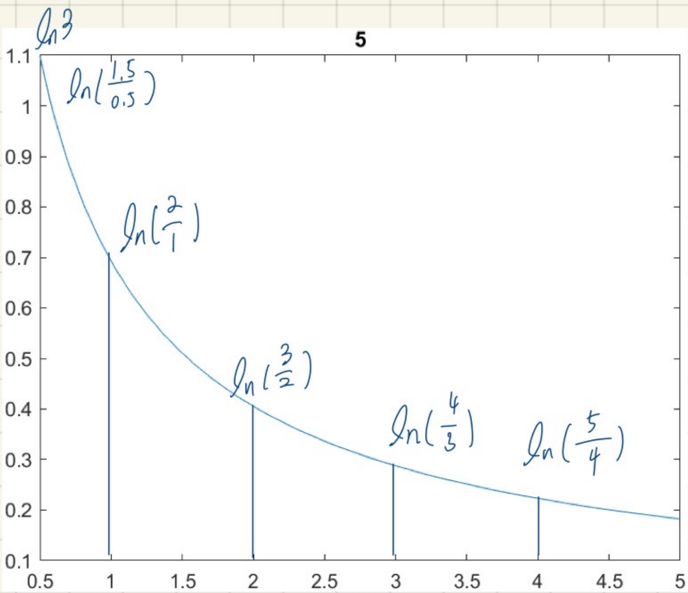
(3) $f(x) = x^3$ is non-decreasing

$\Rightarrow$ it's <u>both quasi-convex and quasi-concave</u>

$\Rightarrow$ it's consistent with the result from fcn-checker

```
convex : no
concave : yes
superconvex : no
superconcave : no
quasiconvex : yes
quasiconcave : yes
```

Problem 5:



let $f(x) = \log\left(\frac{x+1}{x}\right)$

(1) $f'(x) = \frac{x}{x+1} \cdot \frac{1\cdot x - (x+1)\cdot 1}{x^2} = \frac{-1}{(x+1)x}$

$f''(x) = \frac{(x+1)x}{(x+1)^2 x^2} = \frac{1}{(x+1)x} > 0 \Rightarrow$ convex

(2) $\frac{d}{dx}\log(f(x)) = \frac{d}{dx}\left(\log\left(\log\left(\frac{x+1}{x}\right)\right)\right) = \frac{1}{\log(\frac{x+1}{x})} \cdot \frac{-1}{(x+1)x}$

$\frac{d^2}{dx^2}\log(f(x)) = \left(\frac{1}{\log(\frac{x+1}{x})}\right)' \cdot \frac{-1}{(x+1)x} + \frac{1}{\log(\frac{x+1}{x})} \cdot \left(\frac{-1}{(x+1)x}\right)'$

$= \frac{-\left(\frac{-1}{(x+1)x}\right)}{\left(\log(\frac{x+1}{x})\right)^2} \cdot \frac{-1}{(x+1)x} + \frac{1}{\log(\frac{x+1}{x})} \cdot \frac{1}{(x+1)x} > 0$

$\Rightarrow$ super convex

(3) $f'(x) = \frac{x}{x+1} \cdot \frac{1\cdot x - (x+1)\cdot 1}{x^2} = \frac{-1}{(x+1)x} < 0$

$\Rightarrow$ it's non-increasing

$\Rightarrow$ it's both quasi-convex and quasi-concave

$\Rightarrow$ it's consistent with the result from fcn_checker