Computer Assignment # 5

Due: May 26, 2024, 23:59:59

Consider the following unconstrained problem

$$\min_x f(x) = -\sum_{i=1}^{m} \log(1 - \mathbf{a}_i^T \mathbf{x}) - \sum_{j=1}^{n} \log\left(1 - \frac{1}{2}x_j^2\right),$$

where variable $\mathbf{x} \in \mathbb{R}^n$, and **dom** $f = \{\mathbf{x} : \mathbf{a}_i^T \mathbf{x} < 1, i = 1, 2, ..., m, |x_j| \leq 1, j = 1, 2, ..., n\}$. This is the problem of computing the analytic center of the set of linear inequalities

$$\mathbf{a}_i^T \mathbf{x} < 1, i = 1, 2, ..., m, \ ; |x_j| < 1, \ j = 1, 2, ..., n.$$

Note that you can choose $x^{(0)} = 0$ as your initial point. You can generate instances of this problem by choosing $\mathbf{a}_i$ from some distribution on $\mathbb{R}^n$.

(a) Use the gradient method to solve the problem, using reasonable choices for the backtracking parameters, and a stopping criterion of a form $\|\nabla f(\mathbf{x})\|_2 \leq \eta$. Plot the objective function and step length versus iteration number. Experiment with the backtracking parameters α and β to see their effect on the total number of iterations required. Carry these experiments out for several instances of the problem, of different sizes.

(b) Repeat using Newton's method, with stopping criterion based on the Newton decrement λ^2. Use the chain rule to find expressions for $\nabla f(\mathbf{x})$ and $\nabla^2 f(\mathbf{x})$

Submission Policy leftmargin=*

- Place all your scripts inside the folder called "codes".
- Place convergence figures with verbal explanations inside report (word or pdf).
- Put "codes" and report inside zip file called `ca05_xxxxxx.zip`, where `xxxxxx` is your student ID and submit it to e3.