Written and Computer Assignment # 4

Due: May 17, 2024

There is a written part to this assignment which you need to give the solutions for separately from the code. You have to show **ALL** work to get any credit.

Problem 1

Given the nonnegative matrix factorization in the notes (ADMM Example 3).

(a) **(Written)** Assume the $\mathbf{X}$- and $\mathbf{C}$-update steps are separated, using real-valued matrix differentiation in the document "part3_minka2000realvalueddiff.pdf" posted on the course website, derive the closed-form solution $\mathbf{C}^\star$, $\mathbf{X}^\star$, and $\mathbf{R}^\star$ for their respective optimization problem in ADMM Example 3.

(b) **(Written)** Derive the primal and dual feasibility conditions that make your implementation (see below) to work correctly.

(c) Assume data matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ will be given in a separate data file called "pa04.mat" and $P = 7$, implement the ADMM algorithm in Matlab script to find the factors $\mathbf{C} \in \mathbb{R}^{m \times P}$ and $\mathbf{R} \in \mathbb{R}^{P \times n}$. As indicated in the notes, the $\mathbf{X}$- and $\mathbf{C}$-update can be done simultaneously (since this is a convex problem). However, instead of using the closed-form solution above (because it will likely give you rank deficient factors), a better way to implement it is to find each row of $\mathbf{X}$ and $\mathbf{C}$ independently in parallel. Similarly, in the $\mathbf{R}$-update, instead of using your closed-form solution above, you should solve for it column-by-column in parallel. Your implementation must use the varying parameter technique discussed and must used both `for` loop and `parfor` loop when performing optimization for the $\mathbf{X}$, $\mathbf{C}$ and $\mathbf{R}$ update. You should use the flag `PARAF` to turn on (i.e. `PARAF` $= 1$) and off (i.e. `PARAF` $= 0$) the parfor loop. When solving the $\mathbf{X}$ and $\mathbf{C}$ update problem, you can use the *cvx* toolbox from *http://cvxr.com/* to update $\mathbf{X}$ and $\mathbf{C}$ problem simultaneously (row-by-row).

(d) What is the lowest number of ADMM iterations do you need for the algorithm to converge? How "close" is your solution $\mathbf{CR}$ compared to $\mathbf{A}$ in terms of sum of their singular values?