

MMSE Based Channel Estimator

Zhao-Jie, Luo

janny00kevin@gmail.com

Advisor: Professor Carrson C. Fung

National Yang Ming Chiao Tung University

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- ① Carrson C. Fung, and Dmytro Ivakhnenkov, "Model-Driven Neural Network Based MIMO Channel Estimator"
- ② M. Eisen, C. Zhang, L.F.O. Chamon, D.D. Lee and A. Ribeiro, "Learning optimal power allocations in wireless systems," *IEEE Trans. on Signal Processing*, vol. 67(10), pp. 2775-2790, May 2019.
- ③ OpenAI Spinning Up introduction to RL Part 3: Intro to Policy Optimization

Problem Formulation

Our objective is to minimize the expected mean square error:

$$\min_{\hat{\mathbf{h}}} \mathbb{E}_{\mathbf{y}, \mathbf{h}} \left[\left\| \mathbf{h} - \hat{\mathbf{h}} \right\|_2^2 \right]$$

and it can be written in epigraph form as:

$$\begin{aligned} & \min_{t, \hat{\mathbf{h}}} t \\ & s.t. \mathbb{E}_{\mathbf{y}, \mathbf{h}} \left[\left\| \mathbf{h} - \hat{\mathbf{h}} \right\|_2^2 \right] \leq t \end{aligned}$$

Primal-Dual Optimization Method (1)

We use parameterize channel estimator so that $\hat{\mathbf{h}} = \phi(\mathbf{y}; \boldsymbol{\theta})$, with $\boldsymbol{\theta}$ denoting the parameters of the neural network.

Then the Lagrangian function of (15) can be written as

$$\begin{aligned}\mathcal{L}(\hat{\mathbf{h}}, t, \lambda) &= t + \lambda \left(\mathbb{E}_{\mathbf{y}, \mathbf{h}} \left[\|\mathbf{h} - \hat{\mathbf{h}}\|_2^2 \right] - t \right) \\ &= t + \lambda \left(\mathbb{E}_{\mathbf{y}, \mathbf{h}} \left[\|\mathbf{h} - \phi(\mathbf{y}; \boldsymbol{\theta})\|_2^2 \right] - t \right)\end{aligned}$$

Primal-Dual Optimization Method (2)

It is uncertain whether or not the duality gap equals zero.

However, the stationary point of $\mathcal{L}(\hat{\mathbf{h}}, t, \lambda)$ can be found via the KKT conditions by solving for the primal and dual variables alternately using gradient descent and ascent, respectively:

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - \alpha_{\boldsymbol{\theta},k} \lambda_k \nabla_{\boldsymbol{\theta}_k} \mathbb{E} \left[\|\mathbf{h} - \phi(\mathbf{y}; \boldsymbol{\theta}_k)\|_2^2 \right]$$

$$t_{k+1} = t_k - \alpha_{t,k} (1 - \lambda_k)$$

$$\lambda_{k+1} = \left[\lambda_k + \alpha_{\lambda,k} \left(\mathbb{E} \left[\|\mathbf{h} - \phi(\mathbf{y}; \boldsymbol{\theta}_{k+1})\|_2^2 \right] - t_{k+1} \right) \right]_+$$

Policy Gradient (1)

We have the policy gradient theorem:

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\tau}[G(\tau)] = \mathbb{E}_{\tau} \left[\sum_{t=0}^{T-1} G(\tau) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(A_t|S_t) \right]$$

At each time step, $t = 1, \dots, T - 1$:

$$\nabla_{\boldsymbol{\theta}} \mathbb{E}_{\tau}[G(\tau)] = \mathbb{E}_{\tau} [G(\tau) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(A_t|S_t)]$$

And we can estimate the policy gradient with sample mean:

$$\widehat{\nabla}_{\boldsymbol{\theta}} \mathbb{E}_{\tau}[G(\tau)] = \frac{1}{|\mathcal{D}|} \sum_{\mathcal{D}} G(\tau) \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(A_t|S_t)$$

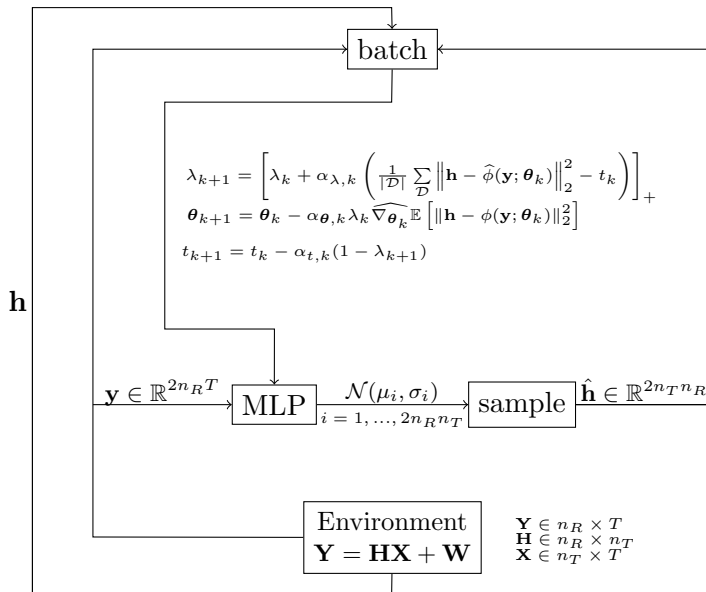
Policy Gradient (2)

Substitute $G(\tau)$, $\pi_{\boldsymbol{\theta}}(A_t|S_t)$ with $\|\mathbf{h} - \phi(\mathbf{y}; \boldsymbol{\theta})\|_2^2$, and $\pi_{\boldsymbol{\theta}}(\hat{h}|\mathbf{y})$, we can get the estimated policy gradient of our problem:

$$\widehat{\nabla}_{\boldsymbol{\theta}} \mathbb{E}_{\mathbf{y}, \mathbf{h}} \left[\|\mathbf{h} - \phi(\mathbf{y}; \boldsymbol{\theta})\|_2^2 \right] = \frac{1}{|\mathcal{D}|} \sum_{\mathcal{D}} \left\| \mathbf{h} - \hat{\phi}(\mathbf{y}; \boldsymbol{\theta}) \right\|_2^2 \nabla_{\boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}} \left(\hat{\mathbf{h}} | \mathbf{y} \right)$$

where $\hat{\phi}(\mathbf{y}; \boldsymbol{\theta}) = \hat{\mathbf{h}}$ is the sampled output of the policy.

Experiment Diagram



Simulation Result

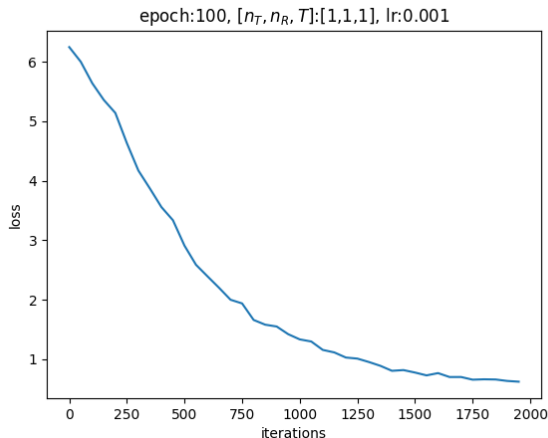


Figure: hidden layer sizes: $[64,32]$, $\mu_H = 5$, $\sigma_H = 0.2$, $\mu_W = 0$, $\sigma_W = 0.1$

Simulation Result

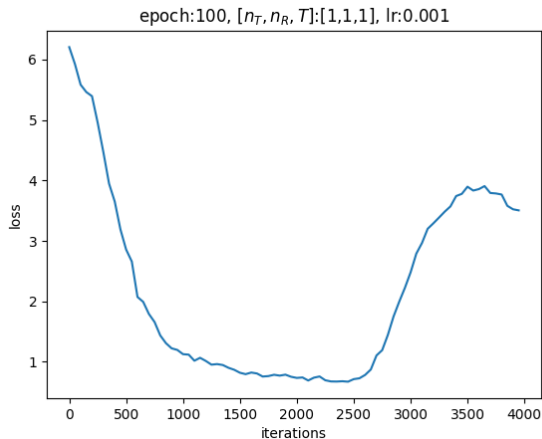


Figure: increase the iterations from 2000 to 4000