Computer Simulation of Communication Systems—Homework 3

Due date: 3/16

1. Generate two i.i.d. sequences $\{w_{cn}\}$ and $\{w_{sn}\}$ of $N = 10^6$ random numbers uniformly distributed in the interval $[-1,\ 1]$. Each of these sequences is passed through a linear filter with impulse response

$$h[n] = (0.9)^n\, u[n] - 0.5 \cdot (0.9)^{n-1}\, u[n-1], \text{ in which } u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases};$$

whose input-output characteristic is given by the recursive relation

$$x_n = 0.9 x_{n-1} + w_n - 0.5 w_{n-1},\ n \geq 1,\ x_0 = 0.$$

Thus we obtain two sequences, $\{x_{cn}\}$ and $\{x_{sn}\}$. The output sequence $\{x_{cn}\}$ modulates the carrier $\cos(\pi n/2)$, and the output sequence $\{x_{sn}\}$ modulates the quadrature carrier $\sin(\pi n/2)$. The bandpass signal $\{y_n\}$ is formed by combining the modulated components as in (2.6.1) in the textbook. Compute and plot the autocorrelation components $\hat{R}_c[m]$ and $\hat{R}_s[m]$ for $|m| \leq 150$ for the sequence $\{x_{cn}\}$ and $\{x_{sn}\}$ respectively. Compute the autocorrelation function $\hat{R}_y[m]$ of the bandpass signal $\{y_n\}$ for $|m| \leq 150$. Use the DFT (or the FFT algorithm) to compute the power spectrum of $\{x_{cn}\}$, $\{x_{sn}\}$, and $\{y_n\}$. Plot these power spectrums and comment on the results.

2. By Monte Carlo simulation, estimate the tail probability $P(m) \triangleq P(Y < 0|m)$, when $Y = m - P$, $m = 5,\ 25, \text{ and } 45$, and P is a Pareto random variable with parameters $x_m = 0.5$ and $\alpha = 2.5$ (see HW2-2.(b) for the formula of CDF). For each case of m:
   (a) Determine the <u>theoretical</u> value of $P(m)$ based on the probability density function of Y.
   (b) Determine the minimal number of samples N needed to <u>accurately estimate</u> $P(m)$ based on the rule of thumb on page 40 of the class slides. Compute the relative error, $\left\{ \dfrac{\left|P(m) - \hat{P}(m)\right|}{P(m)} \right\} \times 100\%$, of your estimate.

Note: MATLAB source code should be included. Explain all your results.