ILLUSTRATIVE PROBLEM

D1

Illustrative Problem 1.1 [Fourier Series of a Rectangular Signal Train] Let the periodic signal $x(t)$, with period T_0, be defined by

$$x(t) = A\Pi\left(\frac{t}{2t_0}\right) = \begin{cases} A, & |t| < t_0 \\ \frac{A}{2}, & t = \pm t_0 \\ 0, & \text{otherwise} \end{cases} \tag{1.2.19}$$

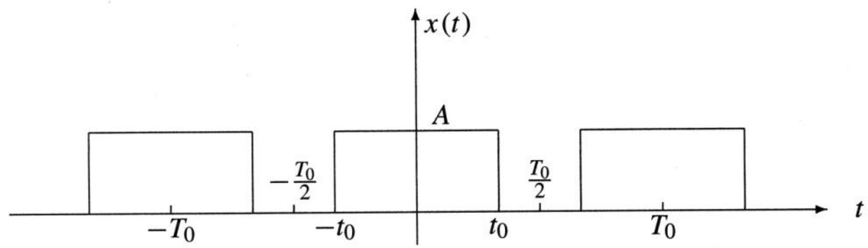


Figure 1.1 The signal $x(t)$ in Illustrative Problem 1.1

for $|t| \leq T_0/2$, where $t_0 < T_0/2$. The rectangular signal $\Pi(t)$ is, as usual, defined by

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ \frac{1}{2}, & t = \pm\frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \tag{1.2.20}$$

A plot of $x(t)$ is shown in Figure 1.1.
Assuming $A = 1$, $T_0 = 4$, and $t_0 = 1$,

1. Determine the Fourier series coefficients of $x(t)$ in exponential and trigonometric form.

2. Plot the discrete spectrum of $x(t)$.

SOLUTION

D1

1. To derive the Fourier series coefficients in the expansion of $x(t)$, we have

$$
\begin{aligned}
x_n &= \frac{1}{4}\int_{-1}^{1} e^{-j2\pi nt/4}\,dt \\
&= \frac{1}{-2j\pi n}\left[e^{-j2\pi n/4} - e^{j2\pi n/4}\right] \\
&= \frac{1}{2}\,\mathrm{sinc}\left(\frac{n}{2}\right)
\end{aligned}
$$

where $\mathrm{sinc}(x)$ is defined as

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

M-FILE

```
% MATLAB script for Illustrative Problem 1.1.
n=[−20:1:20];
x=abs(sinc(n/2));
stem(n,x);
```

D2

ILLUSTRATIVE PROBLEM

Illustrative Problem 1.4 [Filtering of Periodic Signals] A triangular pulse train $x(t)$ with period $T_0 = 2$ is defined in one period as

$$\Lambda(t) = \begin{cases} t+1, & -1 \le t \le 0 \\ -t+1, & 0 < t \le 1 \\ 0, & \text{otherwise} \end{cases} \tag{1.2.31}$$

1. Determine the Fourier series coefficients of $x(t)$.
2. Plot the discrete spectrum of $x(t)$.
3. Assuming that this signal passes through an LTI system whose impulse response is given by

$$h(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases} \tag{1.2.32}$$

plot the discrete spectrum and the output $y(t)$. Plots of $x(t)$ and $h(t)$ are given in Figure 1.10.

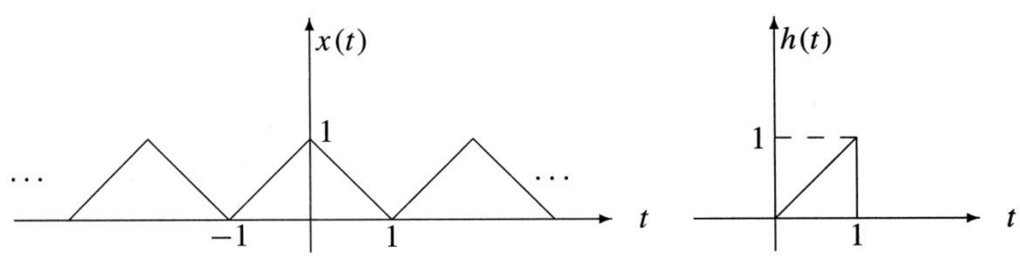


Figure 1.10 The input signal and the system impulse response

SOLUTION

D2

1. We have

$$x_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi nt/T_0}\,dt \tag{1.2.33}$$

$$= \frac{1}{2}\int_{-1}^{1} \Lambda(t)e^{-j\pi nt}\,dt \tag{1.2.34}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \Lambda(t)e^{-j\pi nt}\,dt \tag{1.2.35}$$

$$= \frac{1}{2}\mathcal{F}[\Lambda(t)]_{f=n/2} \tag{1.2.36}$$

$$= \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{n}{2}\right) \tag{1.2.37}$$

where we have used the facts that $\Lambda(t)$ vanishes outside the $[-1, 1]$ interval and that the Fourier transform of $\Lambda(t)$ is $\mathrm{sinc}^2(f)$. This result can also be obtained by using the expression for $\Lambda(t)$ and integrating by parts. Obviously, we have $x_n = 0$ for all even values of n except for $n = 0$.

2. A plot of the discrete spectrum of $x(t)$ is shown in Figure 1.11.

3. First we have to derive $H(f)$, the transfer function of the system. Although this can be done analytically, we will adopt a numerical approach. The resulting magnitude of the transfer function and also the magnitude of $H(n/T_0) = H(n/2)$ are shown in Figure 1.12. To derive the discrete spectrum of the output we employ the relation

$$y_n = x_n H\left(\frac{n}{T_0}\right) \tag{1.2.38}$$

$$= \frac{1}{2}\,\mathrm{sinc}^2\left(\frac{n}{2}\right) H\left(\frac{n}{2}\right) \tag{1.2.39}$$

The resulting discrete spectrum of the output is shown in Figure 1.13.

D2

M-FILE

```
% MATLAB script for Illustrative Problem 1.4.
echo on
n=[-20:1:20];
% Fourier series coefficients of x(t) vector
x=.5*(sinc(n/2)).^2;
% sampling interval
ts=1/40;
% time vector
t=[-.5:ts:1.5];
% impulse response
fs=1/ts;
h=[zeros(1,20),t(21:61),zeros(1,20)];
% transfer function
H=fft(h)/fs;
% frequency resolution
df=fs/80;
f=[0:df:fs]-fs/2;
% rearrange H
H1=fftshift(H);
y=x.*H1(21:61);
```

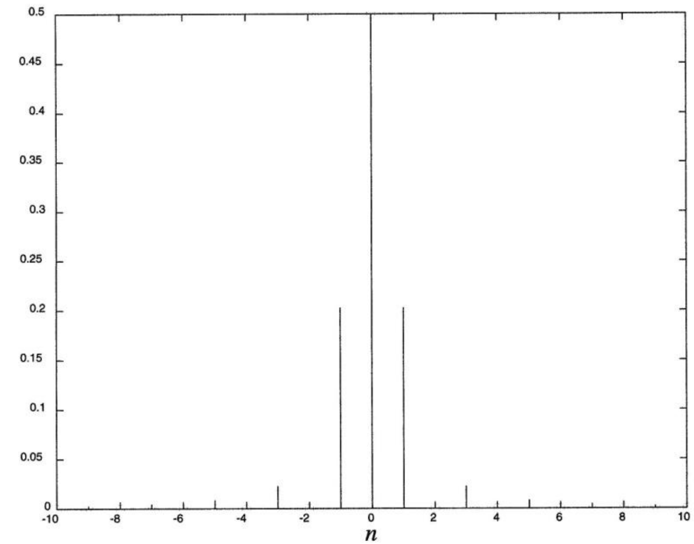


Figure 1.11 The discrete spectrum of the signal

ILLUSTRATIVE PROBLEM

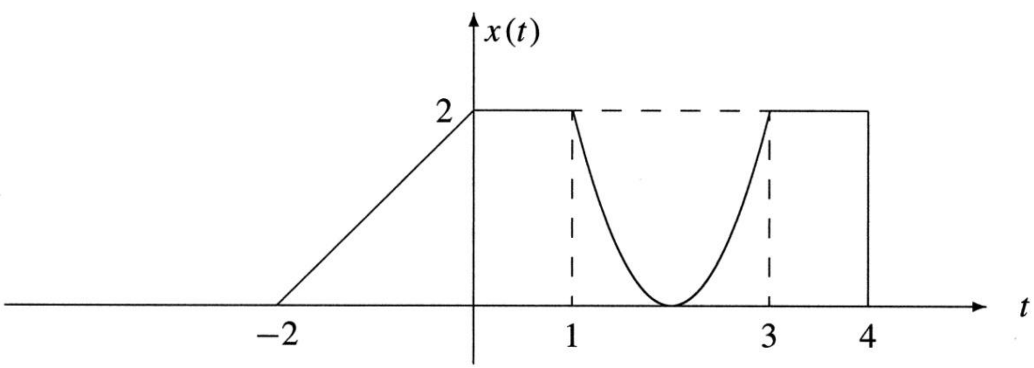
D3

Illustrative Problem 1.7 [LTI System Analysis in the Frequency Domain] The signal $x(t)$ whose plot is given in Figure 1.21 consists of some line segments and a sinusoidal segment.

1. Determine the FFT of this signal and plot it.
2. If the signal is passed through an ideal lowpass filter with a bandwidth of 1.5 Hz, find the output of the filter and plot it.
3. If the signal is passed through a filter whose impulse response is given by

$$h(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases} \tag{1.3.32}$$

plot the filter output.



$$x(t) = \begin{cases} t+2, & -2 \le t \le 0 \\ 1, & 0 < t \le 1 \\ 2+2\cos(0.5\pi t), & 1 < t \le 3 \\ 1, & 3 < t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Figure 1.21 The signal $x(t)$