D1

ILLUSTRATIVE PROBLEM

Illustrative Problem 1.9 [Bandpass to Lowpass Transformation] The signal $x(t)$ is given as

$$x(t) = \operatorname{sinc}(100t)\cos(2\pi \times 200t) \tag{1.5.14}$$

1. Plot this signal and its magnitude spectrum.
2. With $f_0 = 200$ Hz, find the lowpass equivalent and plot its magnitude spectrum. Plot the in-phase and the quadrature components and the envelope of this signal.
3. Repeat part 2 assuming $f_0 = 100$ Hz.

M-FILE

```
function z=analytic(x)
%              z=analytic(x)
%ANALYTIC      returns the analytic signal corresponding to signal x
%
z=hilbert(x);
```

D1

M-FILE

```
function xl=loweq(x,ts,f0)
%               xl=loweq(x,ts,f0)
%LOWEQ          returns the lowpass equivalent of the signal x
%               f0 is the center frequency.
%               ts is the sampling interval.
%
t=[0:ts:ts*(length(x)-1)];
z=hilbert(x);
xl=z.*exp(-j*2*pi*f0*t);
```

M-FILE

```
function [xc,xs]=quadcomp(x,ts,f0)
%               [xc,xs]=quadcomp(x,ts,f0)
%QUADCOMP       Returns the in-phase and quadrature components of
%               the signal x. f0 is the center frequency. ts is the
%               sampling interval.
%
z=loweq(x,ts,f0);
xc=real(z);
xs=imag(z);
```

M-FILE

```
function [v,phi]=env_phas(x,ts,f0)
%               [v,phi]=env_phas(x,ts,f0)
%                     v=env_phas(x,ts,f0)
%ENV_PHAS       returns the envelope and the phase of the bandpass signal x
%               f0 is the center frequency.
if nargout == 2
  z=loweq(x,ts,f0);
  phi=angle(z);
end
v=abs(hilbert(x));
```

D2

M-FILE

```
function [gsrv1,gsrv2]=gngauss(m,sgma)
%    [gsrv1,gsrv2]=gngauss(m,sgma)
%    [gsrv1,gsrv2]=gngauss(sgma)
%    [gsrv1,gsrv2]=gngauss
%              GNGAUSS  generates two independent Gaussian random variables with mean
%              m and standard deviation sgma. If one of the input arguments is missing,
%              it takes the mean as 0.
%              If neither the mean nor the variance is given, it generates two standard
%              Gaussian random variables.
if nargin == 0,
  m=0; sgma=1;
elseif nargin == 1,
  sgma=m; m=0;
end;
u=rand;                                  % a uniform random variable in (0,1)
z=sgma*(sqrt(2*log(1/(1-u))));           % a Rayleigh distributed random variable
u=rand;                                  % another uniform random variable in (0,1)
gsrv1=m+z*cos(2*pi*u);
gsrv2=m+z*sin(2*pi*u);
```

D2

$$F(R) = \begin{cases} 0, & R < 0 \\ 1 - e^{-R^2/2\sigma^2}, & R \geq 0 \end{cases} \tag{2.2.10}$$

is related to a pair of Gaussian random variables C and D through the transformation

$$C = R\cos\Theta \tag{2.2.11}$$

$$D = R\sin\Theta \tag{2.2.12}$$

where Θ is a uniformly distributed variable in the interval $(0, 2\pi)$. The parameter σ^2 is the variance of C and D. Since (2.2.10) is easily inverted, we have

$$F(R) = 1 - e^{-R^2/2\sigma^2} = A \tag{2.2.13}$$

and hence

$$R = \sqrt{2\sigma^2 \ln\left(\frac{1}{1-A}\right)} \tag{2.2.14}$$

where A is a uniformly distributed random variable in the interval (0,1). Now, if we generate a second uniformly distributed random variable B and define

$$\Theta = 2\pi B \tag{2.2.15}$$

then from (2.2.11) and (2.2.12), we obtain two statistically independent Gaussian distributed random variables C and D.

The method described above is often used in practice to generate Gaussian distributed random variables. As shown in Figure 2.5, these random variables have a mean value of zero and a variance σ^2. If a nonzero-mean Gaussian random variable is desired, then C and D can be translated by the addition of the mean value.

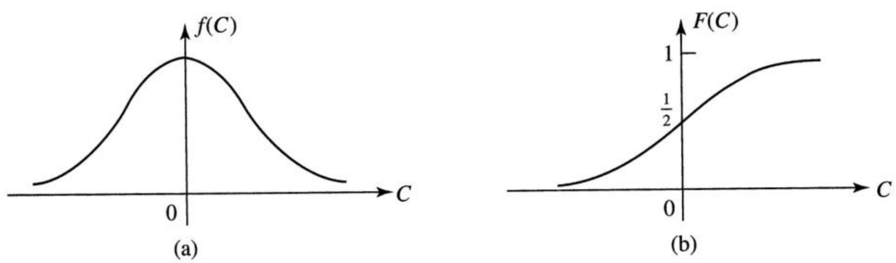

Figure 2.5 Gaussian probability density function and the corresponding probability distribution function

D3

ILLUSTRATIVE PROBLEM

Illustrative Problem 2.2 [Generation of Samples of a Multivariate Gaussian Process] Generate samples of a multivariate Gaussian random process $X(t)$ having a specified mean value $\boldsymbol{m}_x$ and a covariance $\boldsymbol{C}_x$.

SOLUTION

First, we generate a sequence of n statistically independent, zero-mean and unit-variance Gaussian random variables by using the method described in Section 2.2. Let us denote this sequence of n samples by the vector $\boldsymbol{Y} = (y_1, y_2, \ldots, y_n)^t$. Second, we factor the desired $n \times n$ covariance matrix $\boldsymbol{C}_x$ as

$$\boldsymbol{C}_x = \boldsymbol{C}_x^{1/2}(\boldsymbol{C}_x^{1/2})^t \tag{2.3.3}$$

Then, we define the linearly transformed $(n \times 1)$ vector $\boldsymbol{X}$ as

$$\boldsymbol{X} = \boldsymbol{C}_x^{1/2}\boldsymbol{Y} + \boldsymbol{m}_x \tag{2.3.4}$$

Thus the covariance of $\boldsymbol{X}$ is

$$\begin{aligned}
\boldsymbol{C}_x &= E[(\boldsymbol{X} - \boldsymbol{m}_x)(\boldsymbol{X} - \boldsymbol{m}_x)^t] \\
&= E[\boldsymbol{C}_x^{1/2}\boldsymbol{Y}\boldsymbol{Y}^t(\boldsymbol{C}_x^{1/2})^t] \\
&= \boldsymbol{C}_x^{1/2}E(\boldsymbol{Y}\boldsymbol{Y}^t)(\boldsymbol{C}_x^{1/2})^t \\
&= \boldsymbol{C}_x^{1/2}(\boldsymbol{C}_x^{1/2})^t
\end{aligned} \tag{2.3.5}$$

D3

M-FILE

```
function [x] = multi_gp(m,C)
%  [x]=multi_gp(m,C)
%                 MULTI_GP  generates a multivariate Gaussian random
%                 process with mean vector m (column vector) and covariance matrix C.
N=length(m);
for i=1:N,
  y(i)=gngauss;
end;
y=y.';
x=sqrtm(C)*y+m;
```

$$f(\boldsymbol{x}) = \frac{1}{(2\pi)^{n/2}[\det(\boldsymbol{C})]^{1/2}} \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{m})^t \boldsymbol{C}^{-1}(\boldsymbol{x}-\boldsymbol{m})\right]$$