Col Complexity (draft)

Abstract. In this article we propose a rational quadratic source code complexity measure [Blo19] from the perspective of path testing with the conjecture of self-suffcient enclosed scopes, the measure enfolds elementary conditions, nested and cyclic formations.

1 Introduction

A Progam consist of a sequences of instructions. The instructions can be categorized into syntax, logic and arithmetic determining the program control flow.

Software complexity is related with modularity, coupling and cohesion hence quality. 40% to 80% of software costs are emerging due to maintenance and approximatly 40% on fixing defects [SG11].

Analysing source code complexity helps to identify risk, finds potential defects to test critical functionality in detail, increase quality, cohesion and decrease maintainance [RB11].

2 Existing measures

Cyclomatic complexity (CC) has been widely discussed by various authors. The most used metric has been formulated by Thomas J. McCabe in 1976 [McC76].

$$m = e - n + 2 \qquad (1)$$

Where, e = the amount of edges. n = the amount of nodes.

CC is linear in it's nature and correlation with lines of source code (LOC), which is why Graylin JAY et al. [al.09] are suggesting to implement LOC as complexity measure.

Mir Muhammd Suleman Sarwar et al. [MMSS13] are pointing out that it neither takes into account the difference between combined decisions, elementary conditions nor repeating structures. Their adaptation of CC includes loop iterations

$$V(G)* = V(G) + \prod_{i=1}^{n} P_i$$
$$P_i = U_i - L_i + 1$$

Where, P_i = No. of iterations of ith loop. U_i = upper bound of ith loop. L_i = lower bound of ith loop and $V(G)*$ = adjusted cyclomatic complexity for any control flow graph "G". The measure combines control flow and statements exercised.

Tevfik et al. [LB15] are providing a computable measure: asymptotic path complexity covering nested structures and loops.

NPATH [Nej88] from Brian A. Nejmeh. facilitates acyclic path coverage $O(2^n)$ neglecting unfolded conditions (q, Table 1, 2).

2.1 Test Coverage

Test coverage metrics are providing quantitative representation of tested structures. The approach is to maximize coverage while minimizing testing effort. Each condition branches the flow into two control sub-paths, and determines the progression of the programm. The path of an isolated decision d_j thereby is given due to decisions and conditions (MC/DC) [RB11]. Let's define a condition c_j and it's path $\gamma : C \times \Sigma \rightarrow C^*$

$$\Sigma = \{true, false\}$$

$$D \subseteq \{C^*, S, E\}, \quad d_h \in D, \quad d_h \mapsto \Sigma$$

The transition function $alpha$

$$\alpha : D \times \Sigma^* \rightarrow \Sigma \times D$$

The condition

$$C = \{c_0, c_1, c_2, c_3, ..., c_n\}, c_i \mapsto \Sigma$$

$$\varepsilon : C \rightarrow \Sigma \times C$$

and the transition $\vdash$

$$\vdash \subseteq (\Sigma^* \times D \times \Sigma) \times (\Sigma^* \times D \times \Sigma)$$

The possible combinations of conditions on γ are arising through c_j's preceeding and succeeding condition flow. An

acyclic transition path through c_j is described due to passing each condition once

$$B^T_{\lambda j} = (v_h, d_h, b_h), \ldots \vdash (v_k, d_k, b_k) \vdash^* (v_m, d_m, b_m)$$
$$= (c_h, b_h), \ldots \vdash (c_j, b_j) \vdash^* (c_n, b_n)$$

where

$$\alpha(d_m, v_m) \mapsto E$$

with the jacket

$$B_\lambda = (b_h, v_h, d_h) \vdash^* (b_m, v_m, d_m)$$

3 Hypothesis acyclic complexity

Instead of combining arithmetic and logic we propose a metric [Blo19] reflecting the logical test effort of modified condition/decision path coverage [al.01]. The concept of basic paths described by [McC76] neglects loops and nesting. 100% sub-path combination coverage q is intricate due to exponential effort (Table 1). Conditions are providing the logical structure of programs (Listing 1-16). Loops and recursions are construed due to their boundary [RB11] and invariant, which is why their quantitative iteration increases probablility of condition coverage not condition path length. Let's define the inductive start of independent

$$\alpha_i = (c_0, true) \to (c_1), (c_0, false) \to (c_1),$$
$$(c_1, true) \to (E), (c_1, false) \to (E)$$

and dependent branches

$$\alpha_d = (c_0, true) \to (E), (c_0, false) \to (c_1),$$
$$(c_1, true) \to (E), (c_1, false) \to (E)$$

Isolated condition paths are $log(2^n)$ subset of all q paths. The spanning tree of the first branch expands four leaves, the latter three.

Nesting is diminishing the amount of paths, the effort of combinatorial parameter testing is unaffected. The fallacy of subjective perceived complexity increases contrary due to remembering previous conditions (Table 2).


Fig. 1. Upper Bound

Table 1. Condition formation bound

	$O(n)$	$O(2n)$	$O(n^2)$	$O(2^n)$	$O(4^{n-1})$
$\|C\|$	m	q_2	ξ	q	$P(n)$
1	2	$2-2$	$\frac{1}{2}-1$	$2-2$	1
2	3	$3-4$	$1-3$	$3-4$	4
3	4	$4-6$	$\frac{3}{2}-6$	$4-8$	16
4	5	$5-8$	$2-10$	$5-16$	64
5	6	$6-10$	$2\frac{1}{2}-15$	$6-32$	256
6	7	$7-12$	$3-21$	$7-64$	1024
7	8	$8-14$	$3\frac{1}{2}-28$	$8-128$	4096
$\cdots$					
9	10	$10-18$	$4\frac{1}{4}-45$	$10-512$	65536
$\cdots$					
19	20	$20-38$	$9\frac{1}{2}-190$	$20-524288$	$6.87*10^7$
$\cdots$					
49	50	$50-98$	$24\frac{1}{2}-1225$	$50-5*10^{14}$	$7.92*10^{27}$

Listing 1. plain singular

if (−1<1) { } $\varphi_0 = 1$

Listing 2. orthogonal dual

if (3>1 && 0<k) { } $\varphi_{0..1} = 1 + \frac{1}{2}$

Listing 3. orthogonal dual

if (2>1 || 0<k) { } $\varphi_{0..1} = 1 + \frac{1}{2}$

Listing 4. tertiary parallel identical scope

if (2>1 || 0<k || 4>n) { } $\varphi_{0..2} = 1 + \frac{1}{2} + \frac{1}{3}$

Listing 5. tertiary orthogonal

if (2>1 && 7>k && 10>=n) { } $\varphi_{0..2} = 1 + \frac{1}{2} + \frac{1}{3}$

Listing 6. orthogonal dual nested

if (0<1) { $\varphi_0 = 1$
. . .
  if (2>k) { } $\varphi_{0..1} = 1 + 1$
. . .
}

Listing 7. tertiary orthogonal nested

if (0<1 && 4>k) { $\varphi_{0..1} = 1 + \frac{1}{2}$
. . .
  if (11>n) { } $\varphi_{0..2} = 1 + \frac{1}{2} + 1$
. . .
}

Listing 8. tertiary parallel

if (0<1 || 9>k && 1<n) { } $\varphi_{0..2} = 1 + 1 + \frac{2}{3}$

Listing 9. tertiary

if (2>1 && 0<k || 4>n) { } $\varphi_{0..2} = 1 + 1 + \frac{2}{3}$

Listing 10. tertiary orthogonal nested

if (2>1) { $\varphi_0 = 1$
. . .
  if (8>k) { $\varphi_{0..1} = 1 + 1$
. . .
    if (0<n) { } $\varphi_{0..2} = 1 + 1 + 1$
. . .
  }
}

Listing 11. parallel

if (2>1) { } $\varphi_0 = 1$
. . .
if (8>k) { } $\varphi_{0..1} = 1 + 2$

Listing 12. dual parallel

if (0<1) { } $\varphi_0 = 1$
. . .
if (9>k && 1<n) { } $\varphi_{0..2} = 1 + 2 + \frac{1}{2}$

Listing 13. singular nested parallel

if (0<1) { $\varphi_0 = 1$
  . . .
  if (4>k) { } $\varphi_{0..1} = 1 + 1$
  . . .
  if (8>n) { } $\varphi_{0..2} = 1 + 1 + 2$
  . . .
}

Listing 14. orthogonal singular parallel

if (0<1) { $\varphi_0 = 1$
  . . .
  if (8>k) { } $\varphi_{0..1} = 1 + 1$

  . . .
}
if (0<n) { } $\varphi_{0..2} = 1 + 1 + 2$

Listing 15. dual parallel

if (0<1) { } $\varphi_0 = 1$
. . .
if (9>k) { $\varphi_{0..1} = 1 + 2$
  . . .
  if (1<n) { } $\varphi_{0..2} = 1 + 2 + 1$
}

Listing 16. tertiary parallel different scope

if (−4<1) { } $\varphi_0 = 1$
. . .
if (4<k) { } $\varphi_{0..1} = 1 + 2$
. . .
if (8>n) { } $\varphi_{0..2} = 1 + 2 + 3$

Fig. 2. m/ξ
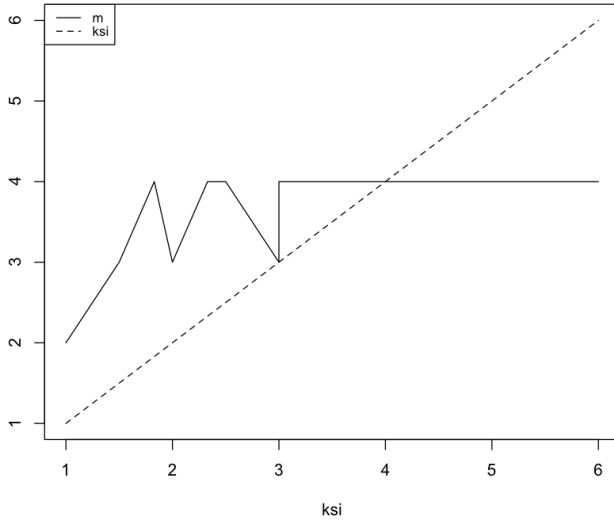
Fig. 3. q/ξ
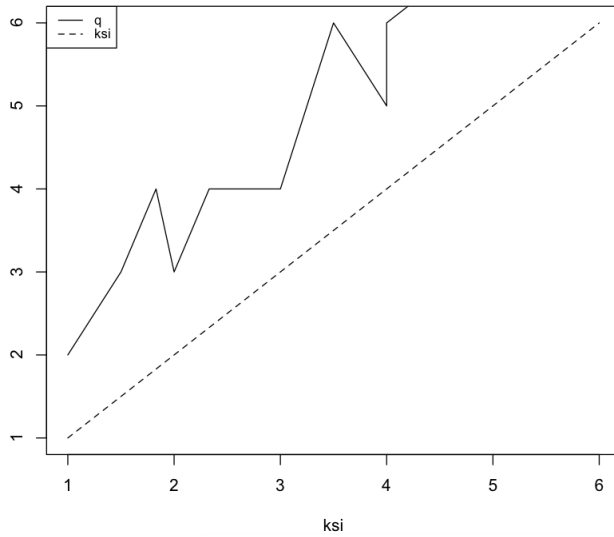
Table 2. Formation complexity

Listing	$|C|$	loc	m	q_2	ξ	q
1	1	1	2	1	1	2
2	2	1	3	3	$1\frac{1}{2}$	3
3	3	1	4	5	$1\frac{1}{2}$	4
4	2	1	3	3	$1\frac{5}{6}$	3
5	2	5	3	3	$1\frac{5}{6}$	3
6	3	1	4	5	2	4
7	3	1	4	5	$2\frac{1}{2}$	4
8	3	5	4	5	$2\frac{2}{3}$	4
9	2	3	3	2	$2\frac{2}{3}$	4
10	3	1	4	5	3	4
11	3	8	4	5	3	4
12	3	4	4	4	$3\frac{1}{2}$	6
13	3	7	4	4	4	5
14	3	6	4	4	4	6
15	3	6	4	4	4	6
16	3	5	4	3	6	8

Fig. 4. q2/ξ

Fig. 5. loc/ξ
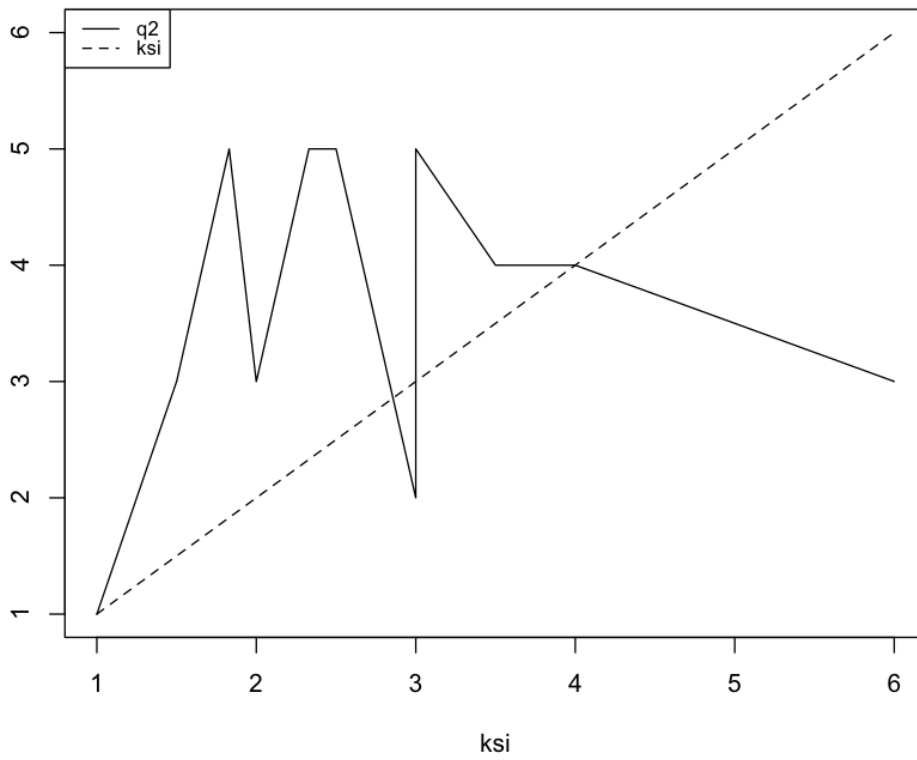
	s	c_0	c_1	e
s		t		
c_0			(t,f)	
c_1				(t,f)
e				
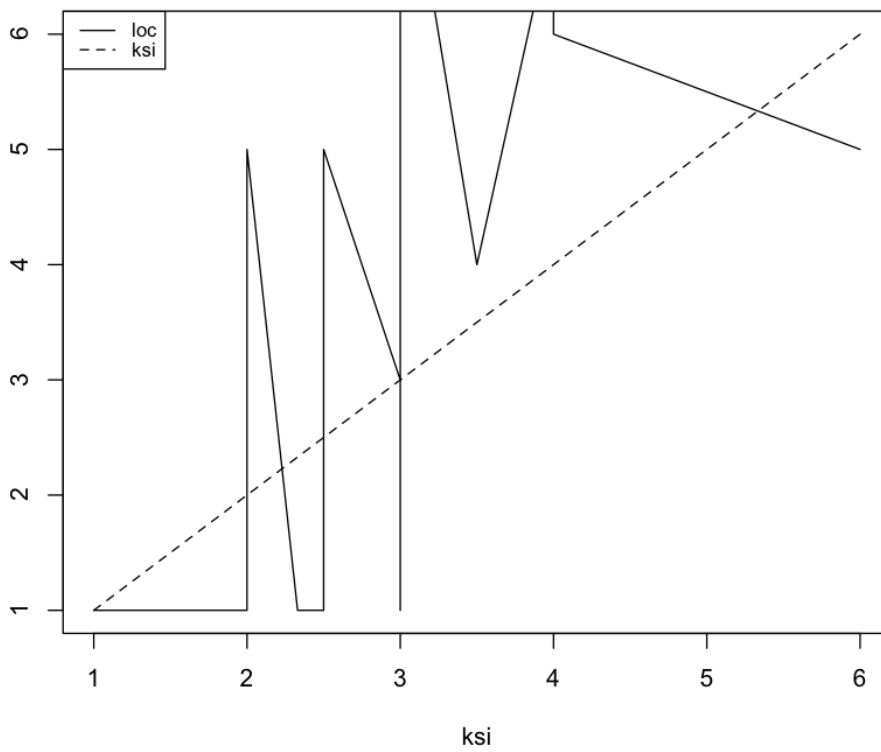
Table 3. α_i flow matrix, $q_2(\alpha_i) = 2$

	s	c_0	c_1	e
s		t		
c_0			f	t
c_1				(t,f)
e				

Table 4. α_d flow matrix, $q_2(\alpha_d) = 3$

The exponential amount of condition path combinations q is parsed into control flow q_2.

$$n = |C|, \quad n \in N$$
$$2^n <= q <= n+1 \tag{2}$$

When every condition is isolated the amount of condensed edges (enclosed scope paths) q_2 in the condition flow matrix represents an intuitive measure (Table 2, 1, 3, 4).

The lower and upper bounds of possible isolated condensed condition paths are

$$2*n >= q_2 >= n+1$$
$$C_e \subset C; \quad m_u = |C_e| + n \tag{3}$$

with $C_e \subset C$ nested conditions. The metric m_u is undifferentiated regarding condition formation. Subpath combinations caused from parallel decision scope (flat-style) are neglected, as well as the subjective memory of decisions, therefor we define the nesting function φ

$$m < \xi < \frac{n(n+1)}{2}$$

The regional col complexity φ of decision d_i depends on the collocations m including the amount of conditions

$$\xi \in \mathbb{C}; \quad c_\wedge \subset d_i; \quad c_\vee \subset d_i; \quad r = \left\lceil \frac{n}{2} \right\rceil$$

$$\xi = \sum_{i=0}^{n-1} \xi_{i+1} - \xi_i = \sum_{i=0}^{n} \varphi(d_i)$$

$$\varphi(d_i) = \frac{m(m+1)}{2} + \left(\sum_{k=1}^{r} \begin{cases} \frac{1+|c_\vee|}{2*|r-k+1|}, & \text{if } c_k\text{=and} \\ \frac{1+|c_\wedge|}{2*|r-k+1|}, & \text{if } c_k\text{=or} \end{cases} \right) i \tag{4}$$

$$|\varphi(d_i)| = \sqrt{ \left(\frac{m(m+1)}{2} \right)^2 + \left(\sum_{k=1}^{r} \begin{cases} \frac{1+|c_\vee|}{2*|r-k+1|}, & \text{if } c_k\text{=and} \\ \frac{1+|c_\wedge|}{2*|r-k+1|}, & \text{if } c_k\text{=or} \end{cases} \right)^2 }$$

Each summand is a unique key on the path, collocations are marked due to triangular numbers, the harmonic approximation differentiates according to positive human perception of invariant boundary test effort of self-sufficient scopes (Table 2).

Listing 17. Decision complexity examples

```
if (a && b && c && d && e && f && g && h) { }
```
$\varphi = 1 + (\frac{1}{8} + \frac{1}{6} + \frac{1}{4} + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8})i = 1 + \frac{19}{12}i, \quad |\varphi| \approx 1.87$

```
if (a || b || c || d || e || f || g && h) { }
```
$\varphi = 1 + (\frac{1}{4} + \frac{1}{3} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{7}{8})i = 1 + \frac{91}{24}i, \quad |\varphi| \approx 3.92$

```
if (a && b || c && d && e && f || g || h) { }
```
$\varphi = 1 + (\frac{1}{2} + \frac{5}{6} + \frac{1}{1} + \frac{2}{1} + \frac{1}{1} + \frac{5}{6} + \frac{5}{8})i = 1 + \frac{163}{24}i, \quad |\varphi| \approx 6.86$

```
if (a && b && c && d || e || f || g || h) { }
```
$\varphi = 1 + (\frac{5}{8} + \frac{5}{6} + \frac{5}{4} + \frac{2}{1} + \frac{1}{1} + \frac{2}{3} + \frac{1}{2})i = 1 + \frac{55}{8}i, \quad |\varphi| \approx 6.94$

```
if (a || b || c && d && e || f || g || h) { }
```
$\varphi = 1 + (\frac{1}{2} + \frac{5}{6} + \frac{5}{4} + \frac{5}{2} + \frac{1}{1} + \frac{2}{3} + \frac{1}{2})i = 1 + \frac{7}{1}i, \quad |\varphi| \approx 7.07$

```
if (a || b && c && d && e || f || g || h) { }
```
$\varphi = 1 + (\frac{1}{2} + \frac{5}{6} + \frac{5}{4} + \frac{5}{2} + \frac{1}{1} + \frac{2}{3} + \frac{1}{2})i = 1 + \frac{29}{4}i, \quad |\varphi| \approx 7.32$

```
if (a) {
    if (b) {
        if (c && d || e && f) {
        }
    }
}
```
$\varphi = (1 + 0i) + (1 + 0i) + (1 + (\frac{1}{2} + \frac{3}{2} + \frac{1}{2})i) = 3 + \frac{5}{2}i, \quad |\varphi| \approx 3.90$

Table 5. Complexity Risk [Cha05]

ξ	risk
1-21	basic program
22-65	intricate, moderate risk
65-	circuitous, high risk

4 Conclusion

The suggested complexity measure (4) expresses the logical test effort of condensed paths, diminishing unapparent complex patterns quantifying control flow into rational numbers.

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