The Fama-French 3-Factor Model and Event Study of TSLA Returns

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Agenda

Introduction

Stock prices are ever-changing and seemingly random. Price changes are highly influenced by the psychology of the investors. The public image of the CEO of a firm may also influence the returns of its stocks.

The Fama-French 3-factor model is given by

$$Z_i = \beta_i Z_M + h\mathrm{HML} + s\mathrm{SMB} + \varepsilon_i.$$

Problem Statement

How does the CAPM compare to the Fama-French 3-factor model, and how do both of these compare to the naïve model, when modeling stock returns, and which is the best? Is there evidence suggesting that the controversies, with which Elon Musk is associated, have had an impact on the returns of the Tesla stock?

Intercept test of the CAPM

We want to test the following null hypothesis against the alternative hypothesis

$$\mathcal{H}_0 : \alpha_i = 0,$$
$$\mathcal{H}_A : \alpha_i \neq 0,$$

on the unrestricted CAPM

$$Z_i = \alpha_i + \beta_i Z_M + \varepsilon_i.$$

To avoid the multiple comparisons problem we collect all our regressions in one SUR model, and rewrite the hypothesis into

$$\mathcal{H}_0 : R\beta_\bullet = \mathbf{0},$$
$$\mathcal{H}_A : R\beta_\bullet \neq \mathbf{0}.$$

Intercept test of the CAPM

We will use the dummy matrix

$$R = I_{30} \otimes \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

We want to perform a Wald test, but the data is not normally distributed. Therefore we rely on asymptotic theory. We found that the parameter estimates are still asymptotically normally distributed even under the assumption of heteroscedasticity. Hence, the Wald test statistic does asymptotically follow the $\chi^2(30)$.

To adjust for heteroscedasiticy we use the HC3 in the Wald test statistic.

Intercept test of the CAPM

Performing the test yielded the following results.

	Wald statistic	p-value	Degrees of freedom
Small-cap	1455.43	0.00	10
Mid-cap	1099.65	0.00	10
Large-cap	2977.18	0.00	10
Combined	5532.26	0.00	30

Table 3.1: Wald test statistics and p-values for the tests of intercepts being zero for the unrestricted CAPMs trained on excess returns for small-cap firms, mid-cap firms, and large-cap firms individually and combined.

Comparison between the FF-3 and CAPM

First we performed a validation procedure to and obtained the following performance metrics.

MAE

	Small	Mid	Large
Naïve model	0.0213	0.0524	0.0207
CAPM	0.0174	0.0182	0.0106
FF-3 model	**0.0164**	**0.0166**	**0.0098**

RMSE

	Small	Mid	Large
Naïve model	0.0300	0.0605	0.0264
CAPM	0.0246	0.0262	0.0151
FF-3 model	**0.0231**	**0.0244**	**0.0140**

R^2_{adj}

	Small	Mid	Large
Naïve model	0.0000	0.0000	0.0000
CAPM	0.2643	0.1936	0.2500
FF-3 model	**0.3084**	**0.2357**	**0.2689**

Table 4.1: Results for running validation using an 80%-20% training-test split on the 30 stocks on naïve, CAPM, and FF-3 models. The R^2_{adj} values are obtained using only the training data.

Comparison between the FF-3 and CAPM

We also performed a Wald test in the same manner as when we tested the intercept of the CAPM. This time we used the following null and alterntaive hypotesis.

$$\mathcal{H}_0 : R\beta_\bullet = \mathbf{0},$$
$$\mathcal{H}_A : R\beta_\bullet \neq \mathbf{0},$$

but with

$$R = I_{30} \otimes \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Comparison between the FF-3 and CAPM

Performing the test yielded the following results.

	Wald statistic	p-value	Degrees of freedom
Small-cap	4824.95	0.00	20
Mid-cap	1983.85	0.00	20
Large-cap	1413.51	0.00	20
Combined	11294.36	0.00	60

Table 4.2: Wald test statistics and p-values for the tests of zero-intercepts with FF-3 models trained on excess returns for small-cap firms, mid-cap firms, and large-cap firms individually and combined.

Event Study

We want to check whether five events have an impact on the returns of TSLA or not. To do this, we investigate the abnormal returns.

$$\mathrm{AR}_t = R_t - \mathbb{E}\left[R_t\right].$$

We use the following test statistic

$$t_{\mathrm{CAR}(T_1,T_2)} = \frac{\mathrm{CAR}(T_1, T_2)}{\sqrt{L\hat{\sigma}^2}},$$

with an event window of length two. Hence, the test statistic is only based on two abnormal returns.

This test statistic follows a t distribution under $\mathcal{H}_0$ given that the returns are normally distributed.

Event Study

We found that the returns of TSLA are not normally distributed. Nevertheless, relaxing the assumption of normally distributed returns we obtain the following results.

Event	CAR	$\hat{\sigma}^2$	t_{CAR}	p-value
Musk offers to buy back TSLA	0.0835	0.0008	2.0343	0.0442
Musk smokes marijuana on JRE	0.0164	0.0010	0.3640	0.7165
Cybertruck's windows crack	-0.0597	0.0009	-1.4163	0.1594
Musk offers to buy Twitter	0.0236	0.0011	0.5109	0.6104
Musk allegedly turned off Starlink	-0.0044	0.0006	-0.1251	0.9007

Table 5.1: Statistics from the event study for each event.

We also supplement with a visual inspection.

Event Study

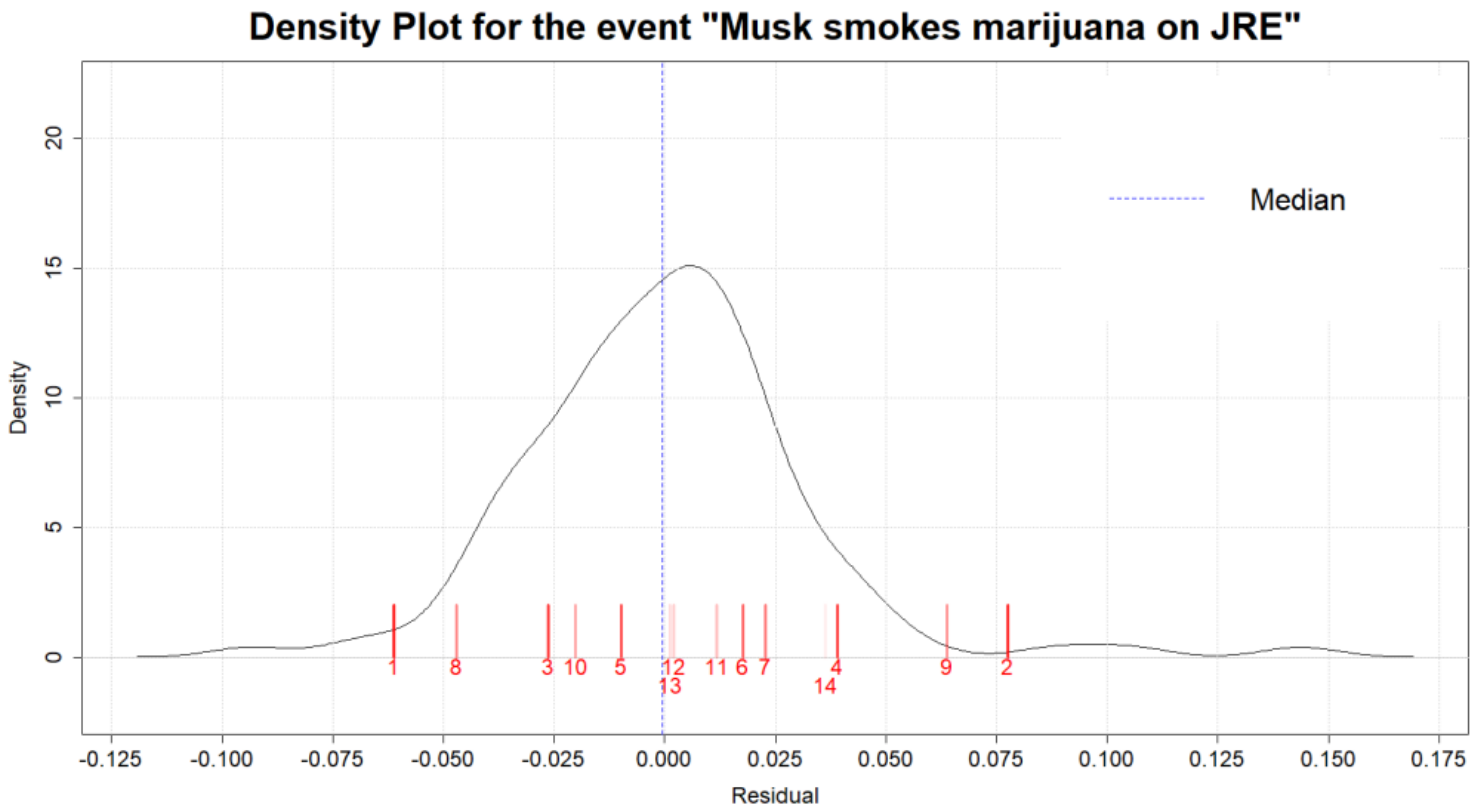

Figure 5.3: Kernel density for the normal returns along with the abnormal returns of TSLA for the event where Elon Musk smoked marijuana on The Joe Rogan Experience.

Discussion and Conclusion

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Discussion

- Long time period
- Test SMB and HML individually
- Few events investigated

Conclusion

- FF3 model outperformed CAPM in all metrics
- Possible to test if an event had an effect

Improvements

- Fama-French 5-factor model
- Factor targeted to Tesla

Testing the parameters associated with HML and SMB individually

In the project, HML and SMB tested together using a single Wald test.

Since then, we have tested each individually. We did this using the following regression equations:

$$Z_i = \alpha_i + \beta_i Z_M + h\mathrm{HML} + \varepsilon_i,$$
$$Z_i = \alpha_i + \beta_i Z_M + s\mathrm{SMB} + \varepsilon_i,$$
$$Z_i = \alpha_i + \beta_i Z_M + h\mathrm{HML} + s\mathrm{SMB} + \varepsilon_i.$$

Testing the parameters associated with HML and SMB

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Performing the tests yields the following results:

	Wald statistic	p-value
HML in CAPM	2944.07	0.00
SMB in CAPM	2615.63	0.00
HML in FF-3	3589.29	0.00
SMB in FF-3	3059.06	0.00

All tests were performed using SUR models composed of 30 regression equations meaning that the test statistics follow $\chi^2(30)$.

Comparing the FF-5 with the FF-3

In the same manner as with the FF-3, we can test the parameters associated with RMW (Robust Minus Weak) and CMA (Conservative Minus Aggresive) from the FF-5 factor model. We use the following regression equations.

$$Z_i = \alpha_i + \beta_i Z_M + h\mathrm{HML} + s\mathrm{SMB} + r\mathrm{RMW} + \varepsilon_i,$$
$$Z_i = \alpha_i + \beta_i Z_M + h\mathrm{HML} + s\mathrm{SMB} + c\mathrm{CMA} + \varepsilon_i,$$
$$Z_i = \alpha_i + \beta_i Z_M + h\mathrm{HML} + s\mathrm{SMB} + r\mathrm{RMW} + c\mathrm{CMA} + \varepsilon_i.$$

Comparing the FF-5 with the FF-3

Performing the tests yield the following results.

	Wald statistic	p-value
RMW in FF-3	484.51	0.00
CMA in FF-3	355.13	0.00
RMW in FF-5	354.33	0.00
CMA in FF-5	486.48	0.00

All tests were performed using SUR models composed of 30 regression equations meaning that the test statistics follow $\chi^2(30)$.

The R^2_{adj} of the FF-5 in different regimes

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In the project, the time period used for calculating the R^2_{adj} for the FF-3 model was long which could cause the low values.

We have calculated R^2_{adj} during the events' estimation windows to shorten the time period. Here are the R^2_{adj} for the FF-3 and the FF-5 for the five events, calculated for small-, mid-, and large-cap firms.

		Ev. 1	Ev. 2	Ev. 3	Ev. 4	Ev. 5
FF-3	Small	0.2630	0.2541	0.3070	0.3558	0.3230
	Mid	0.1764	0.1608	0.3102	0.3553	0.3290
	Large	0.5122	0.4858	0.4642	0.5603	0.3579
FF-5	Small	0.2907	0.2844	0.3118	0.4038	0.3310
	Mid	0.2136	0.1903	0.3229	0.3829	0.3366
	Large	0.5309	0.5060	0.5002	0.5738	0.4106

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Test statistics using the FF-5 model

Event	CAR	$\hat{\sigma}^2$	t_{CAR}	p-value
Musk offers to buy back TSLA	0.0840	0.0008	2.0470	0.0429
Musk smokes marijuana on JRE	0.0169	0.0010	0.3741	0.7090
Cybertruck's windows crack	-0.0646	0.0009	-1.5672	0.1198
Musk offers to buy Twitter	0.0237	0.0010	0.5197	0.6042
Musk allegedly turned off Starlink	-0.0190	0.0005	-0.5908	0.5558

Event	CAR	$\hat{\sigma}^2$	t_{CAR}	p-value
Musk offers to buy back TSLA	0.0835	0.0008	2.0343	0.0442
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Table 5.1: Statistics from the event study for each event.

Event plot using the FF-5 model

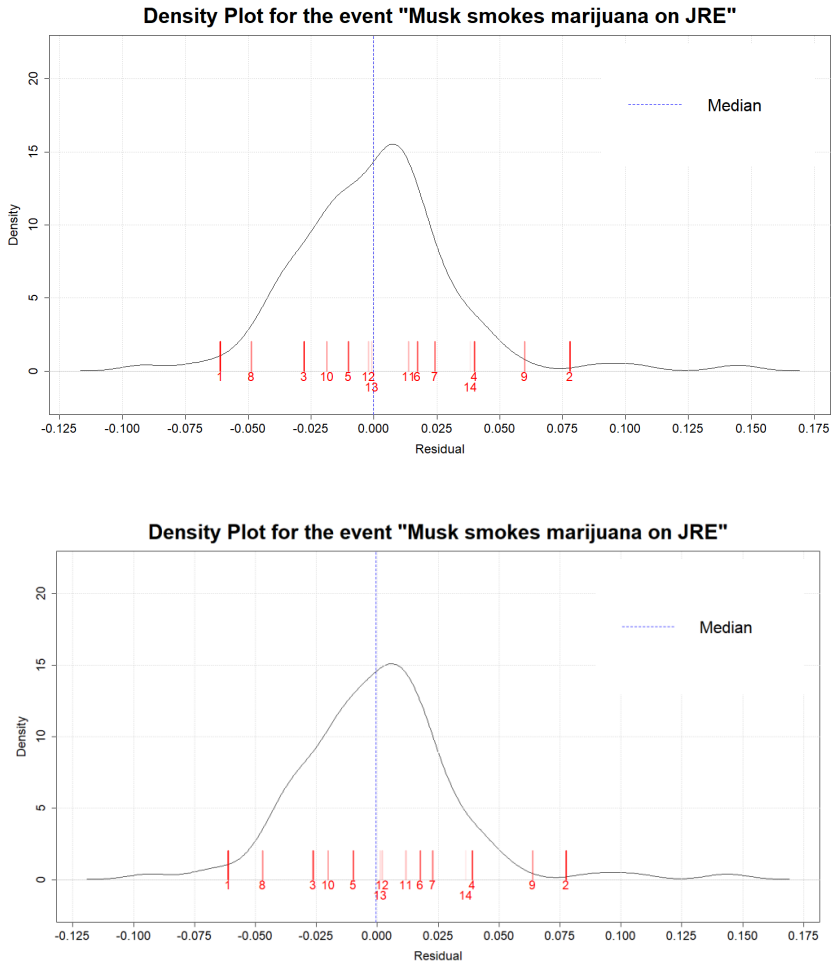

Figure 5.3: Kernel density for the normal returns along with the abnormal returns of TSLA for the event where Elon Musk smoked marijuana on The Joe Rogan Experience.