Preliminary results

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Model

As a reference paper for the life- cycle model I used Kaplan and Violante (2010).

- Economy populated by the continuum of ex-ante same agents indexed by i.
- Unconditional surviving probability π_t, maximal lifespan is T, period is one year (at least now)
- Discount factor β
- CRRA utility $U()$ function of individual consumption $C_{i,t}$

$$\max_{C_{i,t},A_{i,t}} \mathbf{E}_0 \sum_{t=0}^{T} \beta\pi_t U(C_{i,t})$$

Workers:

- Before age R, consumer receives income $\{Y_{i,1}, \dots Y_{i,R}\}$

$$\begin{aligned}
\ln Y_{i,t} &= \kappa_t + y_{i,t} \\
y_{i,t} &= z_{i,t} + \epsilon_{i,t} \\
z_{i,t} &= \rho z_{i,t-1} + \eta_{i,t} \\
\eta_{i,t} &\sim \mathcal{N}(0, \sigma_\eta) \\
\epsilon_{i,t} &\sim \mathcal{N}(0, \sigma_\epsilon) \\
z_{i,0} &\sim \mathcal{N}(0, \sigma_z)
\end{aligned}$$

  I still do not add $\epsilon_{i,t}$, in the original paper ρ is equal 1, however, because of the problems with the pension's calibration I used the DeNardi et al. (2018) calibration for ρ.

- Retired consumer receives the pension $P(y_{i,R})$, depending on median income y_med:

$$P(y_{i,R}) = 0.9 * y_{i,R} \, if \, 0.38 * y_med \geq y_{i,R}$$
$$P(y_{i,R}) = 0.9 * 0.38 * y_{med} + 0.32 * y_{i,R} \, if \, 0.38 * y_med \leq y_{i,R} \leq 1.59 * y_med$$
$$P(y_{i,R}) = 0.9 * 0.38 * y_{med} + 0.32 * 1.59 * y_{med} + 0.15 * y_{i,R} \, if \, 0.38 * y_med \leq y_{i,R} \leq 1.59 * y_med$$

β	0.9700
r	0.0300
σ_z	0.0100
σ_{z0}	0.1500
CRRA	2.0000
ϱ_t	0.0001

- It is simplification from original paper where the pension depend on the average lifetime income
- Consumers may invest in risk-free assets $A_{i,t}$, with interest rate r
- Perfect annuity markets

$$
\begin{aligned}
C_{i,t} + A_{i,t+1} &= (1+r)A_{i,t} + Y_{i,t} \ \ if\, t \leq R \\
C_{i,t} + (\frac{\pi_t}{\pi_{t+1}})A_{i,t+1} &= (1+r)A_{i,t} + P(y_{i,R}) \ \ if\, t > R \\
A_{i,t} &> 0
\end{aligned}
$$

I still not add annuity

Additional features

- Hyperbolic discounting for $\delta = 0.95, 0.9, 0.8, 0.75$
- Sticky expectations with update probability 1/2. The aggregate shocks are:

$$
\begin{aligned}
y_{i,t} &= z_{i,t} + \epsilon_{i,t} + P_t \\
P_{t+1} &= P_t + \varrho_t \\
\varrho_t &\sim \mathcal{N}(0, \sigma_\varrho)
\end{aligned}
$$

Calibration

For the standard LCM parameters I followed Kaplan and Violante (2010), (only for ρ I chose DeNardi parameter). β was calibrated to obtain income - assets ratio equal 2.5 (as in the reference paper the ratio must replicate 95% of the consumers). Pensions were diminished to obtain ratio of the pension income to the worker's income equal to 0.11. The age depended productivity was given by the 3-rd degree polynomial (from the Kaplan and Violante (2012)). The calibrated values are presented in the table:

Preliminary results

I simulate economy for 100 aggregate shock paths, 10 consumers in each. Firstly, I checked the case without sticky expectations and hyperbolic discounting. The number of individuals which undersave was much lower than in the Borsch-Suspan et al. (2010): 18% instead

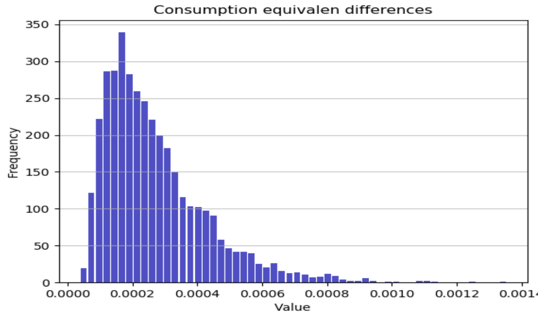

Figure 1: rational consumer consumption equivalent

of 61%. The average consumption equivalent was those who undersave was 0.00024 of the lifetime consumption. However the distribution was skewed and is presented at the figure 1.

Next, I do the same for only sticky expectations. The fraction of undersaving consumers was: 18% and the average consumption equivalent: 0.00027.

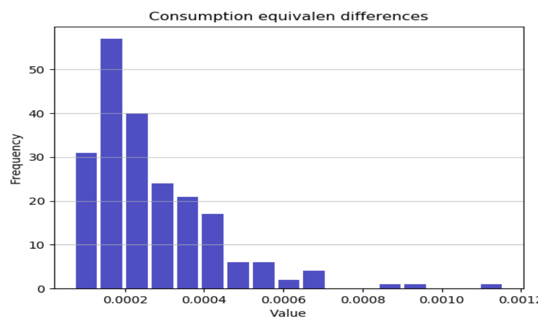

Figure 2: rational consumer consumption equivalent with sticky expectations

Next, the same for only hyperbolic preferences. The fraction of undersaving consumers was: 86% and the average consumption equivalent 0.001.

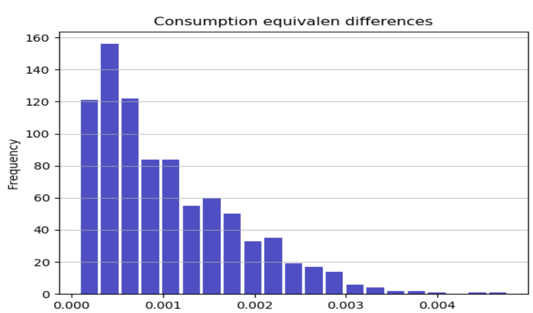

Figure 3: hyperbolic consumer consumption equivalent

Eventually, full model. 87% under save, the average consumption equivalent was 0.001.

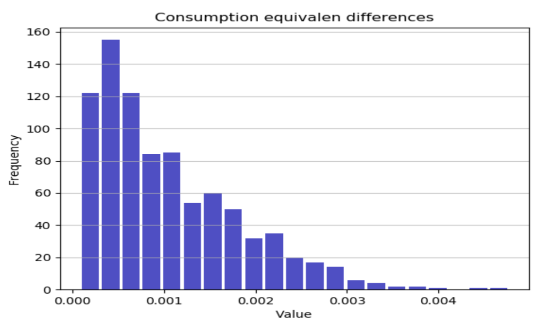

Figure 4: full model consumption equivalent